Curvature, optimal transport, entropy and applications

Groupe de travail

September 28, 2018

Abstract

Purpose of the *groupe de travail* is to study and deeply understand some crucial concept and tool at the interplay of probability, analysis, geometry and related fields, with the motivation of applied problem.

Organization

The *groupe de travail* will be based on a weekly meeting during 9 months. The meetings will take place every Friday afternoon time and place to be decided. We will cover three thematic sections, one for trimester. For each section there will be some "teaching oriented" meeting done by me or Tryphon, and some "research oriented" meeting done by all of us. In the first case we will introduce and present the basic stuffs of a subject while in the second case one of us will choose to study deeper one of the selected paper and present it to the rest of the group. A calendar will be provided.
Hopefully every meeting will be followed by discussions that hopefully will give us new ideas and bring us to new results.

Content

- **Section 1**
  1. **Γ_2-calculus**
     1.1 Markov semigroups; infinitesimal generator; invariant measure ; carré du champs operator; Γ_2- operator; examples.
  2. **Bakry-Emery condition**
     2.1 Definition; Meaning; Geometrical flavor; equivalent formulation; examples.
  3. **Functional inequalities**
     3.1 Log-Sobolev, Poincaré; Concentration; Prekopa- Leindler; Isoperimetric; Talagrand; relation to the Γ_2-operator.
- **Section 2**
  1. **Optimal transport**
     1.1 Monge-Kantorovich problem; Wasserstein distance; Brenier's map; McCann geodesics; dual formulation; Benamou-Brenier formulation.
  2. **Otto Calculus**

3. **Schrödinger problem**

   3.1 Definition of the problem; Schrödinger bridge and entropic cost; dual and entropic Benamou-Brenier formulation; relation to optimal transport; physical meaning.

- **Section 3**

  1. **Geometry on discrete spaces**
  2. **Large deviation**
  3. **Thermodynamics**
  4. **Interpolations**
  5. **Principal components**

Bibliography

- **References for Section 1**

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Selected papers:

- **References for Section 2**

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Selected papers:

- **References for Section 3**

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