# Temporal Logic Falsification via Monte Carlo Tree Search

Lugi Berducci

Alessandro Steri

May 20, 2019

#### Abstract

Simulation-based verification is often the only chance to verify complex hybrid systems and it requires a large amount of time and resources. In particular, the process to find anomalies and faults is called *falsification* because it consists of finding a scenario (a trace of disturbances) that leads the system to violate the requirements, described as formal specifications.

Many research groups are focusing their effort on techniques for reduction of verification time and recent works adopted search algorithms driven by a heuristic called *robustness*.

In this project, we focused on falsification of Temporal Logic Specification using one of the most promising search algorithm: the Monte Carlo Tree Search, famous for being adopted in AlphaGo. Specifically, we tested this approach on the well-known Automatic Transmission model and analysed the obtained result, describing strenghts and difficulties of MCTS algorithm.

## Contents

1	Tw	o-Layered Falsification	<b>2</b>
	1.1	Introduction	2
	1.2	Upper-layer: MCTS	2
	1.3	Lower-layer: Search Algorithms	7
		1.3.1 Random Search: MCTS Left Alone	7
		1.3.2 Hill Climbing: the Greedy	7
		1.3.3 Simulated Annealing: the Impatient	7
2	Exp	perimental phase	8
	2.1	Model and Specifications	8
	2.2	Implementation of Robustness Metric	8
	2.3	The proposed baseline: URS	9
	2.4	Experiments	10
3	Ana	alysis of the results	11
	3.1	Comments	11
	3.2	Hyperparameters	15
4	Cor	nclusion	16

# 1 Two-Layered Falsification

#### 1.1 Introduction

The process of falsification aims to find a counter example to a given specification, such examples typically are *hard to find* errors. In general, the search space is too wide and the process of verification through simulation is expensive, thus a clever strategy to drive the search and properly invest the simulation time is crucial.

Following the work done in [3] we decided to use a two layer strategy. In the top layer, Monte Carlo Tree Search act as a zoomed out view of the search: at each control point, the system quantizes the input space in regions and tries to iteratively explore new regions and give to those region a score mesuring how promising they are (see Section 1.2). In the bottom layer, a search algorithm (see Section 1.3) starts a fine grain exploration in the promising regions coming from the top layer and updates score.

Whilst the MCTS allows to tune the tradeoff between exploration and exploitation at a macroscopic level, the second layer algorithm works only on a few selected regions according to the time advance.

Given the success of MCTS in automated game play, this approach seems to be promising in verification because of its adversarial characterization. In fact, we could interpret the verification problem as if the error is cleverly placed by an adversary and the strategy needs to be clever enough to find it.

## 1.2 Upper-layer: MCTS

The MCTS is based on the iterative building of a tree T = (V, E) where nodes  $v \in V$  are labelled with a score v.s and a counter v.n, and edges in E are labelled with the corresponding actions. At the begin, the tree is initialized as a root node with root.s = Inf and root.n = 0. The execution evolves in different phases described below and represented by the Figure 1.

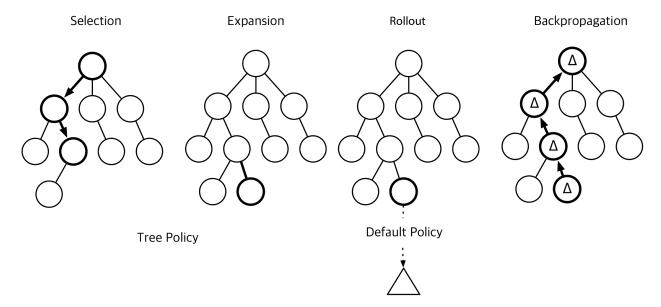


Figure 1: Graphical overview of MCTS phases and theirs behaviors.

**Selection:** Visit the tree starting from the root, moving to the node v which maximize the UCB(v) function explained in the following paragraph. Once a leaf l is reached, if the tree hasn't yet reached its maximum expansion (i.e. the height of the tree corresponds to the control points in the simulation) then go to Expansion(l), otherwise it directly run Rollout(l).

**Expansion:** For each action a available from l we expand the tree T with a new node v having v.s = Inf, v.n = 0 and connected to the node l through an edge labeled a. Finally, we pick a random child c of l, and run Rollout(c).

**Rollout(1)** For each ancestor node of l starting from the root till l itself, the lower layer (see Section 1.3) searches in the region of the selected node (tree policy). Then starting from the leaf l, the remaining simulation is driven by the search (default policy) up till the horizon. At the end, the trace will have a robustness value of s which will be used as an estimate for l and its ancestors by means of Backpropagation (l, s).

**Backpropagation(l,s):** From l till the root, going trough l's ancestors say  $l = a_1, a_2, \ldots, a_n = root$ , update each  $a_i$  as follow:

- $a_i.n = a_i.n + 1$
- $a_i.s = min\{a_i.s, s\}$

UCB(v): In the most general MCTS implementation, the UCB value of the node v is computed as:

$$UCB = v.s + C \cdot \sqrt{\frac{\ln N}{v.n}} \tag{1}$$

where N = root.n hence the total number of visited nodes. The idea behind the parameter C (see Figure 2, 3, 4) is to fine-tune exploration (see Figure 4) and exploitation (see Figure 2).

When considering the falsification problem thus using robustness as heuristic we face two problem:

- Robustness can have positive as well as negative values, while classical UCB is designed to work only with positive value.
- Given a specification, generally it is not possible to define an upper bound to the robustness domain

In order to overcome both the problems, along the line of [3], we used UCT defined as:

$$UCT = 1 - \frac{v.s}{max_v v.s} + C \cdot \sqrt{\frac{\ln N}{v.n}}$$
 (2)

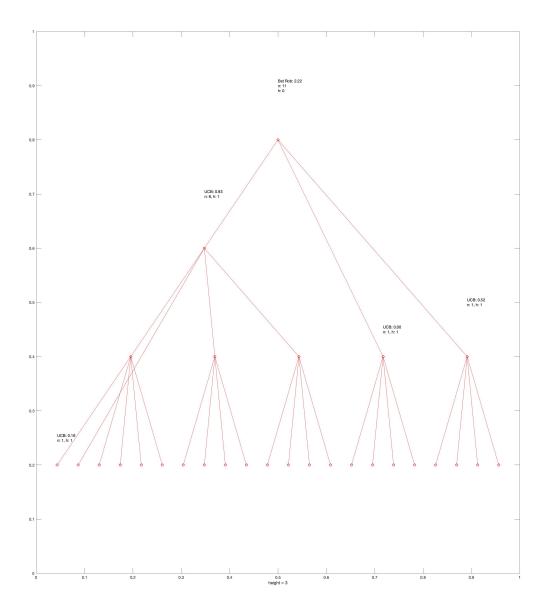


Figure 2: MCTS: Favour exploitation (C=0).

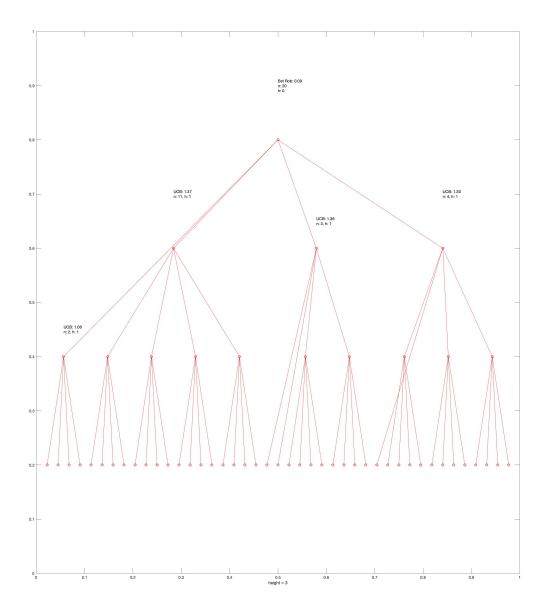


Figure 3: MCTS: Balanced (C = 0.5).

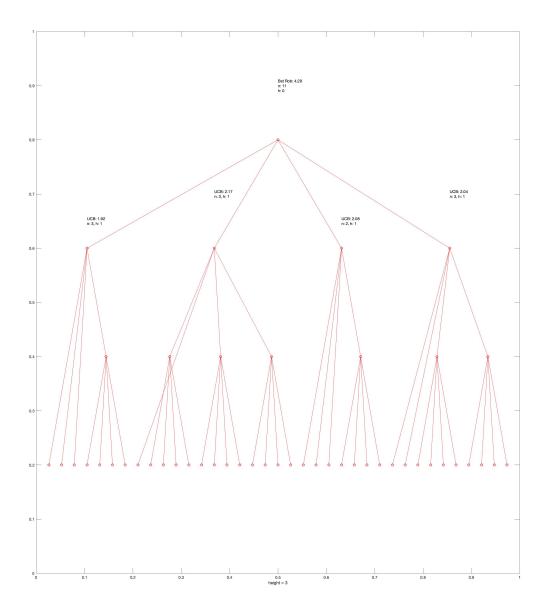


Figure 4: MCTS: Favour exploration (C=1).

#### 1.3 Lower-layer: Search Algorithms

In the very end, the falsification task boils down to a search in the space of the states. We aim to find a falsifying trace of k disturbances, where k is the number of desired control points.

When the MCTS (see Section 1.2) selects a node at depth h from which to start the rollout, the MCTS is basically defining a meta-trace  $\hat{\delta}$ . The first h meta-disturb of  $\hat{\delta}$  ( $\hat{\delta}$  prefix) are regions of the input space in which to restrict the search, and in the remaining part of the trace (suffix) the search is allowed in the whole input space.

One of the following search algorithm is then used to produce a trace  $\delta$  accordingly with the semantic of  $\hat{\delta}$ .

#### 1.3.1 Random Search: MCTS Left Alone

To have a way to evaluate the contribution purely given by the MCTS layer, we decided to implement a fast and dumb search strategy. Given a meta-trace  $\hat{\delta}$ , for each control point k in  $\hat{\delta}$ , RS basically samples u.a.r. a disturbance from the appropriate region for such control point. This result in a very fast algorithm which basically delegates all the search to the upper layer (see Section 2 for comparison).

#### 1.3.2 Hill Climbing: the Greedy

Here, the classical Hill Climbing implementation has been used. The only difference is that, since eventually we need a full trace (a trace having a disturbance for each control point), in the case HC can't find a non pejorative move at a given depth of the search, the less pejorative move available is selected.

Also, to have a stronger falsification power, at the cost of a longer simulation time restart has been implemented but not adopted in the experiments.

On the other hand to speed up the simulation, at the cost of a weaker falsification power it is possible to bound the number of neighbour tested by the algorithm at each step. Clearly this search strategy is way clever and effective than RS. Conversely, the simulation time needed to carry on the falsification process with a non trivial property to falsify is of another order of magnitude (see Section 2 for comparison).

#### 1.3.3 Simulated Annealing: the Impatient

The SA, basically an HC enriched with a notion of temperature has been implemented. As notion of temperature the MCTS budget (hence number of expanded nodes) has been used. The higher the temperature the hardest is to make a pejorative move. The goal here is to further balance exploration and exploitation and be, in terms of speed and performance in the middle of RS ans HC. Note that, since MCTS already implements exploration, the SA introduces some redundancy in the search (a bias towards exploration) that could affect the final result (see Section 2 for comparison).

# 2 Experimental phase

### 2.1 Model and Specifications

The benchmark adopted is the Automatic Transmission by MathWorks, provided in Simulink. It models a vehicle equipped with a trasmission controller and allows the user to change two input signals: the *throttle* and the *brake*.

There are several specifications defined on this model because it is a common benchmark in verification papers. Starting from the reference paper and [1], we selected two specification:

- 1. The engine speed never reaches 120.
- 2. If the vehicle gear is 3 then the speed is always greater than 20.

We selected these two specifications because they are representative of different kind of search. As explained in Section 2.2, the former is easier to falsify because a greedy approach drives towards the decrease of robustness. Conversely the latter requires an higher exploration factor and then a more clever search.

#### 2.2 Implementation of Robustness Metric

In order to understand the difficulty of falsification of the second specification, we need to spend a few words on the computation of robustness metric used to drive the search.

According to the definition of Robustness given in [2], this metric gives us a measure of how the state of the model is far from the falsification. In the first specification, the computation of Robustness is simply the difference between the reference speed and the current speed. Conversely, in the second specification the computation is more complex and can be summarized by the following steps:

- 1. Since the second specification is an implication, we wrote it as an OR.
- 2. The Robustness of the OR operator is the max value of the Robustness computed in the two subformulas.
- 3. The first subformula is the absolute value of the difference between the current gear and the reference gear (3).
- 4. The second subformula is the difference between the current speed and the reference speed (20).

Notice that the value of the second subformula is positive if the current speed is greater than 20, otherwise is negative. Conversely, the value of the first subformula is a small positive integer.

As a result, when the gear is different from the reference one (3) the first subformula dominates the Robustness computation. When the gear is the third one, the value could be dominated by the second subformula only if it is greater than 20 and then positive. Then, the resulting metric space is characterized by long plateau when the current speed is far from 20, and local minima because of the different unit of measurement for gear and speed.

## 2.3 The proposed baseline: URS

The two specifications above have been implemented in an external function in order to maintain a single model file. The Simulink model has been modified with two output blocks, respectively the *speed* and the *gear*. The external function computes the robustness according to the specification, described by a parameter in the configuration file.

The second specification S2 caused too long time to reach falsification and we decided to change its implementation in order to make it suitable to the time and resources that we can use. In particular, one of the main problem of S2 is the different units of measurement that lead to local minima. In fact, whilst speed is between 0 and 100, the gear is an integer value between 1 and 4.

In contrast with the original definition of the Robustness, we proposed to normalize the second subformula of S2 according to the unit scale. Since the speed goes from 0 to 100, we normalize the second subformula dividing by 100. In this way, the Robustness metric is still characterized by long plateau which make the search not trivial but we reduced the number of local minima.

#### Uniform Random Sampling - Robustness Distribution

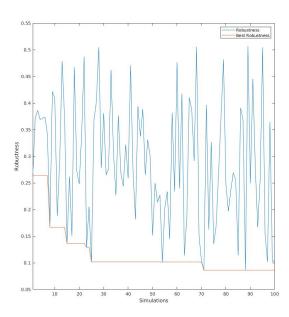


Figure 5: Robustness distribution over 100 simulation of URS search. This random distribution highlights that there is no learning in URS. Moreover, all the values are in [0,1] because we are using the normalized version of specific S2 and is rather likely to reach the third gear at least once during the simulation with random values.

# 2.4 Experiments

In order to evaluate the effectiveness of the proposed falsification framework, we executed a bunch of experiments on the Automatic Transmission model considering several search algorithms: Uniform Random Sampling (URS), MCTS with Random Search (MCTS+RS), MCTS with Hill Climbing (MCTS+HC), MCTS with Simulated Annealing (MCTS+SA).

Since the execution of experiments is based on randomness, we repeated the same experiment 10 times to ensure more confident results.

First of all, we observed that the effectiveness of MCTS is based on its capacity to build the complete tree. In fact, the MCTS algorithm is characterized by a inherent phase of exploration which allows to have a general view of the space (tree width) and the time (tree height). Only when the tree is sufficiently complete, the MCTS algorithm results effective and specializes the search. As a consequence, if the branching factor of the nodes is too high, the number of simulation required to build the entire tree is huge and the MCTS doesn't result effective, compared with the URS.

Thus, in order to exploit the power of MCTS, we tuned the split parameters for the input signals. The Speed signal has been split into 2 regions and the Brake signal has been split into 3 regions, then the branching factor of the tree is 6. In the original paper [3], they adopted a different tuning but we observed that such tuning is not effective compared with the URS algorithm. Moreover, for reason of time and resources, the experiments below have been configured with budget parameter equal to 100, it means that the maximum number of iterations of MCTS is 100 and then this is an upperbound in the number of simulations.

The tables below show the obtained results varying the parameter C which balance the value of exploration and exploitation in the UCB formula. The tables contain the following fields:

- Algo: Search algorithm adopted.
- C: Value of parameter C.
- Avg Trace Falsification: Number of trace before falsification, considering only the tests which falsify the specification.
- Success Rate: Number of tests which falsify the specification.
- Robustness: statistics on robustness value.
- Time: statistics on elapsed time, both time are considered as average of the all number of experiments.

Furthermore, the results varying the input space partitioning are reported in the Table 3 in order to emphasize the influence of hyperparameter tuning.

In order to highlights the different approach of search algorithms adopted, the Figure ?? shows the Robustness over a sample of 5 experiments. For each search algorithm MCTS+RS, MCTS+HC, MCTS+SA, the Figure shows the Robustness obtained and the current best Robustness. Moreover, the Figure shows in the first half the experiments with C=0.0 and in the second half the experiments with C=0.125. In this way, it also highlights the impact of MCTS exploration/exploitation factor on the search algorithm underlying.

Experiments on specification S1

Algo	C	Avg Trace	Success Rate	Robustness			Time (sec)	
Aigo		Falsification		min	avg	std dev	tot	trace
URS		40.0	2/10	-4.07960	4.91882	5.514537	178.014	2.010
MCTS+RS	0.500		0/10	5.30248	8.34921	2.400517	559.916	5.599
MCTS+HC	0.500	94.0	1/10	-1.93401	5.07959	4.386757	1034.225	10.405
MCTS+SA	0.500	24.0	1/10	-0.68394	7.71311	4.152730	891.788	9.781
MCTS+RS	0.250	62.0	4/10	-4.52952	2.80386	4.995340	355.163	4.198
MCTS+HC	0.250	74.4	5/10	-3.63763	2.02058	4.289398	676.262	7.774
MCTS+SA	0.250	74.6	7/10	-10.29078	-2.38069	3.744391	597.866	7.297
MCTS+RS	0.125	46.5	8/10	-6.27352	-1.28520	3.320830	170.078	2.996
MCTS+HC	0.125	68.1	8/10	-4.61933	-0.07330	4.719893	380.664	5.127
MCTS+SA	0.125	48.5	8/10	-6.43350	-0.69363	5.875865	304.758	5.225
MCTS+RS	0.000	65.5	2/10	-4.20689	6.83323	7.070816	325.054	3.510
MCTS+HC	0.000	41.0	4/10	-2.64884	1.95654	3.712926	430.037	5.589
MCTS+SA	0.000	78.0	2/10	-1.51834	4.81908	6.613282	558.383	5.834

Table 1: Comparison between MCTS and URS, given a budget of 100 simulations and varying the parameter C in order to balance exploration and exploitation in the search. The input signals have been split into 2 and 3 region.

# 3 Analysis of the results

In this section, a contextual analysis of the experimental results and of what are the factors which influenced such results. Before analysing the results, we enforce that many problems have been overcome in order to obtain and evaluate the proposed approach.

For example, randomness in Matlab is not always random as one would expect. In fact, at every startup the seed which generate the random sequence is the same, thus we had to further tweak the environment in order to avoid the risk to select a favorable configuration. Moreover, the object orientation in poorly handled in Matlab, especially when you decide to use a Simulink model. In fact, we faced with problems and poor performances caused by scope overloading. To solve these problems, we had to rewrite the all code avoiding object-orientation and only using scripts and global scopes. We did all our best to give structure and cleanless to the code using packages and structs.

#### 3.1 Comments

The first experiments presented consider a fixed input space partitioning (2x3) and have been tested on both specifications S1 and S2, varying the parameter C.

S1 The engine speed never reaches 120. Clearly the robustness function resulting by S1 is proportional to the break signal and inversely proportional to throttle, *i.e.* high value of throttle and low value of brake lead to low robustness. As a consequence, minimizing the robustness should be quite straightforward, even using a greedy search, such as Hill Climbing.

According to this intuition, the best performing experiments are those with a reasonably small value for C = 0.125, as you can see in Table 1. Recall that a small C means to give a bias towards exploitation rather than exploration. Thus, encouraging the exploitation reflects the monotonic

Experiments on specification S2

Algo	C	Avg Trace	Success Rate	Robustness			Time (sec)	
Aigo		Falsification		min	avg	std dev	tot	trace
URS			0/10	0.02080	0.06130	0.03130	408.585	4.086
MCTS+RS	0.500	90.0	2/10	0.00000	0.05430	0.04420	397.635	4.056
MCTS+HC	0.500	53.0	7/10	0.00000	0.00490	0.00840	1039.426	15.517
MCTS+SA	0.500		0/10	0.00071	0.04930	0.04330	681.465	6.815
MCTS+RS	0.250		0/10	0.00661	0.03571	0.02372	572.351	5.723
MCTS+HC	0.250	29.1	9/10	0.00000	0.00023	0.00073	914.175	26.076
MCTS+SA	0.250	46.0	1/10	0.00000	0.06933	0.09205	941.542	9.977
MCTS+RS	0.125	42.7	3/10	0.00000	0.04101	0.03548	275.017	3.325
MCTS+HC	0.125	31.6	8/10	0.00000	0.00388	0.00843	497.749	11.466
MCTS+SA	0.125	76.3	4/10	0.00000	0.05349	0.11098	506.049	5.577
MCTS+RS	0.000	32.0	1/10	0.00000	0.14757	0.16817	473.518	5.113
MCTS+HC	0.000	36.6	5/10	0.00000	0.01148	0.01682	1322.240	19.317
MCTS+SA	0.000		0/10	0.00834	0.16304	0.14714	873.488	8.734

Table 2: Comparison between MCTS and URS, given a budget of 100 simulations and varying the parameter C in order to balance exploration and exploitation in the search. The input signals have been split into 2 and 3 region.

nature of the robustness of S1 and then exploiting promising regions gives back promising results. The results obtained by MCTS and Hill Climbing (MCTS+HC) confirm this intuition. Furthermore, the computational overhead due to the Hill Climbing search is almost negligible. The reason is that the search proceeds almost straight, following the steepest hill of the robustness curve and

Conversely, favouring exploration (e.g. C=0.5) results in extremely poor results. In fact, the falsification rate is even worst than URS and the reason is that most of the budget is wasted in the exploration of the huge state space. Moreover, the overhead of Hill Climbing (MCTS+HC) and Simulated Annealing (MCTS+SA) is quite relevant because exploration of less promising regions leads to difficulties in finding not pejorative neighbours (MCTS+HC presents a net 2x overhead).

**S2** If the vehicle gear is 3 then the speed is always greater than 20. As already said in the Section 2.1, this specification is not trivial at all and a clever strategy is needed in order to do any falsification. Evidence of this is the fact that Uniform Random Sampling (URS) is never able to falsify such requirement, as can be observed from Table 2.

The best performing strategy here is again MCTS and Hill Climbing (MCTS+HC), which is able to falsify S2 specification 9 times out of 10 with C = 0.25.

Note that:

- The C hyper-parameter balances two factors, exploitation and exploitation.
- The exploration factor is not normalized, typically is  $\in [0, 2.5]$ .
- The exploitation is normalized  $\in [0, 1]$ .

then Hill Climbing easily finds not pejorative neighbour.

The ratio among the two factors is in the order of 2.5. Then, using C=0.25, this ratio is mapped into a more reasonable and performing 0.6 factor which favours exploitation and maintains exploration.

Experiments on different partitioning

Algo	Part	Avg Trace	Success Rate	Robustness			Time (sec)	
Aigo		Falsification		min	avg	std dev	tot	trace
URS			0/10	0.02080	0.06130	0.03130	408.585	4.086
MCTS+RS	2x3	90.0	2/10	0.00000	0.05430	0.04420	397.635	4.056
MCTS+HC	2x3	53.0	7/10	0.00000	0.00490	0.00840	1039.426	15.517
MCTS+SA	2x3		0/10	0.00071	0.04930	0.04330	681.465	6.815
MCTS+RS	2x2	70.0	3/10	0.00000	0.06517	0.06197	631.963	6.804
MCTS+HC	2x2	48.7	7/10	0.00000	0.00099	0.00163	1479.660	22.530
MCTS+SA	2x2		0/10	0.00561	0.05776	0.02567	1393.552	13.936

Table 3: Comparison of search algo with fixed C=0.5 and varying the input space partitioning. Test on spec S2.

Finally, in the experiments which falsify the specification, the number of simulated trace is surprisingly small, as reported by the column *Avg Trace Falsification*. This is due the effectiveness of the search algorithms and highlights their performance.

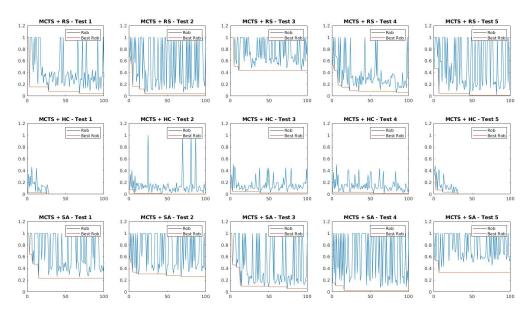
Impact of search algorithms. The previous comments consider the obtained result in a monolithic way, without any consideration of the two-layers search. Considering the details about the Robustness distribution varying the search algorithm allows to understand what is the impact of each search algorithm in the lower level. For this reason, we reported a sample of 5 tests in the Figure ?? varying the search algorithm and presenting the result with C = 0.0 (worst performance) and C = 0.125 (best performance).

First of all, in both the experiments, we can observe a different behaviour between MCTS+HC and MCTS+RS, MCTS+SA. Whilst MCTS+HC remains bounded locally with respect to the Robustness value, the other search algorithms present high peaks. These wide oscillations correspond with a less effective search that has been confirmed by the results.

Considering the experiments with C=0.0, all the algorithms seem to explore their best regions without any exploration and for this reason, they rarely reach the falsification. Even the MCTS+HC continue to search locally without success (3 times over 5). However, when MCTS+HC luckily choose the right down-hill, it go straight and reach falsification (2 times over 5).

Conversely, considering the best tuning with C=0.125, all the algorithms perform better than the previous experiments. The peculiar behaviour of each algorithm is confirmed. In fact, we can easily recognize the shape of the MCTS+HC compared with the others.

### Robustness Monitoring - Test with C=0.0



## Robustness Monitoring - Test with C=0.125

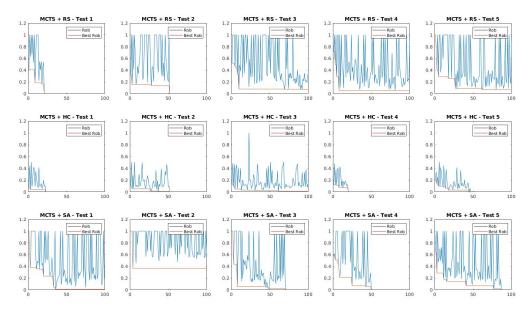


Figure 6: Comparison of robustness monitoring using different search algorithm: MCTS+RS (first line), MCTS+HC (second line), MCTS+SA (third line). Robustness distribution over 100 simulation, picking 5 sample test of falsification on specification S2.

**Execution Times** In conclusion, a brief consideration of the execution time is worth to be made. However, even if the performances for Hill Climbing are quite good even for this non-trivial specification, they come with a higher cost. The *Time* column of Table 2 highlights that the time needed to simulated a single trace with MCTS+HC is 26.076 seconds, compared with 5.723 seconds

needed for MCTS+RS and 9.977 seconds for MCTS+SA. Having a cluster of multiple machines would allow to test multiple configuration of the same experiment and exploit *a posteriori* those configurations which expand the tree, pruning those configurations which waste time on not relevant regions.

Comparing the times of the several MCTS implementations with the Uniform Random Sampling (URS), one can notice that the overhead of MCTS layer is minimal. In fact, the difference between the overall times for URS and MCTS+RS is not relevant and then, we can conclude that the upper layer is performed with a little overhead. Unfortunately, this is only true when using Random Sampling as search algorithm. Any other search algorithm ends up in a more consistent overhead, after all there is no free lunch.

State Space Partitioning Table 3 shows another direction of tests we investigated on. In such experiment, the state space is partitioned in a different way: the throttle and the brake has been divided into two regions (2x2) or in two and three regions respectively (2x3), as in the previous experiments. This may sound as a pointless modification but it opens up the scenario described till now to a new perspective. This new partitioning induces a MCTS tree with a smaller branching factor (4 vs 6). Direct consequences are the following:

- 1. The space partitioning is coarse grained.
- 2. MCTS has an higher view of the search space.
- 3. Lower branching factor allow the MCTS to finish earlier the tree building.
- 4. The effort is moved to the search algorithm in the rollout phase.

The experiments done are only a few and still far to a conclusion. However, they produce a slight improvement in Falsification Rate as well as Avg Trace Falsification. This fact sheds the light on the importance of hyper-parameter tuning because it could lead to a relevant boost in performances (see Section 3.2).

#### 3.2 Hyperparameters

As already remarked, the role of hyper-parameters and the importance of a proper tuning of those is crucial for effectiveness of search algorithm. Having the computational power to do so, a proper grid search in the space of hyper-parameter should be performed to fine-tune the system and fully exploit its potentiality. The hyper-parameters that should be investigated are the following:

- 1. C: in order to balance exploration and exploitation and to better combine MCTS with a given search strategy.
- 2. Space partitioning: in order to control MCTS branching factor.
- 3. Disturbance quantization: in order to control the search space and to avoid its explosion.
- 4. Control Points: in order to define the degree of freedom of the simulation itself.

Clearly a full search is prohibitive thus a clever approach should be devised in order to performs hyper-parameters optimization.

# 4 Conclusion

In this project, we had the chance to explore a new trend in falsification which consists of the application of a sophisticated combination of search algorithms in falsification of Temporal Logic specifications. The proposed MCTS algorithm performs well, even in more challenging scenario as the Specification S2, but it shown that it requires a high number of simulation to compute the search tree because the building is based on exploration of each branch. As a consequence, the application of MCTS on simple specifications (i.e. easy to find an event trace which falsify the specification) doesn't result effective.

In conclusion, we would like to propose a few developments of the presented work:

- 1. Since the tuning of parameter C seems to be determinant, an extension of the algorithm could be its adaptive version which change the value of C according to the obtained results. The design of a learning model could be explored with a reinforcement learning approach.
- 2. The proposed approach doesn't take into account any domain knowledge that could be exploited on a specific model. In fact, we could customize:
  - (a) the ordering of node explored in the Expansion phase. We could infer an ordering to bias toward know-to-be-good regions.
  - (b) the branching factor at different levels of tree because the height of the tree corresponds to the time advancement. We could adopt a low branching factor in less significant period of time and a wide branching factor on more interesting time periods.
- 3. The adopted UCB formula seems to be a bias towards exploration because the two terms are not normalized properly. In fact, the first term is normalized in [0,1] whilst the second term is not and change in [0,3]. We could rafine this formula normalizing the second term with respect to the max value, improving the performance.
- 4. The tree structure of MCTS seems to be adapt to a parallel execution. In particular, the expansion phase at a given layer will expand all node with a 0 score and, up till exist one, a node with such score is selected. Then it is possible to exploit embarrassing parallelism with a master-slave architecture to speed up the expansion phase which is the bottle neck of MCTS. That way, one layer can be attacked at once to reduce the exploration cost and invest in exploration.

The system presented, which eventually obtains good performances, could largely benefits from all these new directions.

# References

- [1] Houssam Abbas Bardh Hoxha and Georgios Fainekos. Benchmarks for temporal logic requirements for automotive systems. *Proceedings of applied verification for continuous and hybrid systems*, 2014.
- [2] Georgios E Fainekos and George J Pappas. Robustness of temporal logic specifications. In Formal Approaches to Software Testing and Runtime Verification, pages 178–192. Springer, 2006.

[3]	Zhenya Zhang, Gidon Ernst, Sean Sedwards, Paolo Arcaini, and Ichiro Hasuo. Two-layered falsification of hybrid systems guided by monte carlo tree search. <i>IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems</i> , 37(11):2894–2905, 2018.