15)

A) $ABCD = 3\sqrt{3}$

B) $V = 6$

C) $ABCD = \frac{2\sqrt{2}}{3}$

D) $EABD = 1$

E) $DEB = \frac{3\sqrt{35}}{35}$

A)

11) $||AB \times AC|| = 1^2 \cdot \frac{\sqrt{3}}{2}$

$||AB \times AC|| = \frac{\sqrt{3}}{2} \cdot 1^2$

12) $2a + 3b + 4c = 9$

$-b - c = -2 \quad b + c = 2$

$a - c = 0 \quad a = c$

$a + b = 2 \quad a = b = c = 1 \quad \vec{x}(1,1,1)$

B) $x(a, b, c) \times (1, 0, 1) = (2, 2, -2)$

$||x|| = \sqrt{6}$; $a^2 + 4 + (a+2)^2 = 6 \quad a = -1 \quad c = 1 \quad x(-1, 2, 1)$

C) $||x|| = \sqrt{3}$ x é ortogonal a v e V e altura com J

$(-1, -1, -1)$

13) $AB \times AC = \begin{vmatrix} i & j & k \\ -1 & 1 & 0 \\ 0 & 1 & 3 \end{vmatrix} = i(1 \cdot 3 - 0 \cdot 1) - j(-1 \cdot 3 - 0 \cdot 0)\, i(3) - j(-3) + k(-1) = (3\ 3\ -1)$

$\sqrt{3^2 + 3^2 + (-1)^2} = \sqrt{19}$

$||BC|| = \sqrt{1^2 + 0^2 + 3^2} = \sqrt{10}$

$\Delta = \frac{1}{2}\sqrt{19} = \frac{1}{2} \cdot \sqrt{10} - \frac{2\sqrt{19}}{\sqrt{10}} = \frac{\sqrt{190}}{\sqrt{100}}$

área ABCD = $\sqrt{42}$ altura relativa é $\frac{\sqrt{190}}{10}$

14)

A) $U \cdot (V \cdot W) = W(U \cdot V) = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$ $U \times W = i(y_2 z_3 - z_2 y_3) - j(x_2 z_3)\, k(x_2 y_3\ y_2 x_3)$

$V \times W = x_1(y_2 z_3 - z_2 y_3) - y_1(x_2 z_3 - z_2 x_3)$

$z_1(x_2 y_3 - y_2 x_3)$

$V \cdot (V \times W) = W \cdot (U \times V)$ F portanto é identidade

B) $(U, V, W) = \begin{vmatrix} 1 & 3 & 2 \\ 0 & 1 & -2 \\ -1 & 2 & 0 \end{vmatrix}$ Expansão $1 \cdot (0+4) - 3 \cdot (0-2) + 2 \cdot (0+1) = 4 + 6 + 2 = 12$

$UVW = UVW = 12$

$(U\ 2W\ V) = 24$

$(U, V, W) = 12$

$(U, W, V) = -12 \quad (U, 3V - 2U, W + 3V) = 36$

$(U\ 2W, V) = 24$

8)

A) $V \cdot U = 1.3 + -1.-1 + 2.1 = 2+1+3 = 6$

$3^2 + (-1)^2 + 1^2 = 9+1+1 = 11$ $\quad proj_U V = \frac{6}{11}(3,-1,1)\left(\frac{18}{11}, -\frac{6}{11}, \frac{6}{11}\right)$

B) $1.-3 + 1.3 + 5.0 = 0$ $\quad (-3)^2 + 1^2 + 0^2 = 10$ $\quad proj_U V \ \frac{0}{10}(-3,1,0) = (0,0,0)$

C) $-1.-2 + 1.1 + 1.2 = +2+2+1 = 5$ $\quad proj \ \frac{5}{9}\left(\frac{10}{9}, \frac{5}{9}, \frac{10}{9}\right)$

$4+4+1 = 9$

D) $1.(-2) + 2.(-4) + 4.(-8) = -2-8-32 = -42$

$(-2)^2 + (-4)^2 + (-8)^2 = 84$

$-\frac{42}{84}(-2,-4,-8) = -\frac{1}{2}(-2,-4,-8) = (1,2,4)$

9) $V \cdot U = 18$ $\quad U \ \|U\|^2 = 9$ $\quad \frac{18}{9}U = 2U = 4,-4,2$

$V \ \|V\|^2 = 45$ $\quad \frac{18}{45}V = \frac{2}{5}V \ \left(\frac{6}{5}, -\frac{12}{5}, 0\right)$

$P = (4,-4,2)$

$q = (-1,-2,-2) = 3$

$U = 3$

$3.3 = 9$ $\quad U \times V = (6,3,-6) = \sqrt{36+9+36} = 9$

10) $\begin{vmatrix} i & j & k \\ 3 & 3 & 0 \\ 5 & 4 & 0 \end{vmatrix} = (0,0,-3) = \sqrt{0^2+0^2+3^2} = 3$

$U = (4,0,-5)$
$r = (1,2,-1)$ $\quad \begin{vmatrix} i & j & k \\ 4 & 0 & -5 \\ 1 & 2 & -1 \end{vmatrix} = (10,2,14) = \sqrt{10^2+2^2+14^2} = \sqrt{300} = 10\sqrt{3}$

$U = (1,-3,1)$ $\quad V = (1,1,4)$

$\begin{vmatrix} i & j & k \\ 1 & -3 & 1 \\ 1 & 1 & 4 \end{vmatrix} = (-13,-3,4) = \sqrt{194}$ $\quad U(2,1,2)$ $\quad V(4,2,4)$ $\quad \begin{vmatrix} i & j & k \\ 2 & 1 & 2 \\ 4 & 2 & 4 \end{vmatrix} (0,0,0) = \sqrt{0^2+0^2+0^2} = 0$

Lista 06

5)

A) $1.-2+0+2 = -2+0+2 = 0 = \theta = \frac{\pi}{2}$ rad

B) $-1.1+1.1+1.1 = -1+1+1$ $\quad u = \sqrt{3}$ $\quad V = \sqrt{3}$ $\quad \cos\theta = \frac{1}{\sqrt{3}} = 1,231$ rad

C) $3.2+3.1+0 = 6+3+0 = 9$ $\quad u = 3\sqrt{2}$ $\quad V = 3$ $\quad \cos\theta = \frac{1}{\sqrt{2}} = \frac{\pi}{4}$ rad

D) $\sqrt{3}.\sqrt{3}+1+0 = 4$ $\quad u = \sqrt{3+1} = 2$ $\quad V = \sqrt{3+1+12} = 4$ $\quad \cos\theta = \frac{4}{2.4} = \frac{1}{2}$ $\frac{\pi}{3}$ rad

6)

A) $(x+1)(x-1)+1(-4)+2 = 0$

$(x^2-1)-4+2 = 0$ $\quad x^2 = 3$ $\quad x = \sqrt{3}$

B) $x.4 + x^2 + 4 = 0$ $\quad 4x + x^2 = 0$ $\quad (x+2)^2 = 0$ $\quad x = -2$

7)

A) $4x - y + 5z$ $\quad 4(2y-3z) - y + 5z = 0$

$x - 2y + 3z$ $\quad 8y - 12y - y + 5z = 0$ $\quad \vec{v}(1,1,1)$

$x + y + z$ $\quad y = z$ $\quad z = -1$

$3y - 2y = -4$ $\quad y = -1$

B) $\begin{vmatrix} i & j & k \\ 2 & 3 & -1 \\ 2 & -4 & 6 \end{vmatrix} = (14,-14,-14)$ $\quad \sqrt{3.14^2} = \sqrt{3}.14$

$\|v\| = \frac{3\sqrt{3}}{\sqrt{3}.14}(14,-14,-14) = (3,-3,-3)$

C) $v.i = 1 > 0$

$v_2.i = 3 > 0$ ambos formam ângulo agudo com i

3) A) $XY = Y - X$

$AB = A - B$ $\quad (5-2,\ 1-4,\ 3-3) = (3,\ -3,\ -6)$

$BC = C - B$ $\quad (0-5,\ -3+1,\ 1+3) = (-5,\ -4,\ 4)$

$CA = A - C$ $\quad (2,\ 4+3,\ 3-1) = (2,\ 7,\ 2)$

B) AB) $\sqrt{9+9+36} = \sqrt{54}$

BC) $\sqrt{25+16+16} = \sqrt{57}$ $\quad BC = CA$

CA) $\sqrt{4+49+4} = \sqrt{57}$ $\quad$ isosceles

C) $M_{AB} = \frac{A+B}{2} = \frac{(7,5,0)}{2} = \frac{7}{2}, \frac{5}{2}, 0$

$AB \cdot CM = 3 \cdot \frac{7}{2} - 3 \cdot \frac{11}{2} + 6 = -6 + 6 = 0$

D) $CA = (2,7,2)$

$CB \cdot CA = 5 \cdot 2 + 4 \cdot 7 + (-4) \cdot 2 = 10 + 28 - 8 = 30$

$\cos\theta = \frac{30}{\sqrt{57} \cdot \sqrt{57}} = \frac{30}{57} \Rightarrow \theta \cong 58{,}3$

E)

$B - A + C - B + A - C = 0$ a soma é nula pois formam um circuito fechado

4) A) $|\,\|\vec{U}\| \cdot \|\vec{V}\| \cdot \cos\theta\,| \leq \|\vec{U}\| \|\vec{V}\|$ igualdade quando $\theta \neq 1$ vetores não paralelos

B) $(U+V) \cdot (U \cdot V) = \|U\|^2 + 2U \cdot V + \|V\|^2$

$\|U+V\| \leq \|U\| + \|V\|$

C) $\|U+V\|^2 - \|U-V\|^2 = \|U\|^2 + 2UV + \|V\|$

$- \|U\|^2 - 2U \cdot 1 = 4\vec{U} \cdot \vec{V}$

GA. LISTA 6

1) A) $\vec{v} = (1,1,1)_e = \sqrt{1^2+1^2+1^2} = \sqrt{3}$

B) $\vec{v} = 3\vec{i} + 4\vec{k} \to \sqrt{3^2+4^2} = \sqrt{25} = 5$

C) $-i + j \to \sqrt{-1^2+1^2} = \sqrt{2}$

D) $u = 4i + 3j - k \to \sqrt{16+9+1} = \sqrt{26}$

2) A) é uma base ortonormal pois os vetores $\vec{e_1}$ $\vec{e_2}$ $\vec{e_3}$ são unitários e mutualmente ortogonais

B) $CD = -\vec{e_2}$ logo $u = -\vec{e_2} + \vec{e_1} \to (1, -1, 0)$

$CB = \vec{e_1}$ $\quad v = e_2 + \vec{e_1} = (1,1,0)$

$w = e_1 + e_2 + e_3 = (1,1,1)$

C) $\vec{u} = (1, -1, 0)$ $\quad F_1 = \frac{1}{\sqrt{2}}(1,-1,0)$

$v = (1, 1, 0)$ $\quad F_2 = \frac{1}{\sqrt{2}}(1,1,0)$

$F_1 \cdot F_2 = \frac{1}{2}(1 \cdot 1 + (-1) \cdot 1 + 0) = \frac{1}{2}(1-1) = 0$ ortogonais

$F_3 = v \cdot w$ $\quad w(1,1,1)$ $\quad u(1,-1,0)$

$$\begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 1 & -1 & 0 \end{vmatrix} = 1 + j - 2k \quad (1,1,-2)$$

$$\frac{1}{\sqrt{6}}(1,1,-2)$$

da normalização F_3 é ortonormal

D)
$$\begin{vmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ 0 & 0 & -\frac{2}{\sqrt{6}} \end{vmatrix}$$

E) $M^t \cdot M = 1$

f_1 f_2 f_3 são ortogonais e unitários

as colunas de M são ortonormais