Quantum Pseudo-Telepathy
The Magic Square Game

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Proposal

Telepathy, the ability of transmitting information from one person's mind to another's, would certainly come in handy in many situations, right? Unfortunately (or not), (to the best of our knowledge) telepathy is not a thing. At least, not according to classical physics. Certain aspects of the quantum realm, however, provide a way of communication that for a layman looks as magical as "true" telepathy. This phenomenon is called quantum *pseudo-telepathy* [1].

Quantum pseudo-telepathy is observed in many contexts, usually described in the format of a game: the "impossible colouring games", based on the Kochen-Specker theorem [1, 3]; the parity games, in which $n \geq 3$ players are given bit-strings and, without communicating to each other, they output one of their bits, winning if their outputs combined obey certain parity conditions [1, 4]; the Deutsch-Jozsa games, where Alice and Bob are given bit strings x and y, and must output bit strings a and b such that $a = b$ if and only if $x = y$ [1, 2]; and, the Magic Square game, the topic of this research[1, 4]. None of these games admit a classical winning strategy (i.e. is not possible to always win), yet they can be won systematically, without any communication, provided that the players share prior entanglement[1].

A *Magic Square* is a 3×3 matrix whose entries are {-1, 1} (or sometimes {0, 1}), with the property that the sum of each row is 1 and the sum of each column is -1. The Magic Square game is defined as follows: a referee Charlie asks Alice to fill the entries of a row $x \in \{0, 1, 2\}$, which he chooses uniformly at random. Charlie, then, asks Bob to fill the entries of a column $y \in \{0, 1, 2\}$, also chosen at random. Alice and Bob win if the product of the row given by Alice is 1, the product of the column given by Bob is -1 and the intersection of the given row and column agrees.

A successful implementation of a quantum winning strategy for any pseudo-telepathy game must convince an observer that something classicaly impossible is happening[1]. Fortunately, proving the classical impossibility of the Magic Square game is extremely easy. Assuming a deterministic strategy, even if Alice and Bob are allowed to talk *before* the game starts, the best that they can do is to construct two grids in advance, one for Alice and another for Bob, and give the entries according to the matrices that they prepared. However, the product of Alice's matrix will always be 1 and the product of Bob's matrix will always be -1, so there must be one cell that does not agree. Probabilistic strategies cannot do better than deterministic [1, Theorem 2], so the Magic Square game cannot be won under classical assumptions.

We will approach this project as follows:

1. Describe the quantum Magic Square game in details;
2. Discuss classical solutions: illustrate with examples and implement them (and make the code available through a jupyter notebook).
3. Demonstrate that no classical strategy can win with probability 1. Show that the best that a classical strategy can achieve is is win 8 out of 9 rounds;
4. Discuss the quantum solution, using entanglement: illustrate with examples;
5. Demonstrate that the quantum strategy can win with probability 1; and
6. Implement the quantum solution in Qiskit (and make the code available through a jupyter notebook).

References

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