On the Secure Compilation of the Constant-Time Policy

Quantitative Information Flow

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Side Channels

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- Outputs of a computer system

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- **Usually unintentional**

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Side Channels

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Cache

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$$X \longrightarrow \text{System} \longrightarrow Y$$

$$\downarrow$$

Time
Cache
...

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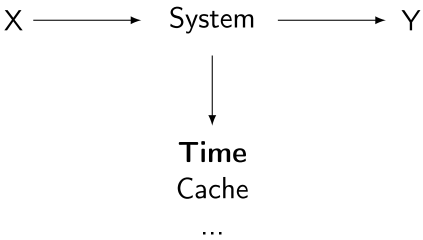

Defense Strategy

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Definition 1 (Constant-Time Programming)

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A program is said to implement a constant-time policy if its memory accesses and control flow do not depend on secret information. In other words, the sequence of instructions and memory accesses must be the same regardless of the secret inputs.

Password Checker

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Example 1

Consider the case of a n-bit password checker

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check : Vect n Bit -> Vect n Bit -> Result
check []          []           = Accept
check (One  :: g) (Zero :: pw) = Reject
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Side channel

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Example 2

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check' : Vect n Bit -> Vect n Bit -> Result
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check' (a :: g) (b :: pw) = ctsel (r && r') Accept Reject
    where r  = check' g pw == Accept
          r' = a == b
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- LLVM's x86-cmov-converter pass replaces cmovs with branches
- **How to prove that the constant-time property is preserved?**

Secure Compilation

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- Barthe, Grégoire, and Laporte, “Secure Compilation of Side-Channel Countermeasures: The Case of Cryptographic “Constant-Time””

Secure Compilation

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- **Constant-time simulations**

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- A program P is composed by a sequence of commands
- **The semantics of P is modelled by labelled transitions of the form**

$$a \xrightarrow{t}^{n} a',$$

where a and a' are states, t is the leakage and n is the number of steps

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$$a \xrightarrow{t}{}^{n} b \wedge a' \xrightarrow{t'}{}^{n} b' \wedge a \,\phi\, a' \implies t = t' \wedge (b \in \mathcal{S}_f \iff b' \in \mathcal{S}_f).$$

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3. $\forall$ source and target states b and β such that $b \approx \beta$, we have that b is a final source state iff β is a final target state

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$$
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a \arrow[r] \arrow[d, dotted, dash, "\approx"'] & b \arrow[d, dotted, dash, "\approx"] \\
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Secure Compilation

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2. $\forall$ pairs of input parameters i and i' such that $i \, \varphi \, i'$, we have that $S(i) \equiv_S S(i')$ and $C(i) \equiv_C C(i')$, where φ is a binary relation on inputs

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& \alpha' \arrow[rr, "\tau"] & & \beta' \\
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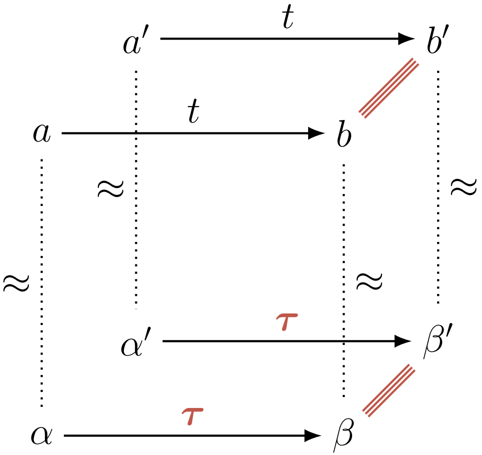
Secure Compilation

- Let $a.\text{cmd}$ and $a'.\text{cmd}$ be $y := A[i] * k$



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- Let $a.\mathrm{cmd}$ and $a'.\mathrm{cmd}$ be $y := A[i] * k$
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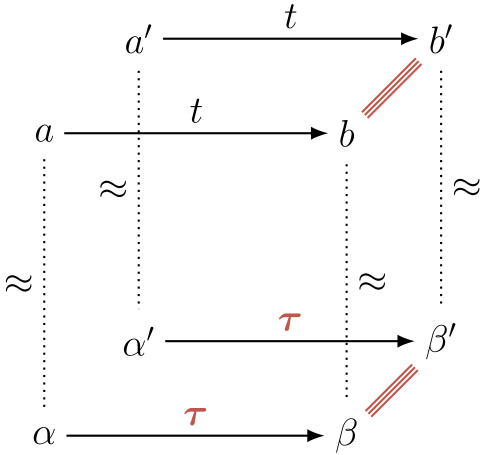

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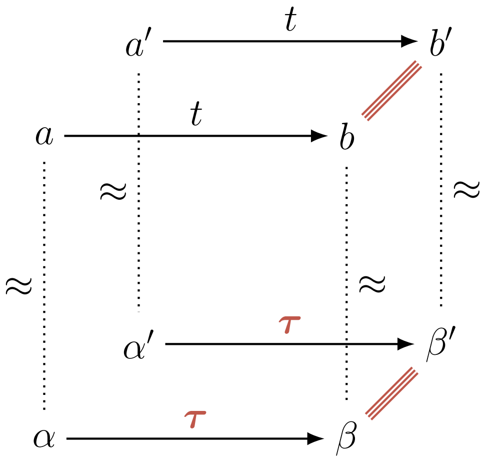

Secure Compilation

- Let a.cmd and a'.cmd be $y := A[i] * k$
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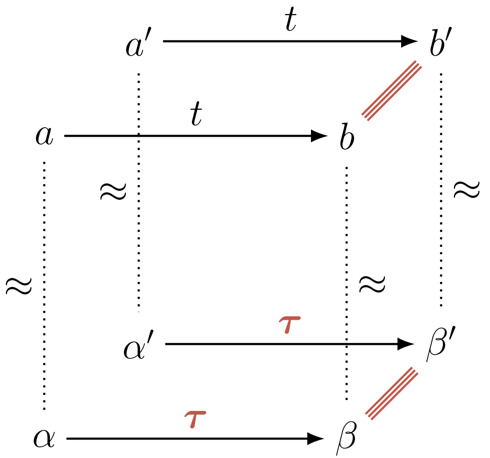

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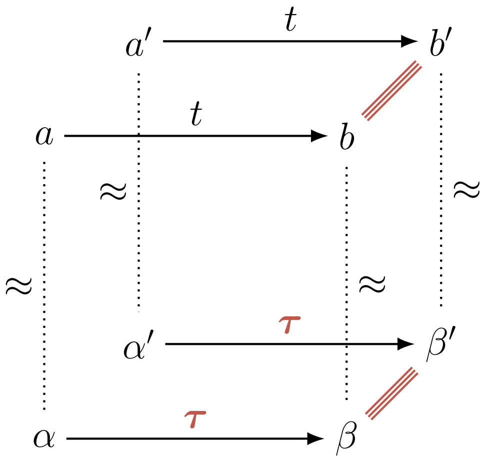

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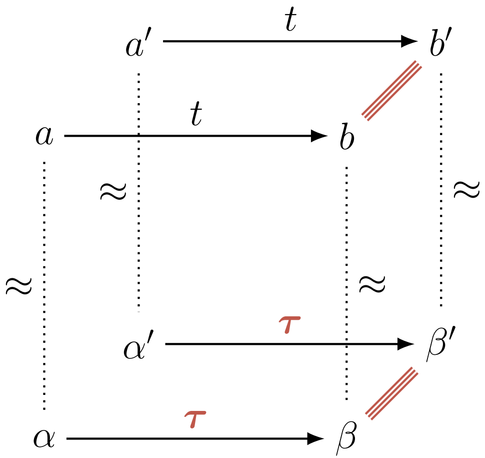
- $t = t' \cdot (A, [i]_{\rho_a})$ and $[i]_{\rho_a} = [i]_{\rho_{a'}}$



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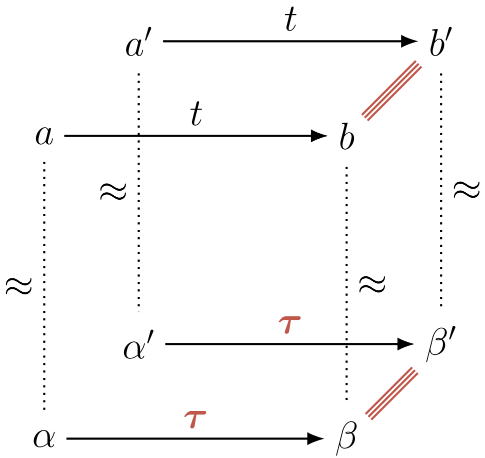

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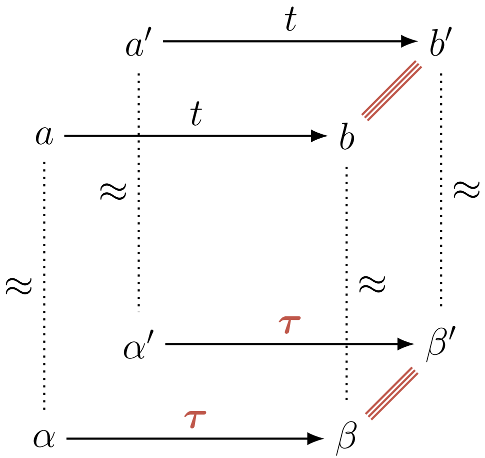
- Suppose k always evaluates to 0
- Then $\alpha.\mathrm{cmd}$ $\alpha'.\mathrm{cmd}$ are $y := 0$
- Similarly, $\beta.\mathrm{cmd}$ and $\beta'.\mathrm{cmd}$ are $z := x$
- $t = t' \cdot (A, [i]_{\rho_a})$ and $[i]_{\rho_a} = [i]_{\rho_{a'}}$
- What is the leakage τ and is it the same in both steps?
- $\tau = \tau'$



Relation to QIF

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- Labelled transitions
- Information-theoretic channels

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- Labelled transitions
- **Leakage as a trace of events**

- Information-theoretic channels
- **Leakage as a real number**

Relation to QIF

- Labelled transitions
- Leakage as a trace of events
- **Constant-time simulation**

- Information-theoretic channels
- Leakage as a real number
- **Refinement**

References

- Alvim, Mário S et al. (2020). *The Science of Quantitative Information Flow*. Springer.
- Barthe, Gilles, Benjamin Grégoire, and Vincent Laporte (2018). "Secure Compilation of Side-Channel Countermeasures: The Case of Cryptographic "Constant-Time"". In: *2018 IEEE 31st Computer Security Foundations Symposium (CSF)*, pp. 328–343. DOI: 10.1109/CSF.2018.00031.