On the Secure Compilation of the Constant-Time Policy

Quantitative Information Flow

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Side Channels

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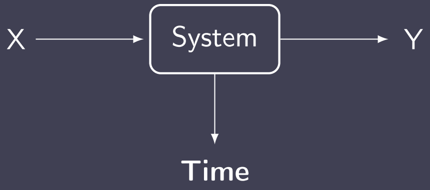

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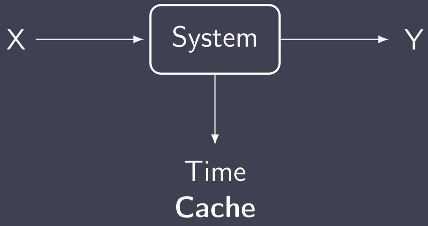

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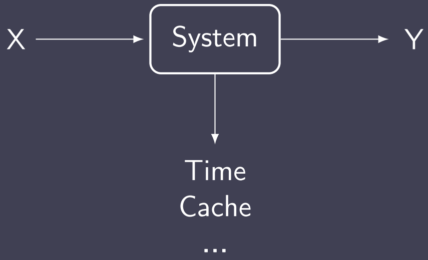

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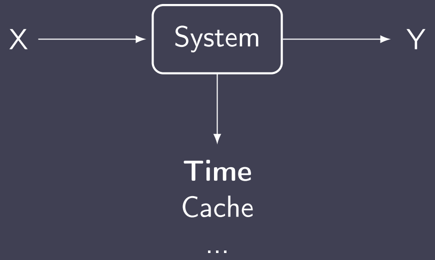

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check' : Vect n Bit -> Vect n Bit -> Result
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check' (a :: g) (b :: pw) = ctsel (r && r') Accept Reject
    where r  = check' g pw == Accept
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- LLVM's x86-cmov-converter pass replaces cmovs with branches
- **How to prove that the constant-time property is preserved?**

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$$a \xrightarrow{t}^{n} a',$$

where a and a' are states, t is the leakage and n is the number of steps

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$$a \xrightarrow{t}{}^{n} b \wedge a' \xrightarrow{t'}{}^{n} b' \wedge a \,\phi\, a' \implies t = t' \wedge (b \in \mathcal{S}_f \iff b' \in \mathcal{S}_f).$$

Secure Compilation

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3. $\forall$ source and target states b and β such that $b \approx \beta$, we have that b is a final source state iff β is a final target state

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$$\begin{array}{ccc} a & \longrightarrow & b \\ \approx \vdots & & \vdots \approx \\ \alpha & \longrightarrow & \beta \end{array}$$

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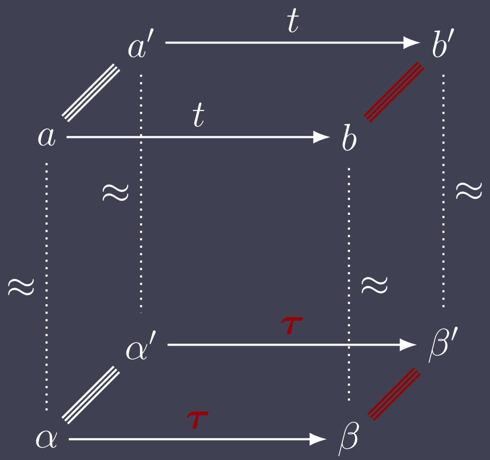

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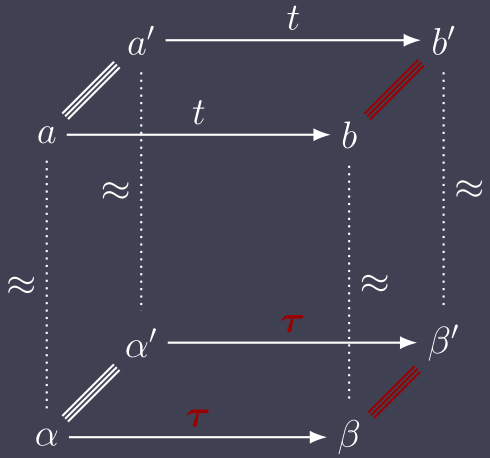

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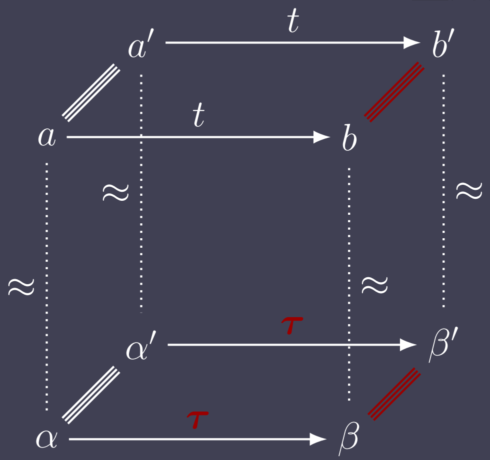
- Let $a.\mathrm{cmd}$ and $a'.\mathrm{cmd}$ be $y := A[i] * k$



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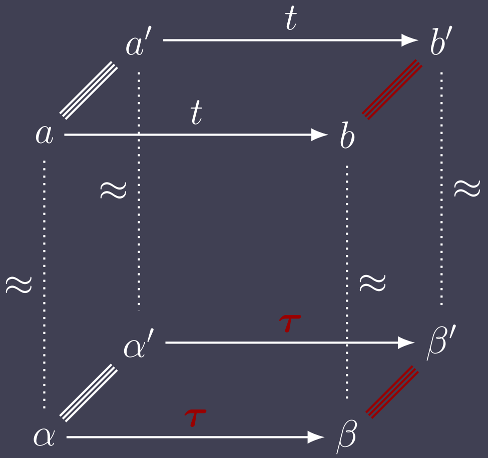

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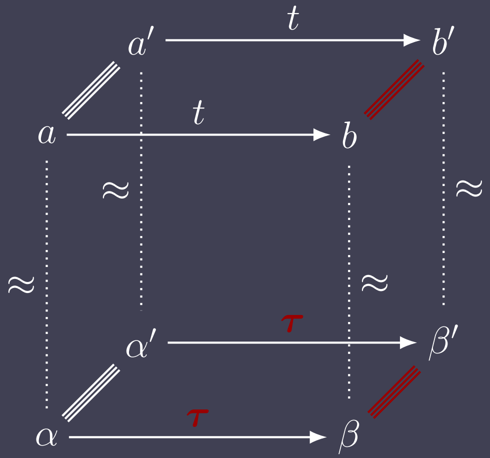

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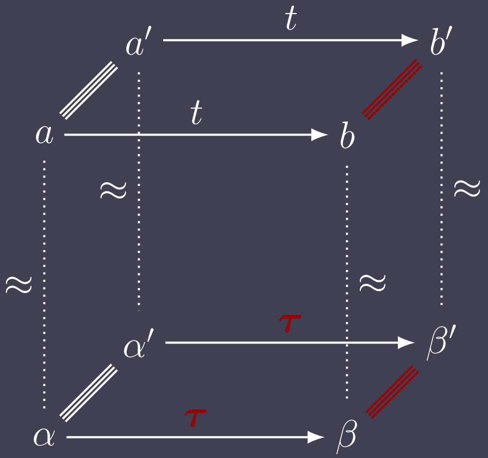

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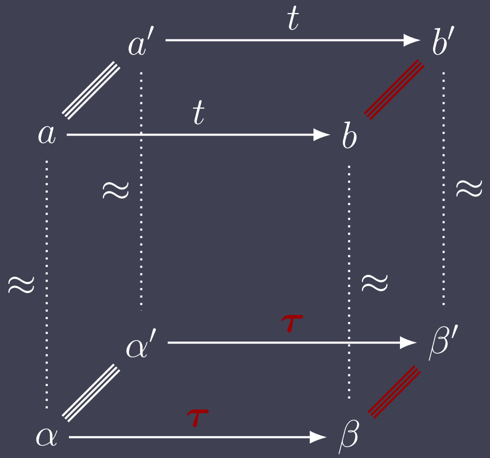

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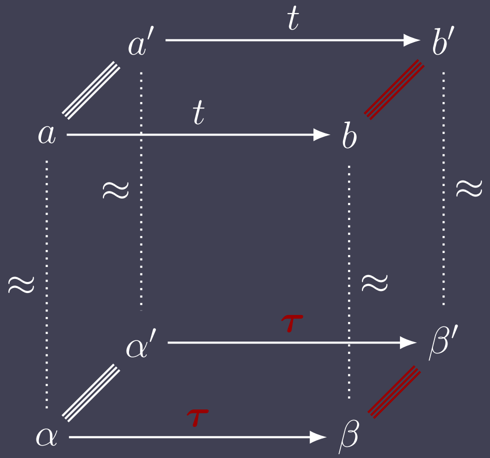

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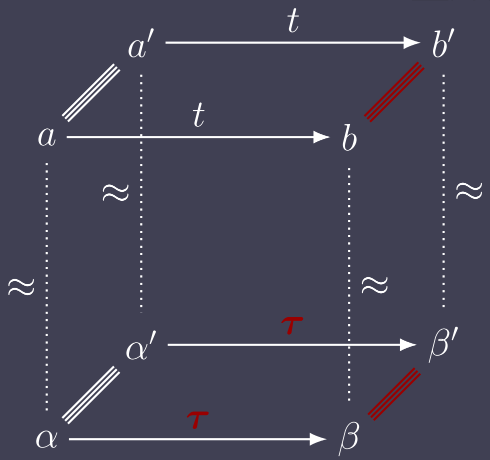

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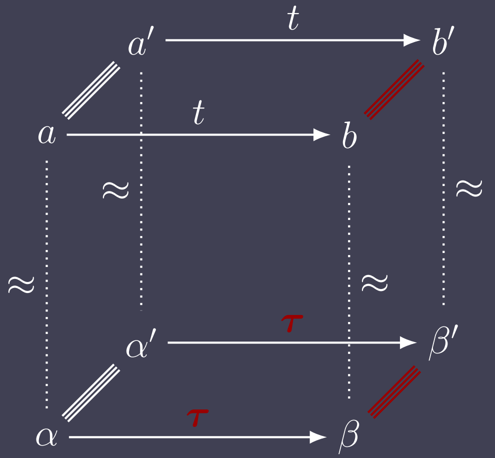

Relation to QIF

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- Labelled transitions
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- Labelled transitions
- **Leakage as a trace of events**

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- **Leakage as a real number**

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- Labelled transitions
- Leakage as a trace of events
- **Constant-time simulation**

- Information-theoretic channels
- Leakage as a real number
- **Refinement**

References

Alvim, Mário S et al. (2020). *The Science of Quantitative Information Flow*. Springer.

Barthe, Gilles, Benjamin Grégoire, and Vincent Laporte (2018). "Secure Compilation of Side-Channel Countermeasures: The Case of Cryptographic "Constant-Time"". In: *2018 IEEE 31st Computer Security Foundations Symposium (CSF)*, pp. 328–343. DOI: `10.1109/CSF.2018.00031`.