RIPRENDI TEOREMI SUI LIMITI

TEOREMA SULLA PERMANENZA DEL SEGNO

se $\lim_{x \to x_0} f(x) = l$ con $l \neq 0$

se $l > 0$ $\exists I_{x_0} : x \in I_{x_0}$ $f(x) > 0$

se $l < 0$ $\exists I_{x_0} : x \in I_{x_0}$ $f(x) < 0$

Dim

$-\varepsilon < f(x) - l < \varepsilon$ $\quad l > 0$

$-\varepsilon + l < f(x) < \varepsilon + l$ $\quad \varepsilon = l$

$0 < f(x) < 2l$ $\quad - \; +$

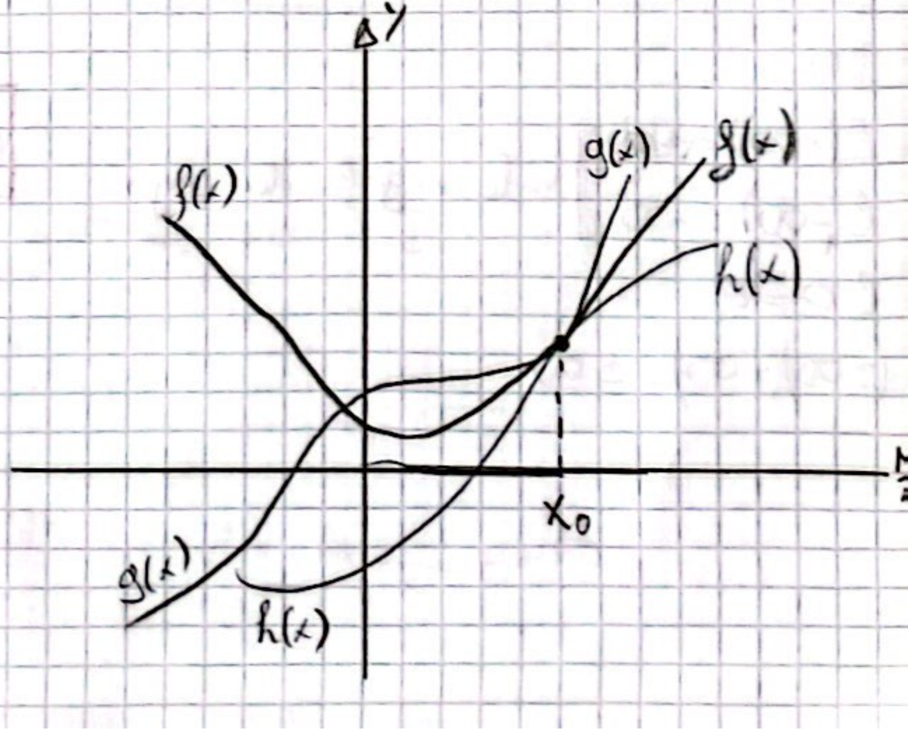
TEOREMA DEL CONFRONTO

$f(x)$ $\quad g(x)$ $\quad h(x) : I_{x_0}$ $\quad h(x) \leq f(x) \leq g(x)$ $\quad x \in I_{x_0}$

se $\lim_{x \to x_0} h(x) = l = \lim_{x \to x_0} g(x)$

$\Rightarrow \lim_{x \to x_0} f(x) = l$

RAPP. GRAFICA



Dim

$\forall \varepsilon > 0 \ \exists \delta_\varepsilon : -\delta_\varepsilon < x - x_0 < \delta_\varepsilon$

$$\left.\begin{array}{l} l-\varepsilon < g(x) < l+\varepsilon \\ l-\varepsilon < h(x) < l+\varepsilon \end{array}\right] \rightarrow \underline{l-\varepsilon} < g(x) \leq \underline{\underline{f(x)}} \leq h(x) < \underline{l+\varepsilon}$$

$$\lim_{x \to x_0} f(x) = l$$

Operazioni con i limiti

Somma algebrica

$$\left.\lim_{\substack{x \to x_0 \\ \infty}} \big(f(x) \pm g(x)\big) = \lim_{\substack{x \to x_0 \\ \infty}} f(x) \pm \lim_{\substack{x \to x_0 \\ \infty}} g(x) \right] \text{se finiti} = l+m$$

$l+m$

$l + \infty = +\infty$

$l - \infty = -\infty$

$+\infty + \infty = +\infty$

$-\infty - \infty = -\infty$

$\boxed{+\infty - \infty \quad \text{F.I.}}$

Prodotto

$$\lim_{\substack{x \to x_0 \\ \infty}} f(x)\, g(x) = \lim_{\substack{x \to x_0 \\ \infty}} f(x) \cdot \lim_{\substack{x \to x_0 \\ \infty}} g(x)$$

$l \cdot m$

$\left.\begin{array}{l} l(+\infty) = +\infty \\ l(-\infty) = -\infty \end{array}\right]$ vale regola dei segni

$l \cdot 0 = 0$

$(\pm\infty)(\pm\infty) = \pm\infty$

$\boxed{0 \cdot \pm\infty \quad \text{F.I.}}$

QUOZIENTE

$$\lim_{x \to x_0 / \infty} \frac{f(x)}{g(x)} = \frac{\lim_{x \to x_0 / \infty} f(x)}{\lim_{x \to x_0 / \infty} g(x)}$$

$\frac{l}{m}$ $\quad \frac{0}{l} = 0$ $\quad \frac{\infty}{\infty} = \text{F.I.}$ $\quad \frac{0}{0} = \text{F.I.}$

$\frac{l}{\pm\infty} = 0$ $\quad \frac{\pm\infty}{l} = \pm\infty$

$\frac{l}{0} = \pm\infty$ $\quad \frac{\pm\infty}{0} = \infty$ $\quad \frac{0}{\pm\infty} = 0$

es

$$\lim_{x \to -\infty} \left(x + \frac{5}{x}\right) = -\infty$$

es

$$\lim_{x \to 0^-} \left(\frac{2x+1}{\sin x}\right) = -\infty$$

$$\lim_{x \to 0^+} \left(\frac{2x+1}{\sin x}\right) = +\infty$$

es

$$\lim_{x \to -\infty} \left(x^4 - x^2 - 9\right) = +\infty - \infty \quad \text{F.I}$$

$$\lim_{x \to -\infty} x^4\left(1 - \frac{1}{x^2} - \frac{9}{x^4}\right) = +\infty$$

es

$$\lim_{x \to +\infty} \left(\sqrt{x+1} - \sqrt{x+2}\right) = +\infty - \infty \quad \text{F.I.}$$

$$\lim_{x \to +\infty} \left(\frac{\left(\sqrt{x+1} - \sqrt{x+2}\right)\left(\sqrt{x+1} + \sqrt{x+2}\right)}{\sqrt{x+1} + \sqrt{x+2}}\right) =$$

$$\lim_{x \to +\infty} \frac{x+1-(x+2)}{\sqrt{x+1} + \sqrt{x+2}} = \frac{-1}{+\infty} = 0$$

es

$$\lim_{x \to 1^+} \left[2\ln(x-1) - \ln(x^2-x)\right] = \mp\infty + \infty \quad \text{F.I.}$$

$$\lim_{x \to 1^+} \cancel{\left[e^{x^2-2x+1} - e^{x^2-x}\right]}$$

$$\lim_{x \to 1^+} \ln\frac{(x-1)^2}{x^2-x} = \ln\frac{(x-1)^{\cancel{2}}}{x\cancel{(x-1)}} \to \lim_{x \to 1^+} \ln\frac{x-1}{x} = -\infty$$