

## Equazioni con valori assoluti

$$|A(x)| = K^{\geq 0} \quad K \in \mathbb{R}$$

Se  $K < 0$  allora equazione IMPOSSIBILE

Def.

$$|a| = \begin{cases} a & \text{se } a \geq 0 \\ -a & \text{se } a < 0 \end{cases}$$

$$A(x) = K \quad \text{se } A(x) \geq 0$$

$$-A(x) = K \quad \text{se } A(x) < 0$$

$\Downarrow$

$$A(x) = -K$$

es

$$|3x-1| = 3$$

$$1) \swarrow 3x-1 = 3 \quad \text{se } 3x-1 \geq 0 \iff x \geq \frac{1}{3}$$

$$3x = 4 \iff x = \frac{4}{3} \geq \frac{1}{3}$$

$$2) 3x-1 = -3 \quad \text{se } 3x-1 < 0 \iff x < \frac{1}{3}$$

$$3x = -2 \iff x = -\frac{2}{3} < \frac{1}{3}$$

quindi...

$$|3x-1| = 3 \iff 3x-1 = \pm 3$$

$$3x = 1 \pm 3$$

$$x = \frac{1 \pm 3}{3} = \begin{cases} \frac{4}{3} \\ -\frac{2}{3} \end{cases}$$

Se

$$|A(x)| = 0 \iff A(x) = 0$$

Se  $k < 0$  l'equazione è impossibile

$$|A(x)| = k < 0 \quad \text{IMPOSSIBILE}$$

$$2) \underbrace{|A(x)|}_{\geq 0} = \underbrace{B(x)}_?$$

$$\begin{cases} B(x) \geq 0 \\ A(x) = \pm B(x) \end{cases}$$

es.

$$|4-x^2| = 4-2x$$

$$\begin{cases} 4-2x \geq 0 \\ 4-x^2 = \pm (4-2x) \end{cases} \begin{cases} x \leq 2 \\ x^2 \pm (4-2x) - 4 = 0 \end{cases} \begin{cases} x \leq 2 \\ x^2 - 2x = 0 \\ x^2 + 2x - 8 = 0 \end{cases} \begin{cases} x \leq 2 \\ x_1 = 0 \vee x_2 = 2 \\ x_1 = -4 \vee x_2 = 2 \end{cases}$$

$$\text{Sol fin: } x = 0 \vee x = 2 \vee x = -4$$

es.

$$4 - \left| x - \frac{3x-9}{x+1} \right| = 0$$

$$4 = \left| x - \frac{3x-9}{x+1} \right|$$

$$x - \frac{3x-9}{x+1} = \pm 4$$

$$\frac{x^2 + x - 3x + 9}{x+1} = \pm \frac{4(x+1)}{x+1}$$

$$x^2 - 2x + 9 = \pm 4(x+1)$$

$$x^2 - 2x + 9 \pm 4(x+1) = 0$$

$$x^2 - 6x + 9 = 0 \Rightarrow x_1 = 3 \vee x_2 = 1$$

$$x^2 + 2x + 13 = 0 \Rightarrow \text{imp.}$$

$$\text{C.E} = \{x \in \mathbb{R} : x \neq -1\}$$

$$\text{Sol fin: } x = 3 \vee x = 1$$

2

$$\left| \frac{x-3}{x^2+4x-5} \right| + \frac{x+1}{x+5} = \frac{1}{x-1}$$

$$\left| \frac{x-3}{x^2+4x-5} \right| = \frac{1}{x-1} - \frac{x+1}{x+5}$$

$$\left| \frac{x-3}{(x+5)(x-1)} \right| = \frac{x+5-x^2+1}{(x-1)(x+5)}$$

$$\left| \frac{x-3}{(x+5)(x-1)} \right| = \frac{-x^2+x+6}{(x-1)(x+5)}$$

$$C.E. = \{x \in \mathbb{R} : x \neq 1 \wedge x \neq -5\}$$

$$\begin{cases} \frac{x^2-x-6}{(x-1)(x+5)} \leq 0 \\ \frac{x-3}{(x+5)(x-1)} = \pm \frac{-x^2+x+6}{(x-1)(x+5)} \end{cases} \quad \begin{cases} -5 < x \leq -2 \vee 1 < x \leq 3 \\ \pm(-x^2+x+6) - x + 3 = 0 \end{cases}$$

$$\begin{cases} -x^2+9=0 \\ x^2-2x-3=0 \end{cases} \quad \begin{cases} -5 < x \leq -2 \vee 1 < x \leq 3 \\ x_1=3 \quad x_2=-3 \\ \cancel{x_3=-1} \quad x_4=3 \end{cases}$$

$$\text{Sol fin: } x=3 \vee x=-3$$

3

$$||2x-5| - |6-3x|| = x$$

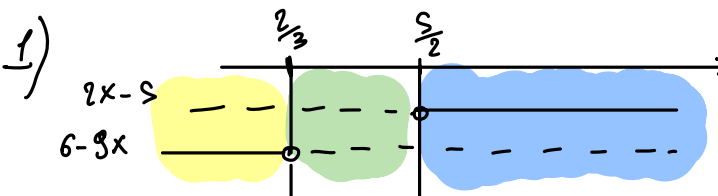
$$|a| = \begin{cases} a, & \text{se } a \geq 0 \\ -a, & \text{se } a < 0 \end{cases}$$

$$x \geq 0$$

$$|2x-5| - |6-3x| = \pm x$$

$$1) |2x-5| - |6-3x| = x$$

$$2) |2x-5| - |6-3x| = -x$$

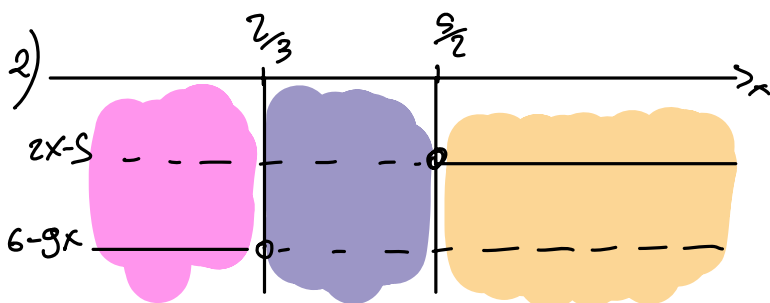
1)  
$$\begin{cases} -2x+5 - (6-3x) = x & \text{se } 0 \leq x \leq \frac{2}{3} \\ 6x-1 = 0 \\ x = \frac{1}{6} \text{ (Acc.)} \end{cases}$$

2ª zona

$$\begin{cases} -2x + 5 - (-6 + 9x) = x & \text{se } \frac{2}{3} < x \leq \frac{5}{2} \\ -12x + 11 = 0 \\ x = \frac{11}{12} \text{ (ACC.)} \end{cases}$$

3ª zona

$$\begin{cases} 2x - 5 - (-6 + 9x) = x \\ -8x + 1 = 0 \\ x = \frac{1}{8} \text{ (NON ACC.)} \end{cases}$$



1ª zona

$$\begin{cases} -2x + 5 - (6 - 9x) = -x \\ 8x - 1 = 0 \\ x = \frac{1}{8} \text{ (ACC.)} \end{cases}$$

$$\text{se } 0 \leq x \leq \frac{2}{3}$$

2ª zona

$$\begin{cases} -2x + 5 - (-6 + 9x) = -x \\ -10x + 11 = 0 \\ x = \frac{11}{10} \text{ (ACC.)} \end{cases}$$

$$\text{se } \frac{2}{3} < x \leq \frac{5}{2}$$

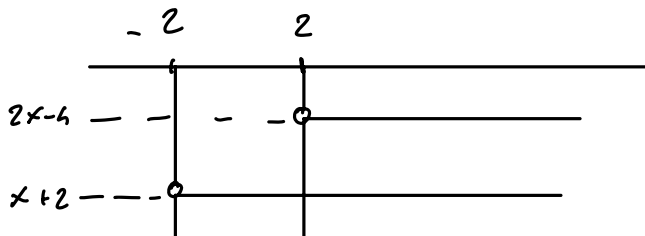
3ª zona

$$\begin{cases} 2x - 5 - (-6 + 9x) = -x \\ -6x + 1 = 0 \\ x = \frac{1}{6} \text{ (NON. ACC.)} \end{cases}$$

Sol fin:  $x = \frac{1}{6}$ ;  $x = \frac{11}{12}$ ;  $x = \frac{1}{8}$ ;  $x = \frac{11}{10}$

es

$$\frac{|2x-4|+1}{|x+2|-1} = x$$



$$1^a \text{ zona } \begin{cases} \frac{-2x+4+1}{-x-2-1} = x \end{cases}$$

$$\text{se } x \leq -2$$

$$2^a \text{ zona } \begin{cases} \frac{-2x+4+1}{x+2-1} = x \\ -2x+5 = x \end{cases} \text{ se } -2 < x \leq 2$$

$$\left( \frac{-2x+5}{x-3} = x \right.$$

$$\left( \frac{x+1}{x+1} = x \right.$$

$$3^{\text{a}} \text{ zona } \left\{ \begin{array}{l} \frac{2x-4+1}{x+2-1} = x \\ \frac{2x-3}{x+1} \end{array} \right.$$