LIMITI

$\lim_{x \to x_0} f(x) = l$ è un limite ~~che tende~~ finito che tende ad un ~~limite~~ valore finito

$x \to x_0$ è p. accomulazione

$\forall \varepsilon > 0 \; \exists \delta_\varepsilon \; \forall x \; |x - x_0| < \delta_\varepsilon \Rightarrow |f(x) - l| < \varepsilon$

$\forall \varepsilon > 0$ PER ogni

δ_ε delta dipendente da ε

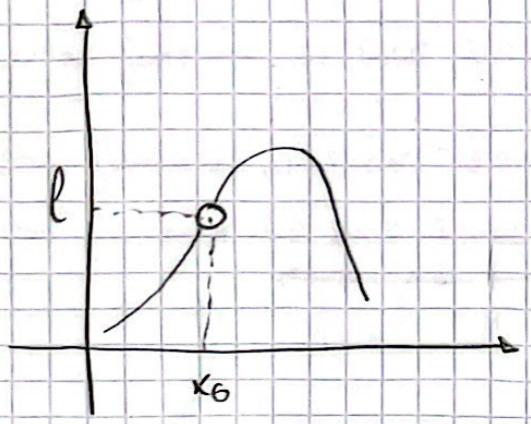
$|x - x_0| < \delta_\varepsilon \Rightarrow -\delta_\varepsilon < x - x_0 < \delta_\varepsilon$

$x_0 - \delta_\varepsilon < x < \delta_\varepsilon + x_0$

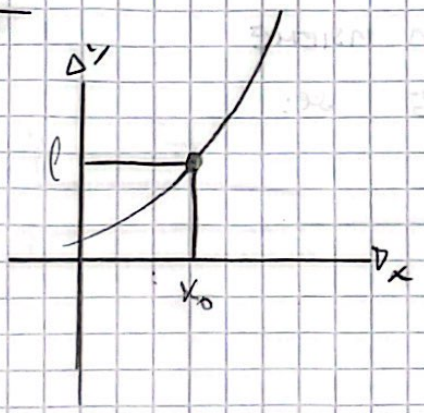
$x \in I_{x_0}$

$|f(x) - l| < \varepsilon \Rightarrow -\varepsilon < f(x) - l < \varepsilon$

$l - \varepsilon < f(x) < \varepsilon + l$



$\lim f(x) = l$



Per ogni $\varepsilon > 0$ esiste un delta dipendente da ε tale che per per $x \; |x - x| < \delta_\varepsilon$

$\lim_{x \to x_0} f(x) = \ell$

$\forall \varepsilon > 0 \ \exists \delta_\varepsilon : |x - x_0| < \delta_3 \Rightarrow |f(x) - \ell| < \varepsilon$ (si ha)

ES

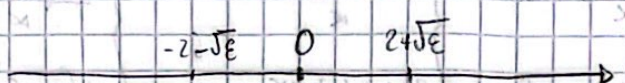
$\lim_{x \to -2} (x^2 - 1) = 3$

$|f(x) - \ell| < \varepsilon \to |x^2 - 1 - 3| < \varepsilon \quad \exists \delta_\varepsilon$

$\begin{cases} x^2 - 4 < \varepsilon \\ x^2 - 4 > -\varepsilon \end{cases} \quad \begin{cases} x^2 < \varepsilon + 4 \\ x^2 > 4 - \varepsilon \end{cases} \quad \begin{cases} -\sqrt{\varepsilon+4} < x < \sqrt{\varepsilon+4} \\ x < -\sqrt{4-\varepsilon} \ \vee \ x > \sqrt{4-\varepsilon} \end{cases}$

$-\varepsilon < f(x) - \ell < \varepsilon$

$\begin{cases} -2 - \sqrt{\varepsilon} < x < 2 + \sqrt{\varepsilon} \\ x < -2 - \sqrt{-\varepsilon} \ \vee \ x > 2 + \sqrt{-\varepsilon} \end{cases}$



$0 < x < 2 + \sqrt{\varepsilon}$

$-2 - \sqrt{\varepsilon} < x < 0$

$\lim_{x \to -2} (x^2 - 1) = 3$

$|x + 2| < \delta_\varepsilon$

$-\delta_\varepsilon < x + 2 < \delta_\varepsilon \to -2 - \delta_\varepsilon < x < \delta_\varepsilon - 2$

RICOMINCIO DA

$\begin{cases} -\sqrt{\varepsilon+4} < x < \sqrt{\varepsilon+4} \end{cases} \rightsquigarrow$

$x < \sqrt{\varepsilon+4} \to x < \sqrt{\varepsilon} \pm 2$

$x > -\sqrt{\varepsilon+4} \quad x > -\sqrt{\varepsilon} \mp 2$