! Riprendi Limiti Notevoli !

- $\lim_{x \to 0} \frac{\sin x}{x} = 1$

- $\lim_{x \to 0} \frac{1-\cos x}{x} = 0 \leadsto \lim_{x \to 0} \frac{(1-\cos x)(1+\cos x)}{(1+\cos x)x} = \lim_{x \to 0} \frac{\sin^2 x}{x(1+\cos x)} =$

$$= \lim_{x \to 0} \overbrace{\frac{\sin x}{x}}^{\to 1} \cdot \overbrace{\frac{\sin x}{1+\cos x}}^{\to 0} = 0$$

- $\lim_{x \to 0} \frac{1-\cos x}{x^2} = \frac{1}{2} \leadsto \lim_{x \to 0} \frac{(1-\cos x)(1+\cos x)}{x^2(1+\cos x)} = \lim_{x \to 0} \frac{\sin^2 x}{x^2} \cdot \frac{1}{1+\cos x} = \frac{1}{2}$

- $\lim_{x \to \pm\infty} \left(1+\frac{1}{x}\right)^x = e$

- $\lim_{x \to 0} (1+x)^{\frac{1}{x}} = e$

- $\lim_{x \to 0} \frac{\ln(1+x)}{x} = 1 \leadsto \lim_{x \to 0} \frac{1}{x}\ln(1+x) = \lim_{x \to 0} \ln\overbrace{(1+x)^{\frac{1}{x}}}^{e} = 1$

- $\lim_{x \to 0} \frac{\log_a(1+x)}{x} = \log_a e$

- $\lim_{x \to 0} \frac{e^x - 1}{x} = 1 \leadsto \left. \begin{array}{l} e^x - 1 = y \\ x = \ln(y+1) \end{array} \right] \nearrow \; x \to 0 \quad \underbrace{e^x - 1 \to 0}_{y \to 0} \quad \lim_{y \to 0} \frac{y}{\ln(y+1)} = \lim \frac{1}{\frac{\ln(y+1)}{y}} = 1$

$$\lim_{x\to 0} \frac{a^x-1}{x} = \lim_{y\to 0} \frac{y}{\log_a(y+1)} = \lim_{y\to 0} \frac{1}{\frac{\log_a(y+1)}{y}} = \frac{1}{\log_a e}$$

$\to \log_a b = \frac{\log_c b}{\log_c a}$

$$= \frac{1}{\frac{\ln e}{\ln a}} = \ln a$$

es

$$\lim_{x\to +\infty} \left(1+\frac{3}{x}\right)^x = \lim \left[\left(1+\frac{3}{x}\right)^{\frac{x}{3}}\right]^3 = e^3$$

es

$$\lim_{x\to -\infty} \left(\frac{x+2}{x+1}\right)^x = 1^{\infty} \text{ F.I.} \quad \left[\lim_{x\to \infty} \left(1+\frac{1}{x}\right)^x = e\right]$$

$$\lim_{x\to -\infty} \left(\frac{x+1+1}{x+1}\right)^x = \lim_{x\to -\infty} \left(\frac{x+1}{x+1} + \frac{1}{x+1}\right)^x = \lim_{x\to -\infty} \left(1+\frac{1}{x+1}\right)^x = \lim_{x\to -\infty} \left(1+\frac{1}{x+1}\right)^{x+1-1} =$$

$$\lim_{x\to -\infty} \frac{\left(1+\frac{1}{x+1}\right)^{x+1}}{\left(1+\frac{1}{x+1}\right)^1} = \frac{e}{1} = e$$