Motional-Stark Effect (MSE)

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Introduction

The *Stark Effect* is the shifting and splitting of the electronic energy levels due to an external electric field. The Motional-Stark Effect (MSE), instead, is the specific case in which the electric field is the Lorentz electric field felt by an atom moving in a magnetic field $\mathbf{B}$ at a velocity $\mathbf{v}$, in its rest frame: $\mathbf{E} = \mathbf{v} \times \mathbf{B}$.

In plasma physics, the Motional-Stark Effect (MSE) serves as a crucial diagnostic tool for determining the poloidal magnetic field $\mathbf{B_P}$ within the plasma. Accurate knowledge of the poloidal magnetic field is essential for understanding plasma stability and the formation of Internal Transport Barriers (ITBs). Additionally, MSE measurements enable inference of the plasma current distribution, since the current density $\mathbf{J}$ is related to the magnetic field $\mathbf{B}$ through Ampère's law:

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}.$$

In TCV, the poloidal magnetic field is determined by the LIUQE (from "EQUIL" spelt backwards) code, which is an equilibrium code based on the resolution of the Grad-Shafranov equation, and on magnetic probes measurements outside of the plasma. However, when using Neutral Beam Heating (NBH) and Electron Cyclotron Resonance Heating (ECRH), the plasma equilibrium velocity is not negligible anymore, and the magnetic field is locally influenced. This explains the need for the Motional-Stark Effect diagnostic, which provides a direct measurement of the local magnetic field inside the plasma, unaffected by external equilibrium assumptions.

A bit of history

The Stark effect was discovered by the German physicist Johannes Stark in 1913, for which he was awarded the nobel price in 1919. During the same year the effect was independently discovered by the Italian physicit Antonio Lo Surdo[1].

Physical principles

The MSE diagnostic is based on the external injection of neutral beam deuterium atoms and, when entering the plasma, they are subject to inelastic collisions with plasma's ions and electrons. The neutrals' electrons, which initially are in their ground state, can reach a higher energy level, E_2 and, when they decay to a lower energy state E_1, they emit a photon with energy $E_\gamma = E_2 - E_1$, and wavelength $\lambda = \frac{hc}{E_\gamma}$.

In particular, of specific interest is the Balmer-α emission of the deuterium atom D_α, which corresponds to the transition between $n = 3$ and $n = 2$, and is characterized by an energy $E_{D_\alpha} = 1.89$ eV and a wavelength $\lambda_{D_\alpha} = 656.279$ nm. This is because the D_α emission has a high emission intensity in the visible spectrum (red light), and it is highly Doppler-shifted with respect to the plasma D_α emission. Moreover, since the neutral's velocity across the magnetic field is high compared to the thermal velocity of the plasma ions, the resulting Lorentz electric field $\mathbf{E_L} = \mathbf{v} \times \mathbf{B}$ is significant. This leads to a measurable splitting and polarization of the D_α, spectral line, which can be used to infer the local magnetic field direction and magnitude inside the plasma.

To analyze the Stark effect in the deuterium atom, it is convenient to solve the non-relativistic Schrödinger Equation using parabolic coordinates. In this framework, the atomic states are described by four quantum numbers $|n, n_1, n_2, m\rangle$: n is the principal quantum number, n_1 and n_2 are parabolic quantum numbers, and m is the magnetic quantum number. The energy splitting caused by the Stark effect, calculated using first-order perturbation theory, is given by[2]:

$$\Delta E^{(1)} = \frac{3}{2} e a_0 n (n_1 - n_2) |\mathbf{E_L}|,$$

where e is the elementary charge, and a_0 is the Bohr radius.

In Figure 1 the lines splitting of the energy levels with principal quantum numbers $n = 2$ and $n = 3$ is shown. For clarity the fine structure splitting is not shown.

The separation of the spectral lines, as shown in Figure 2, is directly proportional to the strength of the electric field[3]:

$$\Delta\lambda \approx \frac{3 e a_0 |\mathbf{E_L}| \lambda_0^2}{2hc},$$

where λ_0 is the unshifted Balmer-α wavelength, h is Planck's constant, and c is the speed of light.

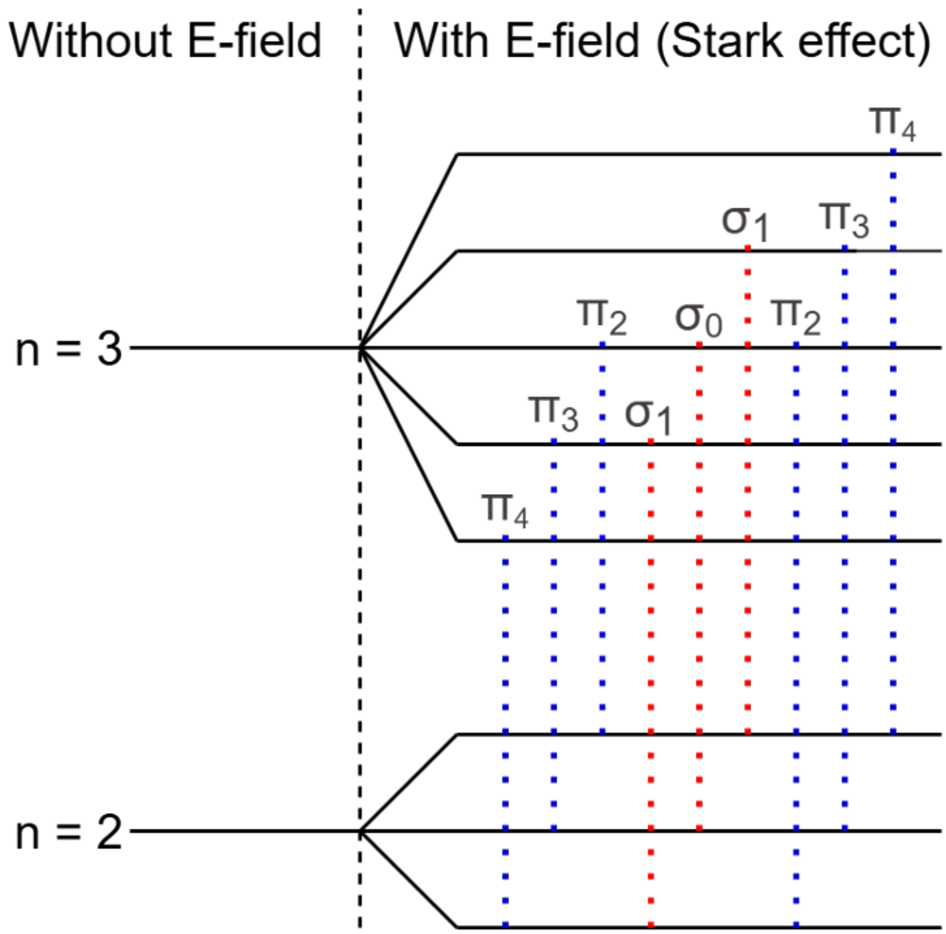

Figure 1: D_α splitting due to MSE[3].

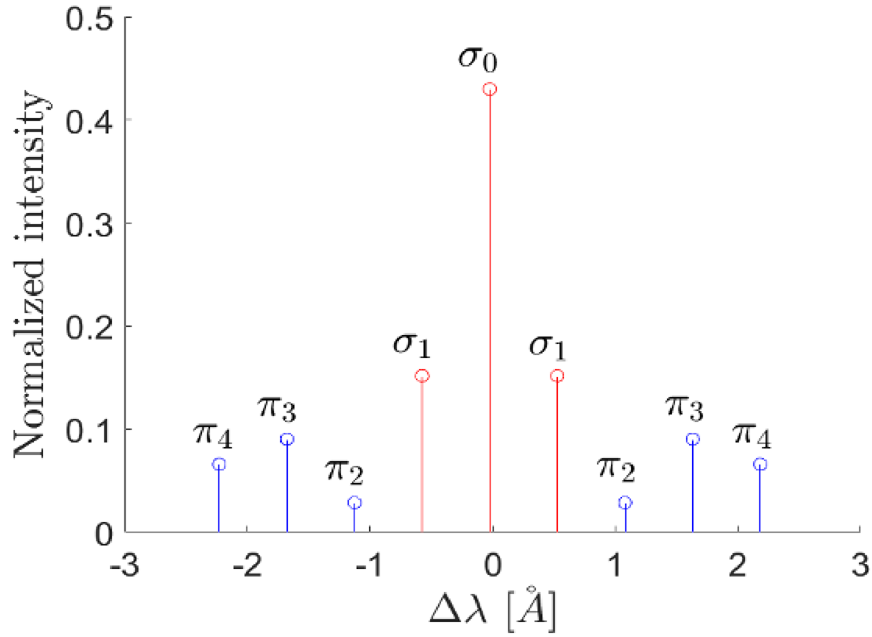

Figure 2: Spectral lines separation of the D_α radiation due to MSE[3].

On TCV

References

1. Wikipedia. Stark effect.
2. Anh-Tai, T. D., Khang, L. M., Vy, N. D., Truong, T. D. H. & Pham, V. N. T. A revisit on the hydrogen atom induced by a uniform static electric field. (2024).
3. Müller, S. Implementation of polarization-resolved motional-stark spectroscopy on TCV. (École Polytechnique Fédérale de Lausanne (EPFL), 2025).