Hermitian adjoint

The Hermitian adjoint of a matrix or operator $\mathbf{A}$, often written as $\mathbf{A}^\dagger$, especially when used in conjunction with the bra-ket notation, is defined as the complex conjugate transpose of $\mathbf{A}$. Mathematically, if $\mathbf{A}$ is an $m \times n$ matrix, then

$$\mathbf{A}^\dagger = (\mathbf{A}^*)^T$$

where $\mathbf{A}^*$ denotes the element-wise complex conjugate and T denotes the transpose.

In quantum mechanics, the Hermitian adjoint plays a crucial role in defining observables and ensuring that physical quantities are real-valued. An operator $\mathbf{A}$ is called Hermitian if $\mathbf{A} = \mathbf{A}^\dagger$. Hermitian operators have real eigenvalues and orthogonal eigenvectors, which are essential properties for representing measurable quantities.

For example, if $\mathbf{A}$ acts on a vector space with basis vectors $|i\rangle$, the matrix elements of the Hermitian adjoint are given by:

$$\langle i|\mathbf{A}^\dagger|j\rangle = \langle j|\mathbf{A}|i\rangle^*$$

This property is widely used in the manipulation of quantum states and operators.