Cosine distance: $\text{cos-dist}(A,B) = 1 - \text{cos-sim}(A,B)$

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$$\Downarrow$$

$$\text{cos-sim}(A,B) = \frac{\langle A, B \rangle}{\|A\| \cdot \|B\|} = \frac{\sum_{i=1}^{n} A_i \cdot B_i}{\sqrt{\sum_{i=1}^{n} A_i^2} \cdot \sqrt{\sum_{i=1}^{n} B_i^2}}$$

triangle inequality:

$$d(a,b) \leq d(a,c) + d(c,b)$$

$$\Downarrow$$

$$\text{cos-dist}(A,B) \leq \text{cos-dist}(A,C) + \text{cos-dist}(C,B)$$

$$\Downarrow$$

$$1 - \text{cos-sim}(A,B) \leq 1 - \text{cos-sim}(A,C) + 1 - \text{cos-sim}(C,B)$$

$$\Downarrow$$

$$1 + \text{cos-sim}(A,B) \geq \text{cos-sim}(A,C) + \text{cos-sim}(C,B)$$

Let $A, B, C \in R^2 \Rightarrow A = (1,0),\ B = (0,1),\ C = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$$\text{cos-sim}(A,B) = \frac{0}{\sqrt{1} \cdot \sqrt{1}} = 0$$

$$\text{cos-sim}(A,C) = \frac{\frac{\sqrt{2}}{2} + 0}{\sqrt{1} \cdot \sqrt{1}} = \frac{\sqrt{2}}{2}\ ; \quad \text{cos-sim}(C,B) = \frac{0 + \frac{\sqrt{2}}{2}}{\sqrt{1} \cdot \sqrt{1}} = \frac{\sqrt{2}}{2}$$

then,

$$1 + 0 \geq \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}$$

$$1 \ngeq \sqrt{2} \simeq 1{,}41\ldots$$

Таким образом, доказано, что косинусное расстояние не подчиняется неравенству треугольника