

Using diversity as an additional-objective in dynamic multi-objective optimization algorithms

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Abstract—This paper investigates the use of additional-objective for solving multi-objective optimization problems in dynamic environment. A number of authors proposed the use of additional-objective for maintaining diversity in static single-objective optimization task; in this paper the work is extended to dealing with the dynamic multi-objective optimization problems. Individual diversity is used as an additional-objective, and a new efficient multi-objective optimization algorithm is proposed. Experimental results clearly indicate that the performance of the proposed algorithm is gratifying.

Keywords—evolutionary algorithms; additional-objectives; multi-objective dynamic problems; multi-objective optimization

I. INTRODUCTION

The real optimization problems are hardly static, and most of them involve multiple objectives. Over the past decade or so, abundance of multi-objective optimization literatures have demonstrated the usefulness of multi-objective evolutionary algorithms (MOEAs) in solving static multi-objective optimization problems (MOPs). Now, there is a growing need for solving dynamic MOPs [1,2].

Multi-objective optimization problems often involve the simultaneous concurrence of conflicting objectives. This interaction results in a set of optimal decision variables, which are known as the Pareto-optimal set (POS), the corresponding optimal objective values are called Pareto-optimal front (POF). Since none of the solutions in this set can be considered as better than the others with respect to all the objectives.

MOEAs are the most powerful tools for solving static MOPs due to their inherent parallelism and ability to exploit the search space by recombination. Different from solving static MOPs, the main goal of dynamic multi-objective optimization is adapt the dynamic multi-objective environment. Therefore, the maintenance of genetic diversity within the population is very important in order to find and approximate the true and dynamic POFs.

A new diversity-preserving evaluation method is presented in this paper, Individual Diversity Evolutionary Method (IDEM), introduces explicitly individual diversity as an additional-objective in the optimization process. The effect of IDEM is to add a useful selection pressure

addressed towards both the Pareto-optimal set and the maintenance of diversity within the population. IDEM can help exploitation in the most promising regions of the search space and keep the exploration of the search space alive. The individual has better diversity measure obtains more genetic chance.

The paper is structured as follows. Section 2 reviews some related work on using additional-objective approve the performance of evolutionary algorithms. In Section 3, a new dynamic multi-objective optimization algorithms based on additional-objective are introduced. Section 4 introduces dynamic multi-objective test problems be chosen and presents the results of proposed algorithm dealing with the dynamic multi-objective problems.

II. RELATED WORK

There are a number of approaches to dealing with dynamic optimization problems, including diversity control, memory-based and multi-population approaches. Without the previous methods, some researchers considered it is possible to introducing a useful additional-objective which can improve ability to compute or approximate the true global optimal solutions (for SOPs) or Pareto-optimal set (for MOPs).

Knowles and Corne proposed multi-objectivization, a concept closely related to the additional-objectives. The authors argue that decomposition into several objectives will remove some of the local optima in the search space [3]. Abbass and Deb proposed to add an additional-objective to promote diversity. Three different artificial objectives were discussed in their work: time-stamp, random, inversion [4]. Jensen proposed the concept of helper-objectives to introduce new objectives for the optimization process. The author considered helper-objectives can guide the search towards solutions containing good building blocks and help the algorithm escape local optima [5,6].

In all of the previous work, the focus was on stationary environments and the object is single-objective optimization problems. With the purpose of solving optimization problems in dynamic environments, Yamasaki introduced a technique to employ the concept of time series as artificial criteria to define dominance relationship within the context of a simple EMO [7]. Bui et al. discussed the use of additional-objectives for solving

single-objective optimization problems in dynamic environments. Specially, they investigated the consideration of the additional-objectives, with the aim of maintaining greater population diversity and adaptability [8]. In terms of promoting diversity, Toffolo and Benini used the sum of the Euclidean distances, which between an individual and all other individuals in the population as an additional-objective, and introduced a new multi-objective evolutionary algorithm based on additional-objective [9].

In this paper, the goal is not only using additional-objective to tackle multi-objective optimization problems, but also to consider the objective functions and even the search space in which the solutions lie.

III. PROPOSED ALGORITHM

A. Additional-objective based on individual diversity

According to the previous references, there are many approaches to design additional-objective. Individual diversity is adopted explicitly as an additional-objective for dynamic EMO problems here, this method is named IDEM. The average of individual's entropy is used to other individuals in the population as a diversity measure, and is used to an additional-objective.

Let D denote the set of genotypic alleles with cardinality s . And let X_1 and X_2 denote the two chromosomes in the genotype of an individual in the population P with size N . The Entropy between X_1 and X_2 is defined as follow:

$$H(X_i, X_j) = \frac{1}{L} \sum_{l=1}^L \left(- \sum_{k=1}^s P_{lk} \log(P_{lk}) \right) \quad (1)$$

where P_{lk} is the rate of k th genotypic allele on l th location.

X_i individual diversity based on entropy is defined as follow:

$$F_{diversity} = \frac{1}{N-1} \sum_{j=1, j \neq i}^N H(X_i, X_j) \quad (2)$$

B. The individual diversity multi-objective optimization evolutionary algorithm

Most work in MOEA is based on the concept of dominance; NSGA2 [10], SPEA2 [11] and PESA2 [12] are the most prominent ones. In this paper, a new dynamic MOEA is proposed by using additional-objective, which named Individual Diversity Multi-objective Optimization Evolutionary Algorithm (IDMOEA). IDMOEA is a framework that is strictly designed around IDEM to exalt its characteristics. Beside additional-objectives, archive method is introduced in IDMOEA. The pseudo-code for IDMOEA is shown in Figure 1.

begin

$t=0$ and the size of archive set $=M$

initialize population P_0 and create empty archive set Q
calculate fitness values and additional-objective value of all individuals in P_0

copy all non-dominated individuals in P_0 to Q

repeat

if environment change **then**

retrieve best individuals from (P_t, Q) to P'_t

else $P'_t = P_t$

genetic operation to P'_t

binary tournament selection

crossover

mutation

replace elite from P_t

update archive set

copy non-dominated individuals in (P'_t, Q) to Q'

if the size of Q' exceeds M **then**

copy the individuals with better additional objective value to Q

else $Q = Q'$

$P_{t+1} = P'_t$

$t = t + 1$

until $t > T$

end

Figure 1. Pseudo-code of IDMOEA

IV. EXPERIMENTAL RESULTS

A. Test Problems

Farina et al. categorized dynamic MOPs into four classes depending on the changed space: decision variable space and objective space, proposed four possible ways a problem can demonstrate a time-varying change [1].

Type I: Static POF_{true} , dynamic POS_{true}

Type II: Dynamic POF_{true} , dynamic POS_{true}

Type III: Dynamic POF_{true} , static POS_{true}

Type IV: Static POF_{true} , static POS_{true}

Based on the categories, Farina et al. proposed a set of dynamic multi-objective optimization test problems. Two problems are chosen to be test problems as follow. Figure 2 & Figure 3 show the dynamic POS and POF of these test problems.

Test problem 1: FDA1, Type I

$$\begin{cases} F_1 = f_1(x_I) = x_I \\ F_2 = g \cdot h \\ g(x_{II}) = 1 + \sum_{x_i \in x_{II}} (x_i - G(t))^2 \\ h(f_1, g) = 1 - \sqrt{f_1 / g} \\ G(t) = \sin(0.5\pi), \quad t = \frac{1}{n_t} \left\lfloor \frac{\tau}{\tau_T} \right\rfloor \\ x_I \in [0, 1], \quad x_{II} \in [-1, 1] \end{cases} \quad (3)$$

Where, τ is the generation counter, τ_T is the number of generation for which t remains fixed, and n_t is the number of distinct steps in t . with $n = 20$, $\tau_T = 5$, $n_t = 10$, $|x_{II}| = 19$.

Test problem 2: FDA5, Type II

$$\left\{ \begin{array}{l} \min_x \quad f_1(x) = (1 + g(x_{II})) \prod_{i=1}^{M-1} \cos\left(\frac{y_i \pi}{2}\right) \\ \min_x \quad f_k(x) = (1 + g(x_{II})) \left(\prod_{i=1}^{M-k} \cos\left(\frac{y_i \pi}{2}\right) \right) \\ \sin\left(\frac{x_{M-k+1} \pi}{2}\right), \quad k = 2 : M-1 \\ \min_x \quad f_M(x) = (1 + g(x_{II})) \sin\left(\frac{y_1 \pi}{2}\right) \\ \text{where} \\ g(x_{II}) = G(t) + \sum_{x_i \in x_{II}} (x_i - G(t))^2 \\ y_i = x_i^{F(t)}, \quad i = 1 : M-1 \\ G(t) = \left\lfloor \sin(0.5\pi t) \right\rfloor, \quad t = \frac{1}{n_t} \left\lfloor \frac{\tau}{\tau_T} \right\rfloor \\ F(t) = 1 + 100 \sin^4(0.5\pi t) \\ x_{II} = (x_M, \dots, x_n), \quad x_i \in [0,1], i = 1 : n \end{array} \right. \quad (4)$$

with $n = 30$, $M = 3$, $\tau_T = 5$, $n_t = 10$, $|x_{II}| = 10$.

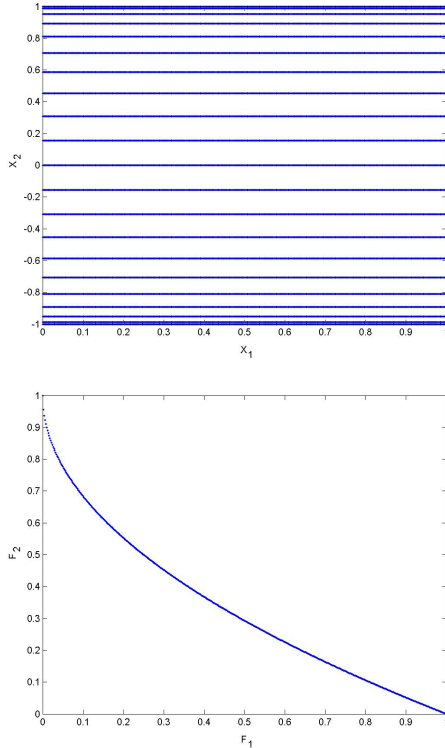


Figure 2. dynamic POS & POF on FDA1 ($\tau = 0:150$)

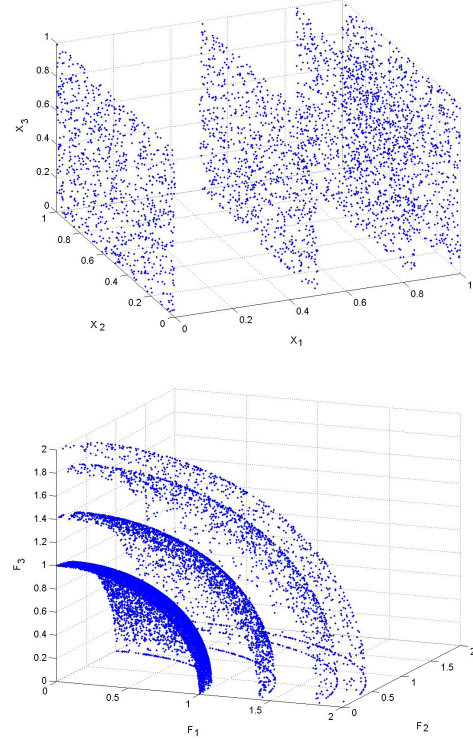


Figure 3. dynamic POS & POF on FDA5 ($\tau = 0,15,35,50$)

B. Performance metrics

Convergence to the POF and diversity of solution in the POS are two main goals in solving a MOP. The two goals are significant all the same in dealing with a dynamic MOP. Since these two goals are distinct to evaluate the performance of a dynamic MOEA, two different metrics are required.

Convergence metric λ : this metric is used to measure the closeness to the POF, which is defined as

$$\lambda = \frac{1}{|Z|} \sum_{z \in Z} \min_{z' \in Z'} \|z - z'\| \quad (5)$$

where Z is an obtained POF with dynamic MOEA, Z' is uniformly distributed POF of the problem.

Diversity metric γ : this metric measures the extent of diversity achieved among the obtained solutions in the objective space, which is defined as

$$\gamma = \frac{1}{N} \sum_{j=1}^N \frac{1}{L} \sum_{l=1}^L \left(- \sum_{k=1}^s P_{lk} \log(P_{lk}) \right) \quad (6)$$

where N is the population size, L is the length of chromosome, s is the cardinality of genotypic alleles, P_{lk} is the rate of k th genotypic allele appear on l th location.

C. Results and Discussion

Test problem 1 and 2 are used to validate the performance of IDMOEA proposed. Parameter Setting: for test problem 1 let population size $N=100$, archive size

$M=50$; for test problem 2 let population size $N=300$, archive size $M=100$; mutation rate $p_m=0.01$, crossover rate $p_c=0.7$, generation=200.

The POF of each problem with IDMOEA after a representative run are shown in Figure 4, the first one is the POF on FDA1, and the second one is the POF on FDA5. In Figure 4, dynamic POFs for FDA1 in 10 time intervals (1 to 10) is shown, dynamic POFs for FDA5 in 4 time intervals (1, 3, 7, 10) is shown. In order to indicate the dynamic POFs as clearly as possible, this paper only show the best 10 and 25 solutions which have better additional-objectives of POF of FDA1 and FDA5, and move the figure along the direction on F_1 appropriately. It can be observed intuitively that IDMOEA performs a gratifying performance. For indicate the changed POF highly stratified, we add a helper-axis in FDA1's POF figure, and add a helper-measure on x axis in FDA5's POF figure (helper-measure=1).

From the experimental results shown in Table 1, it is obviously that IDMOEA can obtain well convergence performance and have a strength ability to maintain population diversity. The results shown in Tables are obtained according ten separate runs, λ is convergence metric with is calculated in the generation before environment changes, γ is the diversity metric in whole run.

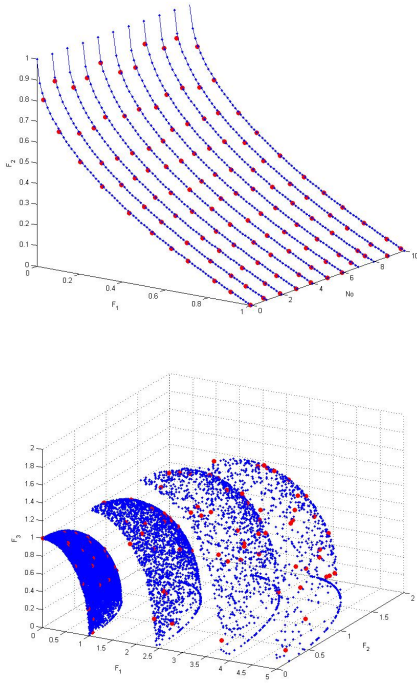


Figure 4. Exact POFs (blue dots) and approximated (red o) with IDMOEA on FDA1 (10 best solutions)& FDA5 (25 best solutions)

TABLE I. EXPERIMENTAL RESULT WITH AVERAGE \pm THE STANDARD ERROR ON FDA1&FDA5

	λ	γ
FDA1	0.0134 ± 0.00175	0.6240 ± 0.0094
FDA5	0.0420 ± 0.0137	0.6515 ± 0.0103

V. CONCLUSION

In this paper, a new dynamic multi-objective optimization evolutionary algorithm is proposed, individual diversity is used as additional-objective. The experimental results on FDA series of benchmark functions shown that this algorithm has better ability to maintain the diversity of the population and adapt the dynamic environment. Moreover, the approximated Pareto fronts obtained by this algorithm are also satisfactory.

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