Control Systems Engineering (EYAG-1005)

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Contenido del Tema

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Armature-controlled DC Motors

The model for the armature-controlled DC motor involves both an electric circuit an a rotational mechanical system. **If no load is attached to the armature** then the models are as shown below:

$$T_m(t) = K_t\, z_a(t)$$

$$v_{bf}(t) = K_b\, \dot{\theta}_a(t)$$

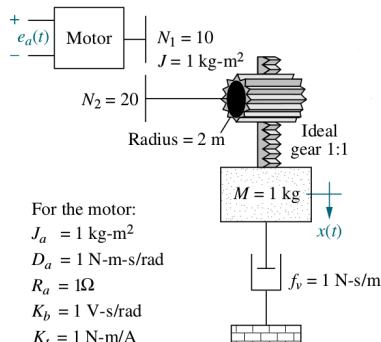$$e_a(t) = R_a\, z_a(t) + L_a\, \dot{z}_a(t) + v_{bf}(t)$$
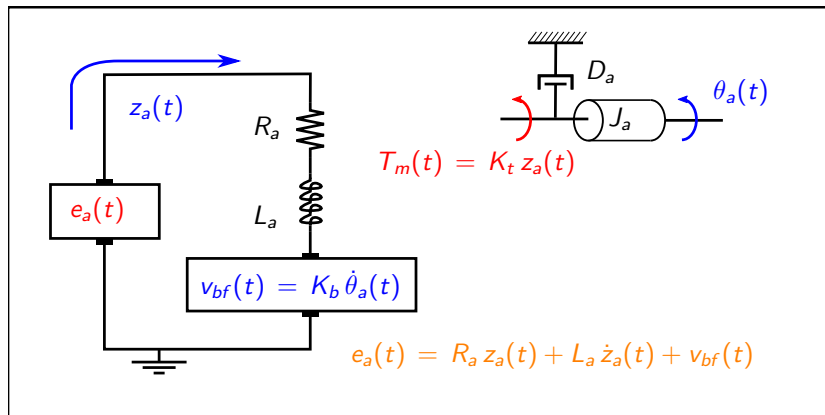
Example
(Nise, Problem 2.46*):

Consider the mechanism on the right, where an armature-controlled DC motor drives a block of mass by means of a system of gears. The input is the supplied voltage $e_a(t)$ and the output is the displacement of the block of mass $x(t)$.
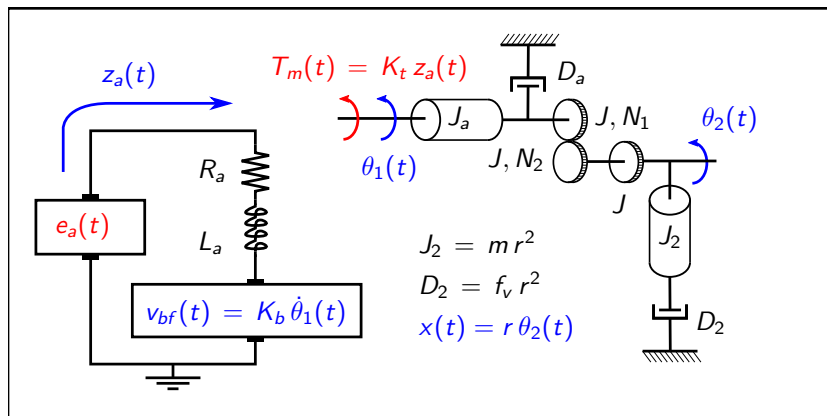
Find the transfer function:

$$G(s) = \frac{X(s)}{E_a(s)}$$

+ $e_a(t)$ Motor | $N_1 = 10$
− | $J = 1$ kg-m^2

$N_2 = 20$

Radius = 2 m | Ideal gear 1:1

$M = 1$ kg

$x(t)$

For the motor:
$J_a = 1$ kg-m^2
$D_a = 1$ N-m-s/rad
$R_a = 1\Omega$
$K_b = 1$ V-s/rad
$K_t = 1$ N-m/A

$f_v = 1$ N-s/m

Note: Each of the three gears shown has moment of inertia $J = 1$ kg-m^2.

Electric circuit and equivalent rotational mechanical system:

Armature-controlled DC Motors

Models:

- Electric circuit:
$$e_a(t) = R_a\, z_a(t) + K_b\, \dot{\theta}_1(t)$$

- Rotational mechanical system:
$$(J_a + J)\, \ddot{\theta}_1(t) + (2J + J_2)\, \ddot{\theta}_2(t) = K_t\, z_a(t) - D_a\, \dot{\theta}_1(t) - D_2\, \dot{\theta}_2(t)$$
$$N_1\, \theta_1(t) = N_2\, \theta_2(t)$$
$$x(t) = r\, \theta_2(t)$$

Armature-controlled DC Motors

Taking the Laplace Transform of both equations on both sides:

- Electric circuit:
$$E_a(s) = R_a Z_a(s) + K_b s \Theta_1(s)$$

- Rotational mechanical system:

$$((J_a + J) s^2 + D_a s) \Theta_1(s) + ((2J + J_2) s^2 + D_2 s) \Theta_2(s) = K_t Z_a(s)$$

$$\Theta_1(s) = (N_2/N_1) \Theta_2(s)$$

$$\Theta_2(s) = (1/r) X(s)$$

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Solving...

- Armature current:

$$Z_a(s) = \frac{E_a(s) - K_b\, s\, \Theta_1(s)}{R_a}$$

- Angles in terms of displacements:

$$\Theta_1(s) = \left(\frac{N_2}{N_1\, r}\right) X(s) \qquad \Theta_2(s) = \left(\frac{1}{r}\right) X(s)$$

- Sum of torques equation:

$$((J_a + J)\, s^2 + D_a\, s)\left(\frac{N_2}{N_1\, r}\right) X(s) + ((2J + J_2)\, s^2 + D_2\, s)\left(\frac{1}{r}\right) X(s)$$

$$= \left(\frac{K_t}{R_a}\right)\left[E_a(s) - K_b\left(\frac{N_2}{N_1\, r}\right) s\, X(s) \right]$$

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- Sum of torques equation (continued):

$$\frac{1}{r} \left[\left(\frac{N_2 \left(J_a + J \right)}{N_1} + 2J + J_2 \right) s^2 + \left(\frac{N_2 \left(D_a + K_b \right)}{N_1} + D_2 \right) s \right] X(s)$$
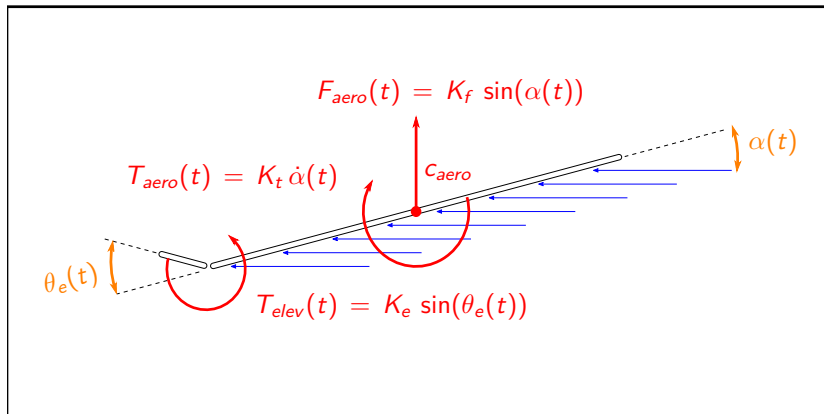$$= \left(\frac{K_t}{R_a} \right) E_a(t)$$

- Finally:

$$G(s) = \frac{r \, K_t}{R_a} \left[\left(\frac{N_2 \left(J_a + J \right)}{N_1} + 2J + J_2 \right) s^2 + \left(\frac{N_2 \left(D_a + K_b \right)}{N_1} + D_2 \right) s \right]^{-1}$$
$$\implies G(s) = \frac{1}{5s^2 + 3s}$$

Contenido del Tema

Model of a simple 2D wing with elevator:

Aerospace Vehicles

Model of a **puller-configuration** fixed-wing aircraft:
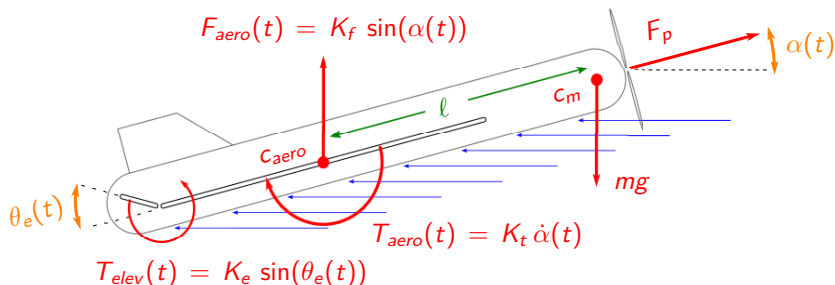
Model of a **pusher-configuration** fixed-wing aircraft:

$$F_{aero}(t) = K_f \sin(\alpha(t))$$

$$T_{aero}(t) = K_t \dot{\alpha}(t)$$

$$T_{elev}(t) = K_e \sin(\theta_e(t))$$