Control Systems Engineering (EYAG-1005): **Unit 02**

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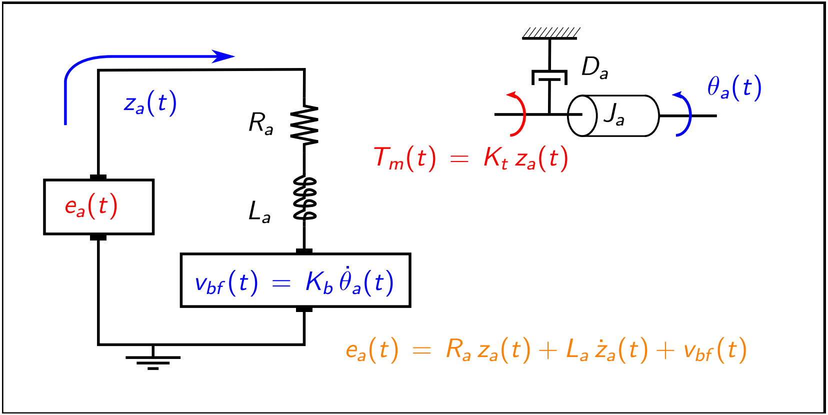
2017 - Term 1

Armature-controlled DC Motors

The model for the armature-controlled DC motor involves both an electric circuit an a rotational mechanical system. **If no load is attached to the armature** then the models are as shown below:

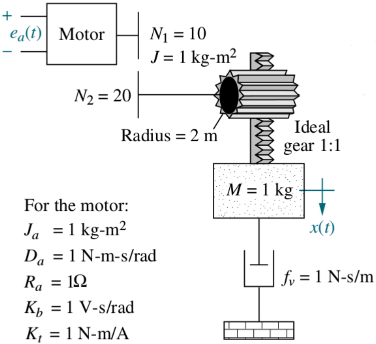


Example
(Nise, Problem 2.46*):

Consider the mechanism on the right, where an armature-controlled DC motor drives a block of mass by means of a system of gears. The input is the supplied voltage $e_a(t)$ and the output is the displacement of the block of mass $x(t)$.

Find the transfer function:

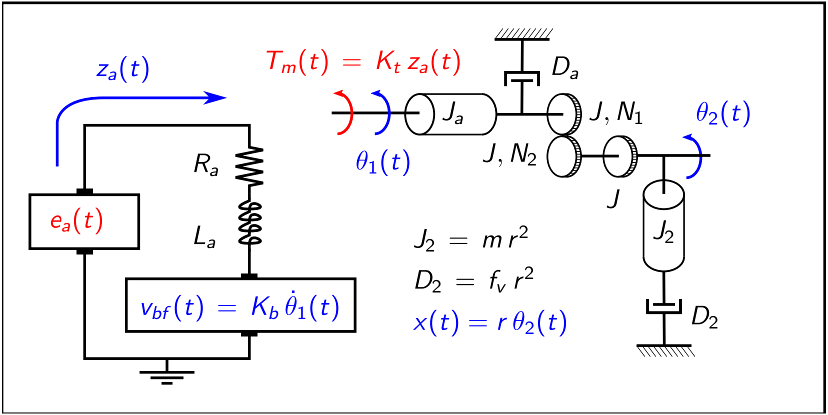
$$G(s) = \frac{X(s)}{E_a(s)}$$



Note: Each of the three gears shown has moment of inertia $J = 1$ kg-m^2.

Armature-controlled DC Motors

Electric circuit and equivalent rotational mechanical system:



Armature-controlled DC Motors

Models:

- Electric circuit:

$$e_a(t) \;=\; R_a\, z_a(t) + K_b\, \dot{\theta}_1(t)$$

- Rotational mechanical system:

$$(J_a + J)\,\ddot{\theta}_1(t) + (2J + J_2)\,\ddot{\theta}_2(t) \;=\; K_t\, z_a(t) - D_a\, \dot{\theta}_1(t) - D_2\, \dot{\theta}_2(t)$$

$$N_1\, \theta_1(t) \;=\; N_2\, \theta_2(t)$$

$$x(t) \;=\; r\, \theta_2(t)$$

Armature-controlled DC Motors

Taking the Laplace Transform of both equations on both sides:

- Electric circuit:

$$E_a(s) \;=\; R_a\, Z_a(s) + K_b\, s\, \Theta_1(s)$$

- Rotational mechanical system:

$$\big(\,(J_a + J)\, s^2 + D_a\, s\,\big)\, \Theta_1(s) + \big(\,(2J + J_2)\, s^2 + D_2\, s\,\big)\, \Theta_2(s) \;=\; K_t\, Z_a(s)$$

$$\Theta_1(s) \;=\; (N_2/N_1)\, \Theta_2(s)$$

$$\Theta_2(s) \;=\; (1/r)\, X(s)$$

- Armature current:

$$Z_a(s) \;=\; \frac{E_a(s) - K_b\, s\, \Theta_1(s)}{R_a}$$

- Angles in terms of displacements:

$$\Theta_1(s) \;=\; \left(\frac{N_2}{N_1\, r}\right) X(s) \qquad\qquad \Theta_2(s) \;=\; \left(\frac{1}{r}\right) X(s)$$

- Sum of torques equation:

$$\begin{aligned} &(\,(J_a + J)\, s^2 + D_a\, s\,) \left(\frac{N_2}{N_1\, r}\right) X(s) + (\,(2J + J_2)\, s^2 + D_2\, s\,) \left(\frac{1}{r}\right) X(s) \\ &= \left(\frac{K_t}{R_a}\right) \left[\, E_a(s) - K_b \left(\frac{N_2}{N_1\, r}\right) s\, X(s) \right] \end{aligned}$$

- Sum of torques equation (continued):

$$\frac{1}{r}\left[\left(\frac{N_2\left(J_a+J\right)}{N_1}+2J+J_2\right)s^2+\left(\frac{N_2\left(D_a+K_tK_b/R_a\right)}{N_1}+D_2\right)s\right]X(s)$$
$$=\left(\frac{K_t}{R_a}\right)E_a(t)$$

- Finally:

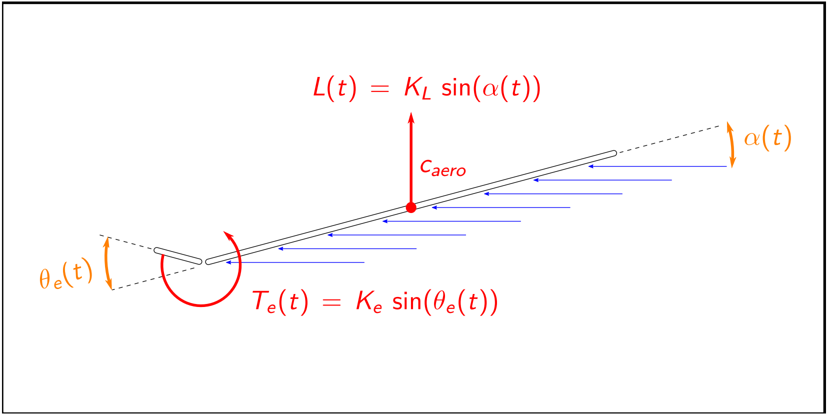
$$G(s)=\frac{\frac{r\,K_t}{R_a}}{\left(\frac{N_2\left(J_a+J\right)}{N_1}+2J+J_2\right)s^2+\left(\frac{N_2\left(D_a+K_tK_b/R_a\right)}{N_1}+D_2\right)s}$$
$$\implies G(s)=\frac{1}{5s^2+4s}=\frac{1/5}{s^2+(4/5)s}=\frac{1/5}{s\,(\,s+(4/5)\,)}$$

Aerospace Vehicles

Introduction to the longitudinal dynamics of fixed-wing aircraft:

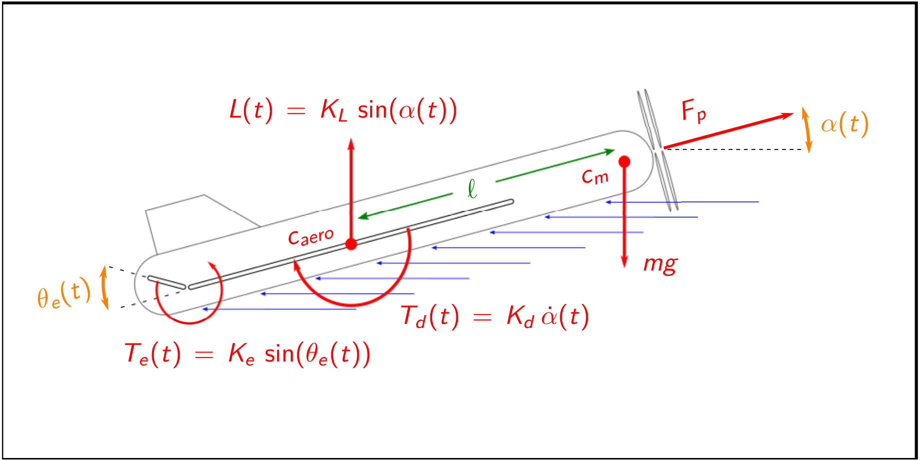
- Consider only aircraft with massless, flat, very wide, rectangular wings.
- Define angle of attack $\alpha(t)$ as the angle between the surface of the wings and the airflow. Also assume vertical elevators are available, and denote the elevator deflection angle as $\theta_e(t)$.
- Assume angle of attack is low, *e.g.*, $-10^\circ \leq \alpha(t) \leq +25^\circ$, so that no wing stalling is possible.
- Define the aerodynamic center c_{aero} as the point where the lift force $L(t)$ acts. Also, assume $L(t)$ is proportional to the sine of the angle of attack.
- Assume elevator generates torque on the wing $T_e(t)$ proportional to the sine of its deflection angle.

Model of a massless, flat, very wide, rectangular wing with elevator:



Aerospace Vehicles

Model of a **puller-configuration** fixed-wing aircraft:



Model of a **pusher-configuration** fixed-wing aircraft: