Control Systems Engineering (EYAG-1005)

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Contenido del Tema

1 Mechanical Systems

2 Electric Circuits

3 Armature-controlled DC Motors

4 Aerospace Vehicles

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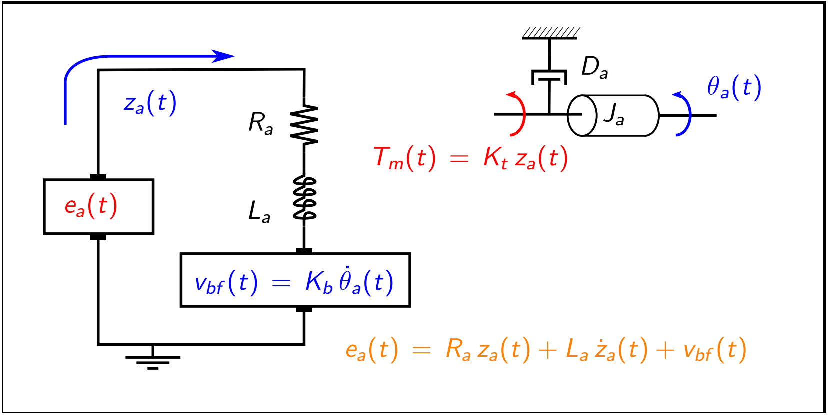
The model for the armature-controlled DC motor involves both an electric circuit an a rotational mechanical system. **If no load is attached to the armature** then the models are as shown below:

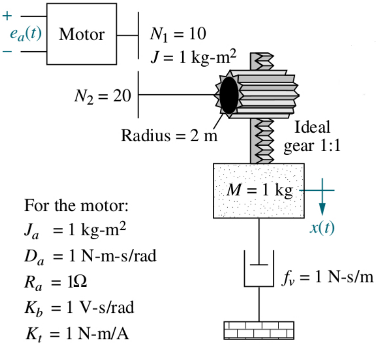


Example
(Nise, Problem 2.46*):

Consider the mechanism on the right, where an armature-controlled DC motor drives a block of mass by means of a system of gears. The input is the supplied voltage $e_a(t)$ and the output is the displacement of the block of mass $x(t)$.

Find the transfer function:

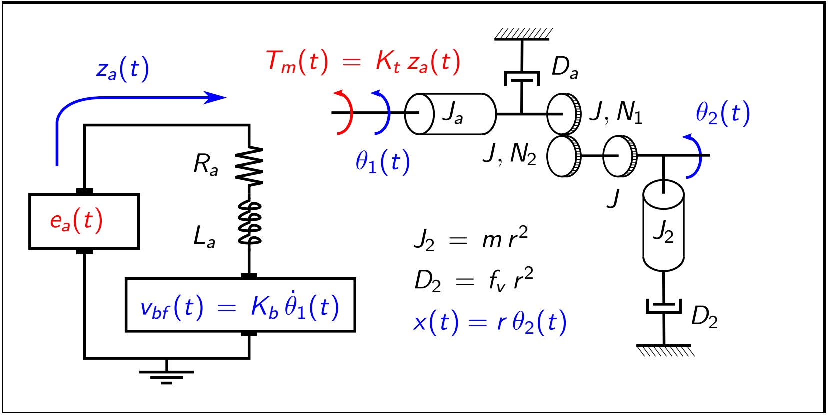
$$G(s) = \frac{X(s)}{E_a(s)}$$



Note: Each of the three gears shown has moment of inertia $J = 1$ kg-m^2.

Armature-controlled DC Motors

Electric circuit and equivalent rotational mechanical system:



Armature-controlled DC Motors

Models:

- Electric circuit:

$$e_a(t) \;=\; R_a\, z_a(t) + K_b\, \dot{\theta}_1(t)$$

- Rotational mechanical system:

$$(J_a + J)\, \ddot{\theta}_1(t) + (2J + J_2)\, \ddot{\theta}_2(t) \;=\; K_t\, z_a(t) - D_a\, \dot{\theta}_1(t) - D_2\, \dot{\theta}_2(t)$$

$$N_1\, \theta_1(t) \;=\; N_2\, \theta_2(t)$$

$$x(t) \;=\; r\, \theta_2(t)$$

Armature-controlled DC Motors

Taking the Laplace Transform of both equations on both sides:

- Electric circuit:

$$E_a(s) \,=\, R_a\, Z_a(s) + K_b\, s\, \Theta_1(s)$$

- Rotational mechanical system:

$$\left((J_a + J)\, s^2 + D_a\, s \right) \Theta_1(s) + \left((2J + J_2)\, s^2 + D_2\, s \right) \Theta_2(s) \,=\, K_t\, Z_a(s)$$

$$\Theta_1(s) \,=\, (N_2/N_1)\, \Theta_2(s)$$

$$\Theta_2(s) \,=\, (1/r)\, X(s)$$

Armature-controlled DC Motors

Solving...

- Armature current:

$$Z_a(s) \;=\; \frac{E_a(s) - K_b\, s\, \Theta_1(s)}{R_a}$$

- Angles in terms of displacements:

$$\Theta_1(s) \;=\; \left(\frac{N_2}{N_1\, r}\right) X(s) \qquad\qquad \Theta_2(s) \;=\; \left(\frac{1}{r}\right) X(s)$$

- Sum of torques equation:

$$\left(\, (J_a + J)\, s^2 + D_a\, s\,\right) \left(\frac{N_2}{N_1\, r}\right) X(s) + \left(\, (2J + J_2)\, s^2 + D_2\, s\,\right) \left(\frac{1}{r}\right) X(s)$$
$$= \left(\frac{K_t}{R_a}\right) \left[\, E_a(s) - K_b \left(\frac{N_2}{N_1\, r}\right) s\, X(s)\,\right]$$

Armature-controlled DC Motors

- Sum of torques equation (continued):

$$\frac{1}{r}\left[\left(\frac{N_2\,(J_a+J)}{N_1}+2J+J_2\right)s^2+\left(\frac{N_2\,(D_a+K_tK_b/R_a)}{N_1}+D_2\right)s\right]X(s)$$
$$=\left(\frac{K_t}{R_a}\right)E_a(t)$$

- Finally:

$$G(s)=\frac{\frac{r\,K_t}{R_a}}{\left(\frac{N_2\,(J_a+J)}{N_1}+2J+J_2\right)s^2+\left(\frac{N_2\,(D_a+K_tK_b/R_a)}{N_1}+D_2\right)s}$$
$$\implies G(s)=\frac{1}{5s^2+4s}=\frac{1/5}{s^2+(4/5)s}=\frac{1/5}{s\,(\,s+(4/5)\,)}$$

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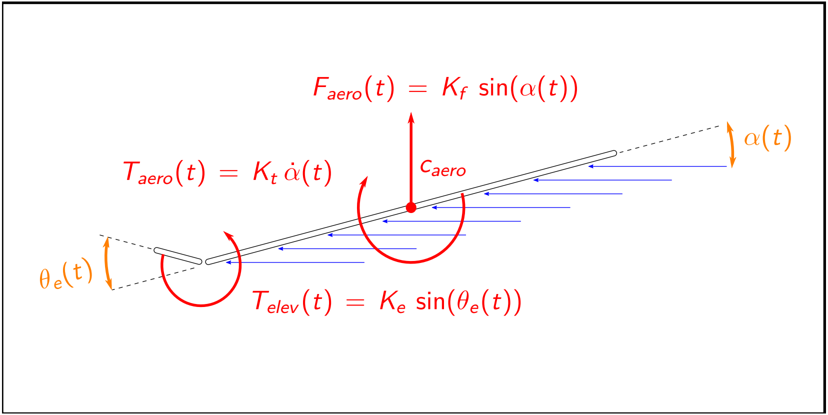
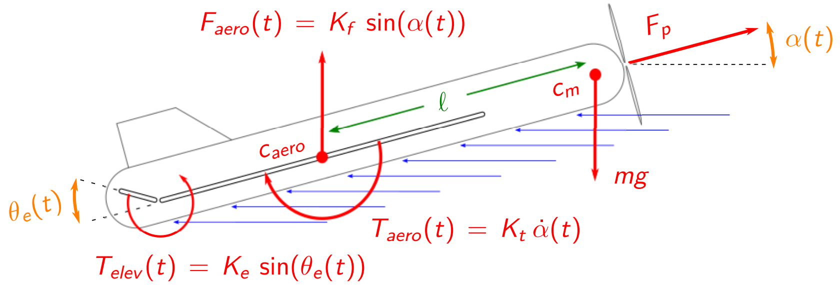
Model of a simple 2D wing with elevator:



Model of a **puller-configuration** fixed-wing aircraft:



Model of a **pusher-configuration** fixed-wing aircraft: