Control Systems Engineering (EYAG-1005): **Unit 03**

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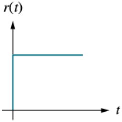
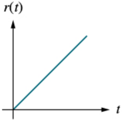
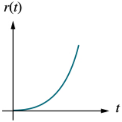
Semester: 2017-T1

1. Steady-state Errors

Steady-state Errors

Recall the reference signals:

TABLE 7.1 Test waveforms for evaluating steady-state errors of position control systems

Waveform	Name	Physical interpretation	Time function	Laplace transform
	Step	Constant position	1	$\frac{1}{s}$
	Ramp	Constant velocity	t	$\frac{1}{s^2}$
	Parabola	Constant acceleration	$\frac{1}{2}t^2$	$\frac{1}{s^3}$

Steady-state Errors

Recall the following result:

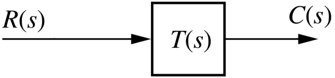
Laplace Transform - Final Value Theorem

For any function $f(t)$ defined for $t \geq 0$ with Laplace Transform $F(s)$ we have that:

$$\lim_{t \to \infty} f(t) \quad = \quad \lim_{s \to 0} s\,F(s)$$

Steady-state Errors

Steady-state errors for feedthrough systems:



$$E(s) \;=\; R(s) - C(s) \;=\; R(s) - T(s)\,R(s)$$

$$\implies E(s) \;=\; R(s)\,[\,1 - T(s)\,]$$

$$\implies e(\infty) \;=\; \lim_{t\to\infty} e(t) \;=\; \lim_{s\to 0}\; s\,R(s)\,[\,1 - T(s)\,]$$

Steady-state Errors

- Step input $r(t) = u(t)$:

$$R(s) = \frac{1}{s} \quad \Longrightarrow \quad e(\infty) = \lim_{s \to 0} 1 - T(s)$$

- Ramp input $r(t) = t\, u(t)$:

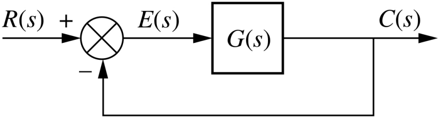
$$R(s) = \frac{1}{s^2} \quad \Longrightarrow \quad e(\infty) = \lim_{s \to 0} \frac{1 - T(s)}{s}$$

- Parabolic input $r(t) = (1/2)\, t^2\, u(t)$:

$$R(s) = \frac{1}{s^3} \quad \Longrightarrow \quad e(\infty) = \lim_{s \to 0} \frac{1 - T(s)}{s^2}$$

Steady-state Errors

Steady-state errors for unity-feedback systems:



$$E(s) \;=\; R(s) - C(s) \;=\; R(s) - G(s)\,E(s)$$

$$\implies E(s)\,[\,1 + G(s)\,] \;=\; R(s)$$

$$\implies E(s) \;=\; \frac{R(s)}{1 + G(s)}$$

$$\implies e(\infty) \;=\; \lim_{t\to\infty} e(t) \;=\; \lim_{s\to 0} \frac{s\,R(s)}{1 + G(s)}$$

Steady-state Errors

- Step input $r(t) = u(t)$:

$$R(s) = \frac{1}{s} \qquad \Longrightarrow \qquad e_{step}(\infty) = \lim_{s \to 0} \frac{1}{1 + G(s)}$$

- Ramp input $r(t) = t\,u(t)$:

$$R(s) = \frac{1}{s^2} \qquad \Longrightarrow \qquad e_{ramp}(\infty) = \lim_{s \to 0} \frac{1}{s\,G(s)}$$

- Parabolic input $r(t) = (1/2)\,t^2\,u(t)$:

$$R(s) = \frac{1}{s^3} \qquad \Longrightarrow \qquad e_{parabolic}(\infty) = \lim_{s \to 0} \frac{1}{s^2\,G(s)}$$

Steady-state Errors

Furthermore, for unity-feedback systems:

- Position constant K_p :

$$K_p = \lim_{s \to 0} G(s) \qquad \Longrightarrow \qquad e_{step}(\infty) = \frac{1}{1 + K_p}$$

- Velocity constant K_v :

$$K_v = \lim_{s \to 0} s\, G(s) \qquad \Longrightarrow \qquad e_{ramp}(\infty) = \frac{1}{K_v}$$

- Acceleration constant K_a :

$$K_a = \lim_{s \to 0} s^2\, G(s) \qquad \Longrightarrow \qquad e_{ramp}(\infty) = \frac{1}{K_a}$$

For non-unity non-unity-feedback systems, simply re-write the system in unity-feedback form: