Control Systems Engineering (EYAG-1005): **Unit 03**

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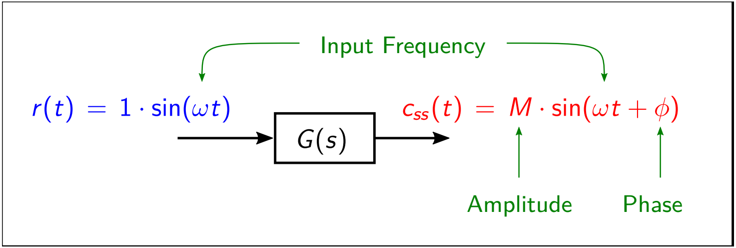
1 Bode Plots

Bode Plots

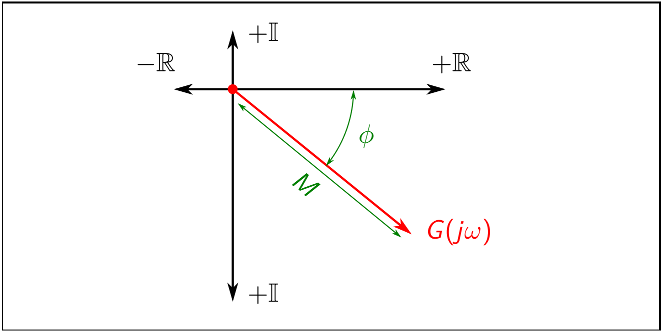
Consider a stable system with transfer function $G(s)$.

- Suppose the input $r(t)$ is a sinusoid with frequency ω and unit amplitude.
- Then the steady-state output $c_{ss}(t)$ must be a a sinusoid with the same frequency ω but with a particular amplitude M and phase ϕ which depend on the transfer function $G(s)$ and on the input frequency ω.



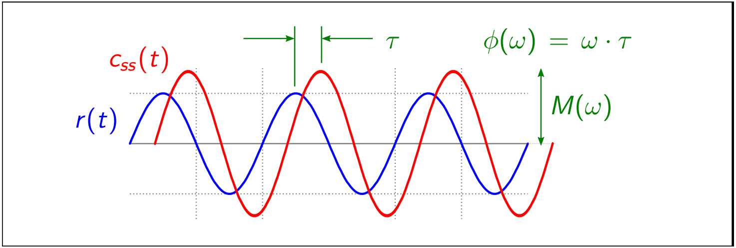
Bode Plots

- Given $G(s)$ and a frequency ω we can evaluate the amplitude and phase of the steady-state output by computing the phasor $G(j\omega)$. In particular:
  - The phasor's magnitude yields the amplitude $M(\omega)$.
  - The phasor's angle with $+\mathbb{R}$ yields the phase $\phi(\omega)$.



Bode Plots

- We can also estimate $M(\omega)$ and $\phi(\omega)$ experimentally:



Bode Plots

- Furthermore, notice that:
  - Amplitude M is always positive.
    - If $M \in (0, 1)$ we get attenuation.
    - If $M = 1$ we get amplitude matching.
    - If $M > 1$ we get amplification.
  - Phase may be negative, zero or postive.
    - If $\phi < 0$ then the output lags the input.
    - If $\phi = 0$ then the output matches the input.
    - If $\phi > 0$ then the output leads the input.

Bode Plots

Bode Plots are diagrams of $M(\omega)$ and $\phi(\omega)$. More precisely, they consist of the following two plots:

- **Magnitude Plot**: Amplitude $M(\omega)$ versus frequency ω.
  - The x-axis is frequency ω in decades, *i.e.*, $x = \log_{10}(\omega)$.
  - The y-axis is amplitude $M(\omega)$ in decibels, *i.e.*, $y = 20 \cdot \log(M(\omega))$.
- **Phase Plot**: Phase $\phi(\omega)$ versus frequency ω.
  - The x-axis is frequency ω in decades, *i.e.*, $x = \log_{10}(\omega)$.
  - The y-axis is phase in degrees, *i.e.*, $y = \phi(\omega)$.

Notice that when sketching Bode Plots by hand, we usually don't draw exactly the functions $M(\omega)$ and $\phi(\omega)$ but instead sketch *asymptotic approximations*.

1. Bode Plots
   - Introduction
   - Elementary Systems

Bode Plots

Bode plot for a simple amplifier: $G(s) = K$

- Magnitude plot is constant at $y = 20 \cdot \log_{10}(K)$ decibels.
- Phase plot is constant at $y = 0$ degree.

Bode Plots

Bode plot for an integrator: $G(s) = \dfrac{1}{s}$

- Phasor as a function of ω :

$$G(j\omega) \;=\; \frac{1}{j\omega} \;=\; -\frac{j}{\omega} \qquad \Longrightarrow \qquad M(\omega) \;=\; \frac{1}{\omega} \quad \& \quad \phi(\omega) \;=\; -90^\circ$$

- Magnitude plot is $y = -20 \cdot \log_{10}(\omega)$ decibels, *i.e.*, it is a line with slope of -20 decibels per decade which hits zero decibels at $\omega = 1$ rad/s.
- Phase plot is constant at $y = -90$ degree.

Bode Plots

Bode plot for a differentiator: $G(s) = s$

- Phasor as a function of ω :

$$G(j\omega) \;=\; j\omega \qquad \Longrightarrow \qquad M(\omega) \;=\; \omega \quad \& \quad \phi(\omega) \;=\; +90^\circ$$

- Magnitude plot is $y = +20 \cdot \log_{10}(\omega)$ decibels, *i.e.*, it is a line with slope of $+20$ decibels per decade which hits zero decibels at $\omega = 1$ rad/s.
- Phase plot is constant at $y = +90$ degree.