Control Systems Engineering (EYAG-1005): **Unit 06:** State Space Models

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Model Definition

Model Variables:

- Input vector $\boldsymbol{u}(t) \in \mathbb{R}^m$
- State vector $\boldsymbol{x}(t) \in \mathbb{R}^n$
- Output vector $\boldsymbol{y}(t) \in \mathbb{R}^\ell$

Model Parameters:

- State matrix $\boldsymbol{A} \in \mathbb{R}^{n \times n}$
- Input matrix $\boldsymbol{B} \in \mathbb{R}^{n \times m}$
- Output matrix $\boldsymbol{C} \in \mathbb{R}^{\ell \times n}$
- Feedthrough matrix $\boldsymbol{D} \in \mathbb{R}^{\ell \times m}$

Model Definition

Model Equations:

- State Equations:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\,\boldsymbol{x}(t) + \boldsymbol{B}\,\boldsymbol{u}(t)$$

- Output Equations:

$$\boldsymbol{y}(t) = \boldsymbol{C}\,\boldsymbol{x}(t) + \boldsymbol{D}\,\boldsymbol{u}(t)$$

Equivalence with Transfer Functions: SS-to-TF

Transfer Function Matrix:

- Compute as:

$$\boldsymbol{G}(s) \;=\; \boldsymbol{C}\,(\,s\boldsymbol{I} - \boldsymbol{A}\,)^{-1}\,\boldsymbol{B} \;+\; \boldsymbol{D}$$

- Entry (i, j) of matrix $\boldsymbol{G}(s)$ is the transfer function from the j^{th} input to the i^{th} output, assuming all other inputs are zero.

Equivalence with Transfer Functions: TF-to-SS

Procedure for Transfer Functions with Constant Numerator, *i.e.:*

$$G(s) \;=\; \frac{C(s)}{R(s)} \;=\; \frac{B}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}$$

1. Re-write the transfer function as an n^{th} order differential equation:

$$\frac{d^n c(t)}{dt^n} \;=\; -\,a_{n-1}\,\frac{d^{n-1}c(t)}{dt^{n-1}} \;-\; \cdots \;-\; a_1\,\frac{dc(t)}{dt} \;-\; a_0\,c(t) \;+\; B\,r(t)$$

2. Assign states to the output and its derivatives:

$$x_1(t) \;=\; c(t)\,, \quad x_2(t) \;=\; \frac{dc(t)}{dt}\,, \quad \cdots\,, \quad x_n(t) \;=\; \frac{d^{n-1}c(t)}{dt^{n-1}}$$

Equivalence with Transfer Functions: TF-to-SS

3. Write the first $n-1$ state equations:

$$\dot{x}_1(t) \;=\; x_2(t)\,, \quad \dot{x}_2(t) \;=\; x_3(t)\,, \quad \cdots\,, \quad \dot{x}_{n-1}(t) \;=\; x_n(t)$$

4. Write the last state equation using the differential equation associated with the transfer function.

$$\dot{x}_n(t) \;=\; -\,a_{n-1}\,\frac{d^{n-1}c(t)}{dt^{n-1}} \;-\; \cdots \;-\; a_1\,\frac{dc(t)}{dt} \;-\; a_0\,c(t) \;+\; B\,r(t)$$

Equivalence with Transfer Functions: TF-to-SS