Control Systems Engineering (EYAG-1005): **Unit 04**

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1 Evan's Root Locus

1. Evan's Root Locus
   - Definition
   - Sketching Rules

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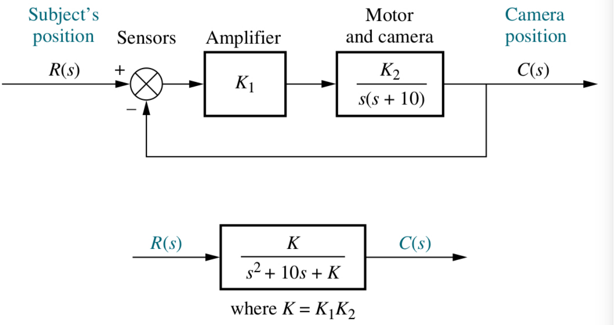
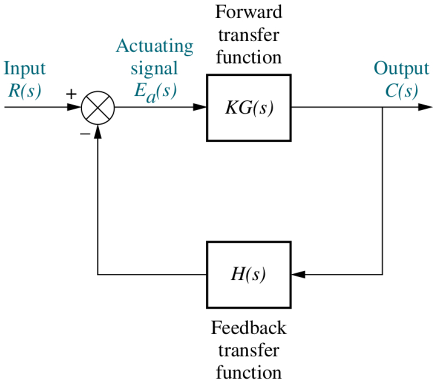
Motivating example:



K	**Pole 1**	**Pole 2**
0	-10	0
5	-9.47	-0.53
10	-8.87	-1.13
15	-8.16	-1.84
20	-7.24	-2.76
25	-5	-5
30	$-5 + j2.24$	$-5 - j2.24$
35	$-5 + j3.16$	$-5 - j3.16$
40	$-5 + j3.87$	$-5 - j3.87$
45	$-5 + j4.47$	$-5 - j4.47$
50	$-5 + j5$	$-5 - j5$

Evan's Root Locus

The root locus concerns the design of closed-loop control systems with the following architecture:



Evan's Root Locus

Definitions:

- Loop gain: K
- Open-loop transfer function: $G(s)\,H(s)$

Objective:

- Sketch the roots of the closed-loop transfer function as the loop gain K ranges from near zero (*i.e.*, $K \to 0^+$) to infinity (*i.e.*, $K \to +\infty$).

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Evan's Root Locus

Root locus sketching rules:

- The root locus is symmetric about the real axis.
- The number of branches, *i.e.*, pole trayectories, equals the number of poles of the open-loop transfer function.
- Each branch begins at an open-loop pole and ends either:
  - At an open-loop zero.
  - At infinity along an asymptote.
- Along the real line, root locus branches can be found to the left of any odd number of real open-loop poles or open-loop zeros.

Evan's Root Locus

- If the root locus has asymptotes, then the number of asymptotes is:

$$(\text{ number of open-loop poles }) \;-\; (\text{ number of open-loop zeros })$$

- If the root locus has asymptotes, then the centroid of the asymptotes is located along the real axis at the point:

$$\sigma_a \;=\; \frac{\sum(\text{open-loop pole locations}) - \sum(\text{open-loop zero locations})}{\text{number of asymptotes}}$$

- If the root locus has asymptotes, then their angles in radians are:

$$\theta_a \;=\; \frac{(2k+1)\,\pi}{\text{number of asymptotes}} \qquad \text{for } k \;=\; 0,\, \pm 1,\, \pm 2,\, \ldots$$