Common language

Common language

A simplicial complex over V is a subset $\Delta \subseteq \mathcal{P}(V)$ closed under contentions

Common language

A vertex.

$$\bullet$$

$$\{\, \bullet \,\}$$

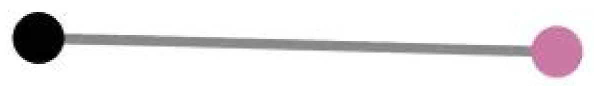
Common language

An edge.

$$\{ \bullet , \bullet \}$$

Common language

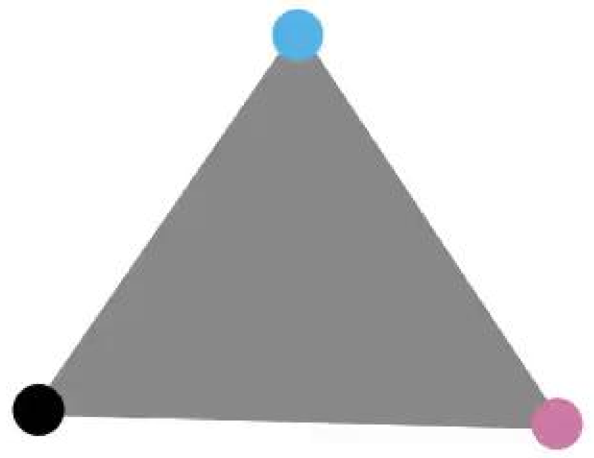
An edge.



$$\{\bullet, \bullet\}$$

Common language

A triangle.



What is a pseudosphere?

What is a pseudosphere?

$\mathbb{P}$: finite

What is a pseudosphere?

$$\mathbb{P} : \text{ finite}$$

$$V_p : \text{ finite for each } \; p \in \mathbb{P}$$

What is a pseudosphere?

$\mathbb{P}$: finite

V_p : finite for each $p \in \mathbb{P}$

Ψ simplicial complex

What is a pseudosphere?

Ψ simplicial complex

Vertices: (p, v) where $v \in V_p, p \in \mathbb{P}$

What is a pseudosphere?

Ψ simplicial complex

Vertices: (p, v) where $v \in V_p, p \in \mathbb{P}$

Simplices: $(p, v), (p, w) \in \sigma \implies v = w$

What is a pseudosphere?

It is the biggest chromatic complex on those vertices.

What is a pseudosphere?

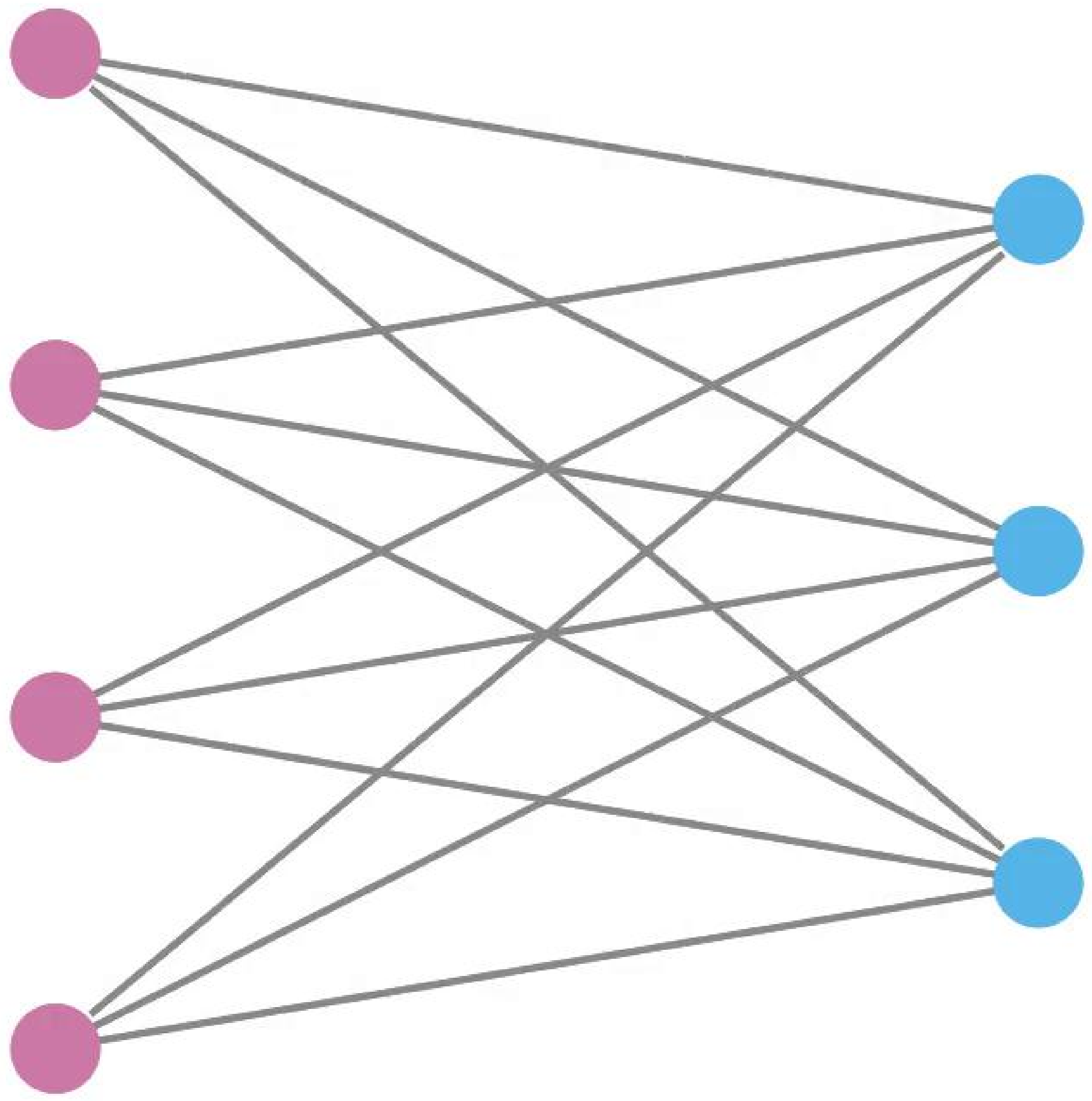
Clear, right?

Some examples

Some examples

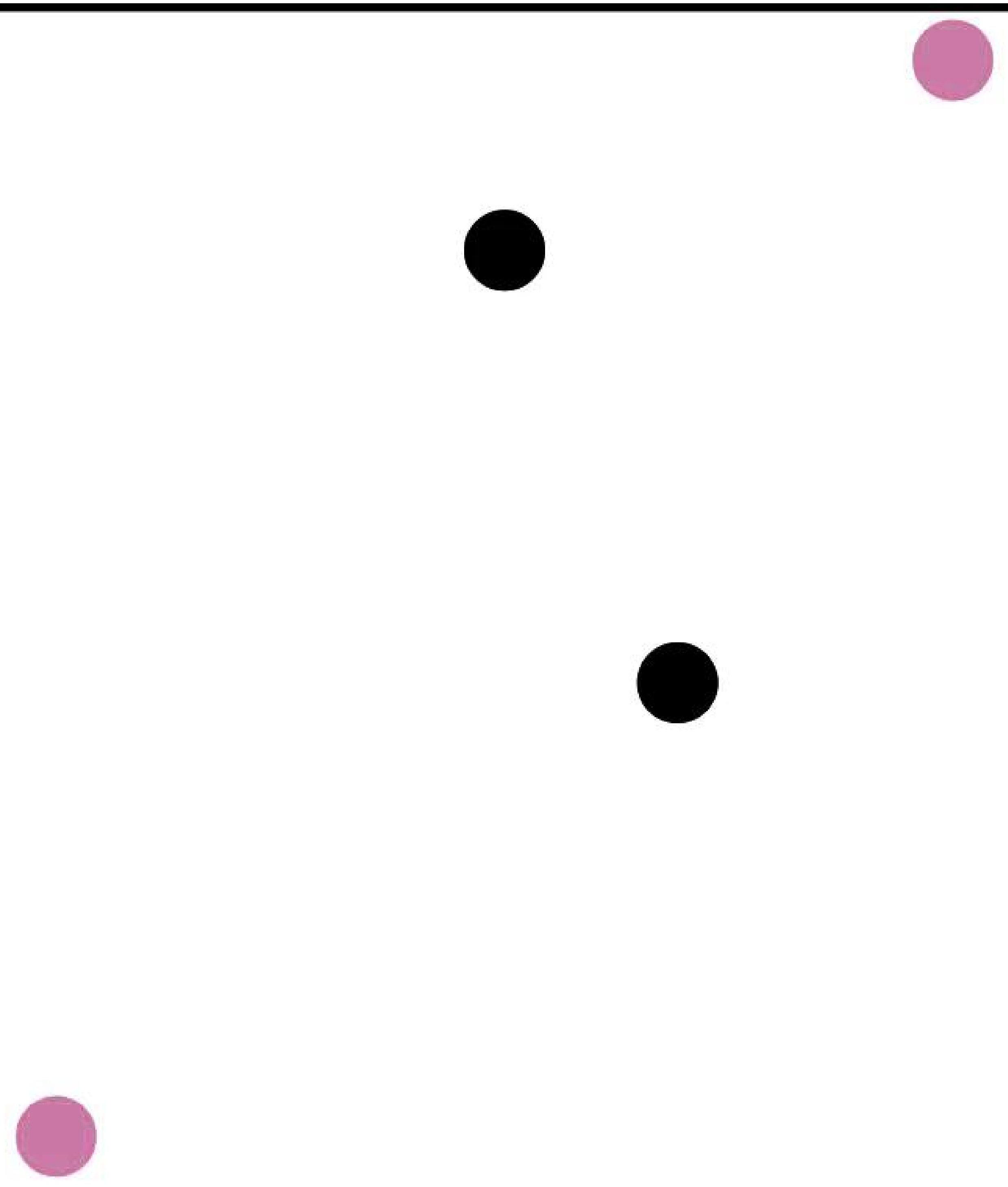
Some examples

Some examples

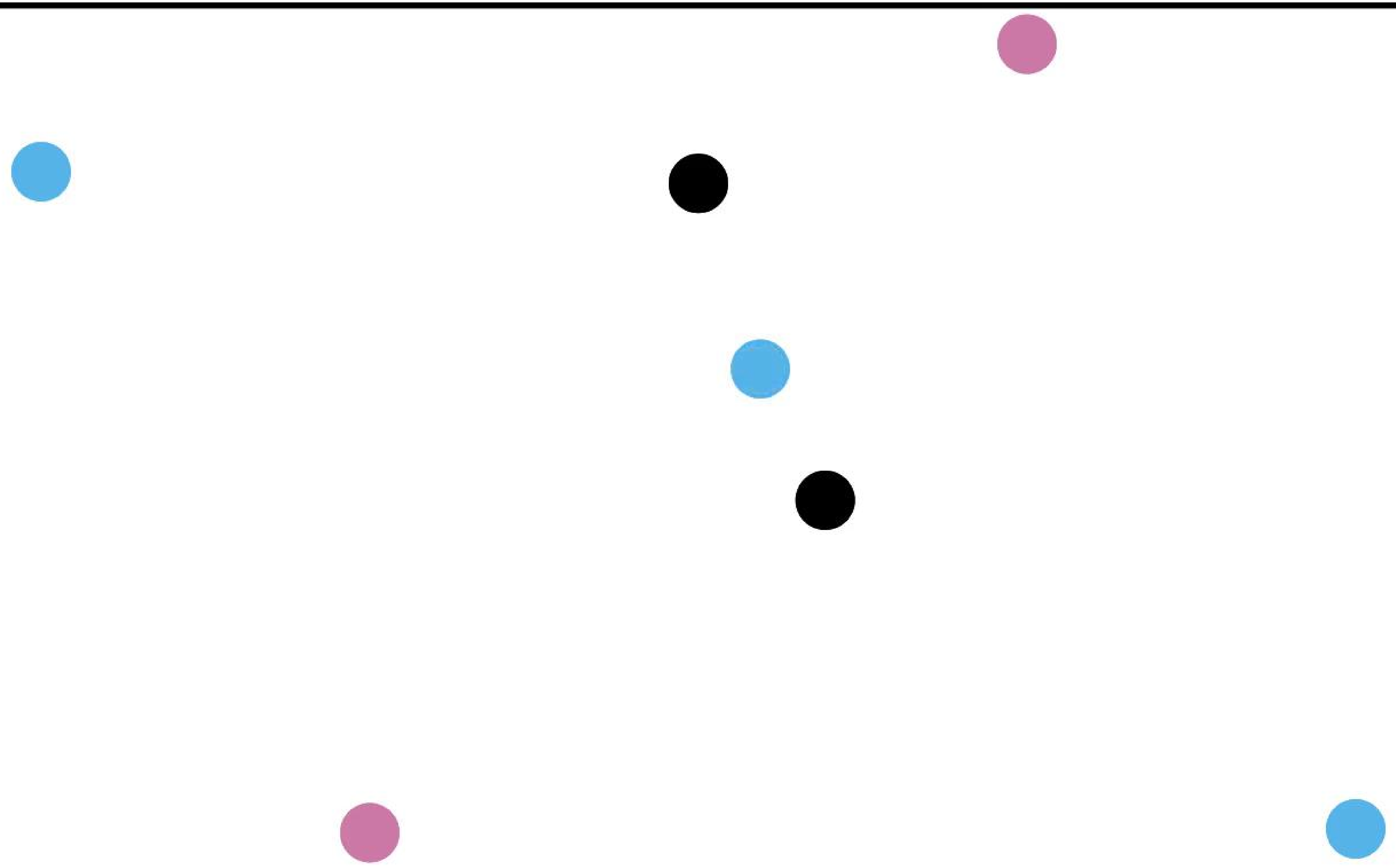


Some examples

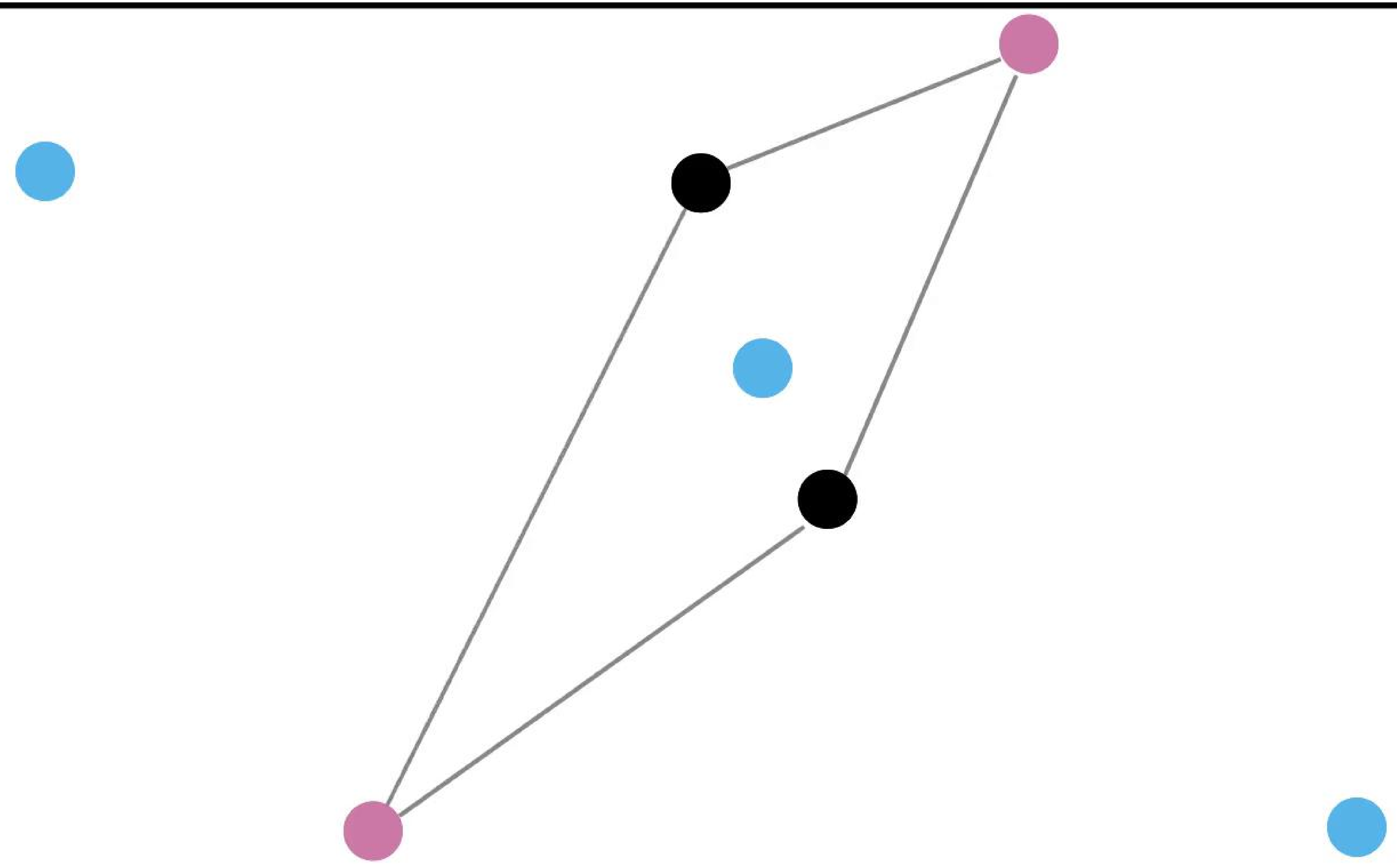
Some examples



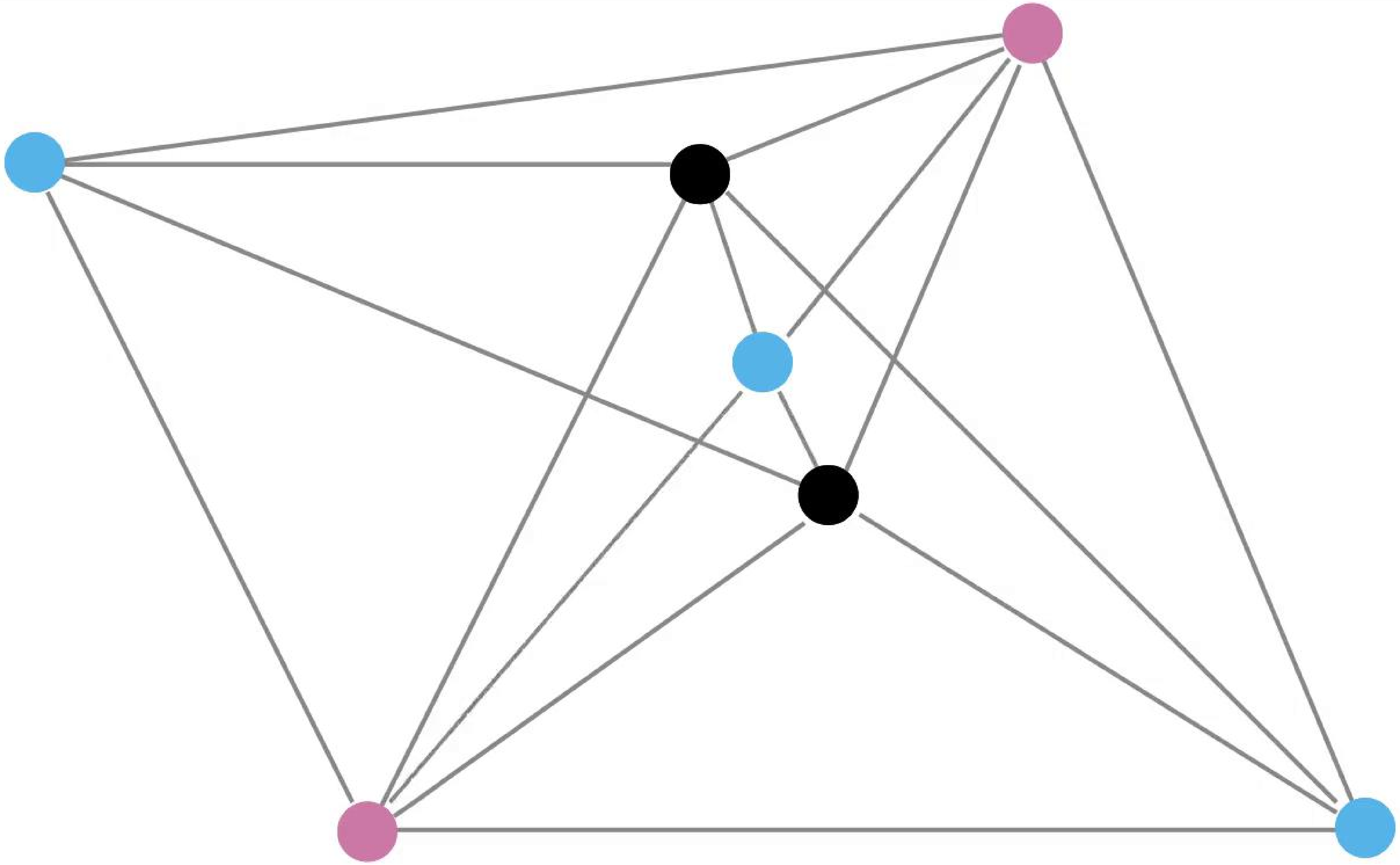
Some examples



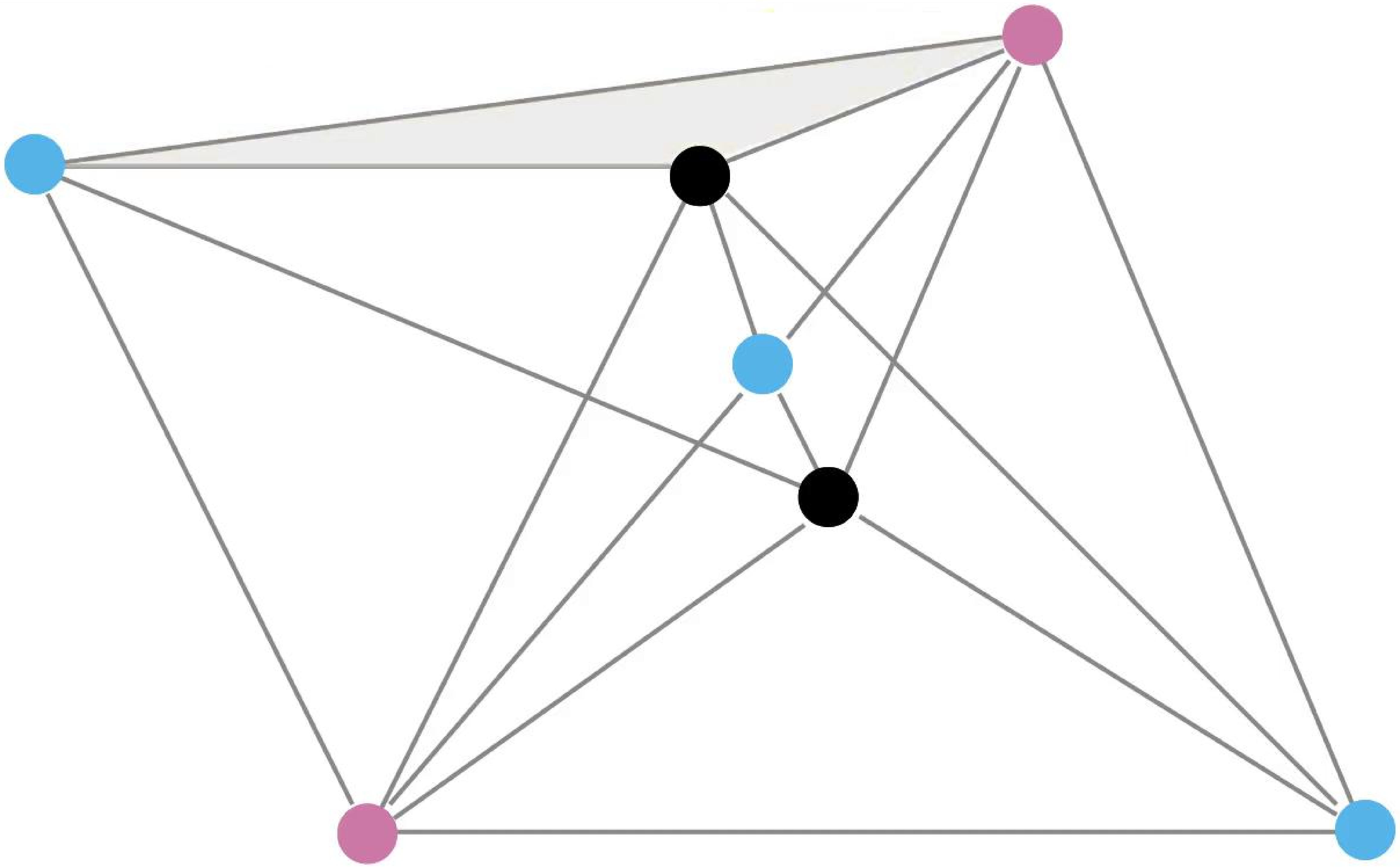
Some examples



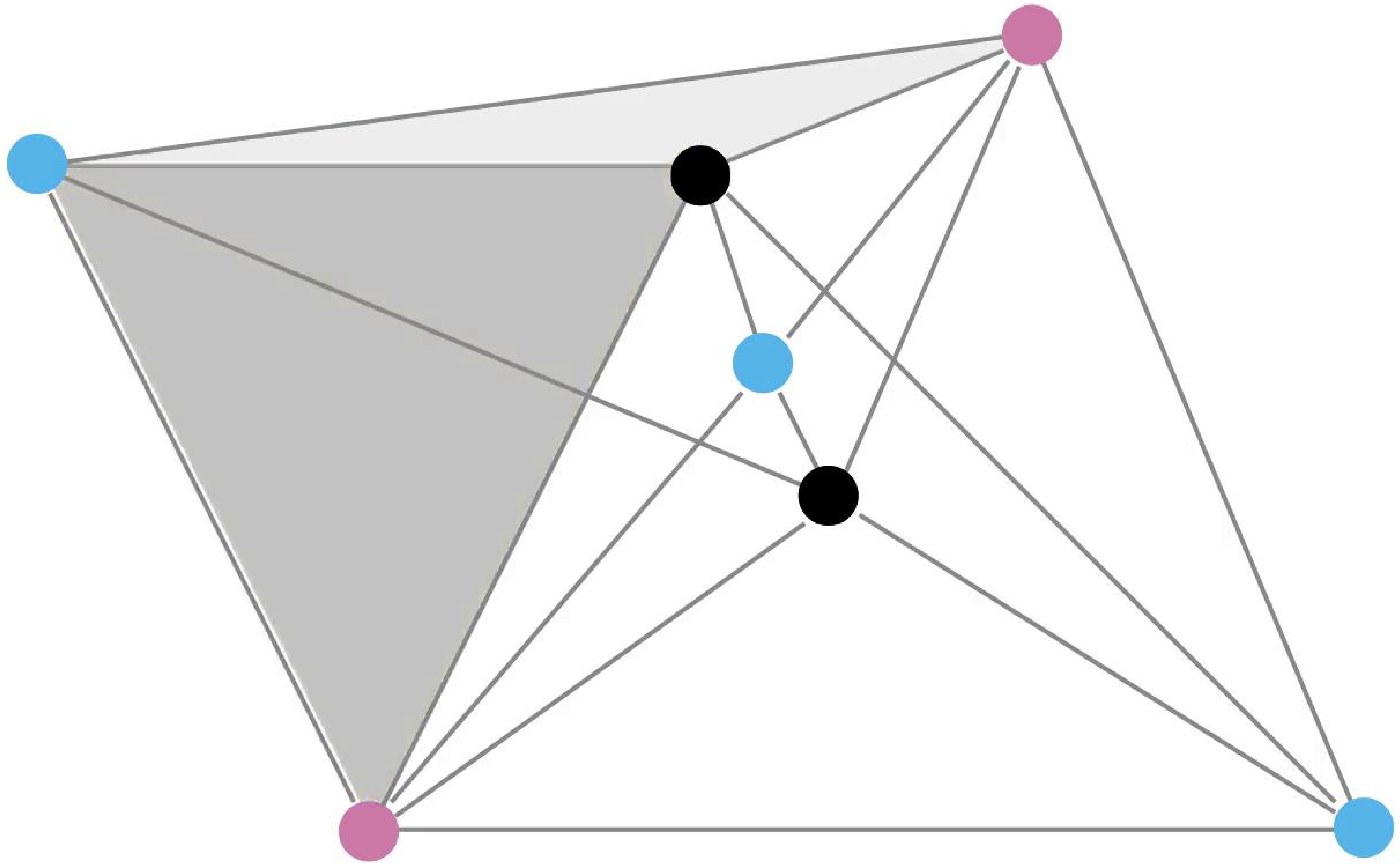
Some examples



Some examples



Some examples



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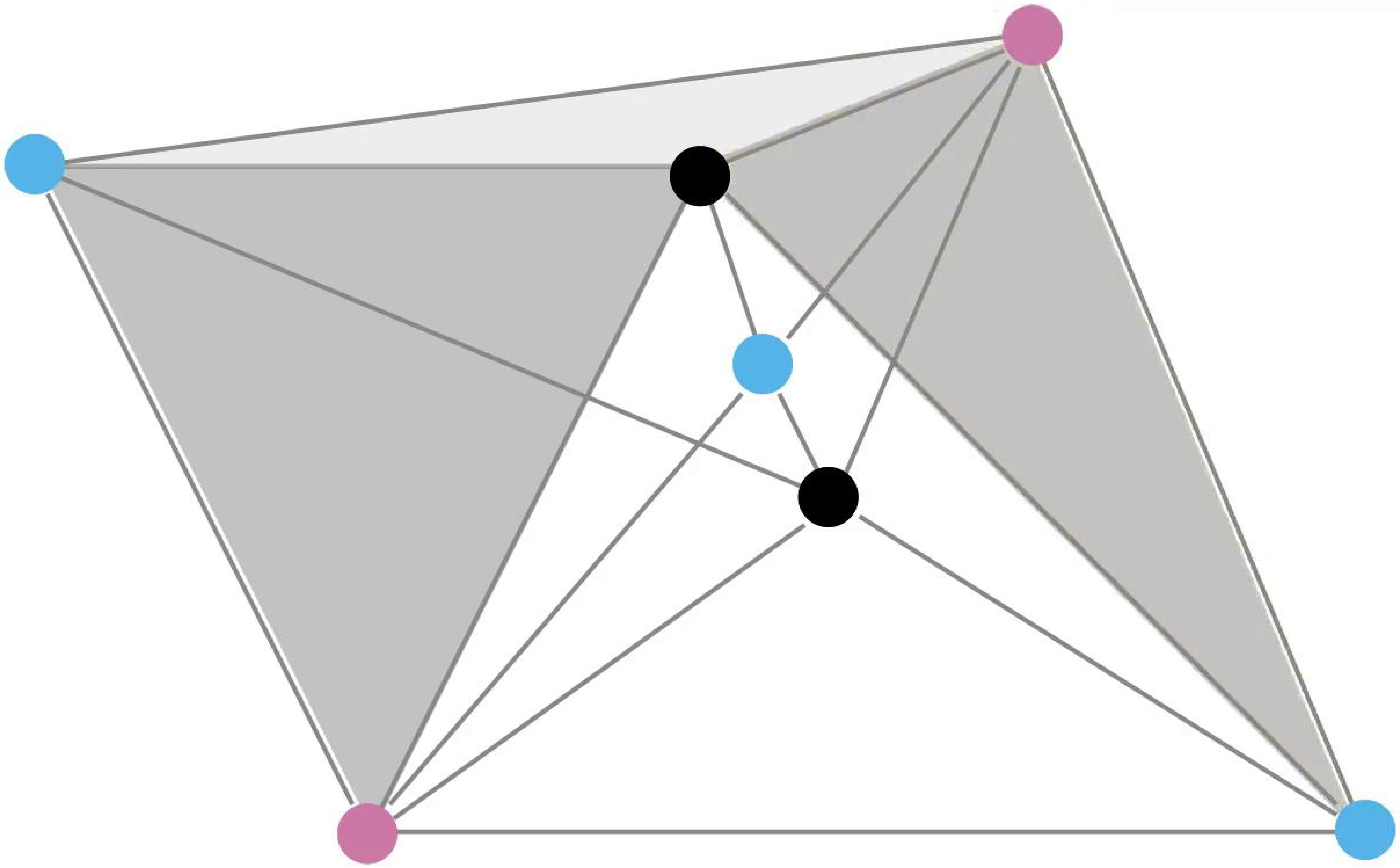

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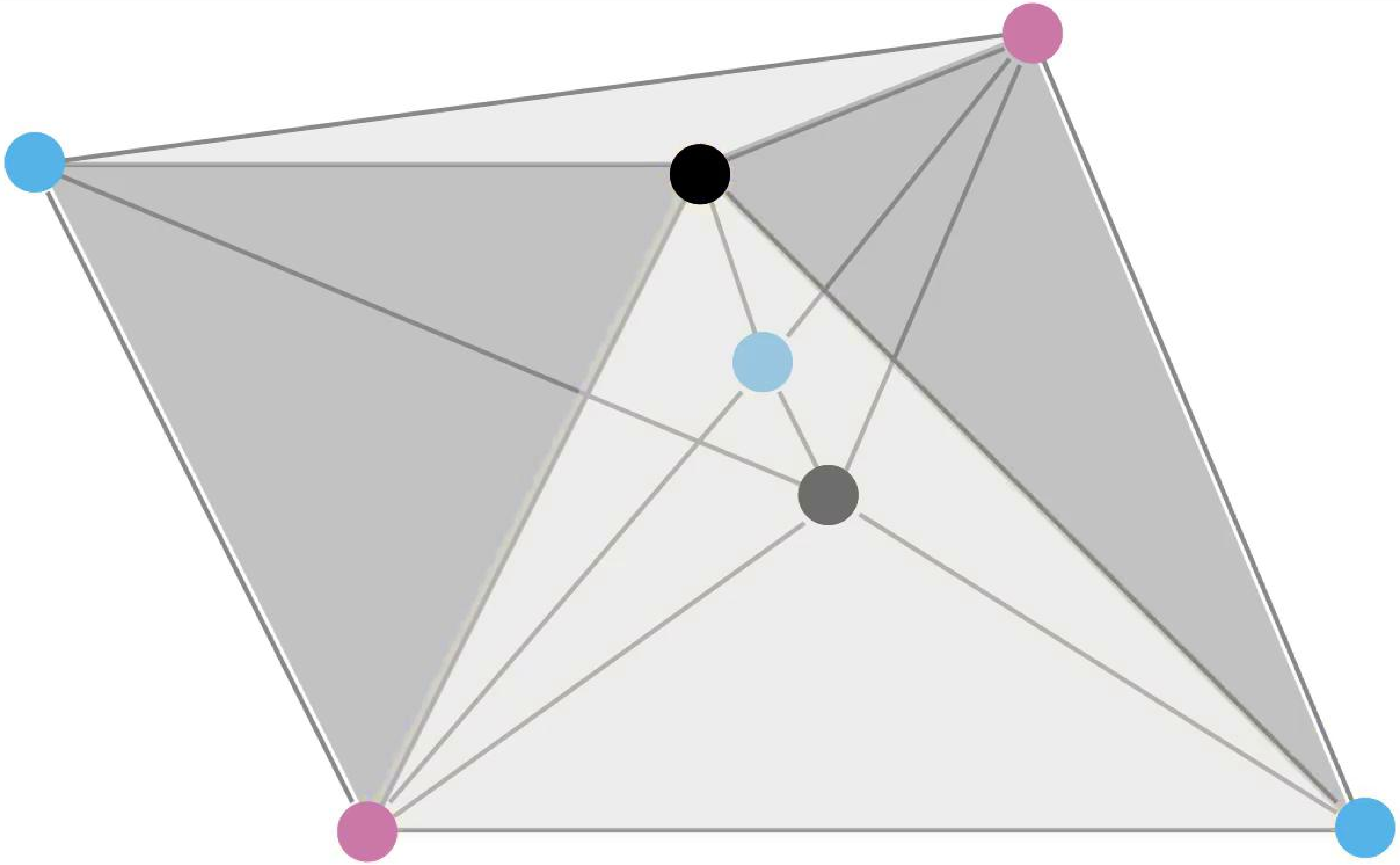


Some examples

Ready to study applications...

Some examples

...maybe not!

Some examples

The knowledge of pseudospheres consisted

of 3 pages of

Distributed Computing

Through Combinatorial Topology!

Some examples

The knowledge of pseudospheres consisted

of 3 pages of

Distributed Computing

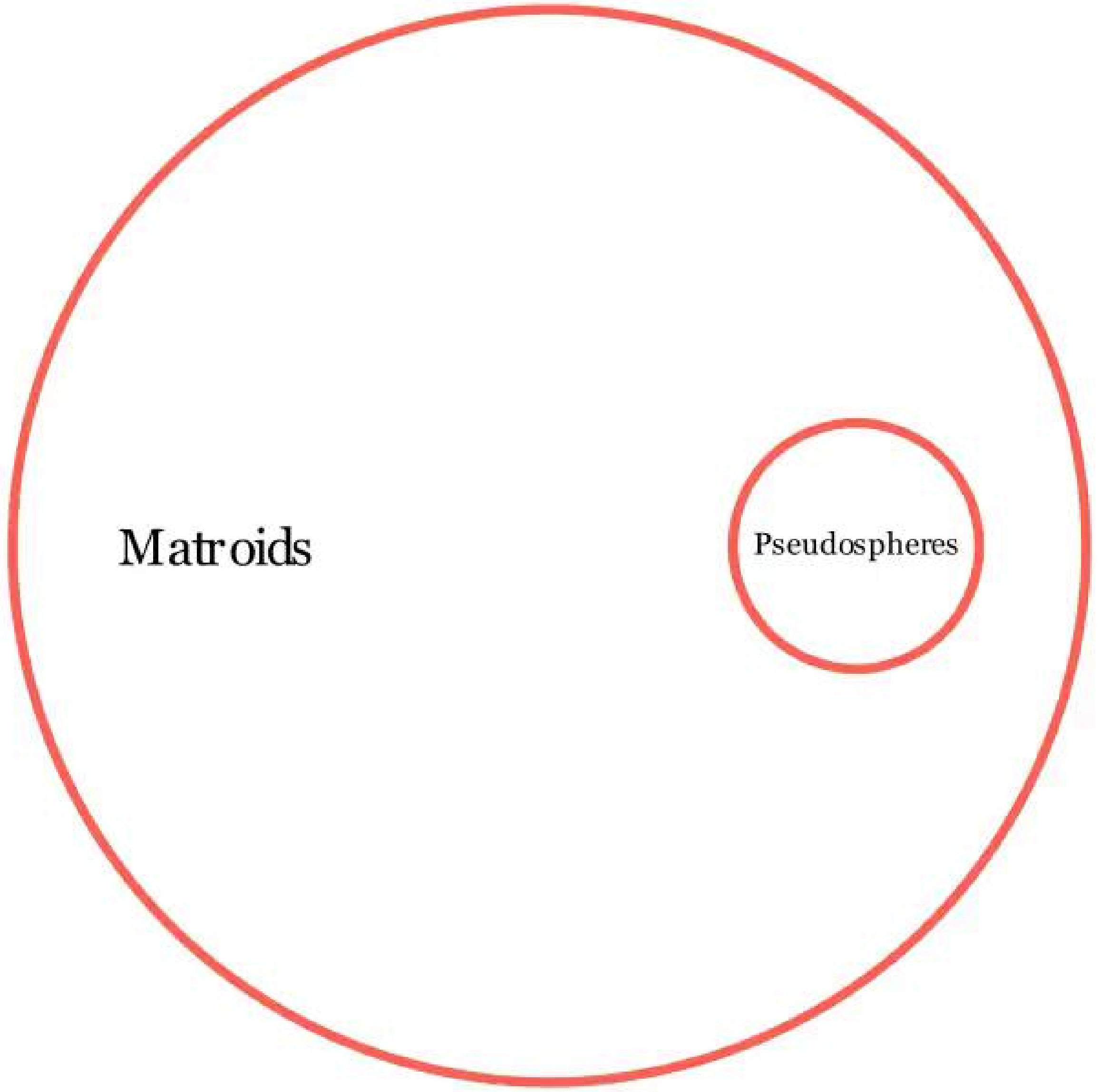
Through Combinatorial Topology!

The first discovery

The first discovery

Pseudospheres

The first discovery



Matroids

Matroids

$$\sigma, \tau \in \Delta$$

$$\dim(\sigma) > \dim(\tau) \implies \exists x \in \sigma \setminus \tau \colon \tau \cup \{x\} \in \Delta$$

Matroids

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Matroids

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Matroids

Matroids

Pseudospheres

are shellable (Theorem 13.3.6 of Herlihy, M., et.al. 2013).

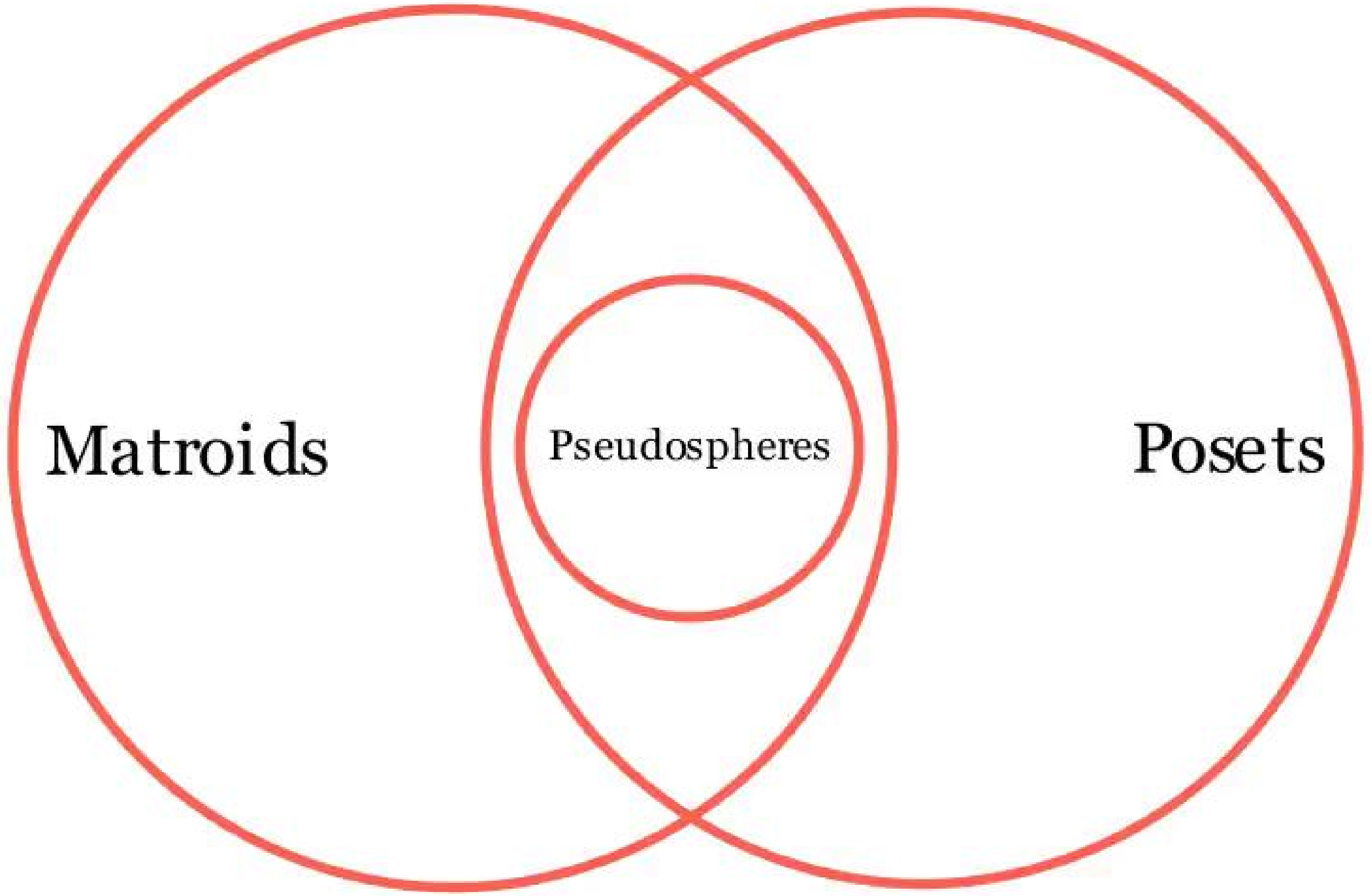
Matroids

Pseudospheres

are shellable because they are matroids.

Which matroids are pseudospheres?

Which matroids are pseudospheres?



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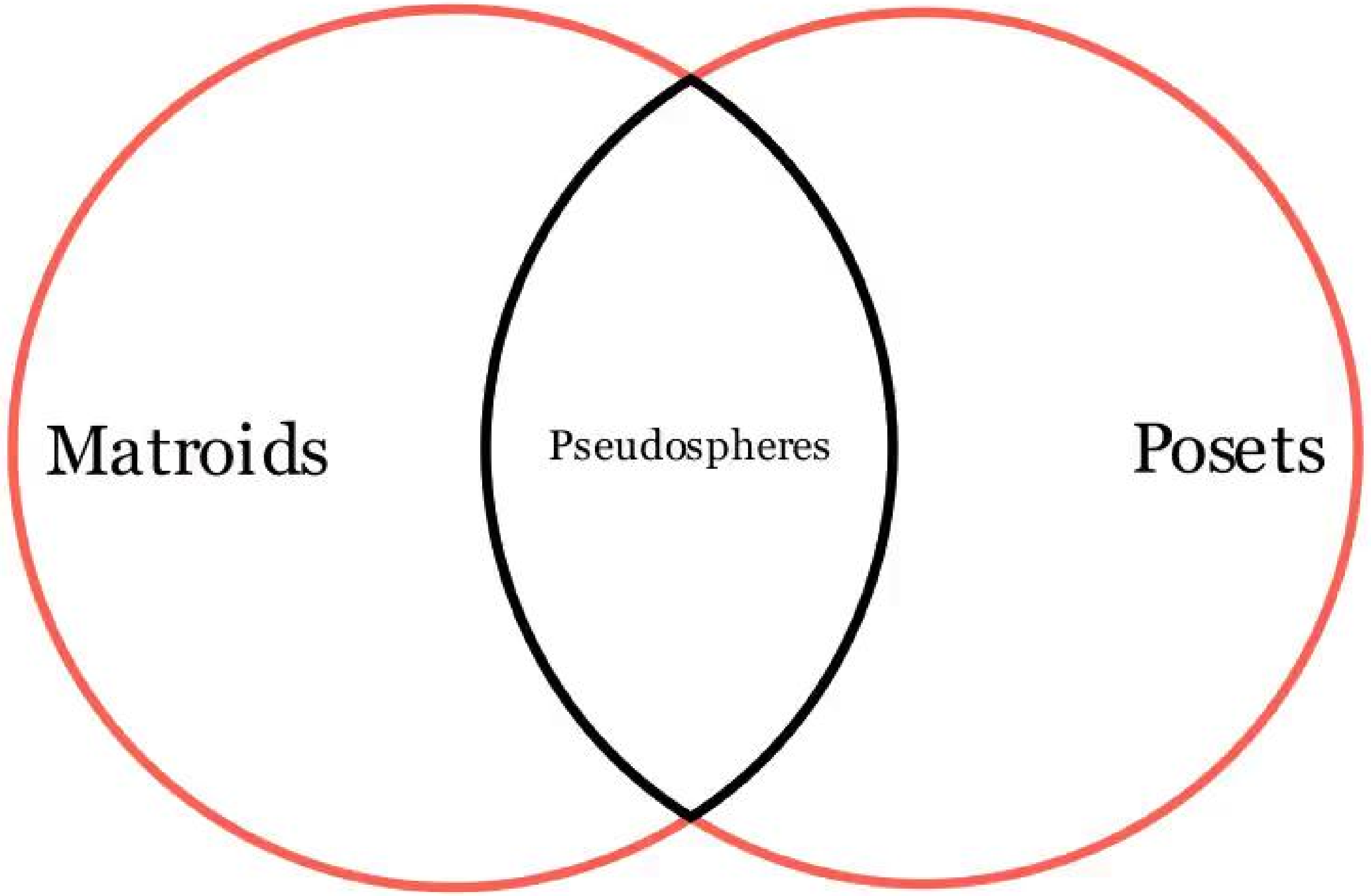

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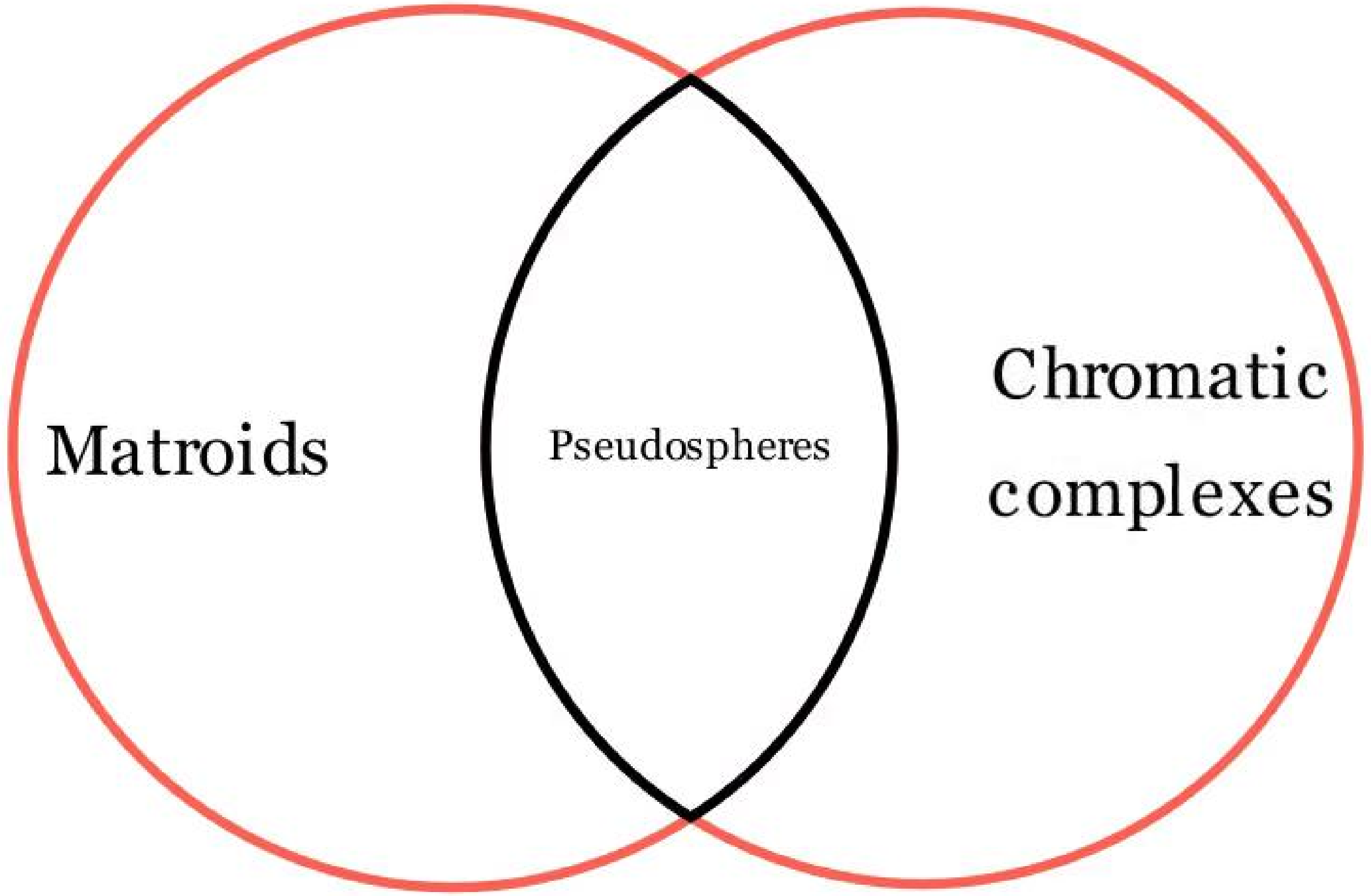


The last characterization

The last characterization

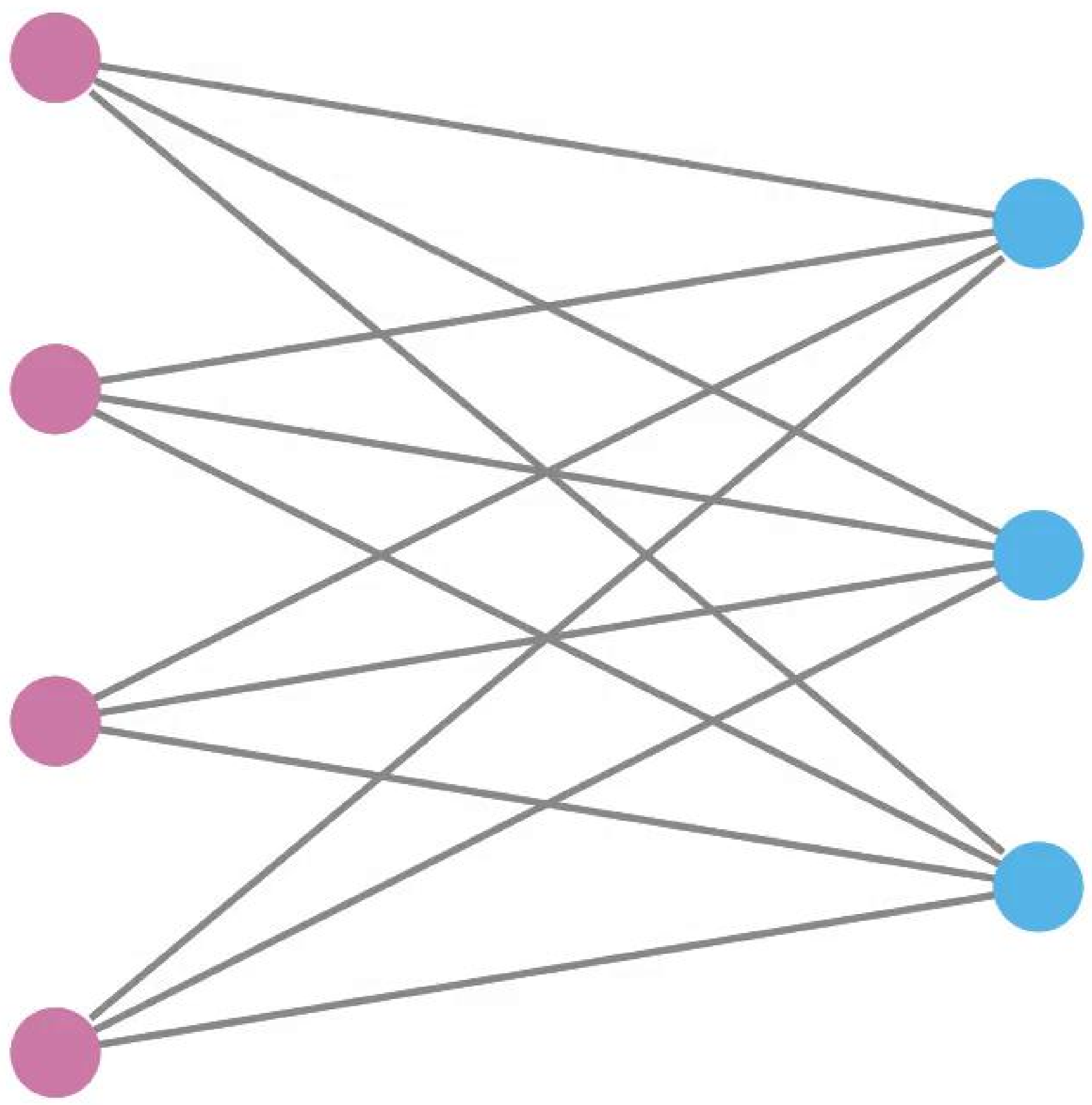
Join

$$\Delta * \Gamma = \{\sigma \cup \tau \mid \sigma \in \Delta, \tau \in \Gamma\}$$

The last characterization

The last characterization

The last characterization



The last characterization

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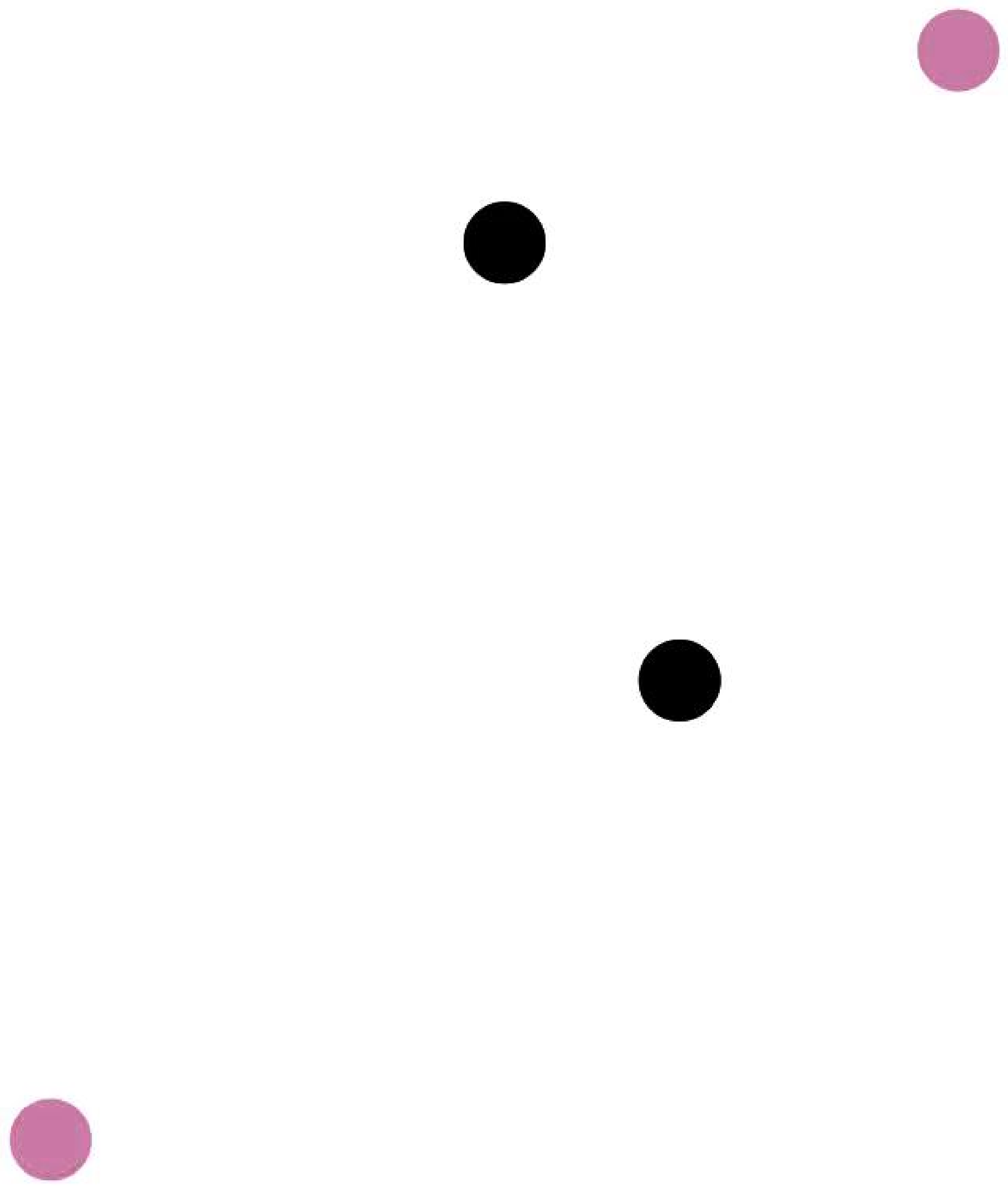

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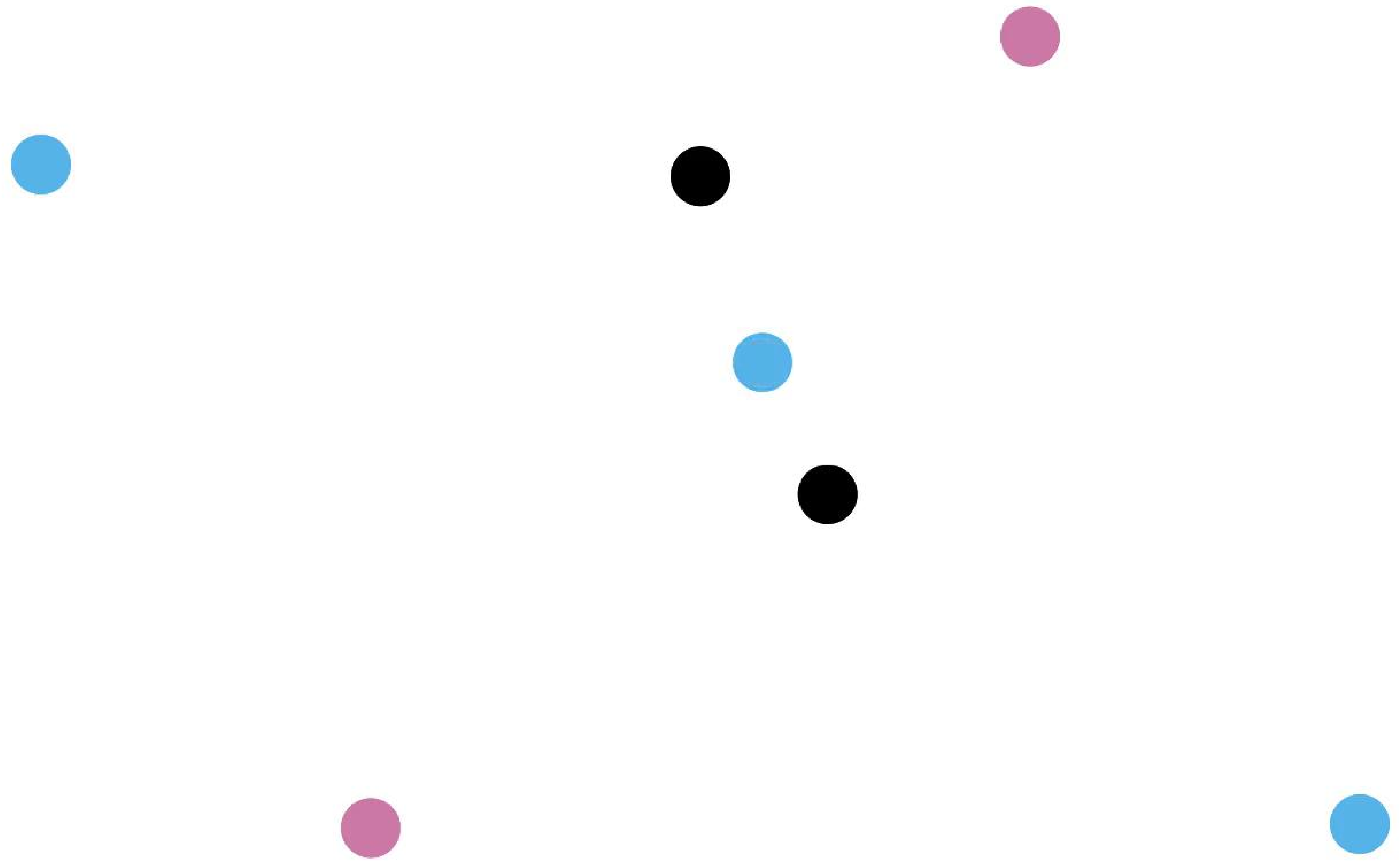

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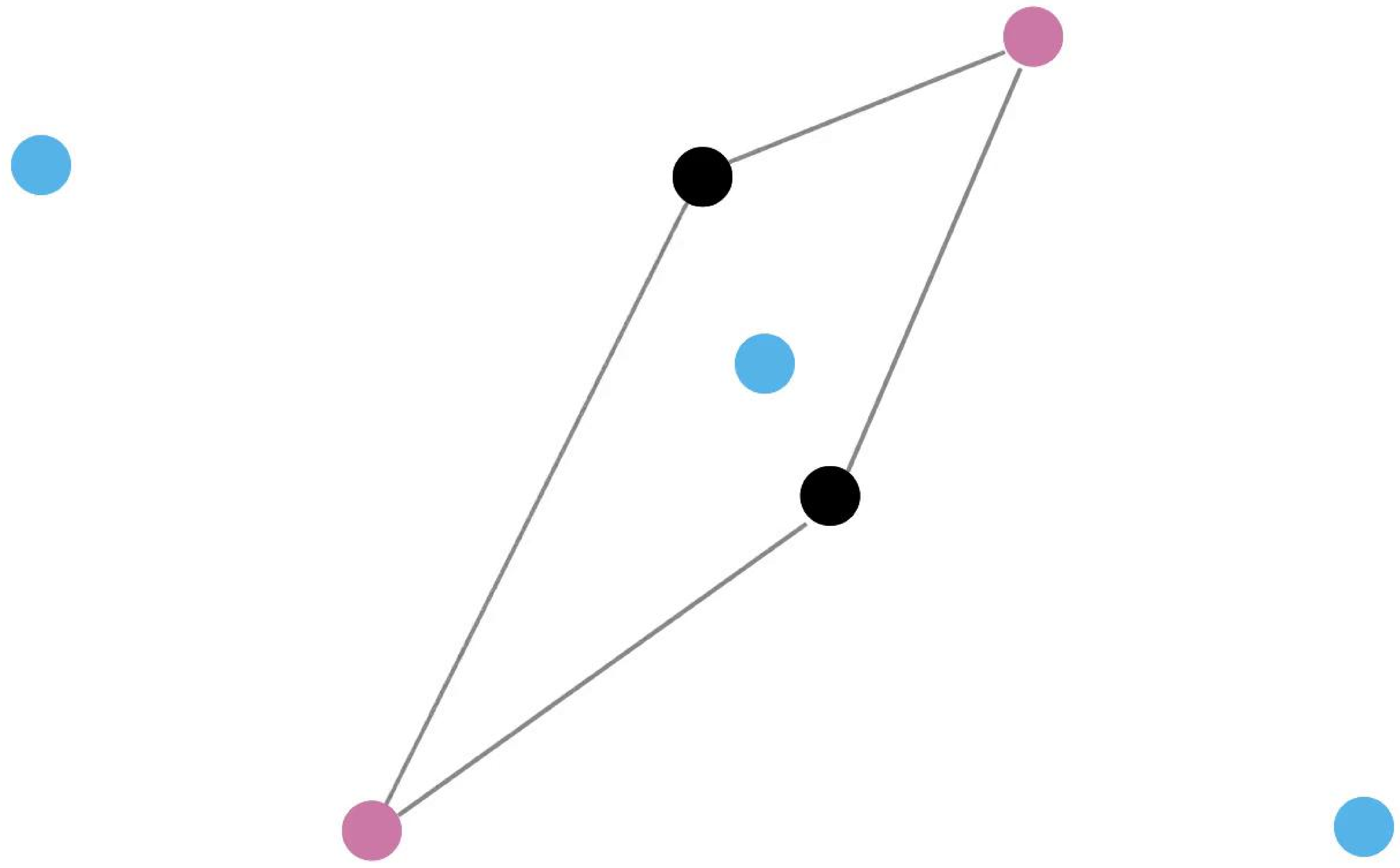

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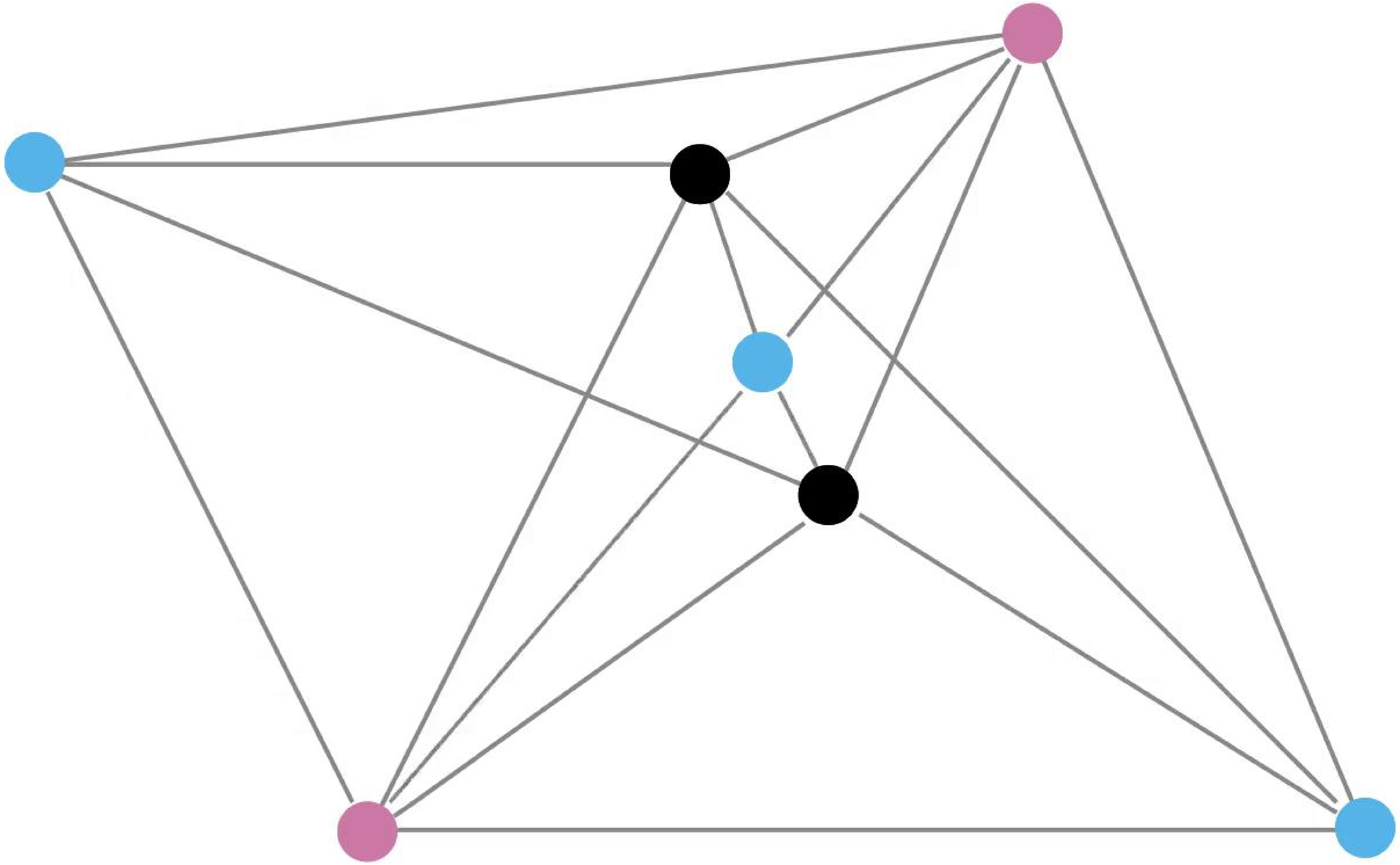

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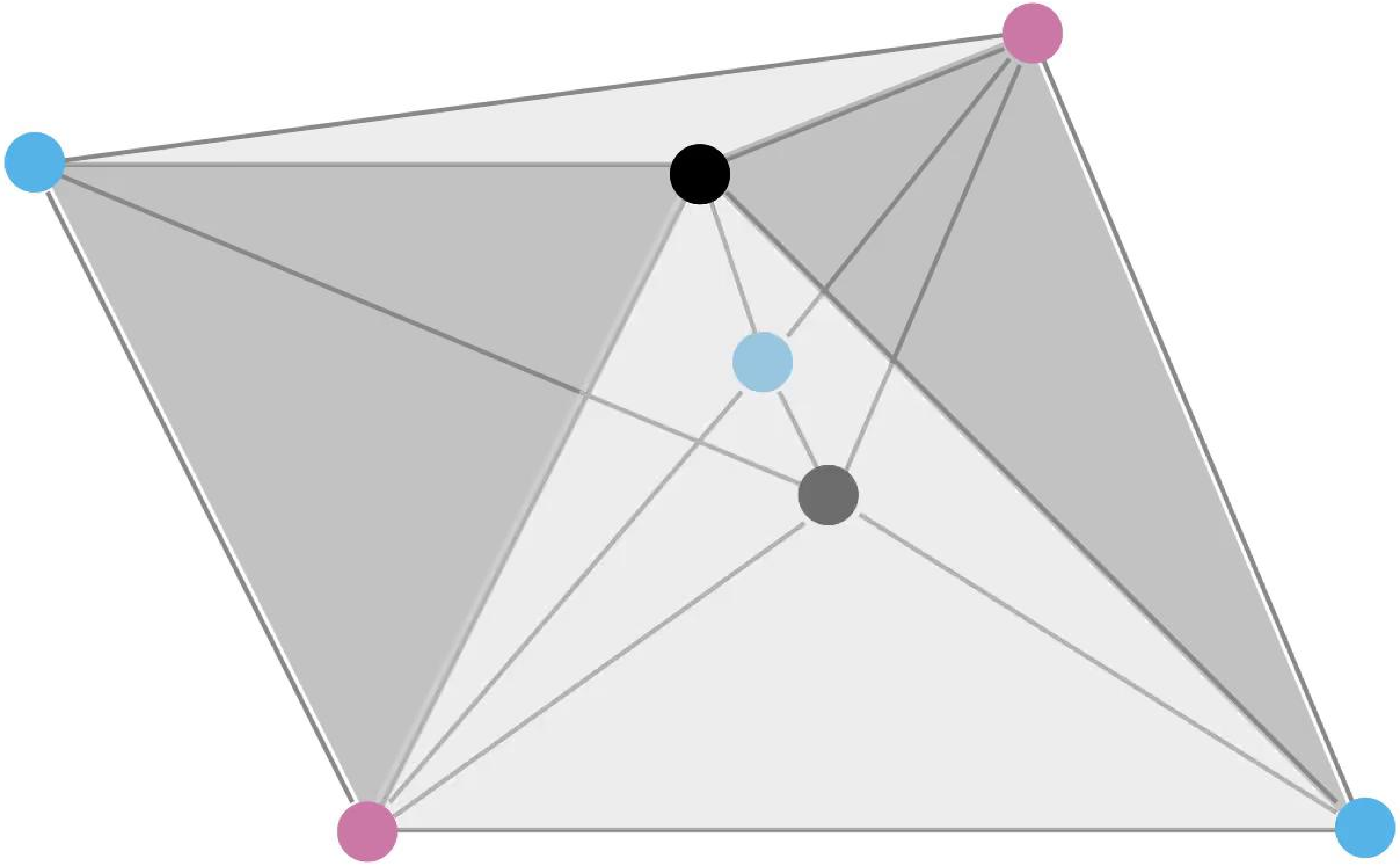


The last characterization

Pseudospheres are precisely finite joins of finite sets.

Groups acting on pseudospheres

Groups acting on pseudospheres

G : a finite discrete group.

Groups acting on pseudospheres

G : a finite discrete group.

$\underbrace{G * \cdots * G}_{n+1}$ is a pseudosphere.

Groups acting on pseudospheres

$$G \text{ acts on } \underbrace{G * \cdots * G}_{n+1} = \Psi_n(G).$$

Groups acting on pseudospheres

$\mathbb{Z}_2$ acts on

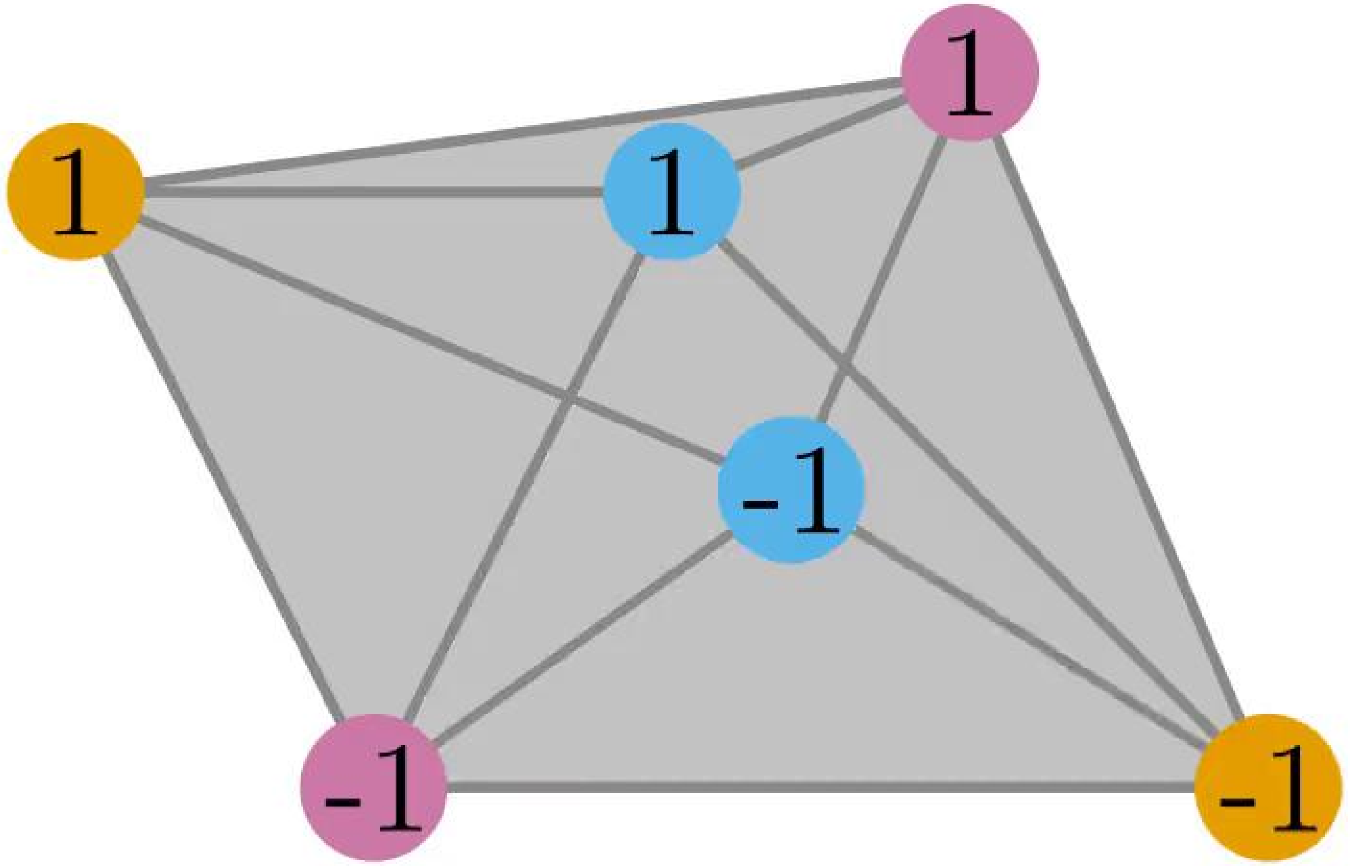
$$\underbrace{\mathbb{Z}_2 * \cdots * \mathbb{Z}_2}_{n+1} = \Psi_n(\mathbb{Z}_2).$$

Groups acting on pseudospheres

$\mathbb{Z}_2$ acts on

$$\underbrace{\mathbb{Z}_2 * \cdots * \mathbb{Z}_2}_{n+1} = S^n.$$

Groups acting on pseudospheres



The Borsuk-Ulam theorem is a theorem about pseudospheres

The Borsuk-Ulam theorem is a theorem about pseudospheres

Borsuk-Ulam

There is no continuous function

$$f\colon S^{n+1} \to S^n$$

preserving the action of $\mathbb{Z}_2$

The Borsuk-Ulam theorem is a theorem about pseudospheres

Borsuk-Ulam

There is no continuous function

$$f : |\Psi_{n+1}(\mathbb{Z}_2)| \to |\Psi_n(\mathbb{Z}_2)|$$

preserving the action of $\mathbb{Z}_2$

The Borsuk-Ulam theorem is a theorem about pseudospheres

Matousek

There is no continuous function

$$f : |\Psi_{n+1}(G)| \to |\Psi_n(G)|$$

preserving the action of G

The Borsuk-Ulam theorem is a theorem about pseudospheres

Tucker

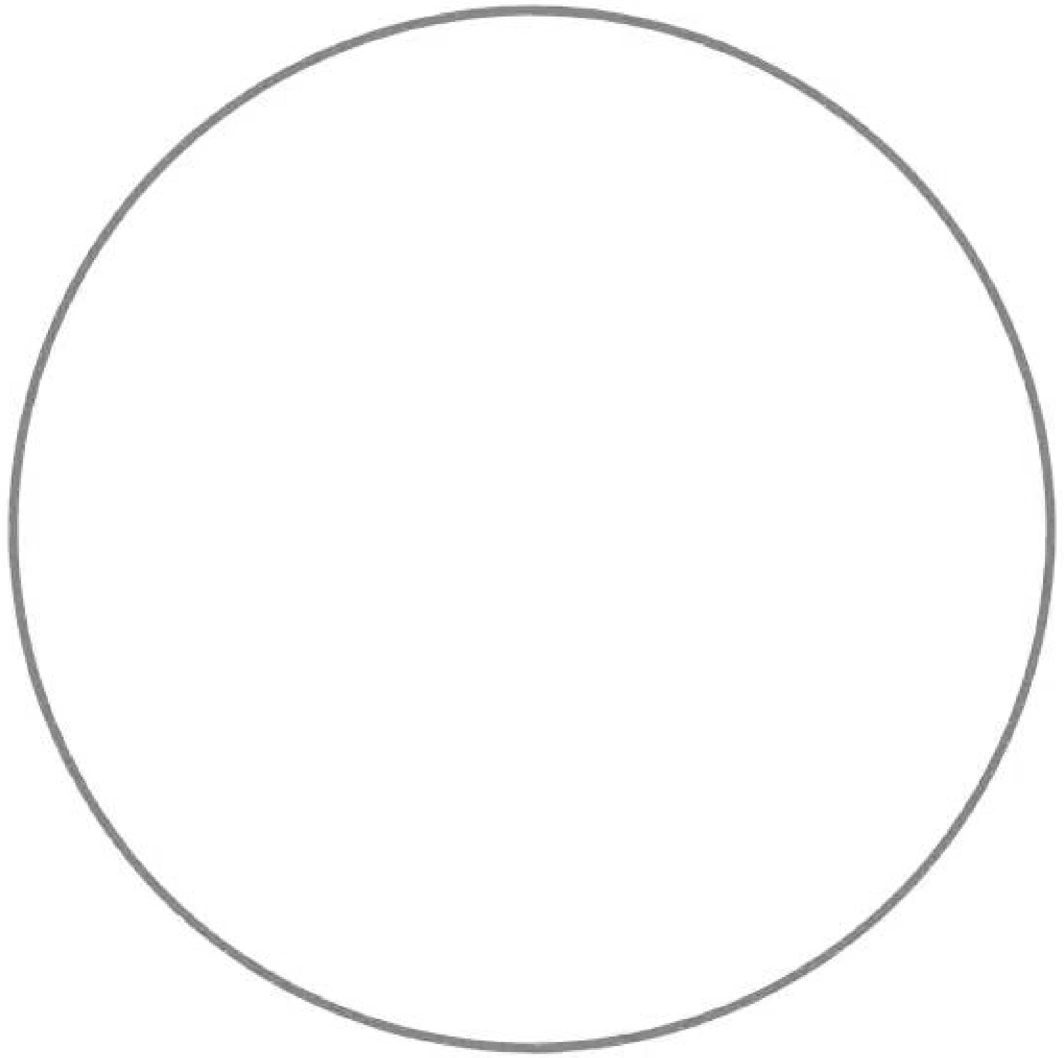
If Δ is a triangulation which is antipodally symmetric on the boundary of B^n, there is no simplicial map

$$f\colon \Delta \to \Psi_{n-1}(\mathbb{Z}_2)$$

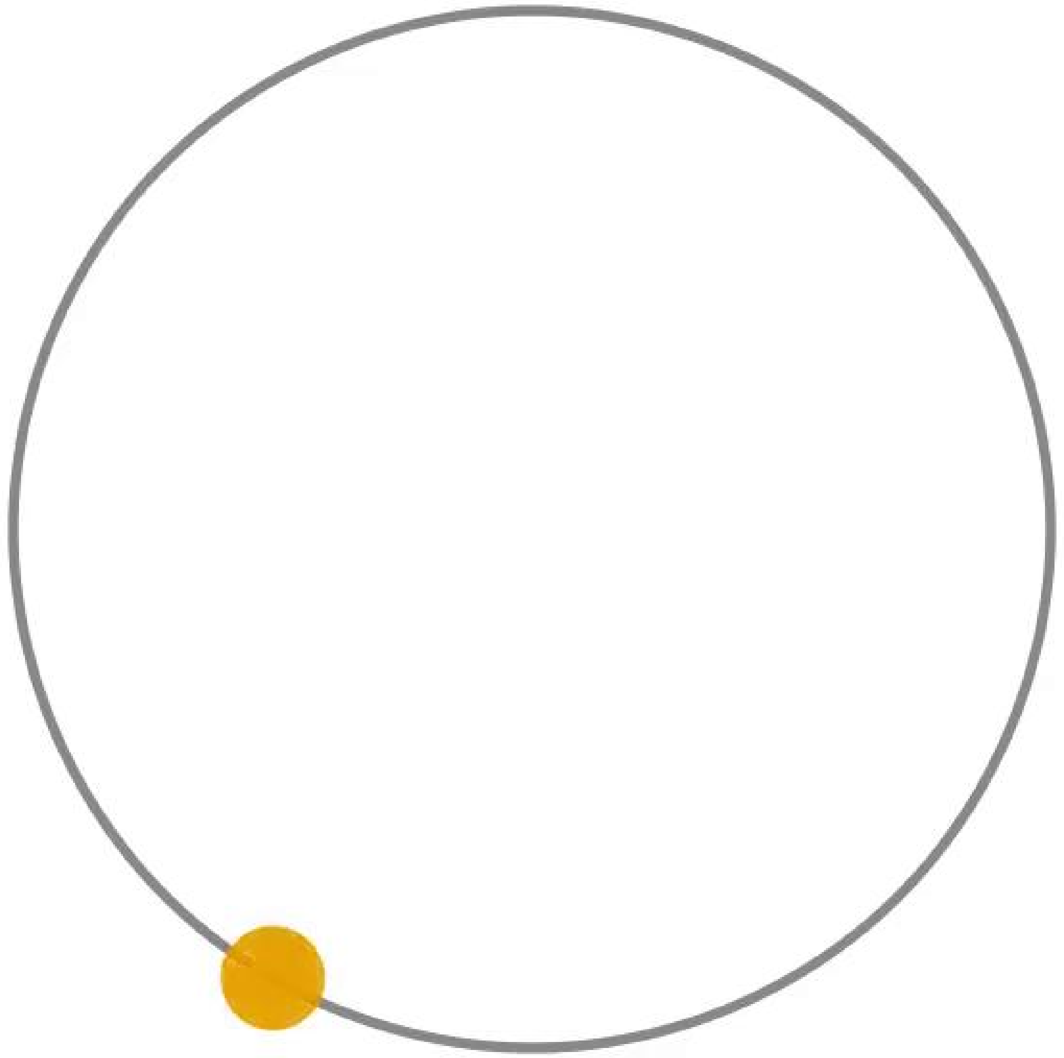
preserving the action of $\mathbb{Z}_2$ on the boundary of B^n

Antipodally symmetric triangulations

Antipodally symmetric triangulations



Antipodally symmetric triangulations



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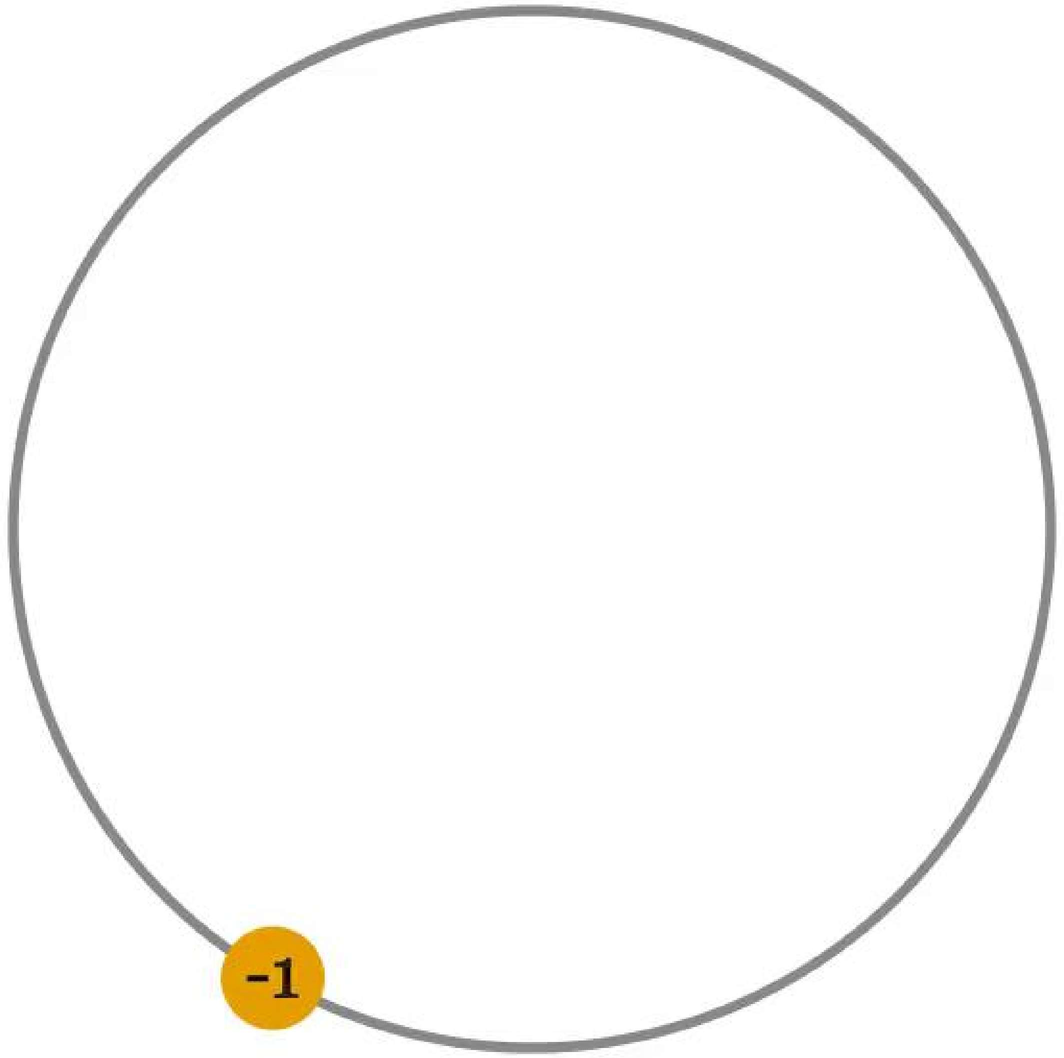
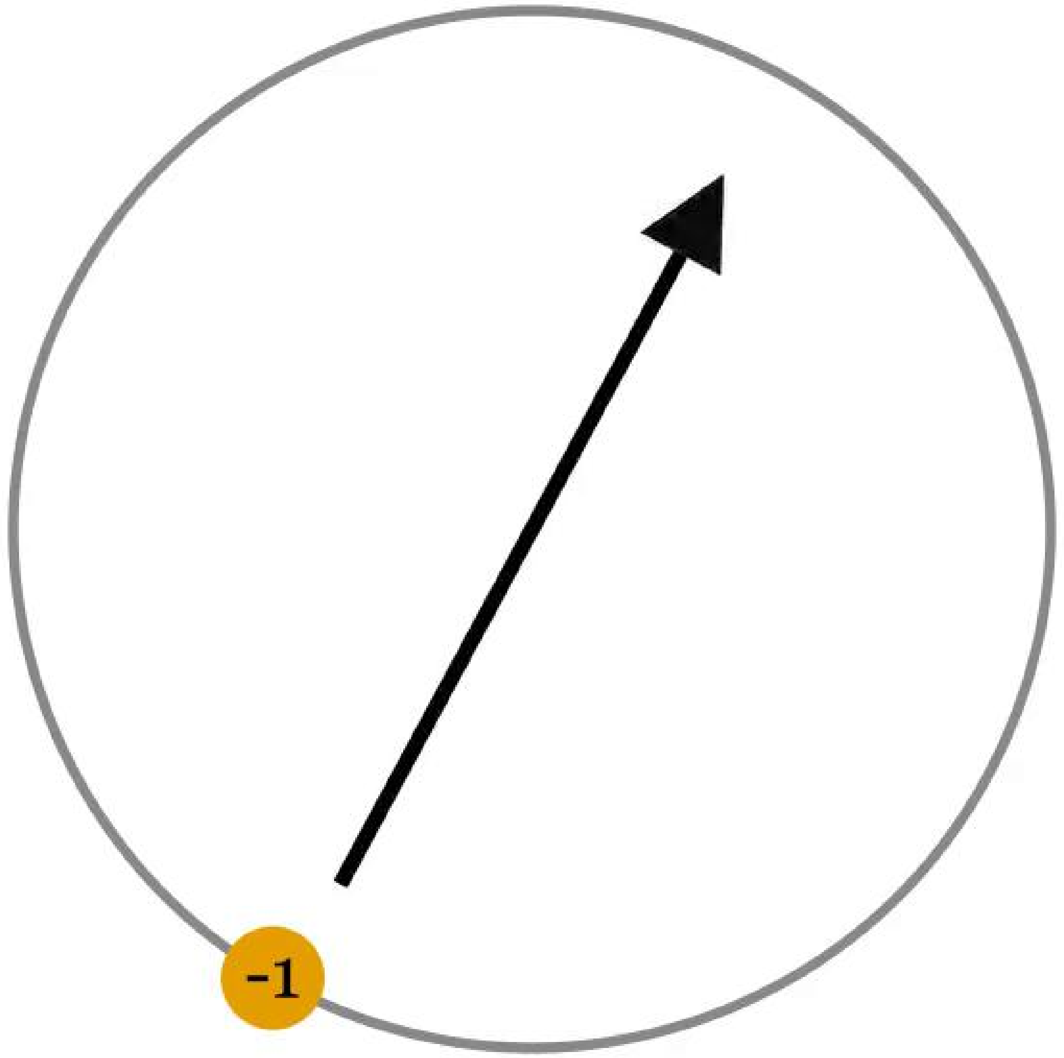


Antipodally symmetric triangulations



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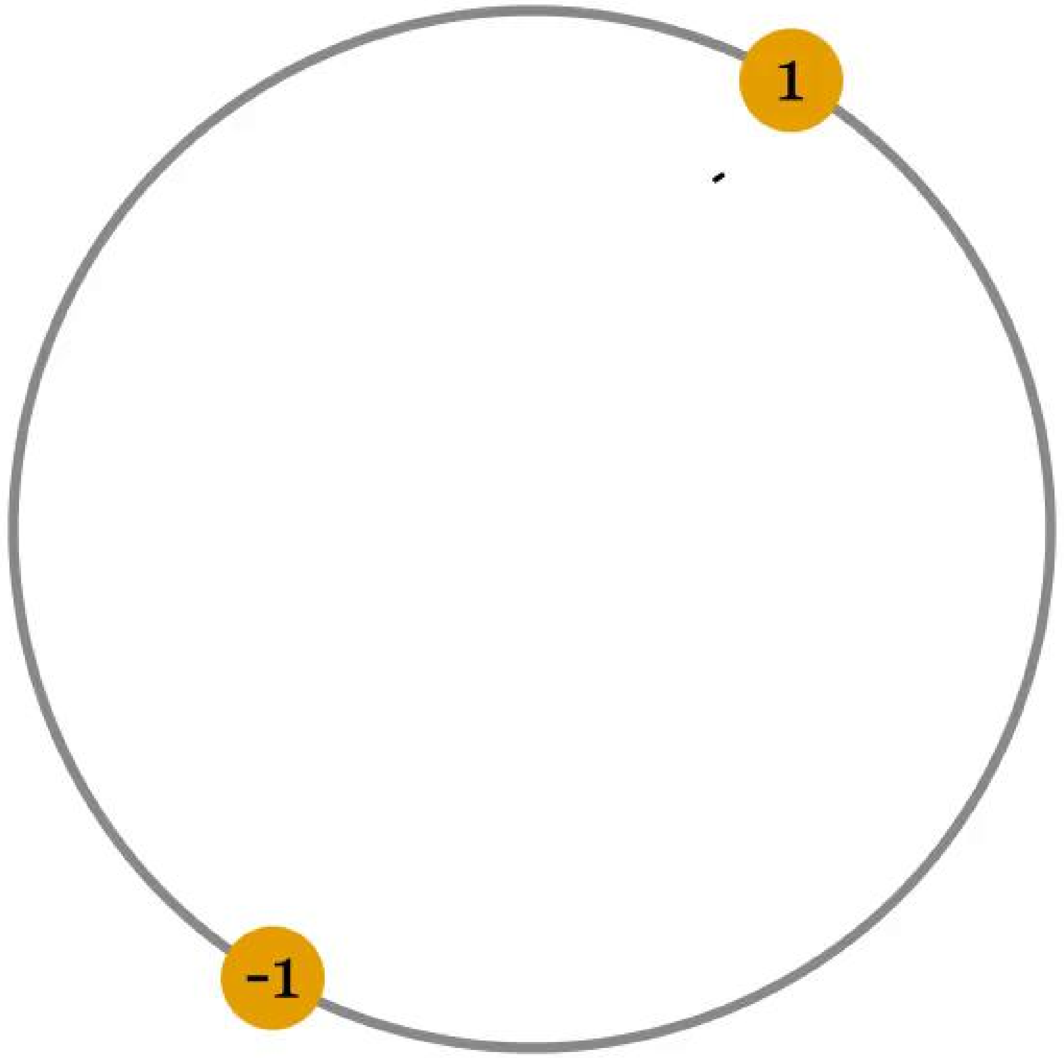

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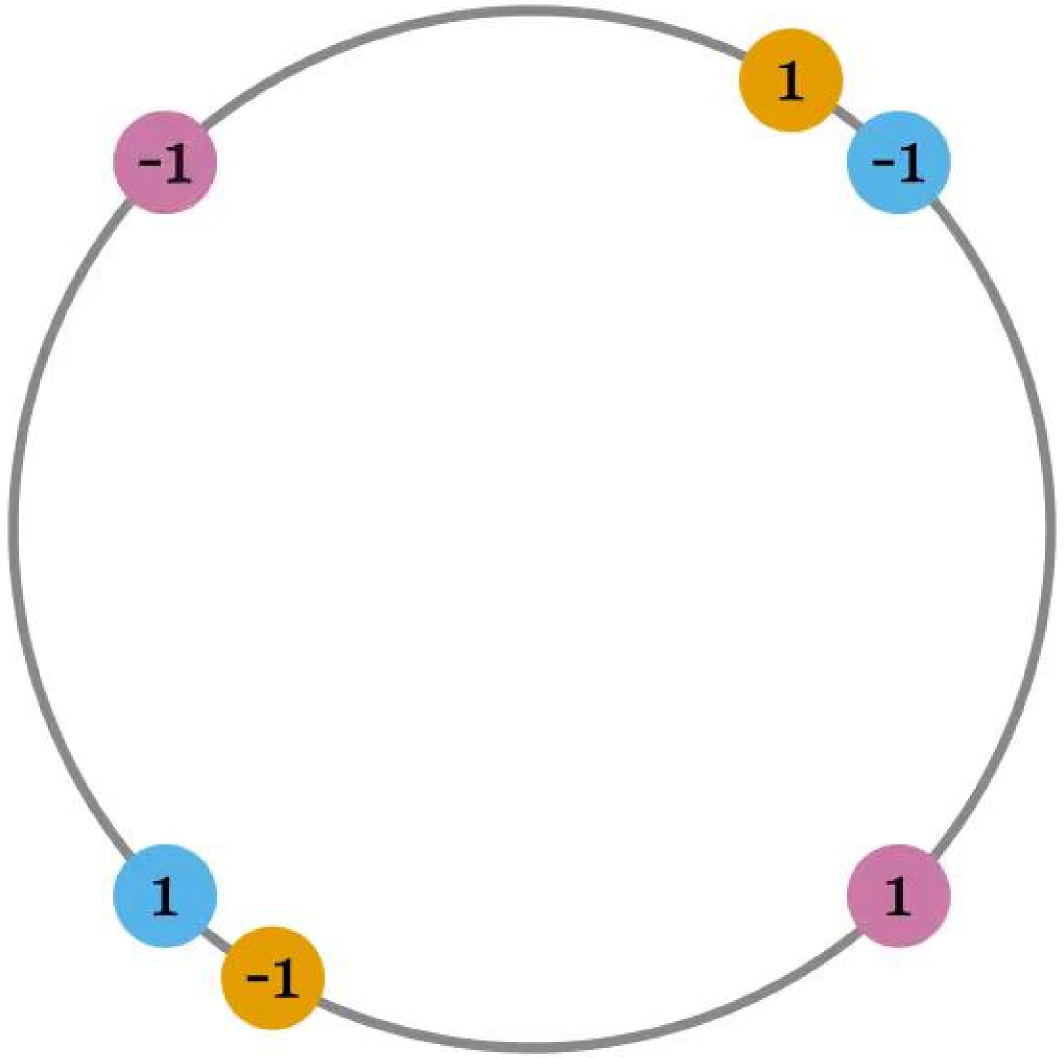

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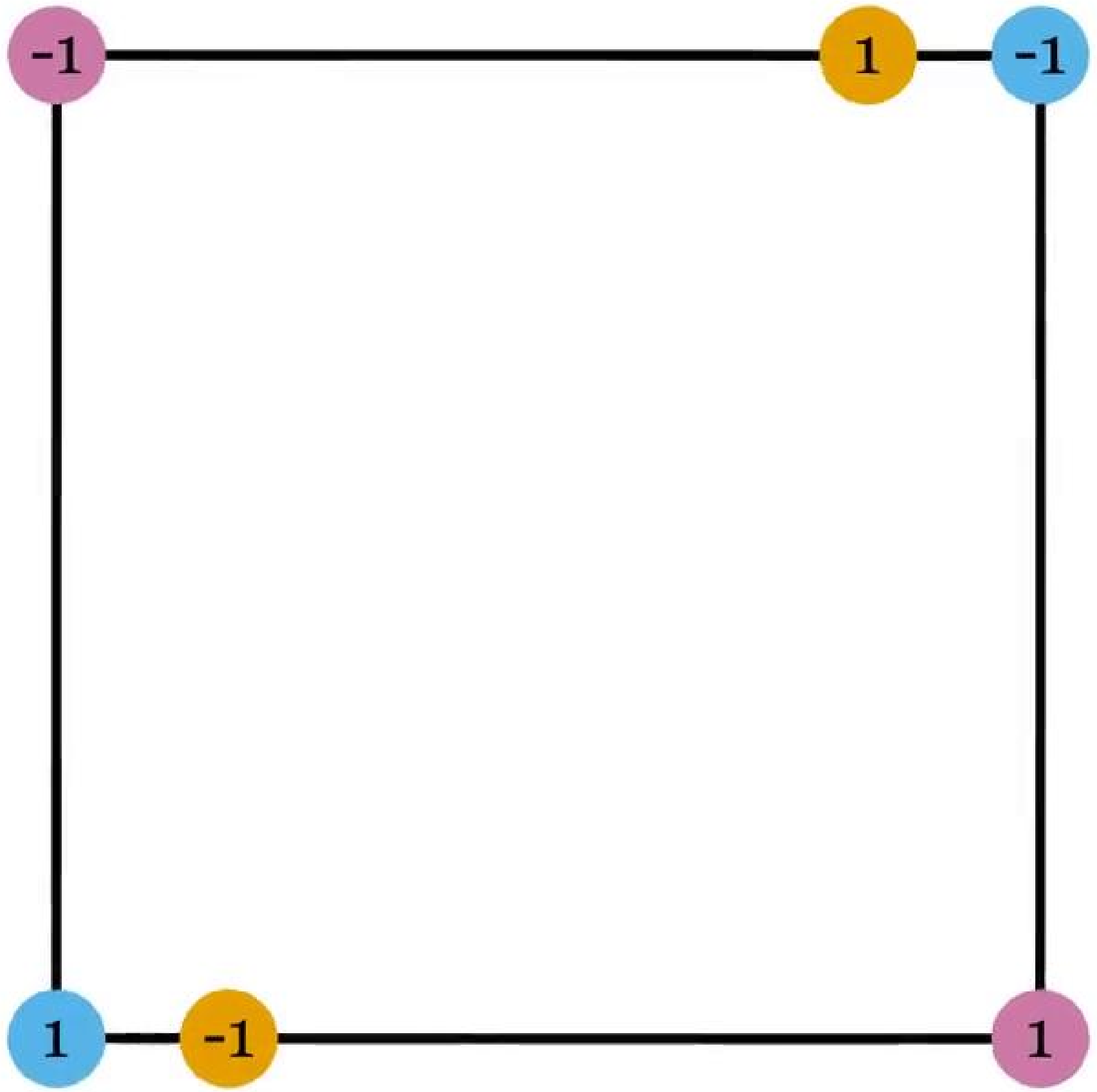

Tucker's lemma is a theorem about pseudospheres!

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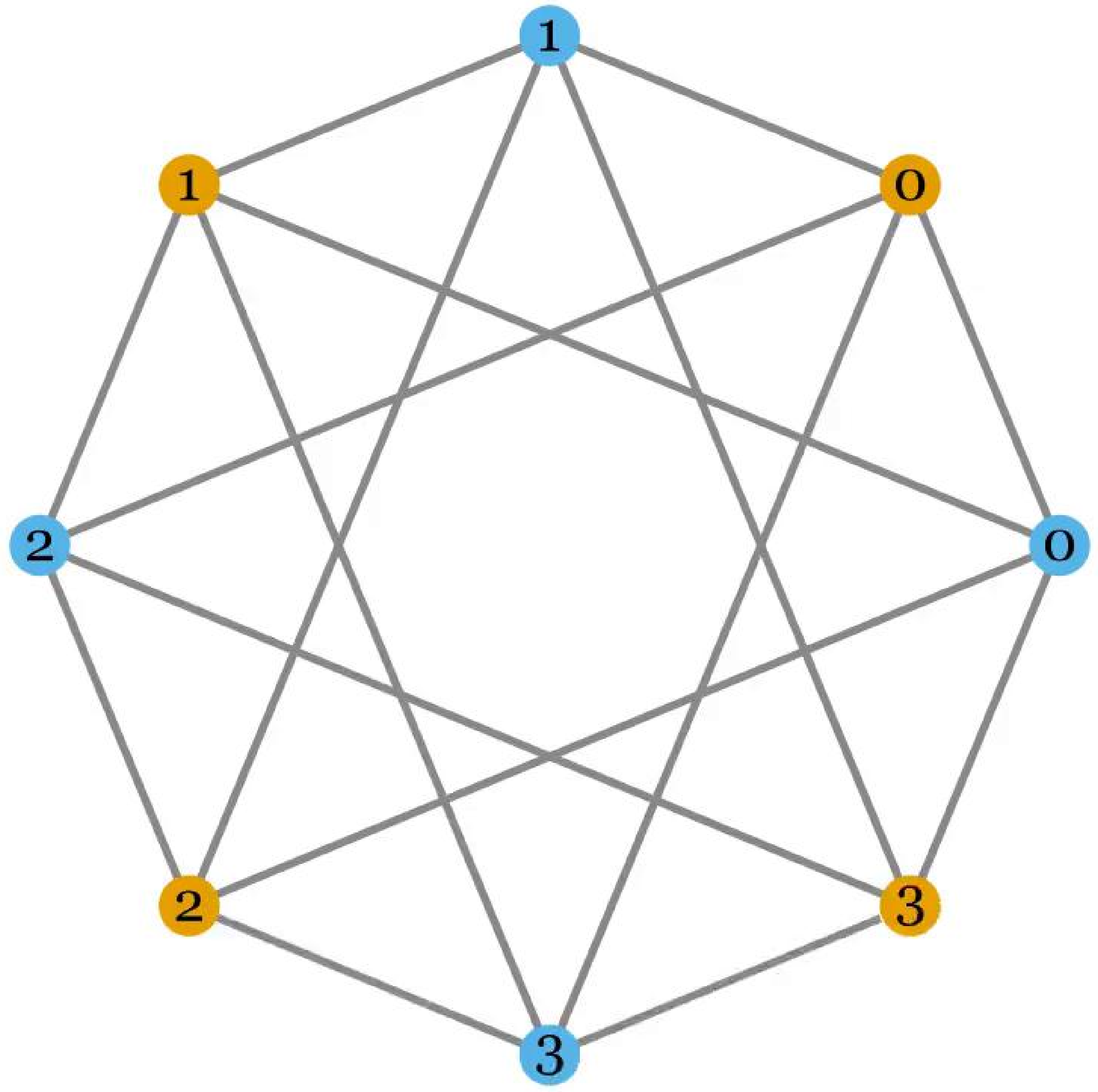

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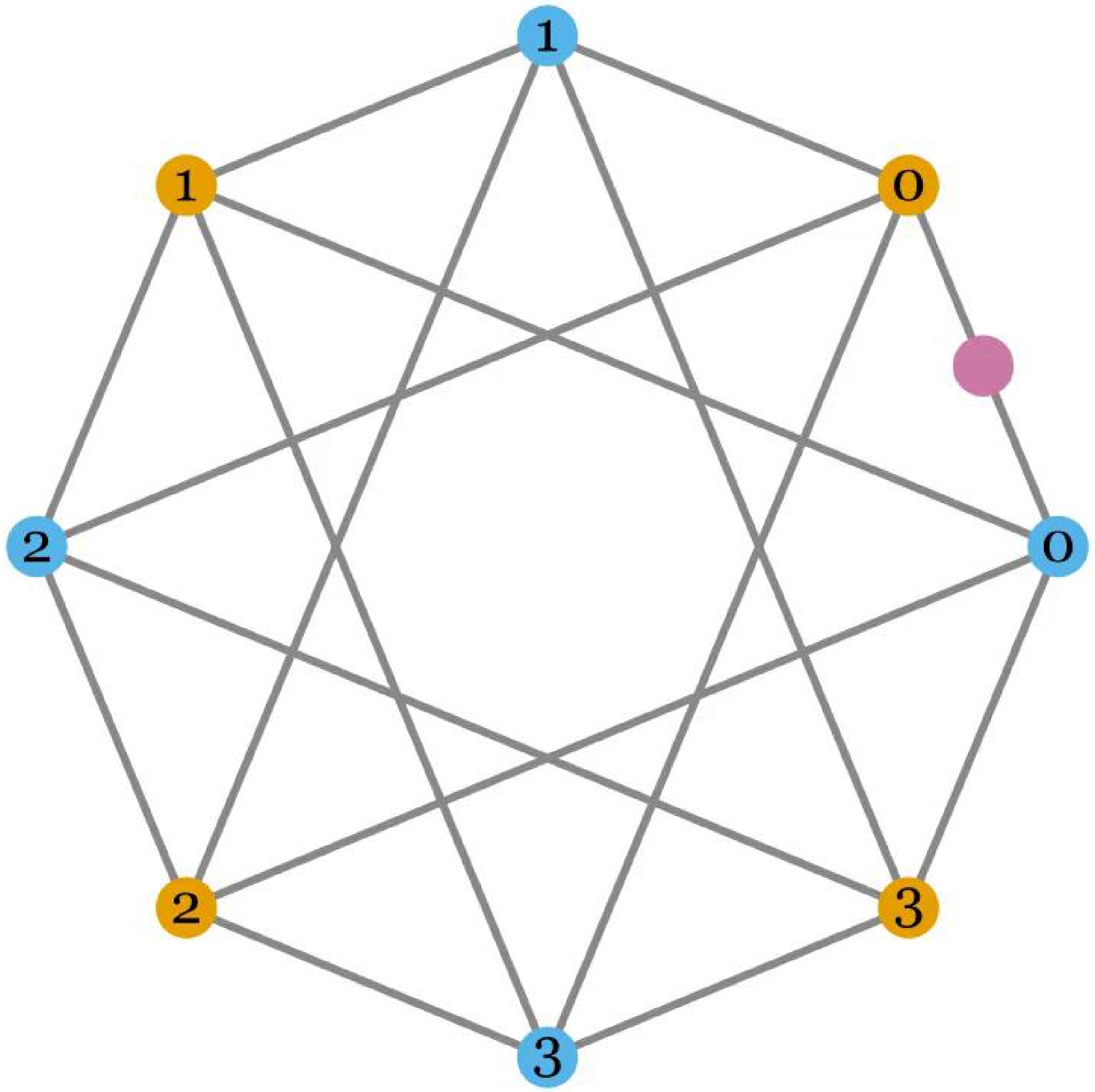

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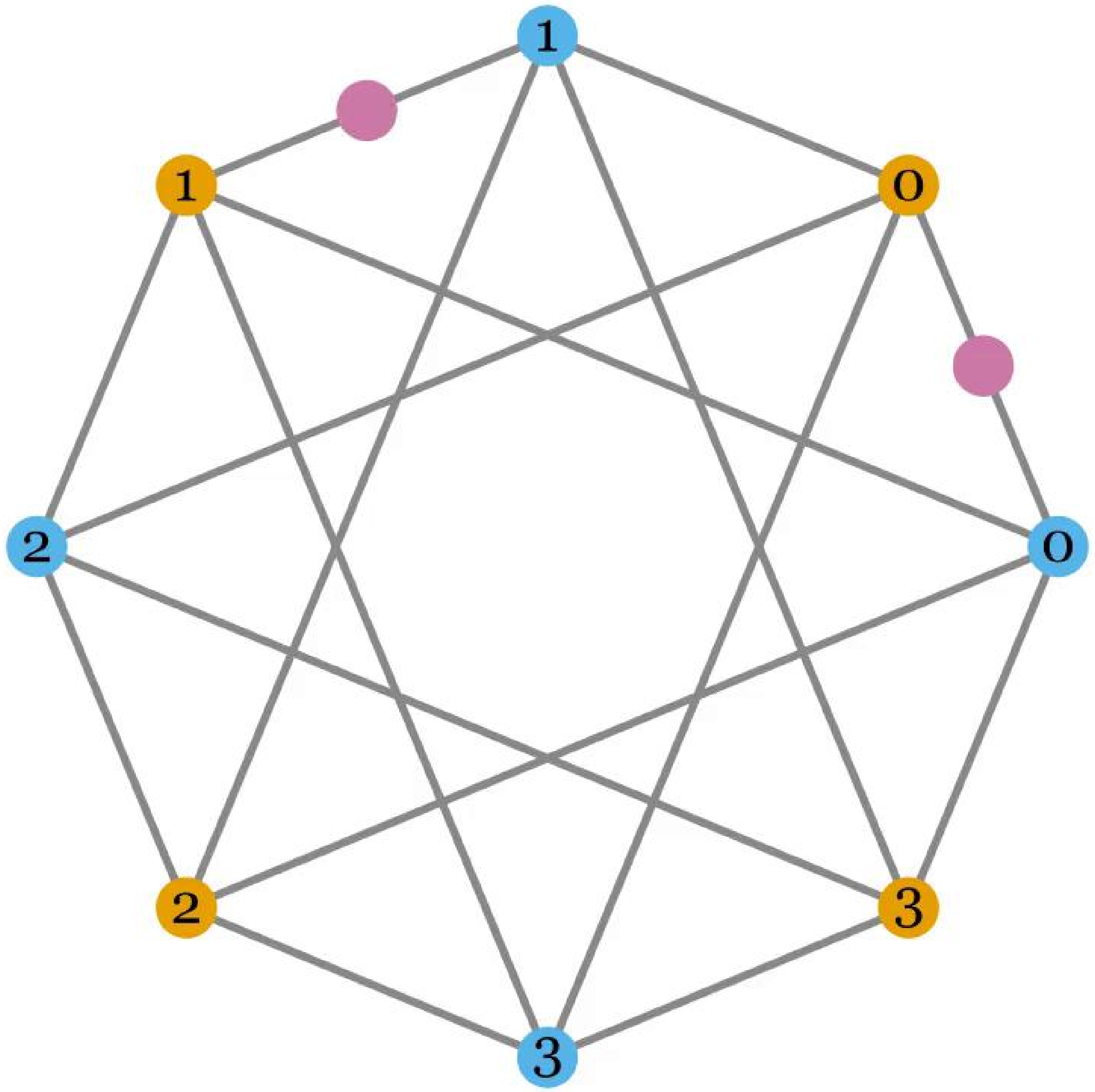

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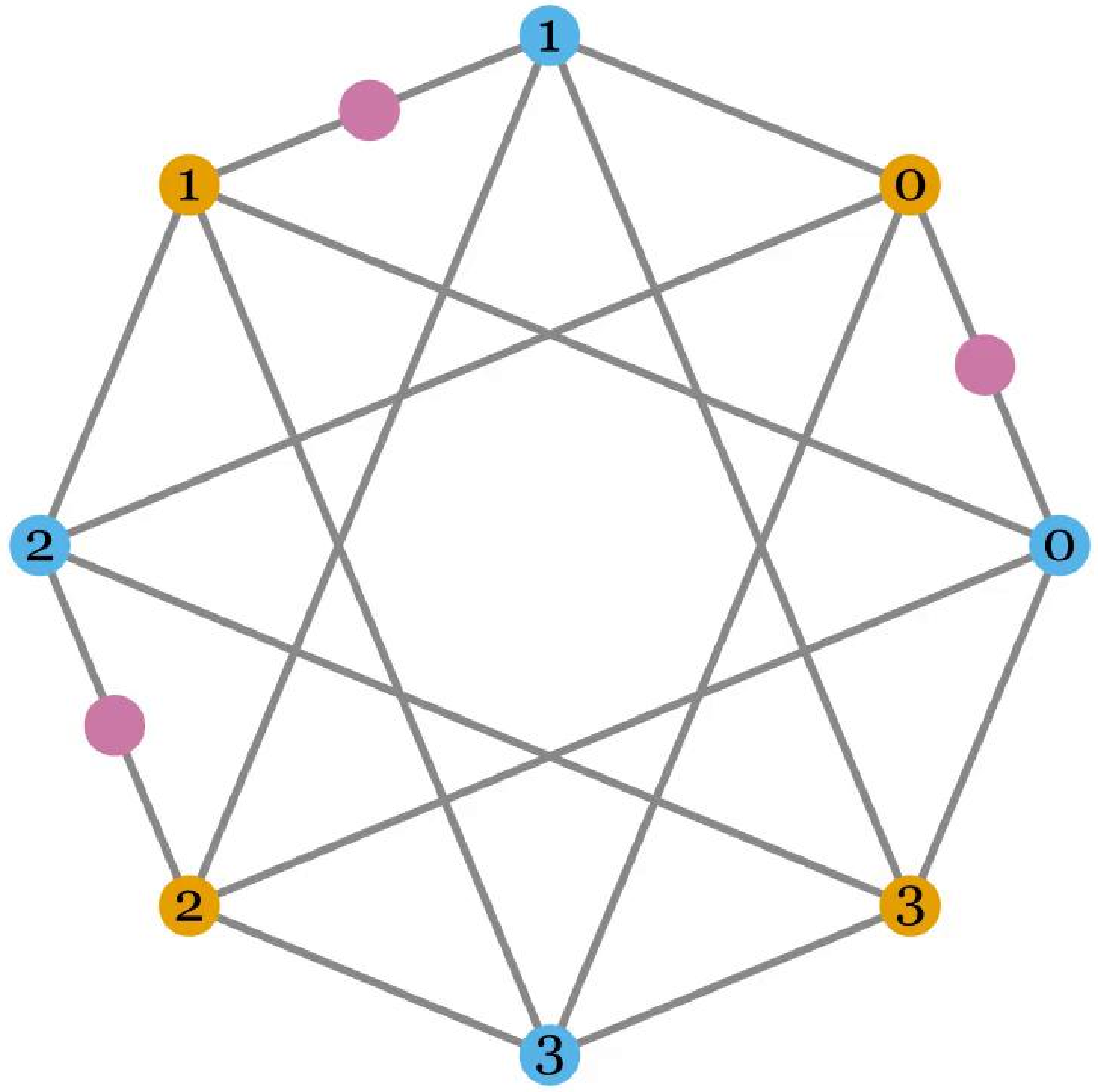

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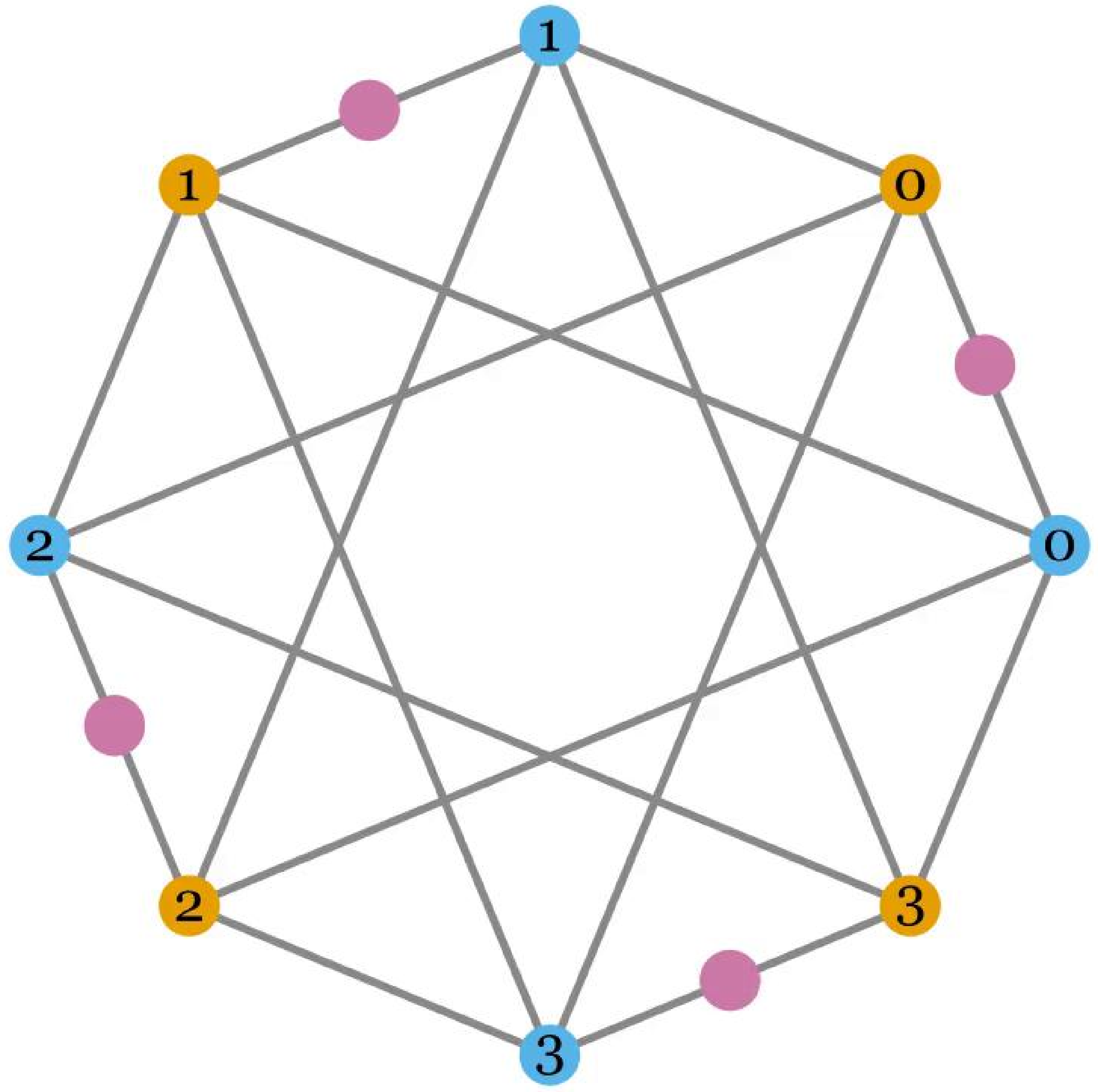

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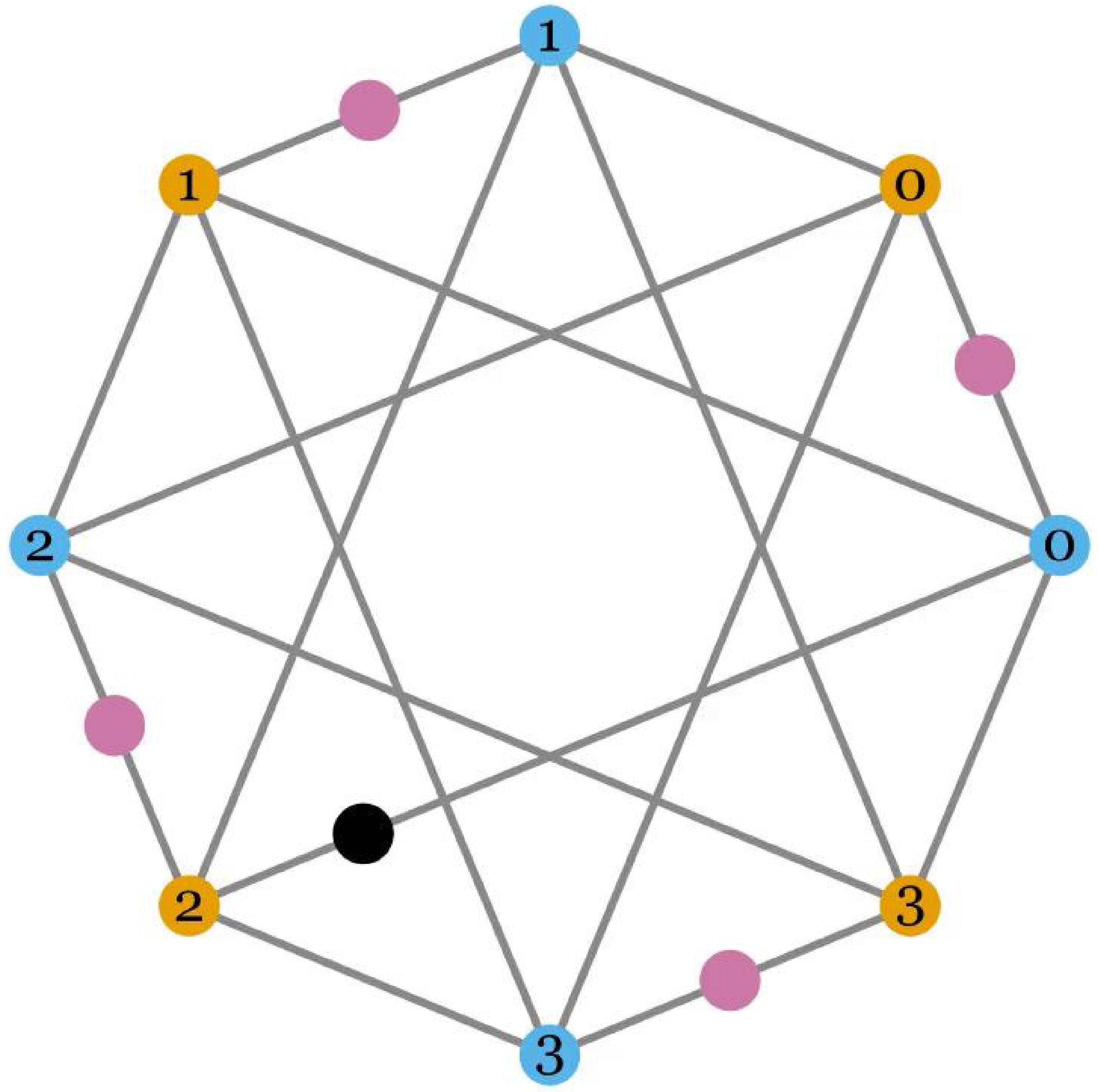

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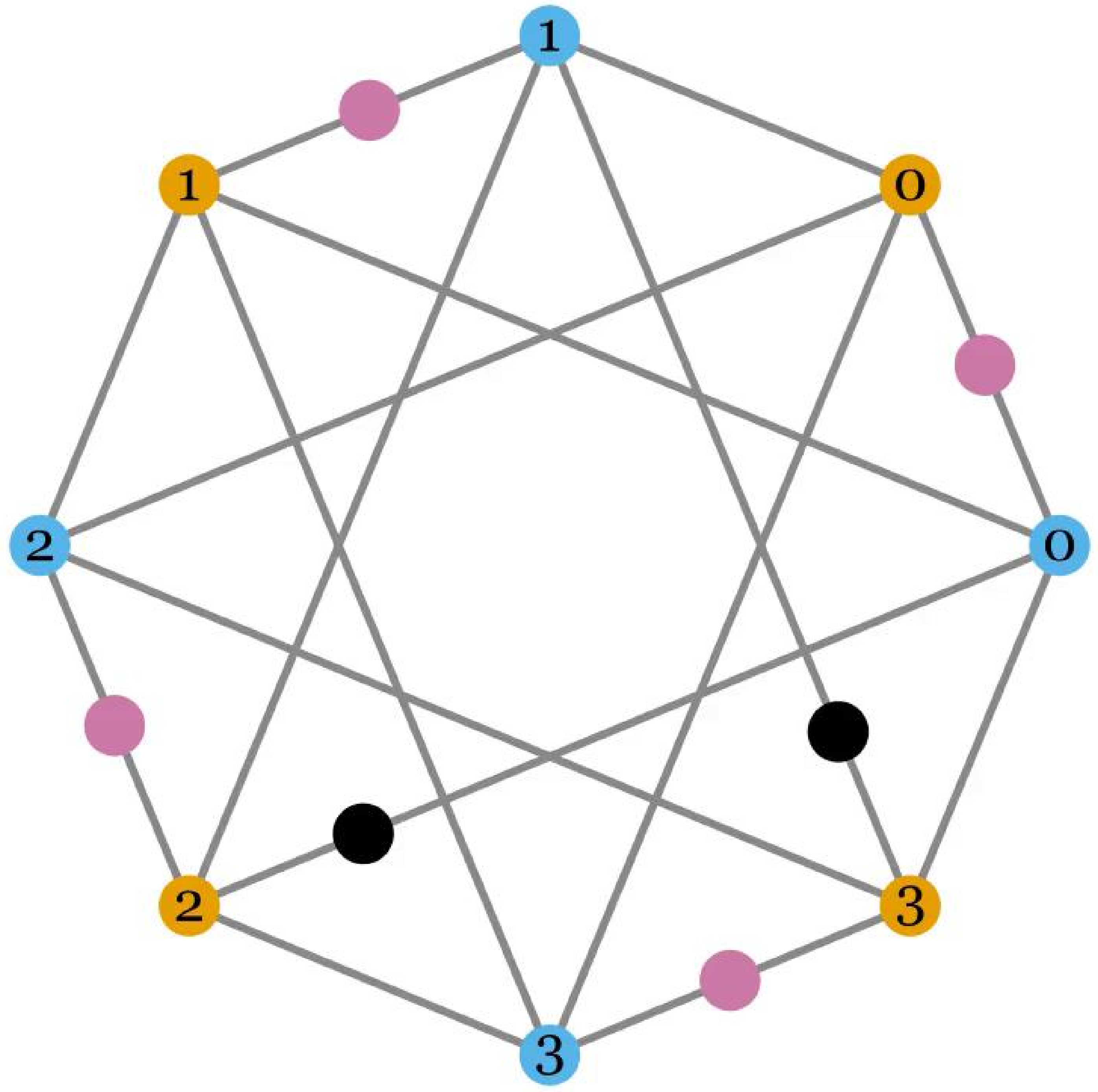

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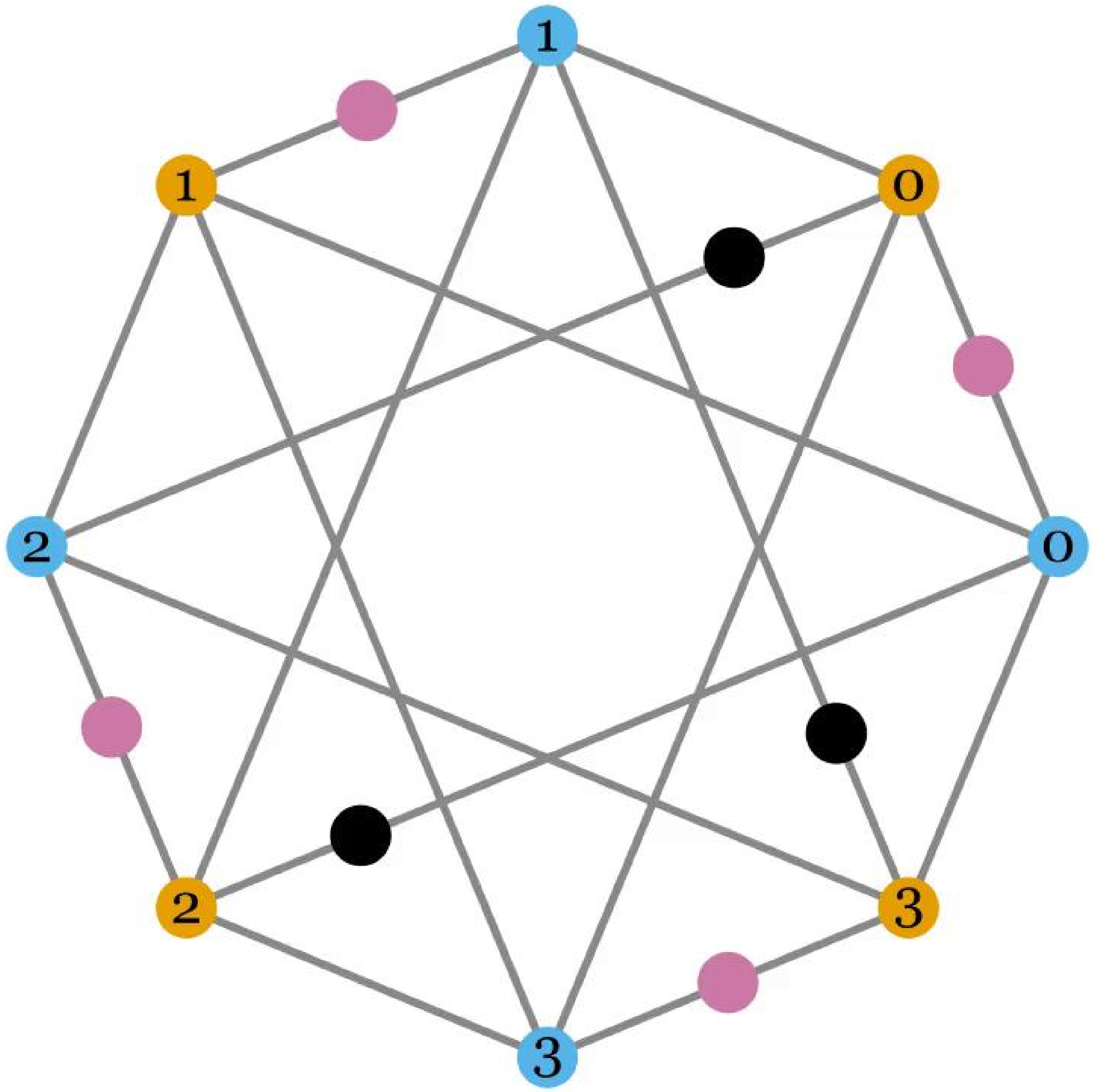

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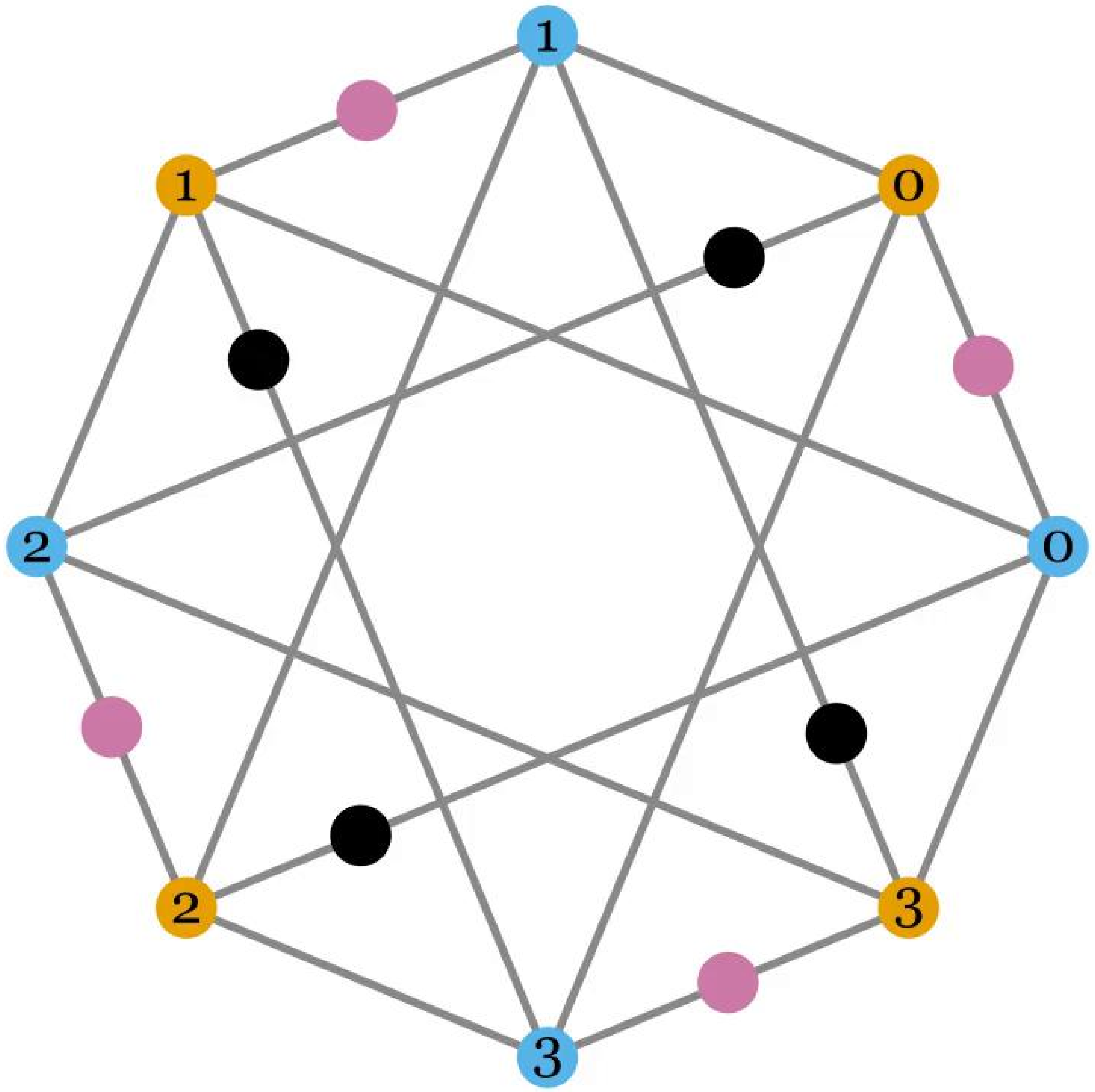


Tucker's lemma is a theorem about pseudospheres!

Tucker's lemma is a theorem about pseudospheres!

If Δ is a triangulation which is antipodally symmetric on the boundary of B^n, there is no simplicial map

$$f : \Delta \to \Psi_{n-1}(\mathbb{Z}_2)$$

preserving the action of $\mathbb{Z}_2$ on the boundary of B^n

Tucker's lemma is a theorem about pseudospheres!

If Δ is a triangulation which is antipodally symmetric on the boundary of $S^{n-1} * x$, there is no simplicial map

$$f : \Delta \to \Psi_{n-1}(\mathbb{Z}_2)$$

preserving the action of $\mathbb{Z}_2$ on the boundary of $S^{n-1} * x$

Tucker's lemma is a theorem about pseudospheres!

If Δ is a subdivision which is $G-$ symmetric on the boundary of $\Psi_{n-1}(G) * x$, there is no simplicial map

$$f : \Delta \to \Psi_{n-1}(G)$$

preserving the action of G on $\Psi_{n-1}(G)$

Tucker's lemma is a theorem about pseudospheres!

Proof

Tucker's lemma is a theorem about pseudospheres!

Matousek

Tucker's lemma is a theorem about pseudospheres!

Matousek

There is no continuous function

$$f : |\Psi_{n+1}(G)| \to |\Psi_n(G)|$$

preserving the action of G

Tucker's lemma is a theorem about pseudospheres!

$$f : |\Psi_{n+1}(G)| \to |\Psi_n(G)|$$

Tucker's lemma is a theorem about pseudospheres!

$$f\colon |\Psi_{n+1}(G)| \to |\Psi_n(G)|$$

$$\Psi_{n+1}(G) = \Psi_n(G) * G = \bigcup_{g \in G} \Psi_n(G) * g$$

Tucker's lemma is a theorem about pseudospheres!

$$f\colon |\Psi_{n+1}(G)| \to |\Psi_n(G)|$$

$$\iff$$

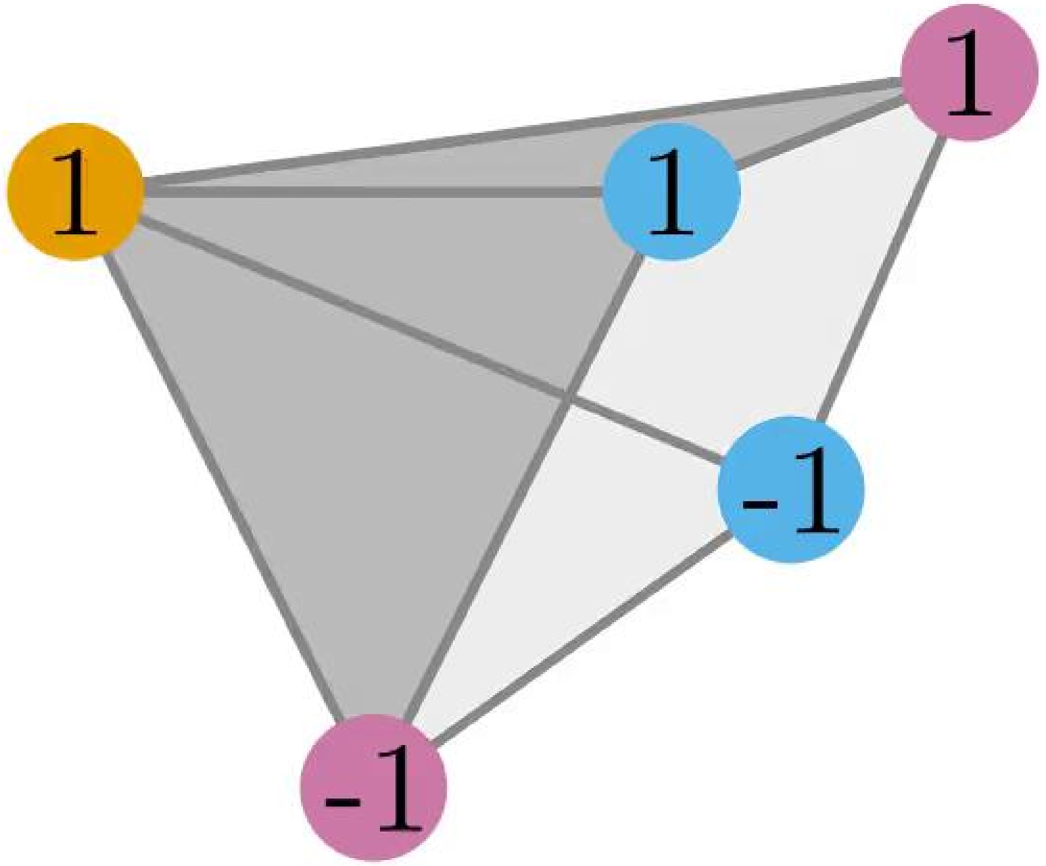
$$\hat{f}\colon |\Psi_n(G) * e| \to |\Psi_n(G)|$$

Tucker's lemma is a theorem about pseudospheres!

$f\colon |\Psi_{n+1}(G)| \to |\Psi_n(G)|$

$$\Longleftrightarrow$$

$\hat{f}\colon |\Psi_n(G) * e| \to |\Psi_n(G)|$



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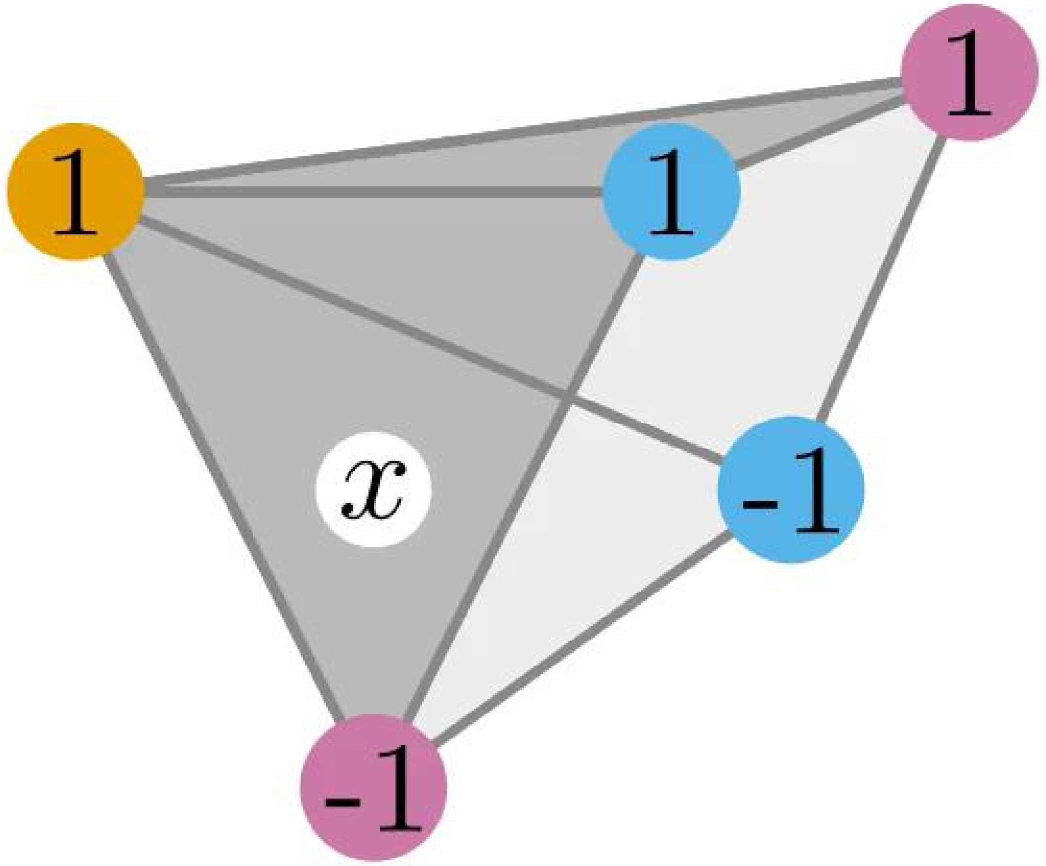

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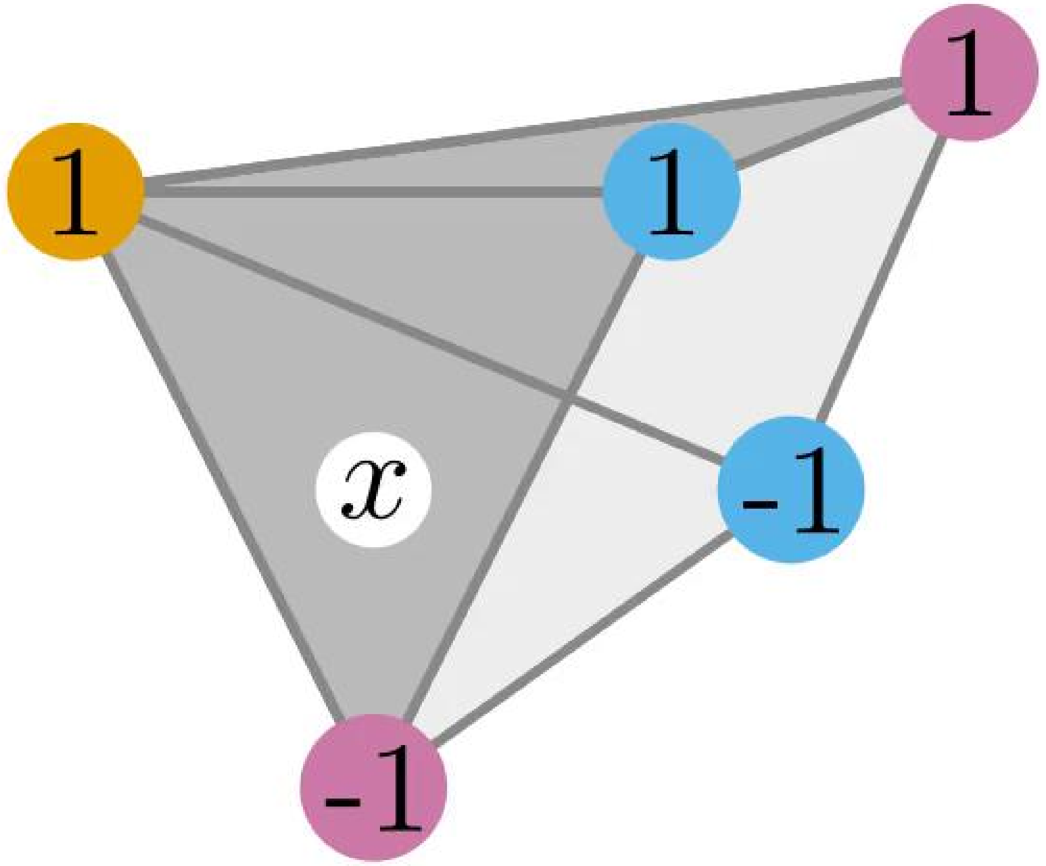
$$f(gx) = gf(x)$$



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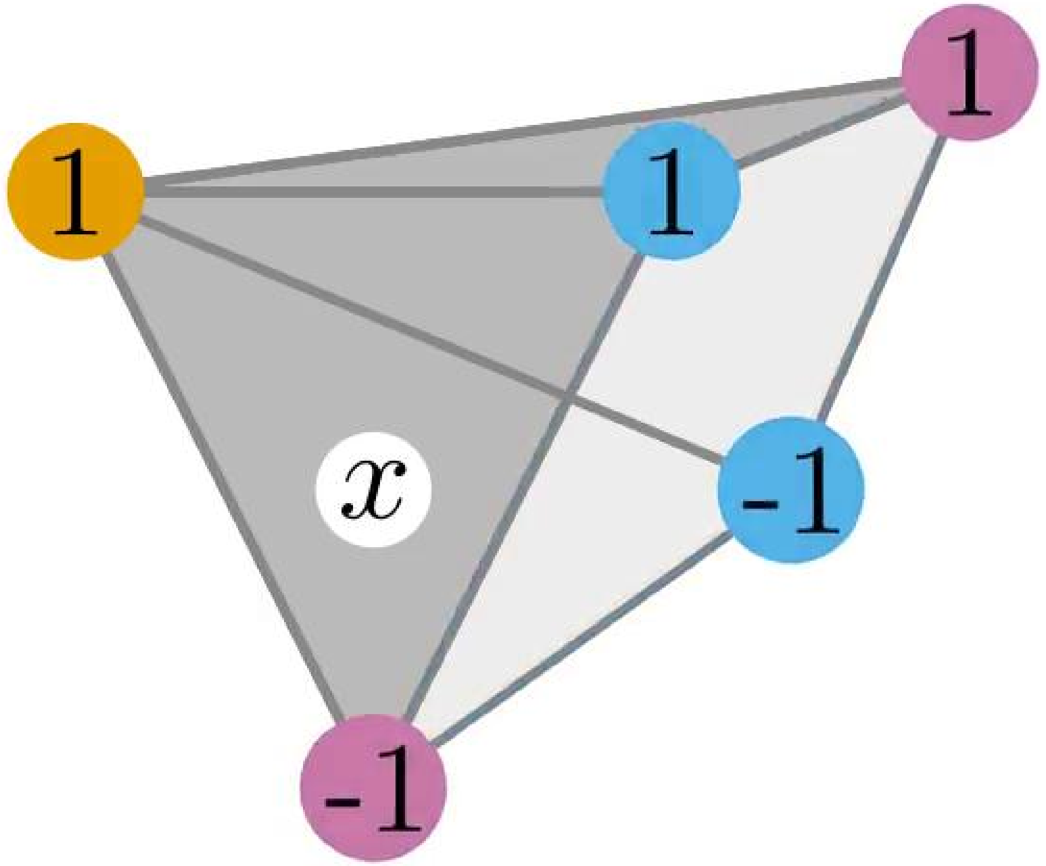

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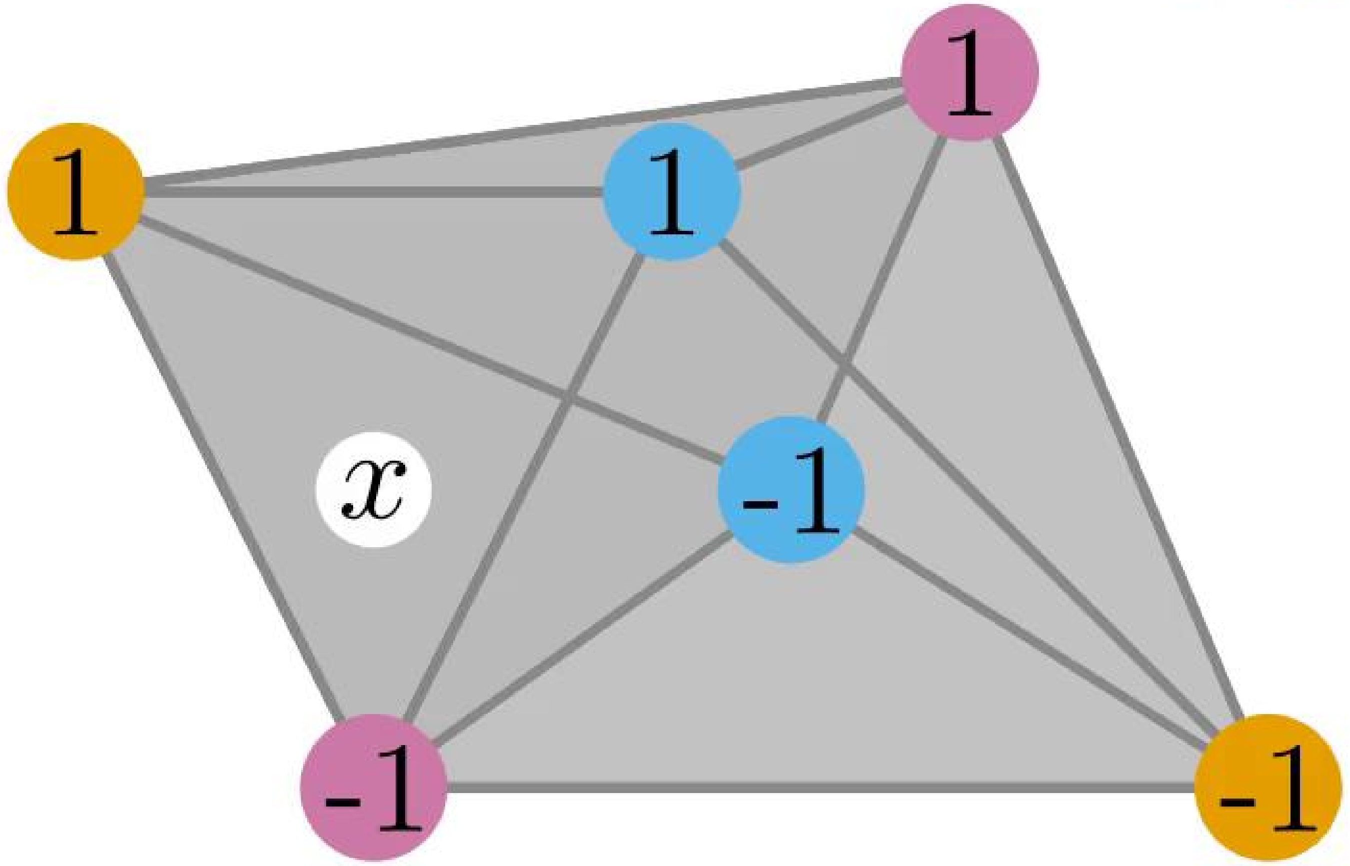

We are almost ready for applications

We are almost ready for applications

A carrier map sends, monotonously, each simplex in I
to a subcomplex of O

We are almost ready for applications

A carrier map is an order preserving function from I into the lattice of subcomplexes of O


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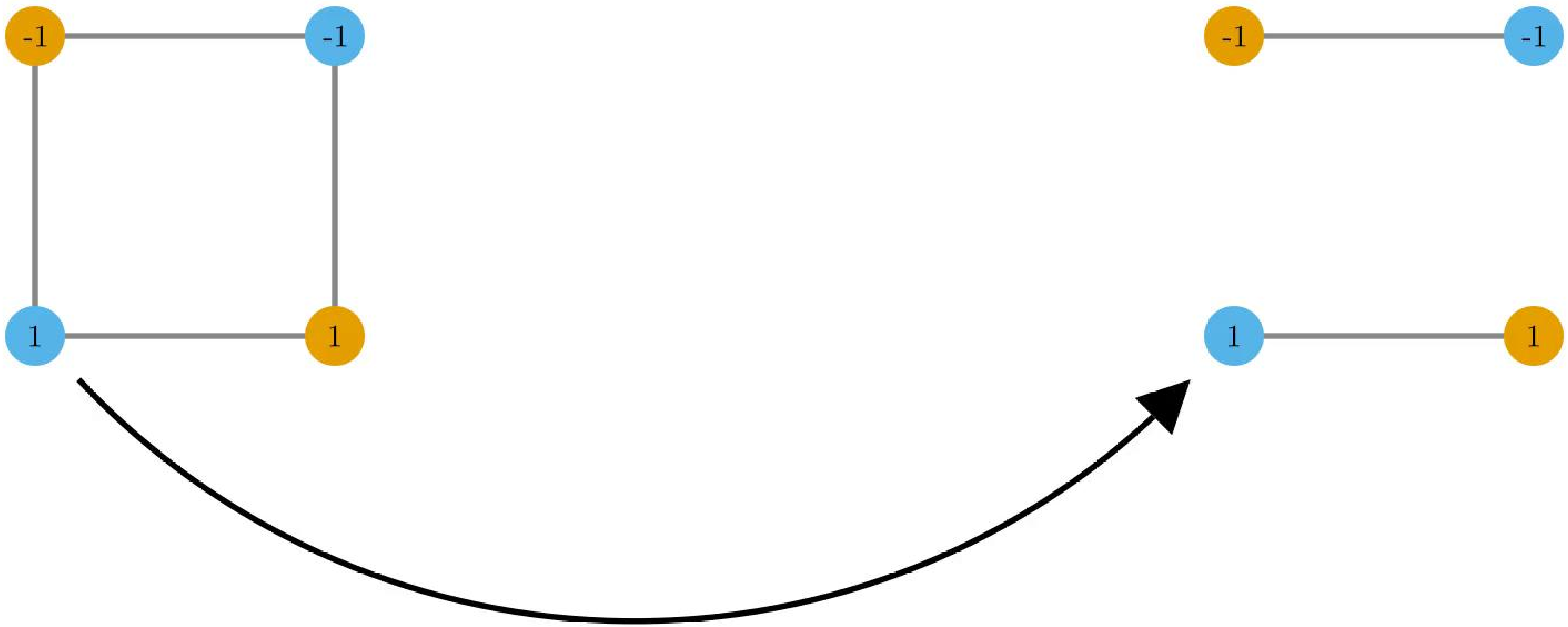
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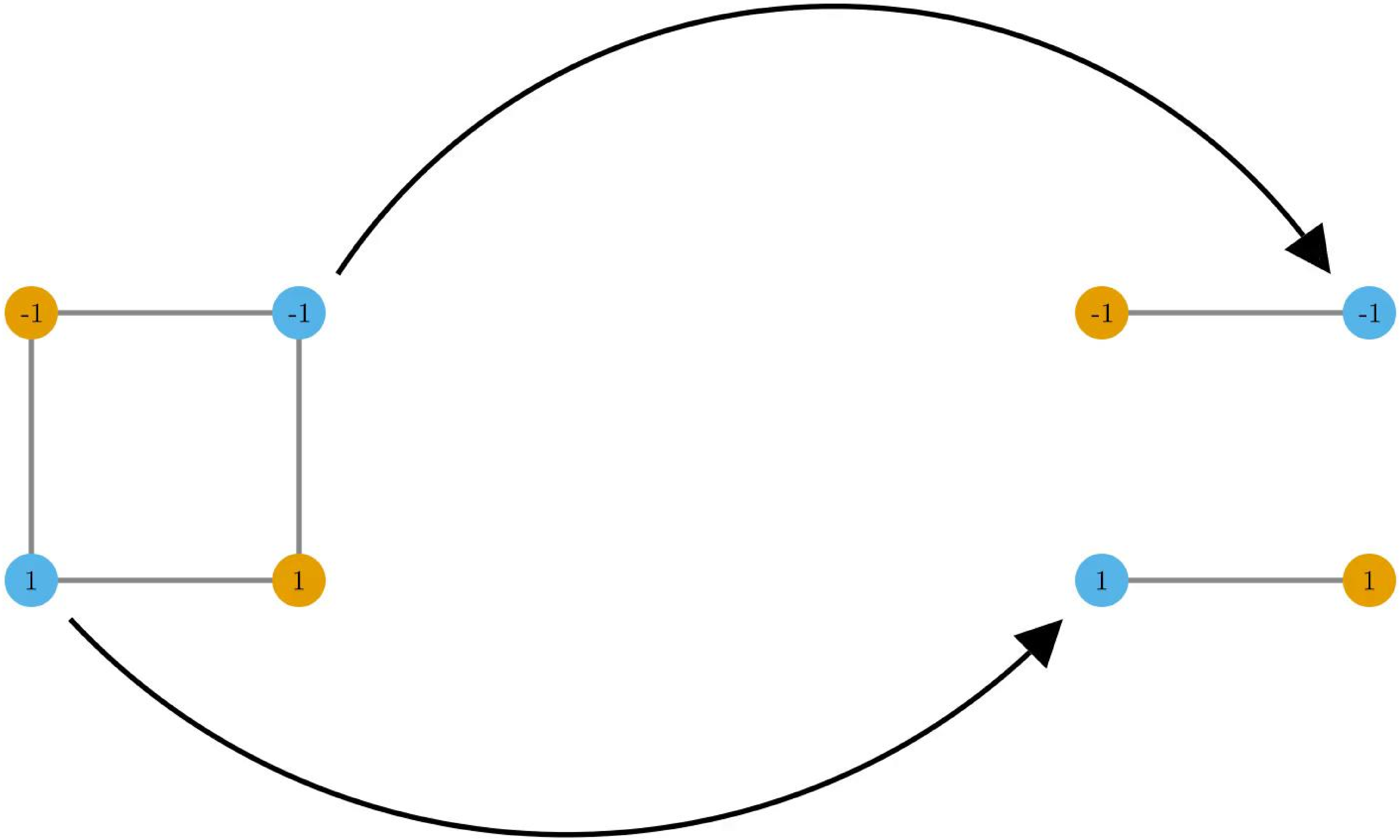

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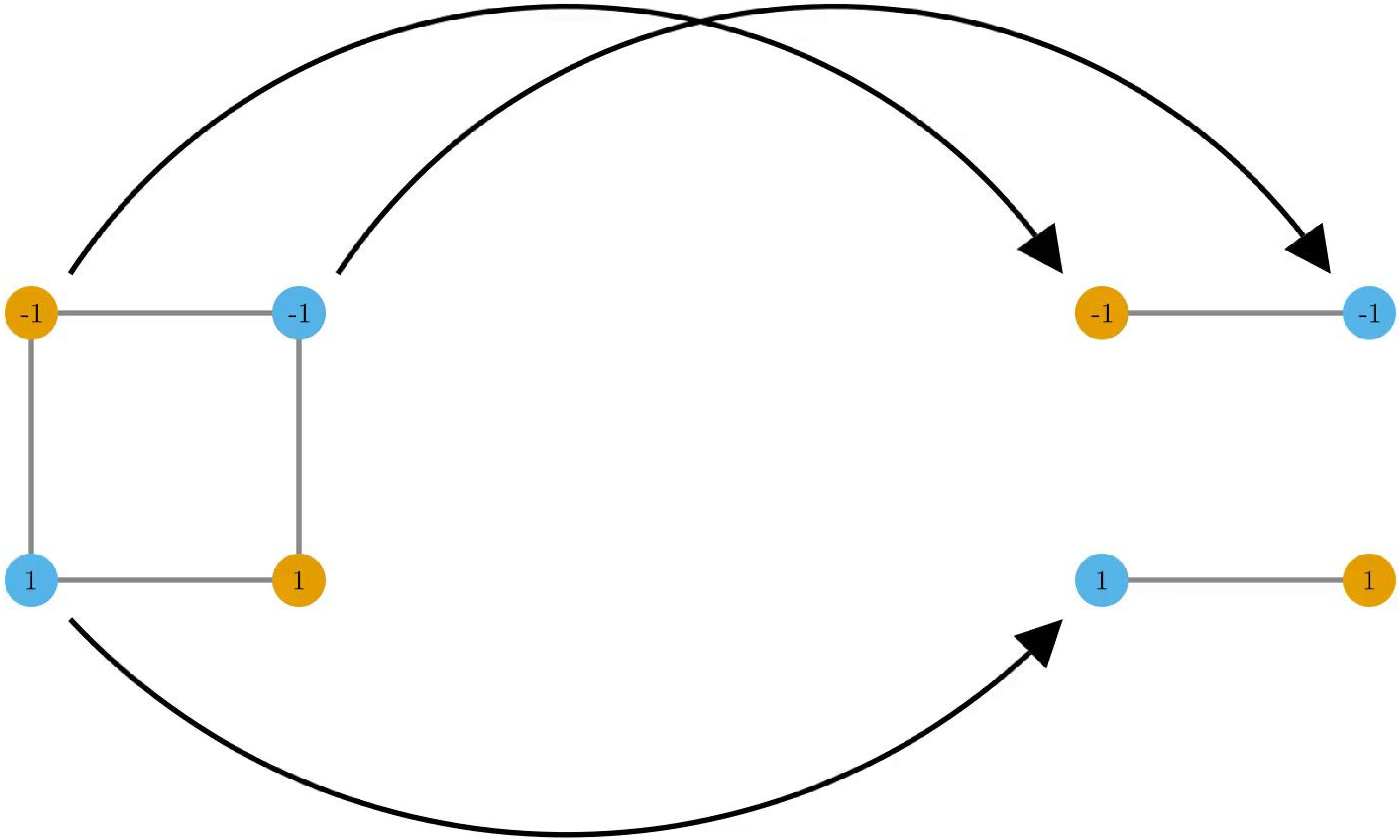
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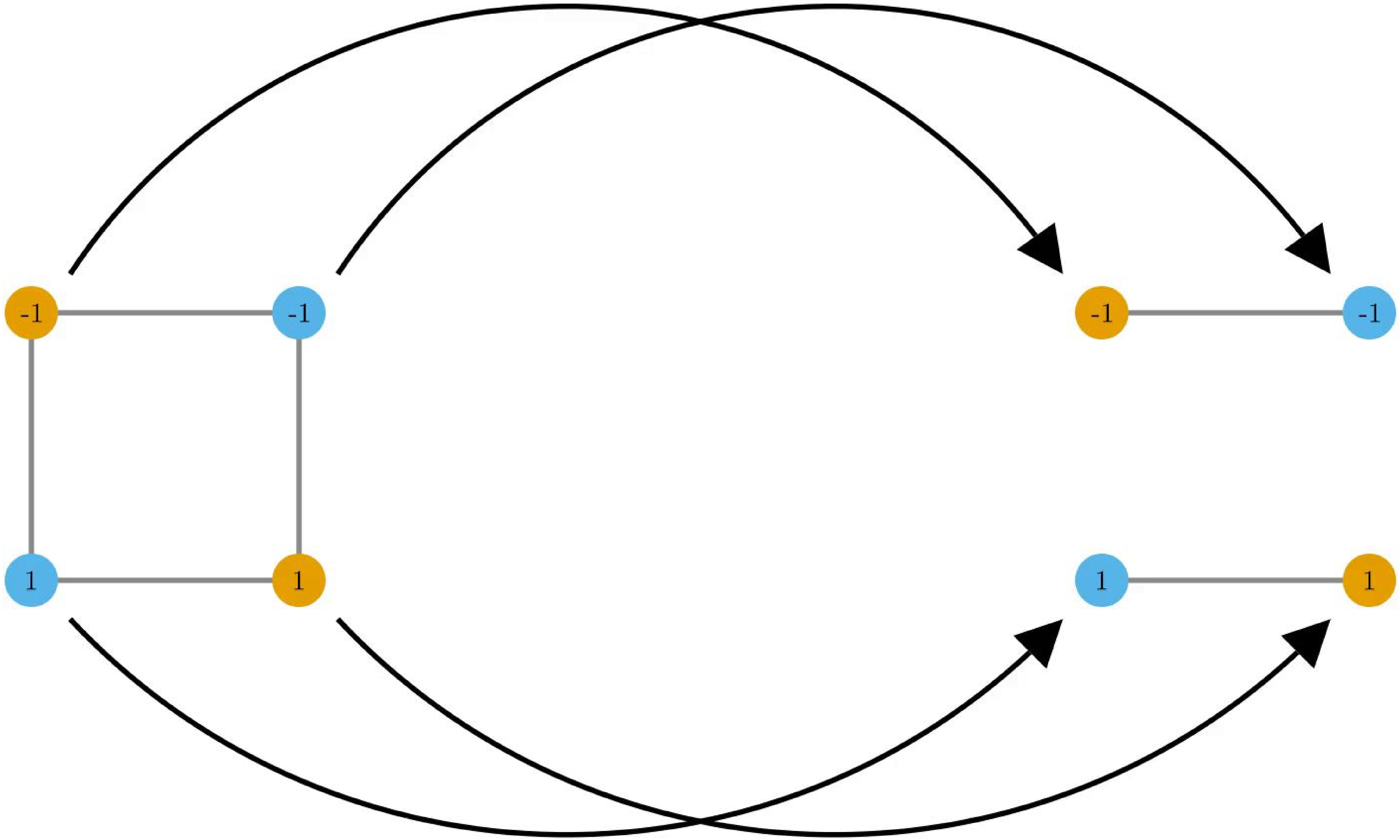
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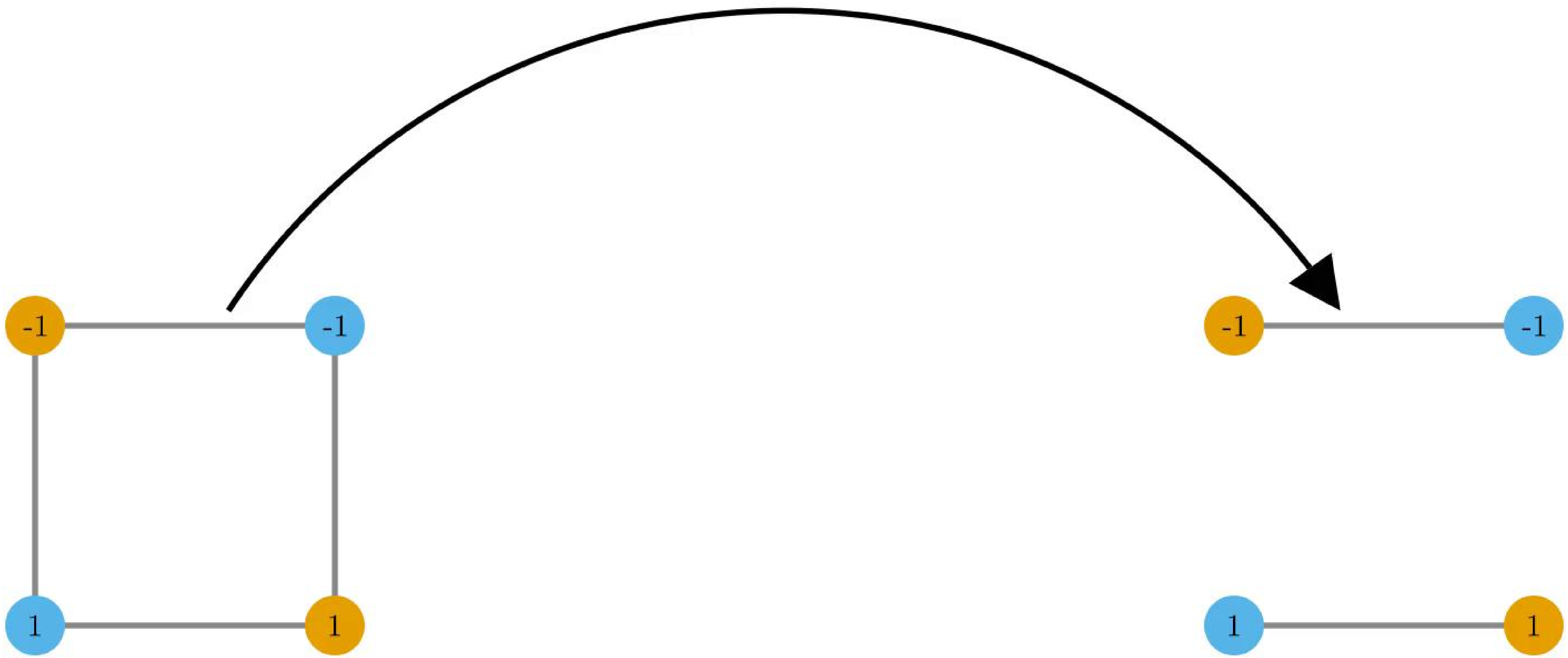

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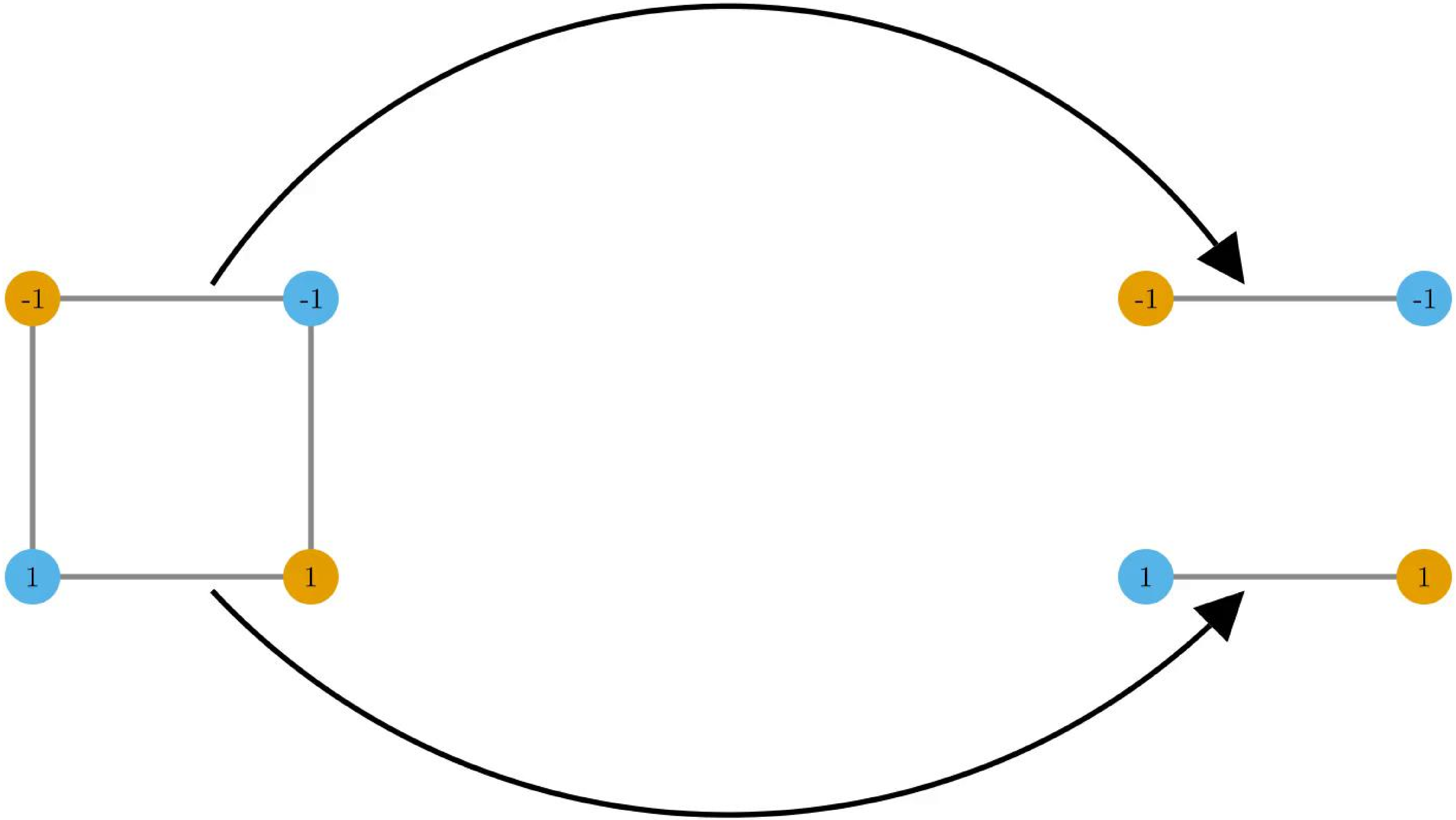

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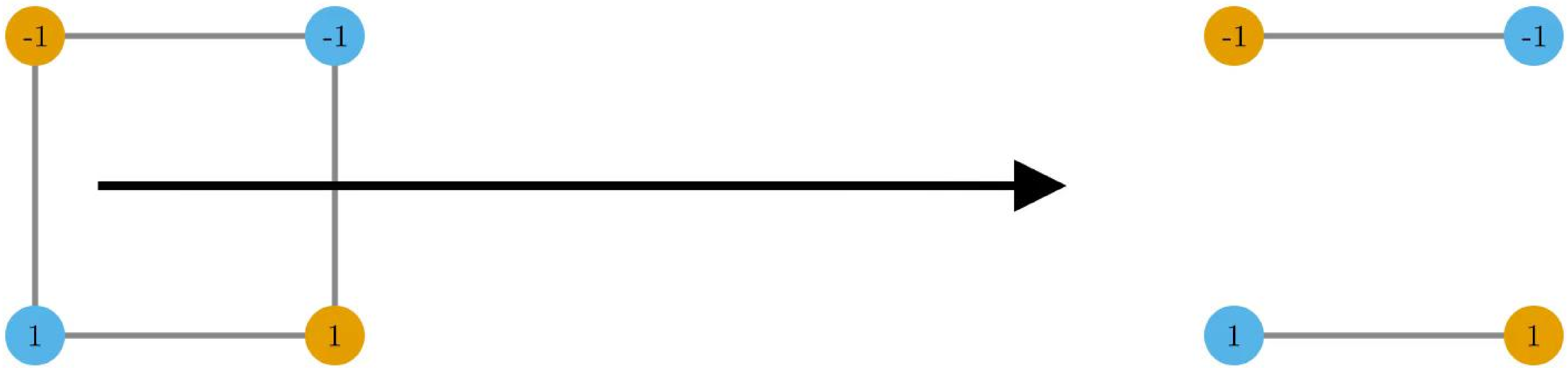

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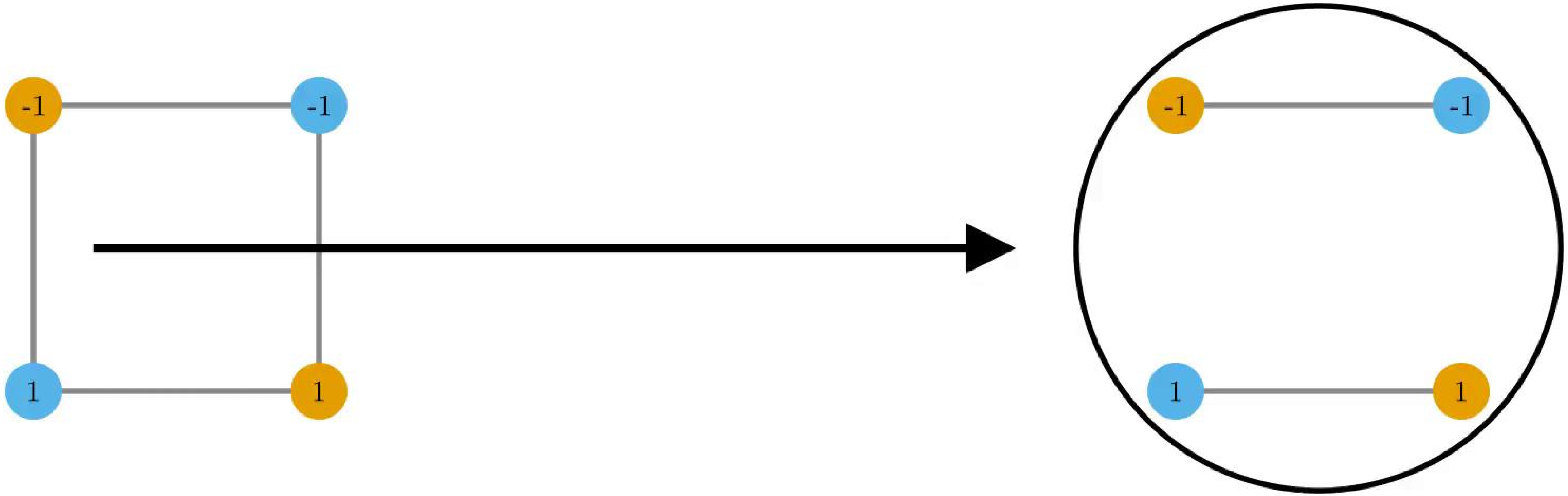

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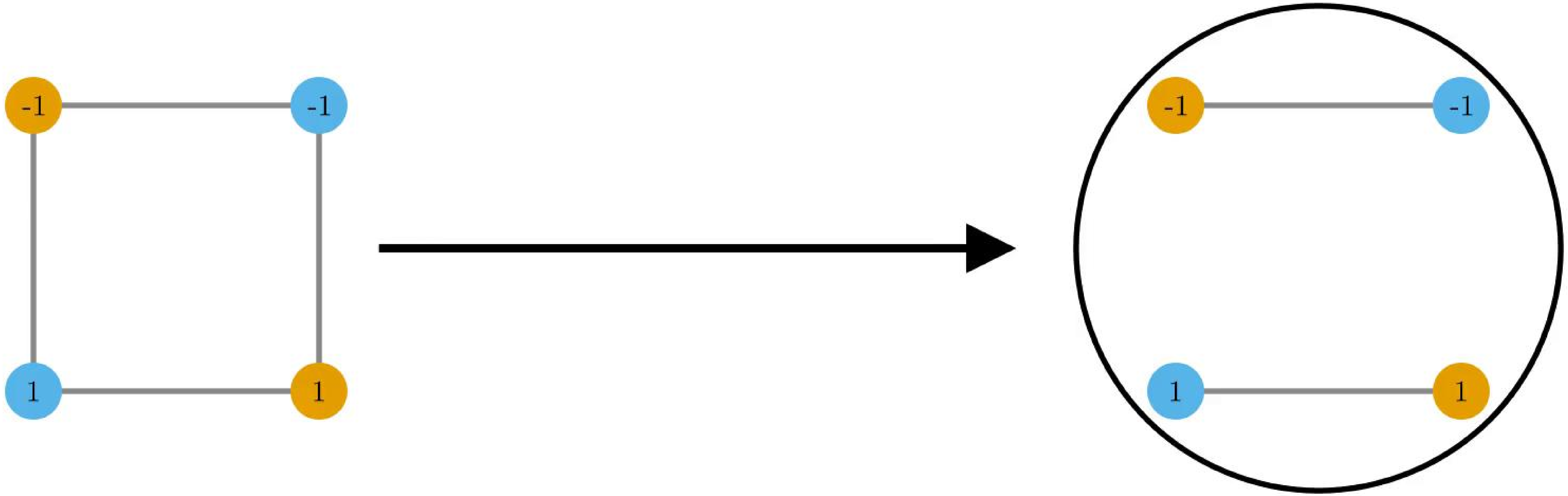
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An abstract theorem about distributed computing

An abstract theorem about distributed computing

$$I \xrightarrow{\mathcal{T}'} SO$$

An abstract theorem about distributed computing

$$
\begin{tikzcd}
I \arrow[rr, "\mathcal{T}'"] \arrow[dr, "\mathcal{E}"'] & & SO \\
& SP &
\end{tikzcd}
$$

An abstract theorem about distributed computing

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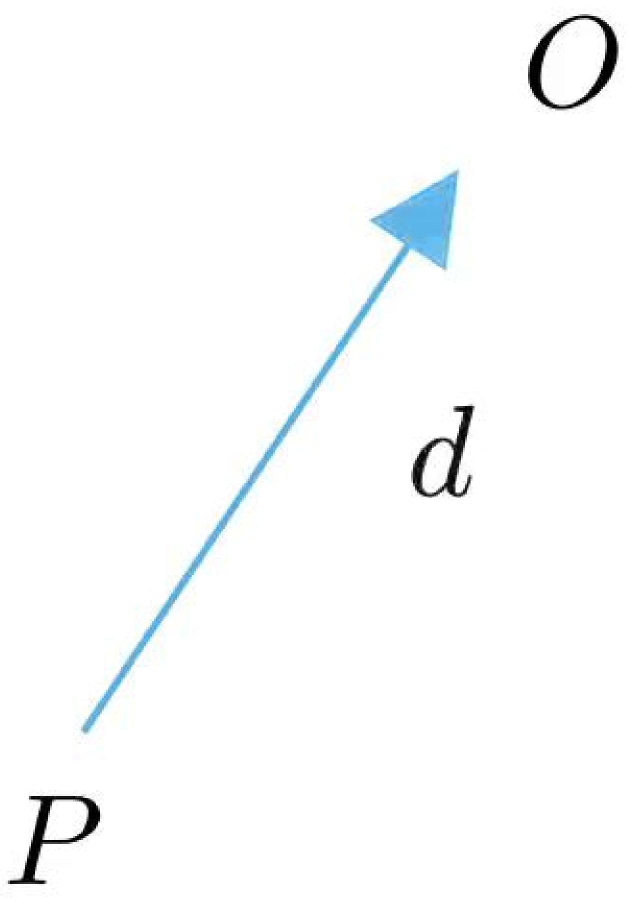
An abstract theorem about distributed computing



An abstract theorem about distributed computing

$$\begin{tikzcd} I \arrow[rr, "\mathcal{T}'"] \arrow[dr, "\mathcal{E}"'] & & SO \\ & SP \arrow[ur, "Sd"'] & \end{tikzcd}$$

An abstract theorem about distributed computing

$$\begin{tikzcd}
I \arrow[rr, "\mathcal{T}'"] \arrow[dr, "\mathcal{E}"'] & \arrow[d, phantom, "\leq"] & SO \\
& SP \arrow[ur, "Sd"'] &
\end{tikzcd}$$

An abstract theorem about distributed computing

$$\begin{tikzcd}
\Psi_n G \arrow[rr, "\mathcal{T}"] \arrow[dr, "\mathrm{WF}"'] & \arrow[d, phantom, "\leq"] & SO \\
& S(\mathrm{WF}(\Psi_n G)) \arrow[ur, "Sd"'] &
\end{tikzcd}$$

An abstract theorem about distributed computing

$$\begin{tikzcd}
I \arrow[rr, "\mathcal{T}'"] \arrow[dr, "\mathrm{WF}"'] & & SO \\
& S(\mathrm{WF}(I)) &
\end{tikzcd}$$

An abstract theorem about distributed computing

G acts freely on I

$$\begin{tikzcd}
I \arrow[rr, "\mathcal{T}'"] \arrow[dr, "\mathrm{WF}"'] & & SO \\
& S(\mathrm{WF}(I)) &
\end{tikzcd}$$

$$
\begin{tikzcd}
I \arrow[ddr] \arrow[rrrd, bend left, "\mathcal{T}'"] & & & \\
& & \Psi_n G \arrow[r, "\mathcal{T}"] \arrow[dd] & SO \\
& S(\mathrm{WF}(I)) & & \\
& & S(\mathrm{WF}(\Psi_n G)) \arrow[uur, "Sd"'] &
\end{tikzcd}
$$

$$
\begin{tikzcd}
I \arrow[rrd, "f'"] \arrow[rrrd, bend left, "\mathcal{T}'"] \arrow[ddr] & & & \\
& & \Psi_n G \arrow[r, "\mathcal{T}"] \arrow[dd] & SO \\
& S(\mathrm{WF}(I)) & & \\
& & S(\mathrm{WF}(\Psi_n G)) \arrow[uur, "Sd"'] &
\end{tikzcd}
$$

$$
\begin{tikzcd}
I \arrow[dr, "f'"] \arrow[drr, bend left, "\mathcal{T}'"] \arrow[dd] & & \\
& \Psi_n G \arrow[r, "\mathcal{T}"] \arrow[dd] & SO \\
S(\mathrm{WF}(I)) \arrow[dr, "Sf'"'] & & \\
& S(\mathrm{WF}(\Psi_n G)) \arrow[uur, "Sd"'] &
\end{tikzcd}
$$

Thank you!