b) $T(n) = 2T(n-1) + n$

$T(1) = 1$

$$T(n) = 2T(n-1) + n$$

$$2T(n-1) = 2^2 T(n-2) + 2(n-1)$$

$$2^2 T(n-2) = 2^3 T(n-3) + 2^2(n-1)$$

$$\downarrow$$

$$T(n) = 2^k T(n-k) + \sum_{i=0}^{k-1} 2^i (n-i)$$

$$= \sum_{i=0}^{k-1} 2^i (n-i)$$

$$= \sum 2^i n - \sum 2^i i$$

$$= n \sum_{i=0}^{k-1} 2^i - \sum 2^i i$$

$$\sum_{i=0}^{n} 2^i + 2^{n+1} = 2^0 + \sum_{i=0}^{n} 2^{i+1}$$

$$\cancel{\sum_{i=0}^{n} 2^i} + 2^{n+1} = 1 + 2\sum_{i=0}^{n} 2^i$$

$$\sum_{i=0}^{n} 2^i = 2^{n+1} - 1$$

$$\sum_{i=0}^{n} 2^i i + 2^{n+1}(n+1) = \cancel{2^0 0} + \sum_{i=0}^{n} 2^{i+1}(i+1)$$

$$\sum_{i=0}^{n} 2^i i + 2^{n+1}(n+1) = 2\sum_{i=0}^{n} i \cdot 2^i + 2\sum_{i=0}^{n} 2^i$$

$$\cancel{\sum_{i=0}^{n} 2^i i} + 2^{n+1}(n) + 2^{n+1} = \cancel{2\sum_{i=0}^{n}} 2^i i + 2.(2^{n+1} - 1)$$

$$\sum_{i=0}^{n} 2^i i = 2^{n+1} . n - 2^{n+1} + 2$$

$$T(n) = n.(2^{n+1} - 1) - (2^{n+1}.n - 2^{n+1} + 2)$$

$$T(n) = \cancel{n.2^{n+1}} - n - \cancel{2^{n+1}.n} + 2^{n+1} - 2$$

$$T(n) = 2^{n+1} - n - 2$$ //

Complexidade $O(2^n)$ //