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1) a) $$\int_3^4\int_1^2 \frac{1}{(x+y)^2}\,dy\,dx = \int_3^4\int_{x+1}^{x+2} u^{-2}\,du\,dx = \int_3^4 -u^{-1}\Big|_{x+1}^{x+2} dx$$

$$u = x+y \Rightarrow du = dy$$

$$\int_3^4 (x+1)^{-1} - (x+2)^{-1}\,dx = \ln(5) - \ln(4) - \ln(6) + \ln(5)$$

$$\boxed{\int_3^4\int_1^2 \frac{1}{(x+y)^2}\,dy\,dx = 2\ln(5) - 2\ln(2) - \ln(6)}$$

b) $\iint_R \frac{xy}{\sqrt{x^2+y^2+1}}\, dA \qquad R=\{(x,y) \mid 0\le x\le 1,\ 0\le y\le 1\}$

$$\int_0^1\int_0^1 \frac{xy}{\sqrt{x^2+y^2+1}}\, dy\,dx = \int_0^1 \int_{1+x^2}^{2+x^2} \frac{xy}{\sqrt{u}}\, \frac{du}{2y}\, dx = \frac{1}{2}\int_0^1\int_{1+x^2}^{2+x^2} x u^{-1/2}\, du\, dy$$

$$u = x^2+y^2+1 \Rightarrow du = 2y\,dy \Rightarrow dy = du/2y$$

$$\frac{1}{2}\int_0^1 2x\sqrt{u}\,\Big|_{1+x^2}^{2+x^2} dx = \int_0^1 x\sqrt{2+x^2} - x\sqrt{1+x^2}\, dx = \int_2^3 \frac{x\sqrt{v}}{2x}\,dv - \int_1^2 \frac{x\sqrt{w}}{2x}\,dw$$

$$v = 2+x^2 \Rightarrow dv = 2x\,dx \Rightarrow dx = dv/2x \qquad w = 1+x^2 \Rightarrow dw = 2x\,dx \Rightarrow dx = dw/2$$

$$\frac{1}{2}\left(\int_2^3 \sqrt{v}\,dv - \int_1^2 \sqrt{w}\,dw\right) = \frac{1}{2}\left(\left[\frac{2v^{3/2}}{3}\right]_2^3 - \left[\frac{2w^{3/2}}{3}\right]_1^2\right) = \frac{1}{3}\left(3\sqrt{3} - 2\sqrt{2} - 2\sqrt{2} + 1\right)$$

$$\boxed{\iint_R \frac{xy}{\sqrt{x^2+y^2+1}}\, dA = \frac{3\sqrt{3} - 4\sqrt{2} + 1}{3}}$$

$$\int_0^{\sqrt{2}/2}\int_x^{2x} x^2\,dy\,dx + \int_{\sqrt{2}/2}^{1}\int_x^{1/x} x^2\,dy\,dx = \int_0^{\sqrt{2}/2} x^2 y\Big|_x^{2x} dx +$$

$$\int_0^{\sqrt{2}/2} 2x^3 - x^3\,dx + \int_{\sqrt{2}/2}^{1} x - x^3\,dx = \int_0^{\sqrt{2}/2} x^3\,dx + \int_{\sqrt{2}/2}^{1} x - x^3\,dx = [$$

$$\frac{1}{16} + \left(\frac{1}{2} - \frac{1}{4}\right) - \left(\frac{1}{4} + \frac{1}{16}\right) = \frac{1}{8}$$

$$\boxed{\iint_R x^2\,dA = \frac{1}{8}}$$

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b) $\iint_D y^2 \, dA \qquad m_1 = 1/1 = 1 \qquad m_2 = -1/3$

$y - 1 = 1(x - 0) \Rightarrow y = x + 1 \Rightarrow x = y - 1$

$y - 2 = -\frac{(x-1)}{3} \Rightarrow y = \frac{7-x}{3} \Rightarrow x = 7 - 3y$

$R = \{(x,y) \mid 1 \leq y \leq 2,\ y - 1 \leq x \leq 7 - 3y\}$

$$\int_1^2 \int_{y-1}^{7-3y} y^2 \, dx \, dy = \int_1^2 y^2 (7 - 3y - y + 1) \, dy = \int_1^2 8y^2 - 4y^3 \, dy = \frac{8y^3}{3} - y^4 \Big|_1^2$$

$$\frac{64}{3} - 16 - \frac{8}{3} + 1 = \frac{11}{3}$$

$$\boxed{\iint_D y^2 \, dA = \frac{11}{3}}$$

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(3) $x-2y+z=1 \qquad x+y=1 \qquad x^2+y=1$

$$\begin{cases} x+y=1 \Rightarrow y=1-x \\ x^2+y=1 \Rightarrow x^2+1-x=1 \Rightarrow x^2-x=0 \end{cases} \begin{matrix} \to x=0 \\ \to x=1 \end{matrix}$$

$$R=\{(x,y) \mid 0 \leq x \leq 1,\ 1-x \leq y \leq 1-x^2\}$$

$$V=\int_0^1 \int_{1-x}^{1-x^2} 1-x+2y\,dy\,dx = \int_0^1 y(1-x)+y^2 \Big|_{1-x}^{1-x^2} dx = \int_0^1 (1-x^2)(1-x)+(1-x^2)^2-2(1-x)^2\,dx$$

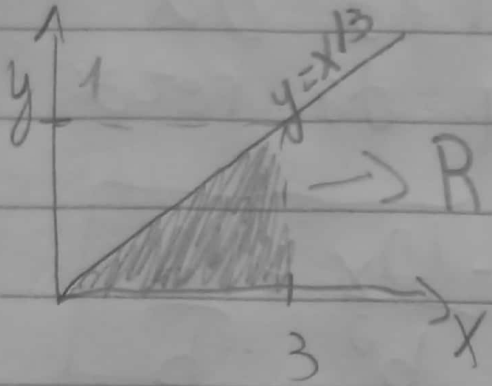
$$\int_0^1 1-x-x^2+x^3+1-2x^2+x^4-2+4x-2x^2\,dx = \int_0^1 x^4+x^3-5x^2+3x\,dx$$

$$V=\frac{x^5}{5}+\frac{x^4}{4}-\frac{5x^3}{3}+\frac{3x^2}{2}\Big|_0^1 = \frac{17}{60}$$

$$\boxed{V=\frac{17}{60}\ \text{u.v.}}$$

$\boxed{\frac{14}{60} \text{ u.v.}}$

(4)



$\rightarrow R = \{(x,y) \mid 0 \le x \le 3,\ 0 \le y \le x/3\}$

$$\int_0^3 \int_0^{x/3} e^{x^2}\,dy\,dx = \int_0^3 y e^{x^2}\Big|_0^{x/3} dx = \frac{1}{3}\int_0^3 x e^{x^2}\,dx$$

$u = x^2 \Rightarrow du = 2x\,dx \Rightarrow dx = du/2x$

$$\frac{1}{3}\int_0^9 \frac{x e^u}{2x}\,du = \frac{1}{6}\int_0^9 e^u\,du = \frac{e^u}{6}\Big|_0^9 = \frac{e^9 - e^0}{6} = \frac{e^9 - 1}{6}$$

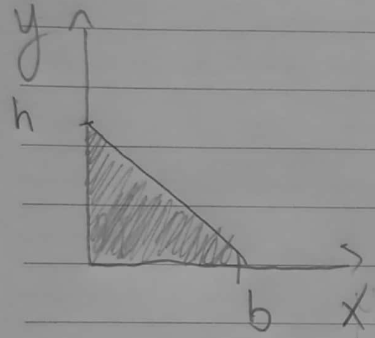
$$\boxed{\int_0^1 \int_{3y}^3 e^{x^2}\,dx\,dy = (e^9 - 1)\frac{1}{6}}$$

5) $R = \{(x,y) \mid 0 \leq x \leq 2,\ 0 \leq y \leq 1\} + \{(x,y) \mid 1 \leq x \leq 2,\ 1 \leq y \leq 2\}$

$$\boxed{\int_0^2 \int_0^1 f(x,y)\,dy\,dx + \int_1^2 \int_1^2 f(x,y)\,dy\,dx = \int_0^1 \int_0^2 f(x,y)\,dx\,dy + \int_1^2 \int_1^2 f(x,y)\,dx\,dy}$$

(6) A solução mais simples é colocar as bases paralelas ao eixo xOy, sendo uma $z=0$ e a outra $z=H$ (H é a altura do prisma).

Assim $V=\iint_D H\,dA$ sendo D a área da base do prisma:



É visível que $0 \leq x \leq b$ e y varia de 0 até a reta.

$m = \frac{\Delta y}{\Delta x} = \frac{-h}{b}$ $\quad y - h = \frac{-h}{b}x$

Por sabermos a área da base podemos simplificar a eq. da reta:

$\frac{bh}{2} = 6 \Rightarrow \boxed{h = \frac{12}{b}}$ $\quad y = \frac{-12/b}{b}x + \frac{12}{b} = \boxed{-\frac{12x}{b^2} + \frac{12}{b}}$

Assim: $D = \{(x,y) \mid 0 \leq x \leq b,\ 0 \leq y \leq -\frac{12x}{b^2} + \frac{12}{b}\}$, portanto:

$$\boxed{V = \int_0^b \int_0^{\frac{12}{b} - \frac{12x}{b^2}} H\,dy\,dx}$$