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1) $y'' + 10y' + 16y = 5x' + 6x \qquad y_0 = 1 \quad y_0' = 2$

$s^2Y(s) - sy_0' - y_0 + 10(sY(s) - y_0) + 16Y(s) = 5(sX(s) - 0) + 6X(s)$

$s^2Y(s) + 10sY(s) + 16Y(s) = 2s + 11 + X(s)[6 + 5s]$

$Y(s)[s^2 + 10s + 16] = (2s + 11) + X(s)[6 + 5s]$

$$Y(s) = \underbrace{\frac{2s+11}{s^2+10s+16}}_{a)} + \underbrace{X(s)\left[\frac{6+5s}{s^2+10s+16}\right]}_{b)}$$

$s^2 + 10s + 16 = 0$

$\Delta = 100 - 64 = 36 \qquad s = \frac{-10 \pm 6}{2} \begin{cases} s_1 = 2 \\ s_2 = -8 \end{cases}$

b) $s^2 + 10s + 16 = (s+2)(s+8)$

a) $\frac{2s+11}{s^2+10s+16} = \frac{A}{s+2} + \frac{B}{s+8} = \frac{As + 8A + Bs - 2B}{(s+2)(s+8)}$ $\qquad A = \frac{-2-12}{-2+8} = \frac{5}{3}$

$B = \frac{-8+12}{-8+2} = \frac{-2}{3}$

b) $\frac{6+5s}{s^2+10s+16} = \frac{A'}{s+2} + \frac{B'}{s+8} = \frac{A's + 8A' + B's + 2B'}{s^2+10s+16}$

$A' = \frac{5(-2)+6}{-2+8} = \frac{2}{3} \qquad B' = \frac{5(-8)+6}{-8+2} = \frac{17}{3}$

(2) $T_0 = 2 \quad \omega_0 = \underline{2\pi} = \pi$

$$Y(s) = \frac{5/3}{s+2} - \frac{2/3}{s+8} + X(s)\left[\frac{2/3}{s+2} + \frac{17/3}{s+8}\right]$$

$$y(t) = \underbrace{\left[\frac{5}{3}e^{-2t} - \frac{2}{3}e^{-8t}\right]\cdot u(t)}_{y_{zi}(t)} + \underbrace{\left(\frac{21}{3}e^{-2t} + \frac{17}{3}e^{-8t}\right) u(t) * x(t)}_{y_{zs}(t)}$$

$$y(t) = \frac{1}{3}\left[\left(-5e^{-2t} + 5e^{-8t}\right)\cdot u(t) + \left(21e^{-2t} + 17e^{-8t}\right)\cdot u(t) * x(t)\right]$$

(2) $T_0 = 2 \quad w_0 = \frac{2\pi}{2} = \pi$

$$a_0 = \frac{1}{2}\int_{-1}^{1} x(t)dt = \frac{1}{2}\int_{-1}^{0} -1dt + \frac{1}{2}\int_{0}^{1} 3dt = \frac{-t}{2}\Big|_{-1}^{0} + \frac{3t}{2}\Big|_{0}^{1}$$

$$a_0 = \frac{-\cancel{0}1}{2} + \frac{3(1-0)}{2} = \frac{1}{2} + \frac{3}{2} = 2$$

$$a_k = \frac{2}{2}\int_{-1}^{1} x(t)\cos(k\pi t)dt = \int_{-1}^{0} -\cos(k\pi t)dt + \int_{0}^{1} 3\cos(k\pi t)dt$$

$$a_k = \frac{-\operatorname{sen}(k\pi t)}{k\pi}\Big|_{-1}^{0} + 3\,\frac{\operatorname{sen}(k\pi t)}{k\pi}\Big|_{0}^{1} = \frac{-0+\operatorname{sen}(-k\pi)}{k\pi} + 3\left[\frac{\operatorname{sen}(k\pi)-0}{k\pi}\right]$$

$$a_k = \frac{1}{k\pi}\left(-\operatorname{sen}(k\pi) + 3\operatorname{sen}(k\pi)\right) = \frac{2\operatorname{sen}(k\pi)}{k\pi} = \frac{2\operatorname{sen}(2m\pi)}{m\pi}$$

$$b_k = \frac{2}{2}\int_{-1}^{1} x(t)\operatorname{sen}(k\pi t)dt = \int_{-1}^{0} -\operatorname{sen}(k\pi t)dt + \int_{0}^{1} 3\operatorname{sen}(k\pi t)dt$$

$$b_k = \frac{\cos(k\pi t)}{k\pi}\Big|_{-1}^{0} - \frac{3\cos(k\pi t)}{k\pi}\Big|_{0}^{1} = \frac{1-\cos(-k\pi)}{k\pi} - 3\left(\frac{\cos(k\pi)-1}{k\pi}\right)$$

$$b_k = \frac{1}{k\pi}\left(1-\cos(k\pi) - 3\cos(k\pi) + 3\right) = \frac{4-4\cos(k\pi)}{k\pi} = \frac{4-4\cos((2m-1)\pi)}{m\pi}$$

$$x(t) = 2 + \frac{1}{m\pi}\sum_{m=1}^{\infty} 2\operatorname{sen}(2m\pi)\cos(m\pi t) + \left(4-4\cos((2m-1)\pi)\right)\operatorname{Sen}(m\pi t)$$

(3) $T_0 = 6 \quad w_0 = \frac{2\pi}{6} = \frac{\pi}{3}$

$$C_k = \frac{1}{6}\int_0^6 x(t)\, e^{\frac{-jk\pi t}{3}} dt = \frac{1}{6}\int_0^1 0\, e^{\frac{-jk\pi t}{3}} dt + \frac{1}{6}\int_1^6 e^{\frac{-jk\pi t}{3}} dt$$

$$C_k = \frac{1}{6} \frac{3e^{\frac{-jk\pi t}{3}}}{-jk\pi}\Bigg|_1^6 = \frac{-1}{2jk\pi} e^{-2jk\pi} - e^{\frac{-jk\pi}{3}}$$

$$C_k = \frac{-1}{2jk\pi}\left[\cos(2k\pi) - j\,sen(2k\pi) - \cos\left(\frac{k\pi}{3}\right) + j\,sen\left(\frac{k\pi}{3}\right)\right]$$

$$C_k = \frac{1 - 0j - \cos\left(\frac{k\pi}{3}\right) + j\,sen\left(\frac{k\pi}{3}\right)}{2jk\pi} = \frac{1 + \cos\left(\frac{k\pi}{3}\right) + j\,sen\left(\frac{k\pi}{3}\right)}{2jk\pi}$$

$$x(t) = \sum_{n=-\infty}^{\infty}\left[\left(\frac{1 + \cos\left(\frac{k\pi}{3}\right) + j\,sen\left(\frac{k\pi}{3}\right)}{2jk\pi}\right) e^{\frac{jk\pi t}{3}}\right]$$

4 $X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt = \int_{-1}^{1} 0{,}5\, e^{-j\omega t}\, dt + \int_{3}^{4} \delta(t-\pi)\, e^{-j\omega t}\, dt$

$$X(\omega) = \frac{0{,}5\, e^{-j\omega t}}{-j\omega}\Bigg|_{-1}^{1} + e^{-j\omega 0} = \frac{-0{,}5}{j\omega}\left(e^{-j\omega} - e^{j\omega}\right) + 1$$

$$X(\omega) = 0{,}5\left(\frac{e^{j\omega} - e^{-j\omega}}{j\omega}\right) + 1 = 0{,}5\left(\frac{\mathrm{sen}(\omega\, 1)}{\omega}\right) + 1$$

$$\boxed{X(\omega) = \mathrm{sinc}(\omega) + 1}$$

5

a) $e^{\beta t}\,\Pi_\pi(t)$

$$X(\omega) = \int_{-\infty}^{\infty} \Pi_\pi(t)\, e^{\beta t} e^{-j\omega t} = \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} e^{(\beta - j\omega)t}\, dt$$

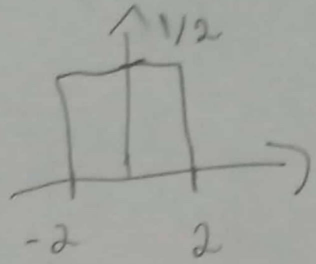
$$X(\omega) = \frac{1}{\pi}\, \frac{e^{(\beta-j\omega)t}}{\beta - j\omega}\Bigg|_{-\pi/2}^{\pi/2} = \frac{e^{\frac{(\beta-j\omega)\pi}{2}} - e^{\frac{(\beta+j\omega)\pi}{2}}}{\pi(\beta - j\omega)}$$

$$X(\omega) = \frac{e^{\frac{\beta\pi}{2}}\, e^{\frac{-j\omega\pi}{2}} - e^{\frac{\beta\pi}{2}}\, e^{\frac{j\omega\pi}{2}}}{\pi\beta - \pi j\omega}$$

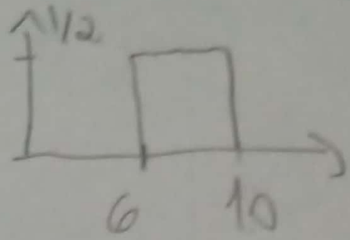
$$\boxed{X(\omega) = \frac{e^{\beta\pi/2}\, 2j\, \mathrm{sen}(\omega\pi/2)}{\pi(\beta - j\omega)}}$$

b) $x(t) = \Pi_4(2t-8)$

$\Pi_4(2t)$:



$\Pi_4(2t-8)$



$$\Pi_4(t) \Leftrightarrow 2\,\text{sinc}\left(\frac{4w}{2}\right)$$

$$\Pi_4(2t) \Leftrightarrow \frac{1}{2}\,\text{sinc}(w)$$

$$\Pi_4(2t-8) \Leftrightarrow \frac{e^{-8jw}}{2}\,\text{sinc}(w)$$

$$\boxed{X(w) = \frac{e^{-8jw}\,\text{sinc}(w)}{2}}$$

(6) $A = \begin{bmatrix} -3 & -2 \\ 1 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$C = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ $D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$$H(s) = C\left[sI - A\right]^{-1} B + D$$

$$H(s) = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \Bigg(\underbrace{\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -3 & -2 \\ 1 & 0 \end{bmatrix}}_{\Delta} \Bigg)^{-1}$$

$\det \Delta = s^2 - 3s + 2$

$$H(s) = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \frac{1}{|\det \Delta|} \begin{bmatrix} s & 2 \\ -1 & s-3 \end{bmatrix}$$

$$H(s) = \frac{1}{s^2-3s+2} \begin{bmatrix} s & 2 \\ s-1 & s-1 \end{bmatrix} = \begin{bmatrix} \frac{s}{s^2-3s+2} & \frac{2}{s^2-3s+2} \\ \frac{s-1}{s^2-3s+2} & \frac{s-1}{s^2-3s+2} \end{bmatrix}$$

$$\boxed{H_{12}(s) = \frac{2}{s^2-3s+2}}$$

$$H_{12}(s) = \frac{2}{s^2-3s+2}$$

$$s^2-3s+2=0$$

$$\Delta = 9-4\cdot1\cdot2 = 1$$

$$s = \frac{-3 \pm 1}{2} \begin{cases} \Rightarrow s_1 = -1 \\ \Rightarrow s_2 = -2 \end{cases}$$

$$H_{12}(s) = \frac{2}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

$$\begin{cases} A+B=0 \Rightarrow B=-A \\ 2A+B=2 \Rightarrow A=2 \\ \quad B=-2 \end{cases}$$

$$H_{12}(s) = \frac{2}{s+1} - \frac{2}{s+2}$$

$$\boxed{h_{12}(t) = 2e^{-t} - 2e^{-2t}}$$