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① a) $$\int_0^1\int_0^1\int_0^{\sqrt{1-z^2}} \frac{z}{y+1}\,dx\,dz\,dy = \frac{\ln(2)}{3}$$

$$\int_0^1\int_0^1 \frac{z}{y+1} x\Big|_0^{\sqrt{1-z^2}} dz\,dy = \int_0^1\int_0^1 \frac{z\sqrt{1-z^2}}{y+1}\,dz\,dy$$

$$\boxed{u = 1-z^2 \Rightarrow du = -2z\,dz \Rightarrow dz = -du/2z}$$ → substituição

$$\int_0^1\int_1^0 \frac{z\sqrt{u}}{y+1}\left(\frac{-1}{2z}\right)du\,dy = -\frac{1}{2}\int_0^1\int_1^0 \frac{\sqrt{u}}{y+1}\,du\,dy = -\frac{1}{2}\int_0^1 \frac{1}{y+1}\,\frac{2u^{3/2}}{3}\Big|_1^0 dy$$

$$-\frac{1}{2}\cdot\frac{2}{3}\int_0^1 \frac{(0-1)}{y+1}\,dy = \frac{1}{3}\int_0^1 (y+1)^{-1}\,dy = \frac{\ln|y+1|}{3}\Big|_0^1 = \frac{\ln(2)-\ln(1)}{3}$$

$$\boxed{\int_0^1\int_0^1\int_0^{\sqrt{1-z^2}} \frac{z}{y+1}\,dx\,dz\,dy = \frac{\ln(2)}{3}}$$

b) $\int_0^{\sqrt{\pi}} \int_0^{x} \int_0^{xz} x^2 \operatorname{sen}(y)\,dy\,dz\,dx$

$$\int_0^{\sqrt{\pi}} \int_0^{x} -x^2\cos(y)\Big|_0^{xz} dz\,dx = \int_0^{\sqrt{\pi}} \int_0^{x} -x^2\big(\cos(xz)-1\big)\,dz\,dx$$

$$= \int_0^{\sqrt{\pi}} \int_0^{x} x^2 - x^2\cos(xz)\,dz\,dx = \int_0^{\sqrt{\pi}} zx^2 - \frac{x^2\operatorname{sen}(xz)}{x}\Big|_0^{x} dx$$

$$= \int_0^{\sqrt{\pi}} x^3 - 0 - x\operatorname{sen}(x^2) + x\operatorname{sen}(0)\,dx = \int_0^{\sqrt{\pi}} x^3 - x\operatorname{sen}(x^2)\,dx = \int_0^{\sqrt{\pi}} x^3 dx - \int_0^{\sqrt{\pi}} x\operatorname{sen}(x^2)\,dx$$

$$= \frac{x^4}{4}\Big|_0^{\sqrt{\pi}} - \int_0^{\sqrt{\pi}} x\operatorname{sen}(x^2)\,dx = \frac{\pi^2}{4} - \int_0^{\sqrt{\pi}} x\operatorname{sen}(x^2)\,dx = \frac{\pi^2}{4} - \int_0^{\pi} \frac{x\operatorname{sen}(u)\,du}{2x}$$

$\boxed{u = x^2 \Rightarrow du = 2x\,dx \Rightarrow dx = du/2x}$ -> substituição

$$= \frac{\pi^2}{4} - \frac{1}{2}\int_0^{\pi} \operatorname{sen}(u)\,du = \frac{\pi^2}{4} - \frac{1}{2}\cos(u)\Big|_0^{\pi} = \frac{\pi^2}{4} - \frac{1+1}{2} = \frac{\pi^2}{4} - 1$$

$$\boxed{\int_0^{\sqrt{\pi}} \int_0^{x} \int_0^{xz} x^2 \operatorname{sen}(y)\,dy\,dz\,dx = \frac{\pi^2}{4} - 1}$$

c) $\iiint_E \frac{z}{x^2+z^2}\,dV$ $\quad E=\{(x,y,z)\mid 1\leq y\leq 4,\ y\leq z\leq 4,\ 0\leq x\leq z\}$

$$\int_1^4\int_y^4\int_0^z \frac{z}{x^2+z^2}\,dx\,dz\,dy=\int_1^4\int_y^z\int_0^z \frac{1}{x^2+z^2}\,dx\,dz\,dy$$

$\boxed{u=x/z \Rightarrow du=dx/z \Rightarrow dx=z\,du}$ → substituição

$$=\int_1^4\int_y^z\int_0^1 \frac{1}{z(u^2+1)}\,du\,z\,dy=\int_1^4\int_y^4 tg^{-1}(u)\Big|_0^1\,dz\,dy=\int_1^4\int_y^4\frac{\pi}{4}\,dz\,dy$$

$$=\int_1^4 \frac{\pi z}{4}\Big|_y^4\,dy=\int_1^4 \frac{4\pi}{4}-\frac{y\pi}{4}\,dy=\pi y-\frac{\pi y^2}{8}\Big|_1^4=4\pi-\pi-\left(\frac{16\pi-\pi}{8}\right)$$

$$=3\pi-\frac{15\pi}{8}=\frac{9\pi}{8}$$

$$\boxed{\iiint_E \frac{z}{x^2+z^2}\,dV=\frac{9\pi}{8}}$$

d) $\iiint_E \operatorname{sen}(y)\,dV$ $\qquad z=x \quad (0,0,0),\ (\pi,0,0),\ (0,\pi,0)$

$$\int_0^{\pi}\int_0^{\pi}\int_0^{x} \operatorname{sen}(y)\,dz\,dy\,dx = \int_0^{\pi}\int_0^{\pi} z\operatorname{sen}(y)\Big|_0^{x} dy\,dx = \int_0^{\pi}\int_0^{\pi} x\operatorname{sen}(y)\,dy\,dx$$

$$= \int_0^{\pi} -x\cos(y)\Big|_0^{\pi} dx = -\int_0^{\pi} x(-1-1)\,dx = 2\int_0^{\pi} x\,dx = x^2\Big|_0^{\pi} = \pi^2$$

$$\boxed{\iiint_E \operatorname{sen}(y)\,dV = \pi^2}$$

2a) $-1 \leq x \leq 1 \qquad x^2 \leq y \leq 1 \qquad 0 \leq z \leq 1-y$

$$V = \int_{-1}^{1}\int_{x^2}^{1}\int_{0}^{1-y} dz\,dy\,dx = \int_{-1}^{1}\int_{x^2}^{1} 1-y\,dy\,dx = \int_{-1}^{1} y - \frac{y^2}{2}\Big|_{x^2}^{1} dx$$

$$V = \int_{-1}^{1} 1-x^2-\left(\frac{1}{2}-\frac{x^4}{2}\right)dx = \int_{-1}^{1} \frac{1}{2}-x^2+\frac{x^4}{2}\,dx = \frac{x}{2}-\frac{x^3}{3}+\frac{x^5}{10}\Big|_{-1}^{1}$$

$$V = \left(\frac{1}{2}-\left(\frac{-1}{2}\right)\right)-\left(\frac{1}{3}-\left(\frac{-1}{3}\right)\right)+\left(\frac{1}{10}-\left(\frac{-1}{10}\right)\right) = 1-\frac{2}{3}+\frac{1}{5} = \boxed{\frac{8}{15}}$$

$$E = \{(x,y,z) \mid -1 \leq x \leq 1, x^2 \leq y \leq 1, 0 \leq z \leq 1-y\} = \{(x,y,z) \mid 0 \leq y \leq 1, -\sqrt{y} \leq x \leq \sqrt{y}, 0 \leq z \leq 1-y\}$$

$$\boxed{V = \int_{-1}^{1}\int_{x^2}^{1}\int_{0}^{1-y} dz\,dy\,dx = \int_{0}^{1}\int_{-\sqrt{y}}^{\sqrt{y}}\int_{0}^{1-y} dz\,dx\,dy = \frac{8}{15}\,u.v.}$$

b) $0 \le z \le y^2$ $\quad 0 \le x \le 1 \quad -1 \le y \le 1$

$E = \{(x,y,z) \mid 0 \le x \le 1, -1 \le y \le 1, 0 \le z \le y^2\}$

$$V = \int_0^1 \int_{-1}^1 \int_0^{y^2} dz\,dy\,dx = \int_0^1 \int_{-1}^1 y^2\,dy\,dx = \int_0^1 \frac{y^3}{3}\Big|_{-1}^1 dx = \frac{1}{3}\int_0^1 1+1\,dx = \frac{2x\big|_0^1}{3} = \frac{2}{3}$$

$$\boxed{V = \int_0^1 \int_{-1}^1 \int_0^{y^2} dz\,dy\,dx = \int_{-1}^1 \int_0^1 \int_0^{y^2} dz\,dx\,dy = \frac{2}{3} \text{ u.v.}}$$

c) $0 \leq x \leq 1 \quad 0 \leq y \leq 1-x \quad 0 \leq z \leq \cos\left(\frac{\pi x}{2}\right)$

$$E = \{(x,y,z) \mid 0 \leq x \leq 1, 0 \leq y \leq 1-x, 0 \leq z \leq \cos\left(\frac{\pi x}{2}\right)\}$$

$$E = \{(x,y,z) \mid 0 \leq y \leq 1, 0 \leq x \leq 1-y, 0 \leq z \leq \cos\left(\frac{\pi x}{2}\right)\}$$

$$V = \int_0^1 \int_0^{1-x} \int_0^{\cos\left(\frac{\pi x}{2}\right)} dz\,dy\,dx = \int_0^1 \int_0^{1-x} \cos\left(\frac{\pi x}{2}\right) dy\,dx = \int_0^1 \cos\left(\frac{\pi x}{2}\right)$$

$$\boxed{\begin{array}{ll} u = 1-x & dv = \cos\left(\frac{\pi x}{2}\right) dx \\ du = -dx & v = \frac{2}{\pi} \operatorname{sen}\left(\frac{\pi x}{2}\right) \end{array}} \rightsquigarrow \text{partes}$$

$$V = u\,v\Big|_0^1 - \int_0^1 v\,du = \frac{2}{\pi} \operatorname{sen}\left(\frac{\pi x}{2}\right)(1-x)\Big|_0^1 - \int_0^1 \frac{-2}{\pi} \operatorname{sen}\left(\frac{\pi x}{2}\right) dx$$

$$V = \frac{2}{\pi}\left(1 \cdot 0 - 0 \cdot 1\right) + \int_0^1 \frac{2}{\pi} \operatorname{sen}\left(\frac{\pi x}{2}\right) dx = \frac{2}{\pi}\left[-\frac{2}{\pi} \cos\left(\frac{\pi x}{2}\right)\right]_0^1$$

$$V = \frac{-4}{\pi^2}\left(\cos\left(\frac{\pi}{2}\right) - \cos(0)\right) = \frac{-4}{\pi^2}(0-1) = \frac{4}{\pi^2}$$

$$\boxed{V = \int_0^1 \int_0^{1-x} \int_0^{\cos\left(\frac{\pi x}{2}\right)} dz\,dy\,dx = \int_0^1 \int_0^{1-y} \int_0^{\cos\left(\frac{\pi x}{2}\right)} dz\,dx\,dy = \frac{4}{\pi^2}}$$

d) $x+z=6 \quad y+z=6 \quad z \geq 0$

$x-z=0 \quad y-z=0$

$\begin{cases} x+z=6 \Rightarrow 2x=6 \Rightarrow x=3 \\ \cancel{y}-z=0 \Rightarrow x=z \end{cases}$

$\begin{cases} y+z=6 \Rightarrow 2y=6 \Rightarrow y=3 \\ y-z=0 \Rightarrow y=z \end{cases}$

$E=\{(x,y,z) \mid 0 \leq z \leq 3,\ z \leq x \leq 6-z,\ z \leq y \leq 6-z\}$

$$V=\int_0^3\int_z^{6-z}\int_z^{6-z} dx\,dy\,dz=\int_0^3\int_z^{6-z} 6-z-z\,dy\,dz=\int_0^3\int_z^{6-z} 6-2z\,dy\,dz$$

$$=\int_0^3 (6-2z)\,y\Big|_z^{6-z} dz=\int_0^3 (6-2z)(6-z-z)\,dz=\int_0^3 (6-2z)^2\,dz=\int_0^3 36-24z+4z^2\,dz$$

$$=36z-12z^2+\frac{4z^3}{3}\Big|_0^3=36\cdot 3-12\cdot 9+4\cdot 9=36$$

$$\boxed{V=\int_0^3\int_z^{6-z}\int_z^{6-z} dx\,dy\,dz=36\,u.v.}$$

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(3) $E = \{(x,y,z) \mid 0 \le x \le 2,\ 0 \le z \le 4-x^2,\ 0 \le y \le x\}$

$E = \{(x,y,z) \mid 0 \le x \le \sqrt{4-z},\ 0 \le z \le 4,\ 0 \le y \le \sqrt{4-z}\}$

$$\int_0^4 \int_0^{\sqrt{4-z}} \int_0^{\sqrt{4-z}} \frac{\operatorname{sen}(2z)}{4-z}\,dx\,dy\,dz = \int_0^4 \int_0^{\sqrt{4-z}} \frac{\sqrt{4-z}\,\operatorname{sen}(2z)}{4-z}\,dy\,dz$$

$$= \int_0^4 \frac{(4-z)\operatorname{sen}(2z)}{4-z}\,dz = \int_0^4 \operatorname{sen}(2z)\,dz = \left.\frac{-\cos(2z)}{2}\right|_0^4$$

$$\boxed{\int_0^2 \int_0^{4-x^2} \int_0^{x} \frac{\operatorname{sen}(2z)}{4-z}\,dy\,dz\,dx = \frac{1-\cos(8)}{2}}$$