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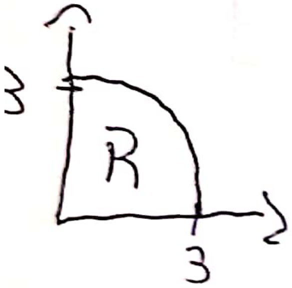
① a) $\iint_R \sqrt{9-x^2-y^2}\,dA = \frac{9\pi}{2}$ $\quad R=\{(x,y) \mid x^2+y^2=9 \text{ e } (x,y)\in 1^{o} \text{ quadrante}\}$



$$0\le x^2+y^2\le 9 \Rightarrow 0\le r^2\le 9 \Rightarrow \boxed{0\le r\le 3}$$

$$\boxed{0\le\theta\le\frac{\pi}{2}}$$

$$\iint_R \sqrt{9-x^2-y^2}\,dA = \int_0^3\int_0^{\pi/2} r\sqrt{9-r^2}\,d\theta\,dr = \int_0^3 \frac{\pi}{2} r\sqrt{9-r^2}\,dr = \frac{\pi}{2}\int_0^3 r\sqrt{9-r^2}\,dr$$

$$u=9-r^2 \Rightarrow du=-2r\,dr \Rightarrow dr = du/-2r$$

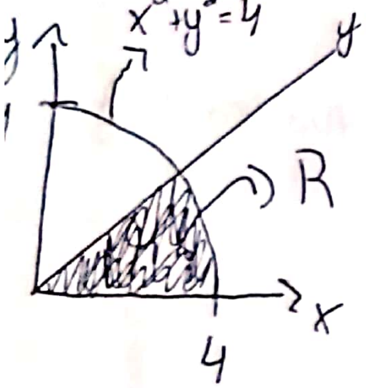
$$=\frac{\pi}{2}\int_9^0 \frac{r\sqrt{u}}{-2r}\,du = \frac{-\pi}{4}\int_9^0 u^{1/2}\,du = \frac{-\pi}{4}\left[\frac{2u^{3/2}}{3}\right]_9^0 = \frac{-\pi}{4}\left(0-\frac{2\cdot 9^{3/2}}{3}\right) = \frac{9\pi}{2}$$

$$\boxed{\iint_R \sqrt{9-x^2-y^2}\,dA = \frac{9\pi}{2}}$$

b) $$\iint_R \frac{1}{1+x^2+y^2}\,dA = \frac{\pi \ln(5)}{8}$$

$y=0 \quad y=x \quad x^2+y^2=4$

1º quadrante



$y=x$ divide o primeiro quadrante de forma igualitária $\Rightarrow \boxed{0 \le \theta \le \frac{\pi}{4}}$

$0 \le x^2+y^2 \le 4 \Rightarrow 0 \le r^2 \le 4 \Rightarrow \boxed{0 \le r \le 2}$

$R = \{(r,\theta) \mid 0 \le r \le 2,\ 0 \le \theta \le \frac{\pi}{4}\}$

$$\iint_R \frac{dA}{1+x^2+y^2} = \int_0^2 \int_0^{\pi/4} \frac{r}{1+r^2}\,d\theta\,dr = \frac{\pi}{4}\int_0^2 \frac{r}{1+r^2}\,dr$$

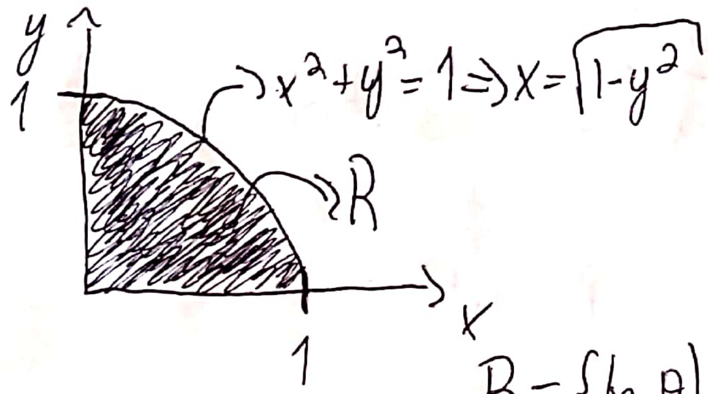
$u = 1+r^2 \Rightarrow du = 2r\,dr \Rightarrow dr = du/2r$

$$= \frac{\pi}{4}\int_1^5 \frac{r}{u}\frac{du}{2r} = \frac{\pi}{8}\int_1^5 u^{-1}\,du = \frac{\pi}{8}\ln|u|\Big|_1^5 = \frac{\pi}{8}(\ln(5)-\ln(1)) = \frac{\pi}{8}\ln(5)$$

$$\boxed{\iint_R \frac{1}{1+x^2+y^2}\,dA = \frac{\pi \ln(5)}{8}}$$

2) I) $\int_0^1 \int_0^{\sqrt{1-y^2}} \cos(x^2+y^2)\,dx\,dy = \frac{\pi}{8}\operatorname{sen}(1)$

$0 \le x^2+y^2 \le 1 \Rightarrow 0 \le r^2 \le 1 \Rightarrow \boxed{0 \le r \le 1}$



θ cobre o 1º quadrante

$\hookrightarrow \boxed{0 \le \theta \le \frac{\pi}{2}}$

$R = \{(r,\theta) \mid 0 \le r \le 1,\ 0 \le \theta \le \frac{\pi}{2}\}$

$$\int_0^1 \int_0^{\sqrt{1-y^2}} \cos(x^2+y^2)\,dx\,dy = \int_0^1 \int_0^{\pi/4} r\cos(r^2)\,d\theta\,dr = \frac{\pi}{4}\int_0^1 r\cos(r^2)\,dr$$

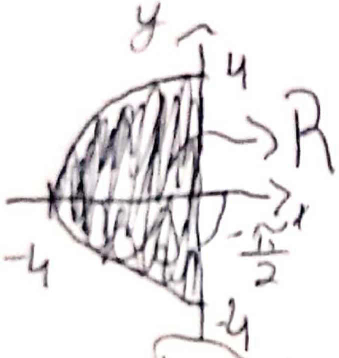
$u = r^2 \Rightarrow du = 2r\,dr \Rightarrow dr = du/2r$

$$= \frac{\pi}{4}\int_0^1 r\cos(u)\frac{du}{2r} = \frac{\pi}{8}\int_0^1 \cos(u)\,du = \frac{\pi}{8}[\operatorname{sen}(u)]_0^1 = \frac{\pi \operatorname{sen}(1)}{8}$$

$$\boxed{\int_0^1 \int_0^{\sqrt{1-y^2}} \cos(x^2+y^2)\,dx\,dy = \frac{\pi \operatorname{sen}(1)}{8}}$$

II) $\int_{-4}^{0}\int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} 3x\,dy\,dx = 128$

Dos limites de y: $y=\pm\sqrt{16-x^2} \Rightarrow y^2=16-x^2 \Rightarrow x^2+y^2=16 \Rightarrow r^2=16 \Rightarrow r=4$



$\rightarrow R=\{(r,\theta)\mid 0\le r\le 4, \frac{3\pi}{2}\le\theta\le\frac{\pi}{2}\}$

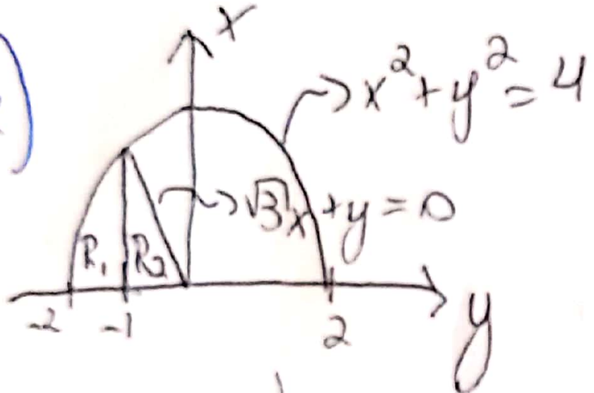
$x=r\cos(\theta) \Rightarrow 3x=3r\cos(\theta)$

$\int_{-4}^{0}\int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} 3x\,dy\,dx = \int_{\frac{3\pi}{2}}^{\pi/2}\int_{0}^{4} 3r^2\cos(\theta)\,dr\,d\theta = \int_{\frac{3\pi}{2}}^{\pi/2}\cos(\theta)\,r^3\Big|_0^4 d\theta = 64\int_{\frac{3\pi}{2}}^{\frac{\pi}{2}}\cos(\theta)\,d\theta$

$=64\left[\operatorname{sen}(\theta)\right]_{\frac{3\pi}{2}}^{\frac{\pi}{2}} = 64(1+1)=128$

$$\boxed{\int_{-4}^{0}\int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} 3x\,dy\,dx = 128}$$

3 a)



$R_2 = \{(x,y) \mid -1 \le x \le 0,\ 0 \le y \le -x\sqrt{3}\}$

$R_2 = \{(x,y) \mid 0 \le y \le \sqrt{3},\ -1 \le x \le -\frac{y}{\sqrt{3}}\}$

$(-1)^2 + y^2 = 4$

$y^2 = 3 \rightarrow y = \sqrt{3}$

$$\iint_{R_2} 2y\,dA = \int_{-1}^{0}\int_{0}^{-x\sqrt{3}} 2y\,dy\,dx = \int_{-1}^{0} y^2\Big|_{0}^{-x\sqrt{3}} dx = \int_{-1}^{0} 3x^2\,dx$$

$$= x^3\Big|_{-1}^{0} = (0^3 - (-1)^3) = 1$$

$$\boxed{\iint_{R_2} 2y\,dA = \int_{-1}^{0}\int_{0}^{-x\sqrt{3}} 2y\,dy\,dx = \int_{0}^{\sqrt{3}}\int_{-1}^{-y/\sqrt{3}} 2y\,dx\,dy = 1}$$

b) $\sqrt{3}x + y = 0 \Rightarrow \sqrt{3}r\cos(\theta) + r\,sen(\theta) = 0$

$sen(\theta) = -\sqrt{3}\cos(\theta) \Rightarrow tg(\theta) = -\sqrt{3} \Rightarrow \boxed{\theta = \frac{5\pi}{6}}$

$R = \{(r,\theta) \mid 0 \leq r \leq 2, \frac{5\pi}{6} \leq \theta \leq \pi\}$

$$\iint_{R_1 \cup R_2} x^2 + y^2\,dA = \int_0^2 \int_{\frac{5\pi}{6}}^{\pi} r \cdot r^2\,d\theta\,dr = \int_0^2 r^3\theta \Big|_{\frac{5\pi}{6}}^{\pi} dr = \int_0^2 r^3 \cdot \frac{\pi}{6}\,dr = \frac{\pi}{6}\frac{r^4}{4}\Big|_0^2$$

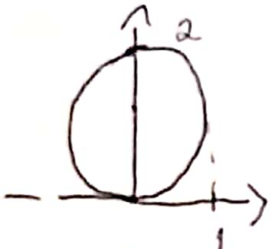
$$= \frac{\pi}{6} \cdot \frac{16}{4} = \frac{4\pi}{6} = \frac{2\pi}{3}$$

$$\boxed{\iint_{R_1 \cup R_2} x^2 + y^2\,dA = \frac{2\pi}{3}}$$

(4) a) $f(x,y)=\sqrt{x^2+y^2}=f(r\cos(\theta), r\,sen(\theta))=\sqrt{r^2}=|r|$

$g(r,\theta)=rf(r\cos(\theta), r\,sen(\theta))=r^2$ $\boxed{g(r,\theta)=r^2}$

$x^2+y^2=2y \Rightarrow x^2+y^2-2y+1=1 \Rightarrow x^2+(y^2-1)=1$



$x^2+y^2=2y \Rightarrow r^2=2r\,sen(\theta) \Rightarrow \boxed{r=2sen(\theta)}$

$R=\{(r,\theta) \mid 0\le\theta\le\pi,\ 0\le r\le 2sen(\theta)\}$

$$V=\int_0^{\pi}\int_0^{2sen(\theta)} g(r,\theta)\,dr\,d\theta=\int_0^{\pi}\int_0^{2sen(\theta)} r^2\,dr\,d\theta=\int_0^{\pi}\frac{r^3}{3}\Big|_0^{2sen(\theta)}d\theta=\frac{8}{3}\int_0^{\pi}sen^3(\theta)\,d\theta$$

$$\int sen^3(x)\,dx=\int sen(x)\cdot sen^2(x)\,dx=\int sen(x)(1-\cos^2(x))\,dx$$

$$u=\cos(x)\Rightarrow du=-sen(x)dx\Rightarrow dx=\frac{-du}{sen(x)}$$

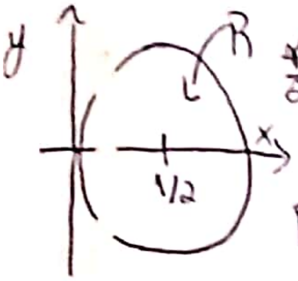
$$\int sen^3(x)dx=\int\frac{sen(x)(1-u^2)du}{-sen(x)}=\int u^2-1\,du=\frac{u^3}{3}-u=\boxed{\frac{\cos^3(x)}{3}-\cos(x)}$$

$$V=\frac{8}{3}\left[\frac{\cos^3(x)}{3}-\cos(x)\right]_0^{\pi}=\frac{8}{3}\left[\left(\frac{-1-1}{3}\right)-(-1-1)\right]=\frac{8}{3}\left(2-\frac{2}{3}\right)=\frac{32}{9}$$

$$\boxed{V=\frac{32}{9}\,u.v}$$

b) $f(x,y) = 1 - x^2 - y^2 = 1 - r^2$ $\quad \boxed{g(r,\theta) = r - r^3}$

$$x^2 - x + y^2 = 0 \Rightarrow x^2 - x + \frac{1}{2} + y^2 = \frac{1}{2} \Rightarrow \left(x - \frac{1}{4}\right)^2 + y^2 = \frac{1}{2}$$



$$\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$$

$$x^2 - x + y^2 = 0 \Rightarrow x^2 + y^2 = x \Rightarrow r^2 = r\cos(\theta) \Rightarrow r = \cos(\theta)$$

$$R = \left\{(r,\theta) \mid -\frac{\pi}{2} \le \theta \le \frac{\pi}{2},\ 0 \le r \le \cos(\theta)\right\}$$

$$V = \iint_R g(r,\theta)\, dA = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{\cos(\theta)} r - r^3 \, dr\, d\theta = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2(\theta)}{2} - \frac{\cos^4(\theta)}{4}\, d\theta$$

$$\int \cos^2(\theta)\, d\theta = \int \frac{\cos(2\theta)+1}{2}\, d\theta = \boxed{\frac{1}{2}\left(\frac{\mathrm{sen}(2\theta)}{4} + \theta\right)}$$

$$\begin{cases} \cos(2\theta) = \cos^2(\theta) - \mathrm{sen}^2(\theta) \\ \mathrm{sen}^2(\theta) + \cos^2(\theta) = 1 \end{cases} \Rightarrow \cos(2\theta) = 2\cos^2(\theta) - 1 \Rightarrow \boxed{\cos^2(\theta) = \frac{\cos(2\theta)+1}{2}}$$

$$\int \cos^4(\theta)\, d\theta = \cos^3(\theta)\,\mathrm{sen}(\theta) + 3\int \cos^2(\theta)\,\mathrm{sen}^2(\theta)\, d\theta$$

$u = \cos^3(\theta) \qquad dv = \cos(\theta)$

$du = -3\cos^2(\theta)\,\mathrm{sen}(\theta) \qquad v = \mathrm{sen}(\theta)$

$$\int \cos^2(\theta)\,\mathrm{sen}^2(\theta)\, d\theta = \int \frac{1 - \cos(4\theta)}{8}\, d\theta = \frac{1}{8}\int 1 - \cos(4\theta)\, d\theta = \frac{1}{8}\left[\theta - \frac{\mathrm{sen}(4\theta)}{4}\right]$$

$$\left(\cos^2(\theta)\,\mathrm{sen}^2(\theta) = \frac{1 - \cos(4\theta)}{8}\right)$$

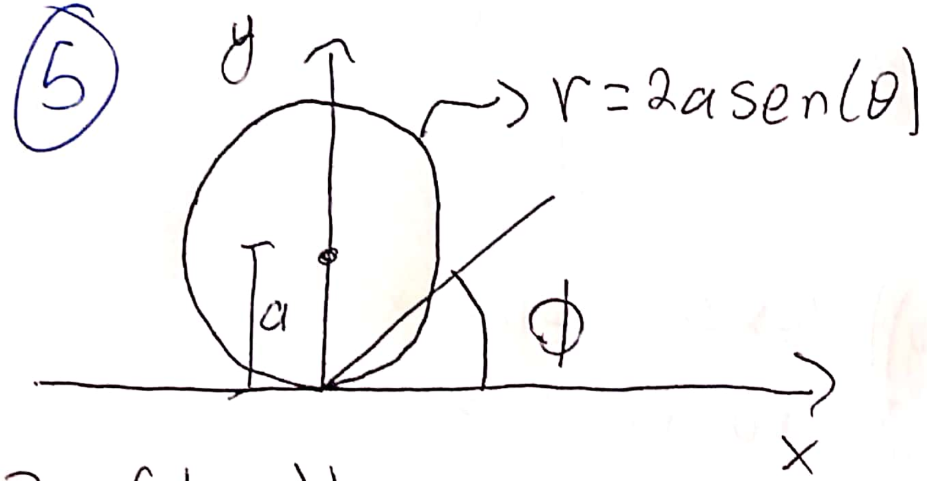
$$\boxed{\int \cos^4(\theta)\, d\theta = \cos^3(\theta)\,\mathrm{sen}(\theta) + \frac{3}{8}\left(\theta - \frac{\mathrm{sen}(4\theta)}{4}\right)}$$

$$V = \left[\frac{1}{4}\left(\cancel{\frac{\mathrm{sen}(2\theta)}{4}}^{\to 0} + \theta\right) - \frac{1}{4}\left(\cancel{\cos^3(\theta)\,\mathrm{sen}(\theta)}^{\to 0} + \frac{3}{8}\left(\theta - \cancel{\frac{\mathrm{sen}(4\theta)}{4}}^{\to 0}\right)\right)\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$V = \frac{1}{4}\left[\theta - \frac{3\theta}{8}\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{1}{4}\left[\frac{5\theta}{8}\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{5}{32}\,\theta\Big|_{-\pi/2}^{\pi/2} = \frac{5\pi}{32}$$

$$\boxed{V = \frac{5\pi}{32}\ u.v.}$$

5



$$R = \{(r,\theta) \mid 0 \leq r \leq 2a\,sen(\theta),\ 0 \leq \theta \leq \phi\}$$

$$A = \iint_R dA = \int_0^{\phi}\int_0^{2a\,sen(\theta)} dr\,d\theta = \int_0^{\phi} 2a\,sen(\theta)\,d\theta = 2a\int_0^{\phi} sen(\theta)\,d\theta$$

$$A = 2a\left[-\cos(\theta)\right]_0^{\phi} = 2a(-\cos(\phi)+1) = 2a(1-\cos(\phi))$$

$$\boxed{A = 2a(1-\cos(\phi))}$$