



PRACTICE 2

OPTIMAL DESIGN OF A STORAGE UNIT

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Introduction

The goal of this practice is the optimisation on a component level of a heat storage system based on a Phase Change Material (PCM), its peculiarity is to store thermal energy through the phase change.

In particular, the PCM initially solid receives heat and when its temperature reaches the melting point (in our case 60 °C) it becomes liquid. In this stage the PCM is able to store heat, this heat is stored through the variation of the enthalpy of the material and the energy stored can be given back easily by cooling down the material through the solidification process.

Regarding the charging and discharging process, we can associate a sensible heat and a latent heat: the sensible heat occurs during the charge when we are heating up the material but we have not reached the melting point yet or when we are overheating the material and during the discharge when we are cooling down the PCM until the solidification point or if we want to overcool the material, while the latent heat is the heat strictly associated with the phase change (when the temperature of the PCM remains constant because the phase change is occurring) which is also the heat the we store and release when it's needed.

- Charging Phase

Initially the PCM is cold and its average temperature increases with time, in this case we have sensible heat, as shown in figure 1. When the material reaches 60 °C the phase change occurs and theoretically this phase is isothermal, but since the PCM is not a pure material in our case the temperature will slightly increase during the melting, during this process we have latent heat.

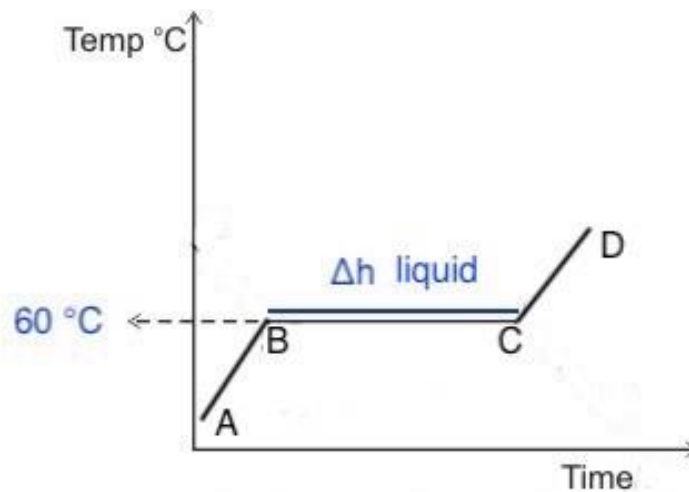


Figure 1 - Charging Phase

- Discharging Phase

In this case we start from a liquid that cools down with time, leaking sensible heat. When the solidification temperature is reached the temperature remains almost constant (slightly decreasing). Like in the charging phase, in this process we have latent heat shown in figure 2.

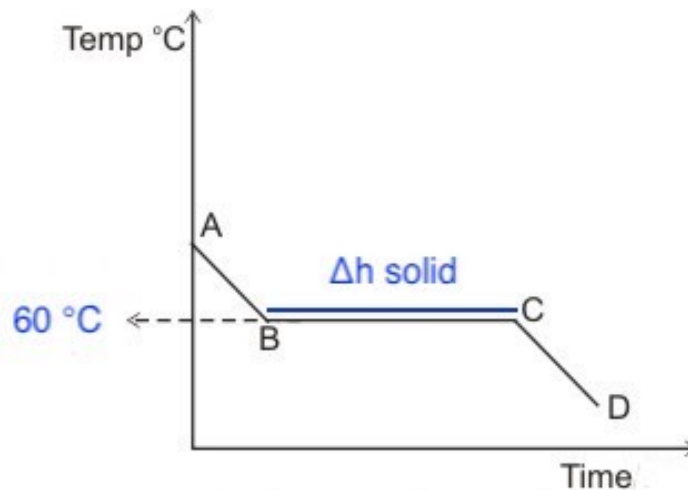


Figure 2 - Discharging Phase

One important thing to say is that Δh_{solid} is equal to Δh_{liquid} , so in the discharging phase we have the same amount of enthalpy that we have previously stored.

The PCM system is often coupled with a district heating system in which hot water heats up, thanks to an heat exchanger, a second circuit of water that goes into the building that

needs hot water. In parallel to the heat exchanger we can consider our PCM system: when there is no need for hot water in the building, the water of the primary circuit feeds the circuit of the PCM, at this point the material heats up and becomes liquid. On the other end when hot water is needed, the water of the second circuit enters the storage unit where it heats up, cooling down the PCM, and comes back into the building.

Geometry of the system

The storage unit has a shell-and-tube structure shown in figure 3, where the fluid passes within the tubes and the PCM is located in the volume outside them. In the analysis we consider a portion of a single tube with the corresponding external cylindrical volume of PCM.

Because of the poor conductivity of the PCM material, heat transfer has been enhanced by installing four Y-shaped fins on the external surface of the pipe.

Single tube portion	
internal diameter	0,01 m
external diameter	0,15 m
length	0,5 m

Properties of the materials					
PCM		Water		Fins (aluminum)	
Conductivity	0,15 W/mK	Density	990 kg/m ³	Conductivity	180 W/mK
Density	800 kg/m ³	Specific heat	5185 J/kgK	Density	2750 kg/m ³
Melting T	56-64°C	Conductivity	0,6 W/mK	Specific heat	880 J/kgK
Specific heat	840 J/kgK	Dynamic viscosity	0,0008 kg/ms		
Latent heat capacity	200 kJ/kg				

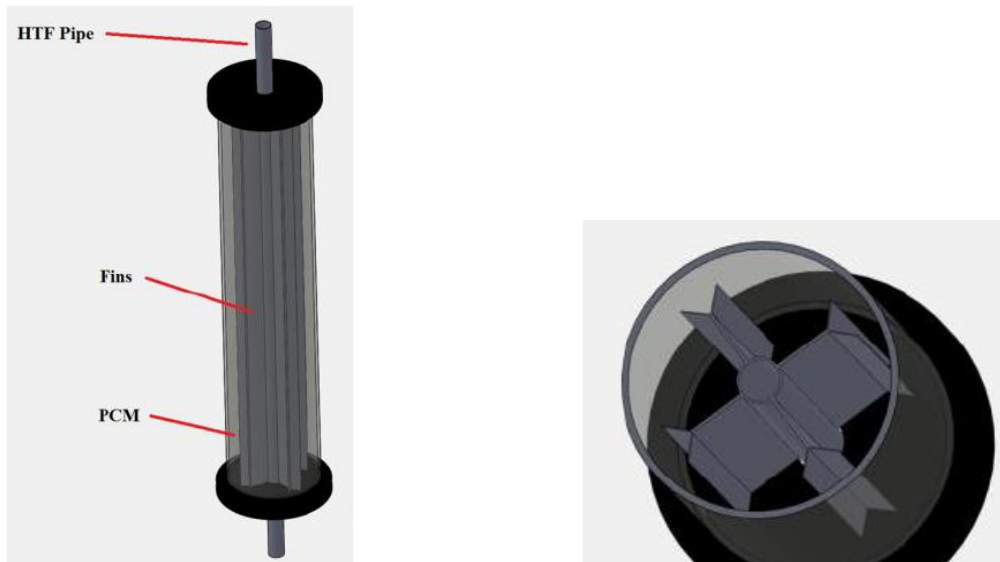


Figure 3 – Geometry of the system, 3D view and 2D horizontal cross section

The aim of this practice is to obtain an optimized design of the fins, assuming a simplified approach, because of the transient nature of the process and the complexity of the heat transfer occurring in the PCM, so the path has been divided in four steps.

Steady-state optimization

The first step consists in a steady-state optimization, performed by considering a 2D horizontal cross section:

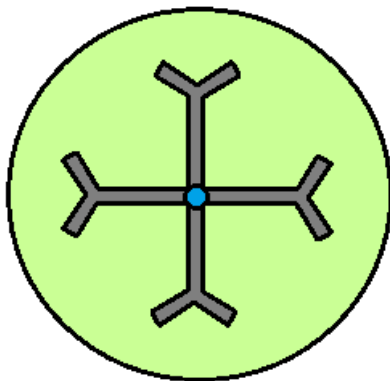


Figure 4 – 2D horizontal cross section

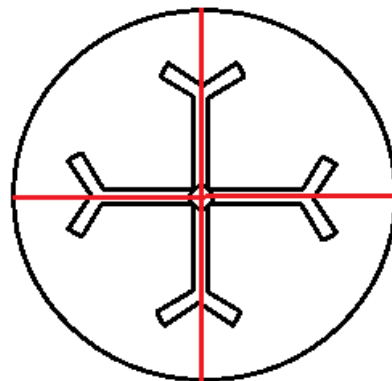


Figure 5 – 2D horizontal cross section, symmetry lines

In figure 4 is represented the cross-section scheme obtained by cutting the pipe horizontally at middle length, so we can assume the temperature of the water (reported in blue in figure 4) as $37,5^{\circ}\text{C}$.

In order to simplify our analysis we proceeded by considering the fact of having symmetries, in particular dividing the circle represented in figure 4 in four parts (as in figure 5) we expect that they will work in the same way.

Proceeding in an analogous way we can subdivide again each slice in two parts, so each sector will be 1/8 of the total cross section area.

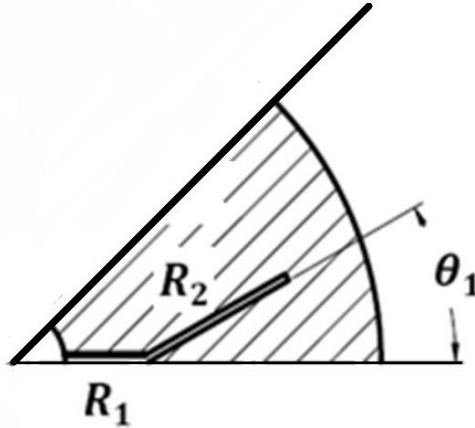


Figure 6 – Portion of the system considered for the analysis

Considering the discharging phase of the PCM, in this step we would like that the latent heat of the PCM is exchanged in a proper time, about 60 minutes, so in order to obtain steady state we need to introduce a corresponding source term.

$$\phi = \frac{\Delta h}{\Delta \tau}$$

In that way we are considering a sort of steady state, where the PCM is on average at a certain temperature, and water too.

In that phase it is like the PCM is generating heat, which can be transferred both to the fin, which then transfers it to the water, or directly to the water, we have a difference in terms of thermal conductivity between the two paths.

From the text is assumed that the temperature of the internal boundary (water pipe) is at the same temperature of the water, 37,5°C (Dirichlet boundary condition), from the boundary to the water we have convective heat transfer $\phi_{convective}$.

In order to obtain that a ΔT between PCM and water is required, so it assumes the form:

$$\phi_{convective} = h(T - T_{\infty})$$

The difference of temperature is assumed to be small enough so that we can assume T almost equal to 37,5°C ($T_{\infty} = 37,5^{\circ}C$), in that way the h coefficient has to be very high, but in that way we can choose to not consider the water in our domain.

What we obtain is that in our system we consider only the PCM and the aluminum alloy of the fin, we do not model the water.

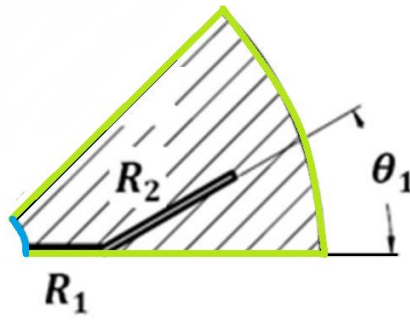


Figure 7 – System boundaries

In figure 7 the blue area is the one where we have imposed Dirichlet boundary condition, while the green boundaries are adiabatic surfaces (the upper and lower ones because they are symmetry axis, the right one because there is not a T gradient perpendicular to the surface, so it is adiabatic).

Because of the adiabatic surfaces represented in green, the heat flux is entirely supplied to the water.

Coming back to the aim of this first step, optimizing the design of the system (selecting the optimal shape of the fin), it is necessary to maximize k, the thermal coefficient, in order to obtain the best Φ possible.

$$\Phi = k * A * (T_{PCM} - T_{water})$$

Unfortunately k is a quite complex term, accounting both for material and geometry, but maximizing k is the same of minimizing temperature difference.

The flux is fixed and the water temperature is 37°C so we have to minimize the average temperature of the PCM, which becomes our objective function.

We decide to use the method of the quadratic approximation. It is an indirect method that can be used for a single independent variable and convex objective function.

To approximate the objective function we used a second order polynomial. We used a system of three equations, calculated at the beginning, at the end of the domain and at an intermediate point, so as to be able to calculate the three coefficients a, b, c necessary to define the equation of the parabola.

$$F = a * x^2 + b * x + c$$

F is the Temperature and x is the angle θ . Thanks to the use of Comsol we calculated the temperature for three θ values (at the two boundaries and at almost the center of the interval) equal to 0°, 30° and 65°, finding respectively the temperatures of 171,21°C, 102,55°C and 148 °C.

We solve the system and we find the coefficients.

a	b	c
0,0550806	-3,93408	171,21

As we know, to calculate the minimum of a function, it is necessary to set the first derivative with respect to x equal to zero. In this way we found the minimum

$$x = -\frac{b}{2a}$$

And we find $\theta = 35,71^\circ$

Here we report the graph in figure 8, made using other theta values.

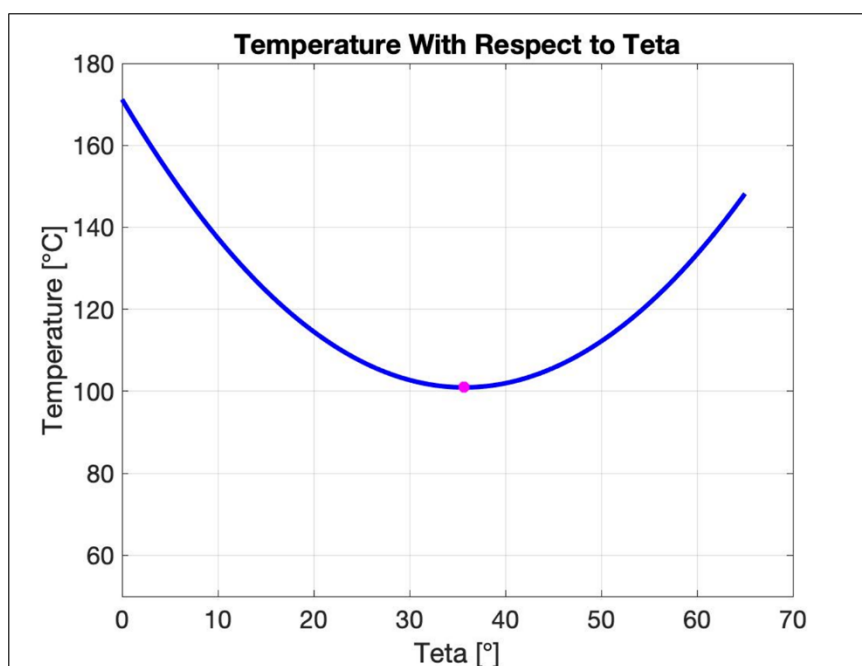


Figure 8 – T_{pcm} vs $Teta$ curve, Quadratic approximation

The temperature in the real case of PCM is around 60°C , even if in our case is quite higher, about $100,96^\circ\text{C}$ (calculated using the parabolic function). This can happen because of the fact that, even if the design of the system we found is the optimal one, it could be not able to exchange the amount of heat we desire in the conditions we impose: in our case, in order to exchange the amount of heat in 60 minutes, with that optimized geometry, the average temperature of the PCM required is higher than the real value, so we could change something else (more material, different geometry of the system, larger time interval,...), because we can't do better acting only on the angle θ .

TRANSIENT

The transient lasts 3600 s, where the initial temperature of the discharging phase is considered at 75°C in the PCM.

In the model it is not considered a heat source as in the steady-state, but the heat release is considered through the properties set in the fluid. The phase change of the heat release is associated with a temperature range of 4 °C, (corresponding to the half range between 64-56 °C, because this is the parameter in the equation). The specific heat of the solid phase corresponds to the specific heat of the liquid; the Gaussian expression is the following:

$$c_p(T) = c_{pS} + \frac{L}{\Delta T_{SL} * \sqrt{\pi}} * e^{-\frac{(T-T_{av})^2}{\Delta T_{SL}}}$$

Where c_{pS} is the specific heat of the PCM, ΔT_{SL} is the condensing/melting temperature half range and T_{av} is the average temperature in the phase change range. The figure 9 represents the Gaussian function corresponding to the latent heat.

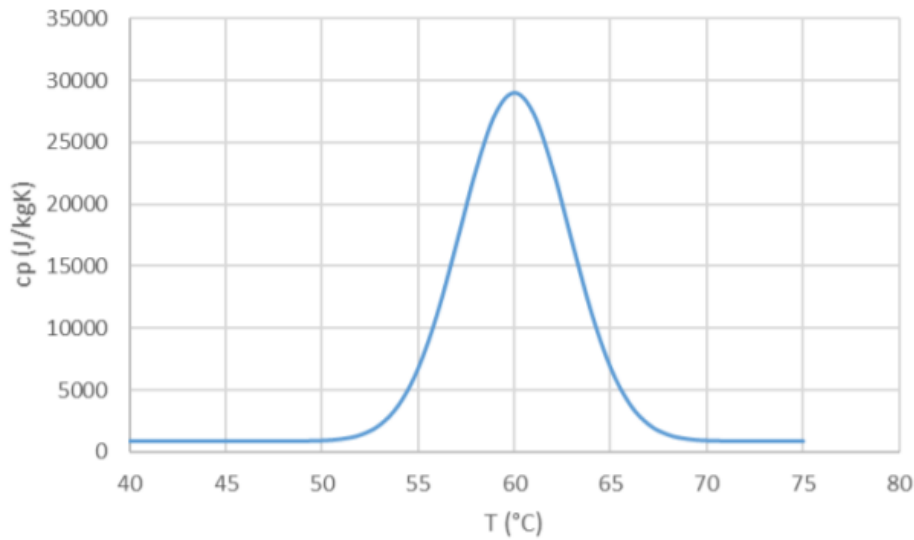


Figure 9 - Equivalent specific heat for the PCM

At 64 °C the condensing phase starts till 56°C when the solid phase is reached. The latent heat is associated with isothermal phase change for pure fluids, while sensible heating is associated with temperature variation. During the entire process

$$q = q_{sensible} + q_{latent} = \int_{T_2}^{T_1} c_{pL} * dT + \int_{T_4}^{T_3} c_{pS} * dT + \int_{T_3}^{T_2} c_{peq} * dT$$

The specific heat considers a Gaussian function during the phase change, where the area is the integral of Cp and it is equal to the enthalpy variation of the phase change. In the equation we will consider half of the temperature range.

In this part of the practice we expect to optimize two design variables: the angle θ and the radius R1 (length of the first branch). The volume for the thin fins is fixed, so the variation of R1 must respect this constraint. The objective function to maximize is the total heat transfer during the process.

Considering the geometry, the total volume is at 75 °C. Starting with the transient, the heat transfer is exchanged with the water. The heat flux must be integrated along the time of the transient.

The thermal energy is represented by the formula $Q_{thermal} = \int_0^{3600} \Phi * dt$

In order to find the maximal value, we can use another approach.

From the first thermodynamic law $\Phi - W_\tau = \frac{\partial U^\tau}{\partial \tau} + \sum_{k=1}^k G * h_k^\tau$

The control surfaces experience no mass flow $\Phi = \frac{\partial U^\tau}{\partial \tau}$

$Q = \left| \int_0^{3600} \Phi * dt \right| = \Delta U$ the variation of internal energy corresponds to
 $\Delta U = U_{in} - U_{out} = M_c \Delta T = M_c * (T_{in} - T_{min})$

The minimum temperature can be found by a conjugate gradient method.

The system we have to solve considers multiple independent variables. The minimum can be reached through subsequent search in perpendicular directions. For every iteration we applied the golden section method whether for the angle or for the branch.

Under the “conjugate method” are associated various methods, only differing by the selection of the distance to move at each iteration. For the Fletcher and Reeves method, the distance is obtained using an approximation of the objective function throw third order polynomial.

The “Golden section method” is applied for a single independent variable and convex objective function. It is an iterative method proceeding by steps referring to figure 10:

1) Definition of the domain $[a, b]$ $b > 2$

2) Consider two additional points

$$c = a + s$$

$$d = b - s$$

$$\text{where } s = t * (b - a) \text{ and } t = \frac{2}{1 + \sqrt{5}}$$

3) If $F(d) < F(c)$ the domain is defined with this operation

$$a = a$$

$$b = d$$

4) If $F(d) < F(c)$ the domain is defined with this operation

$$a = d$$

$$b = b$$

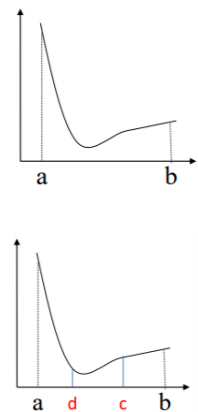


Figure 10

These passages are repeated till the minimum of the function is found.

In our case, the steps of the optimization are in the following tables.

Step 1 (θ)									
		a	b	c	d			s	t
		0,00	65,00	40,17	24,83	fd>fc		40,17	0,62
Tpcm		57,06	56,48	56,60	56,88				
		a	b	c	d			s	t
		24,83	65,00	49,66	40,17	fd>fc		24,83	0,62
Tpcm		56,88	56,48	56,55	56,60				
		a	b	c	d			s	t
		40,17	65,00	55,52	49,66	fd<fc		15,34	0,62
Tpcm		56,60	56,48	56,18	56,55				
		a	b	c	d			s	t
		49,66	65,00	59,14	55,52	fd>fc		9,48	0,62

Tpcm		56,55	56,48	55,48	56,18					
		a	b	c	d				s	t
		55,52	65,00	61,38	59,14	fd<fc			5,86	0,62
Tpcm		56,18	56,48	55,78	55,48					
		a	b	c	d				s	t
		55,52	61,38	59,14	57,76	fd>fc			3,62	0,62
Tpcm		56,18	55,78	55,48	55,70					
		a	b	c	d				s	t
		57,76	61,38	59,99	59,14	fd<fc			2,24	0,62
Tpcm		55,70	55,78	55,57	55,48					
		a	b	c	d				s	t
		59,14	61,38	60,52	59,99	fd<fc			1,38	0,62
Tpcm		55,48	55,78	55,64	55,57					
		a	b	c	d				s	t
		59,14	60,52	59,99	59,67	fd<fc			0,86	0,62
Tpcm		55,48	55,64	55,57	55,53					
		a	b	c	d				s	t
		59,14	59,99	59,67	59,47	fd<fc			0,53	0,62
Tpcm		55,48	55,57	55,53	55,49					
θ	59,14	°								
Tpcm	55,48	°C								

Step 2 (R1)										
		a	b	c	d				s	t
		0,0160	0,0500	0,0370	0,0290	fd<fc			0,021	0,62
Tpcm		57,1700	59,4960	57,9110	57,4580					
		a	b	c	d				s	t
		0,0160	0,0370	0,0290	0,0240	fd<fc			0,013	0,62
Tpcm		57,1700	57,9110	57,4580	57,0160					
		a	b	c	d				s	t
		0,0160	0,0290	0,0240	0,0210	fd<fc			0,008	0,62
Tpcm		57,1700	57,4580	57,0160	55,4760					
		a	b	c	d				s	t
		0,0160	0,0240	0,0210	0,0191	fd>fc			0,005	0,62
Tpcm		57,1700	57,0160	55,4760	55,8920					
		a	b	c	d				s	t
		0,0191	0,0240	0,0221	0,0210	fd<fc			0,003	0,62
Tpcm		55,8920	57,0160	55,8850	55,4760					
		a	b	c	d				s	t
		0,0191	0,0221	0,0210	0,0202	fd>fc			0,002	0,62
Tpcm		55,8920	55,8850	55,4760	55,5760					

		a	b	c	d				s	t
		0,0202	0,0221	0,0214	0,0210	fd<fc			0,001	0,62
Tpcm		55,5760	55,8850	55,7740	55,4760					
		a	b	c	d				s	t
		0,0202	0,0214	0,0210	0,0207	fd>fc			0,001	0,62
Tpcm		55,5760	55,7740	55,4760	55,5320					
		a	b	c	d				s	t
		0,0207	0,0214	0,0211	0,0210	fd<fc			0,0004	0,62
Tpcm		55,5320	55,7740	55,5960	55,4760					
		a	b	c	d				s	t
		0,0207	0,0211	0,0210	0,0209	fd>fc			0,0003	0,62
Tpcm		55,5320	55,5960	55,4760	55,5240					
R1 (larg)	0,02096	m								
R2 (larg)	0,04004	m								
Tpcm	55,48	°C								

Step 3 (θ)										
		a	b	c	d				s	t
		0,00	65,00	40,17	24,83	fd>fc			40,17	0,62
Tpcm		60,602	56,478	56,664	56,7840					
		a	b	c	d				s	t
		24,83	65,00	49,66	40,17	fd<fc			24,83	0,62
Tpcm		56,7840	56,478	56,689	56,664					
		a	b	c	d				s	t
		24,83	49,66	40,17	34,31	fd>fc			15,34	0,62
Tpcm		56,7840	56,689	56,664	56,693					
		a	b	c	d				s	t
		34,31	49,66	43,79	40,17	fd<fc			9,48	0,62
Tpcm		56,693	56,689	56,666	56,664					
		a	b	c	d				s	t
		34,31	43,79	40,17	37,93	fd<fc			5,86	0,62
Tpcm		56,693	56,666	56,664	56,507					
		a	b	c	d				s	t
		34,31	40,17	37,93	36,55	fd>fc			3,62	0,62
Tpcm		56,693	56,664	56,507	56,678					
		a	b	c	d				s	t
		36,55	40,17	38,79	37,93	fd<fc			2,24	0,62
Tpcm		56,678	56,664	56,662	56,51					
		a	b	c	d				s	t
		36,55	38,79	37,93	37,40	fd>fc			1,38	0,62
Tpcm		56,678	56,662	56,507	56,671					
		a	b	c	d				s	t
		37,40	38,79	38,26	37,93	fd>fc			0,86	0,62

Tpcm		56,671	56,662	56,506	56,507					
		a	b	c	d				s	t
		37,93	38,79	38,46	38,26	fd<fc			0,53	0,62
Tpcm		56,507	56,662	56,666	56,506					
		a	b	c	d				s	t
		37,93	38,46	38,26	38,14	fd<fc			0,33	0,62
Tpcm		56,507	56,666	56,506	56,505					
		a	b	c	d				s	t
		37,93	38,26	38,14	38,06	fd>fc			0,20	0,62
Tpcm		56,507	56,506	56,505	56,507					
θ	38,14	°								
Tpcm	56,505	°C								

Step 4 (R1)										
		a	b	c	d				s	t
		0,0050	0,0160	0,0118	0,0092	fd<fc			0,0068	0,6180
Tpcm		55,3230	56,1390	55,5300	55,2090					
		a	b	c	d				s	t
		0,0050	0,0118	0,0092	0,0076	fd<fc			0,0042	0,6180
Tpcm		55,3230	55,5300	55,2090	55,1410					
		a	b	c	d				s	t
		0,0050	0,0092	0,0076	0,0066	fd>fc			0,0026	0,6180
Tpcm		55,3230	55,2090	55,1410	55,1650					
		a	b	c	d				s	t
		0,0066	0,0092	0,0082	0,0076	fd>fc			0,0016	0,6180
Tpcm		55,1650	55,2090	55,1400	55,1410					
		a	b	c	d				s	t
		0,0076	0,0092	0,0086	0,0082	fd<fc			0,0010	0,6180
Tpcm		55,1410	55,2090	55,1500	55,1400					
		a	b	c	d				s	t
		0,0076	0,0086	0,0082	0,0080	fd<fc			0,0006	0,6180
Tpcm		55,1410	55,1500	55,1400	55,1130					
		a	b	c	d				s	t
		0,0076	0,0082	0,0080	0,0078	fd>fc			0,0004	0,6180
Tpcm		55,1410	55,1400	55,1130	55,1140					
		a	b	c	d				s	t
		0,0078	0,0082	0,0081	0,0080	fd=fc			0,0002	0,6180
Tpcm		55,1140	55,1400	55,133000	55,113000					
R1 (larg)	0,0080	m								
R2 (larg)	0,0530	m								

Tpcm	55,113	°C								
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For the fifth step, in particular, we decided to maintain the starting interval that goes from 0 degrees to 65 degrees even if our R1 is so low that an angle of 65 degrees is impossible because the fin would get out of the computational domain.

We decided to go with such a wide interval of θ because we did not want to miss out the minimum of Tpcm if this was to occur for an angle near to 65 degrees (but also compatible to the computational domain), moreover the first value of “s” is so high that the obtained c and d are compatible with the domain so we had no trouble calculating the respective Tpcm.

Step 5 (θ)										
		a	b	c	d				s	t
		0,000	65,000	40,172	24,828	fd>fc			40,17	0,62
Tpcm		60,292	-	55,131	55,569					
		a	b	e	f				s	t
		24,828	65,000	49,656	40,172	fd<fc			24,83	0,62
Tpcm		55,569	-	57,770	55,131					
		a	b	g	h				s	t
		24,828	49,656	40,172	34,311	fd>fc			15,34	0,62
Tpcm		55,569	57,770	55,131	55,299					
		a	b	i	l				s	t
		34,311	49,656	43,795	40,172	fd<fc			9,48	0,62
Tpcm		55,299	57,770	55,812	55,131					
		a	b	m	n				s	t
		34,311	43,795	40,172	37,933	fd<fc			5,86	0,62
Tpcm		55,299	55,812	55,131	55,125					
		a	b	o	p				s	t
		34,311	40,172	37,933	36,550	fd>fc			3,62	0,62
Tpcm		55,299	55,131	55,125	55,203					
		a	b	q	r				s	t
		36,550	40,172	38,789	37,933	fd>fc			2,24	0,62
Tpcm		55,203	55,131	55,115	55,125					
		a	b	s	t				s	t
		37,933	40,172	39,317	38,789	fd>fc			1,38	0,62
Tpcm		55,125	55,131	55,100	55,155					
		a	b	u	v				s	t
		38,789	40,172	39,644	39,317	fd<fc			0,86	0,62
Tpcm		55,155	55,131	55,110	55,100					
		a	b	w	x				s	t
		38,789	39,644	39,317	39,115	fd<fc			0,53	0,62
Tpcm		55,155	55,110	55,100	55,09					
		a	b	y	z				s	t
		38,789	39,317	39,115	38,990	fd>fc			0,33	0,62

Tpcm		55,155	55,100	55,090	55,107					
		a	b	aa	bb				s	t
		38,990	39,317	39,192	39,115	fd<fc			0,20	0,62
Tpcm		55,107	55,100	55,117	55,090					
θ	39,115	°								
Tpcm	55,090	°C								

Step 6 (R1)										
		a	b	c	d				s	t
		0,0050	0,0160	0,0118	0,0092	fd<fc			0,0068	0,6180
Tpcm		55,4840	56,1240	55,4620	55,1310					
		a	b	c	d				s	t
		0,0050	0,0118	0,0092	0,0076	fd<fc			0,0042	0,6180
Tpcm		55,4840	55,4620	55,1310	55,1050					
		a	b	c	d				s	t
		0,0050	0,0092	0,0076	0,0066	fd>fc			0,0026	0,6180
Tpcm		55,4840	55,1310	55,1050	55,2090					
		a	b	c	d				s	t
		0,0066	0,0092	0,0082	0,0076	fd>fc			0,0016	0,6180
Tpcm		55,2090	55,1310	55,0920	55,1050					
		a	b	c	d				s	t
		0,0076	0,0092	0,0086	0,0082				0,0010	0,6180
Tpcm		55,1050	55,1310	55,0990	55,0920	fd<fc				
R1 (larg)	0,0082	m								
R2 (larg)	0,0528	m								
Tpcm	55,092	°C								

Through this method we obtain the minimum temperature of the PCM, where the system is composed by PCM and the fins. In the figure 11 we graphically represent the steps of the conjugate method.

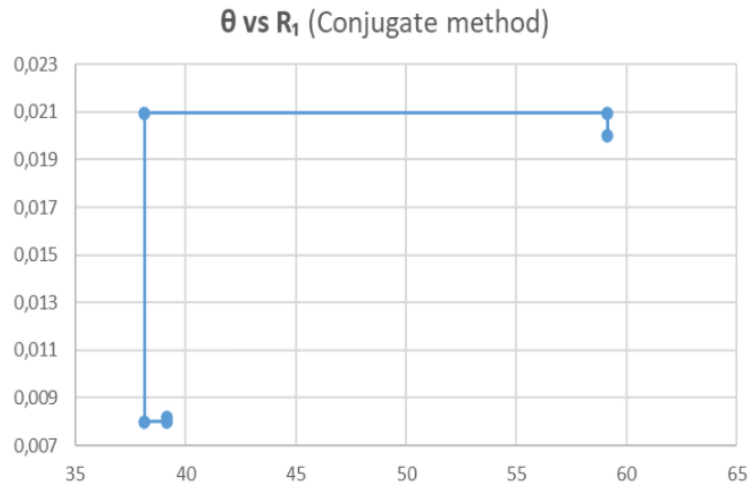


Figure 11 - Conjugate method

In the following point we consider the system without the fins: the goal now is to evaluate the thermal conductivity k of the homogeneous material, obtaining a final temperature as much as similar to the previous evolution. The other properties remain the same. The final value of the homogeneous conductivity is 10.2 W/mK. The figure 12 represents the comparison between the temperature values for homogeneous conductivity and the baseline one.

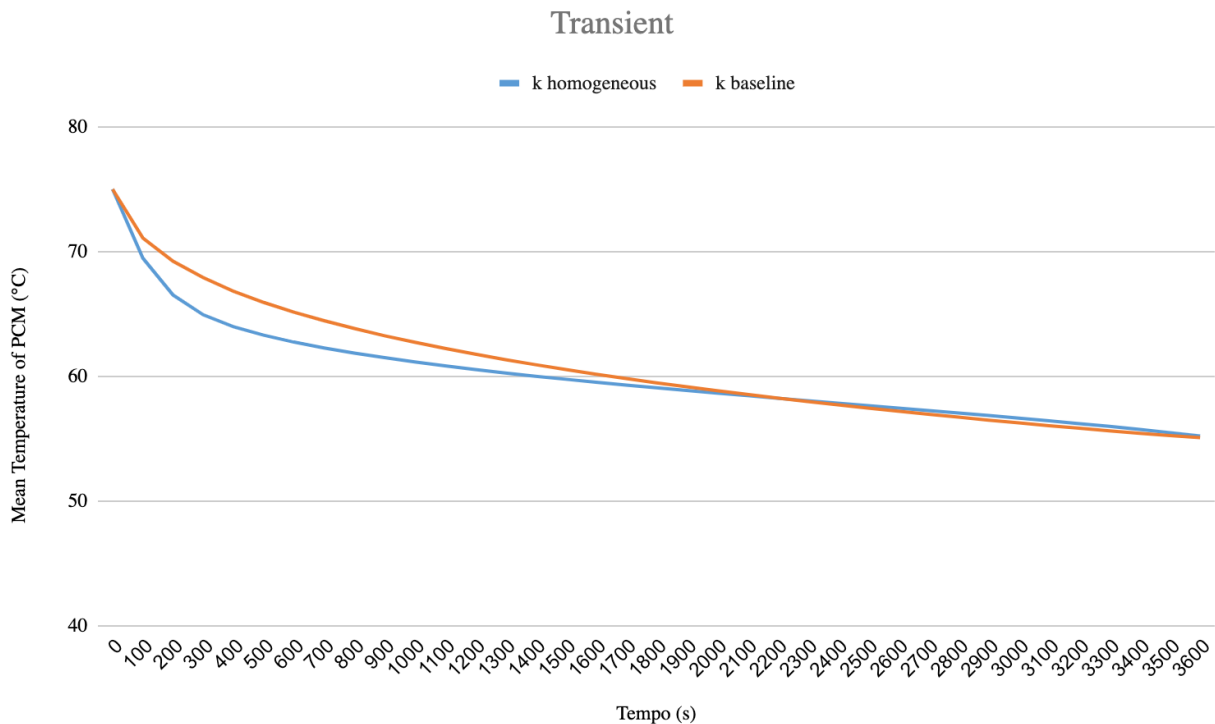


Figure 12 - Conductivity Homogeneous vs Baseline

2D VERTICAL CROSS SECTION

In this part the vertical cross section is taken into account: the analysis will consider the domain composed by 2D axial-symmetric as reported in the figure 13.

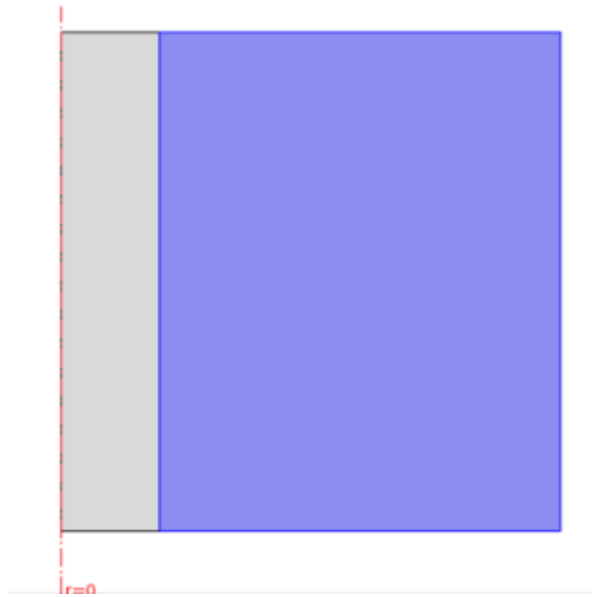


Figure 13 - Axial symmetric cross section

The water flows into the pipe from the bottom to the top, while the blue part of the figure represents the ensemble of the PCM and the fins, obtained at the previous step in order to remove the fins and consider a homogeneous environment, with the k value we found before. This hypothesis lets us perform the 2D analysis, instead of considering a 3D structure.

In this step the PCM (consisting in the ensemble PCM + fins) is considered as steady, so only the thermal problem will be solved for it, because there is not any velocity. For the water we will solve both fluid dynamic and thermal problem, finally the expected result is the water temperature at the outlet section.

The Fourier equation is the following

$$\rho * c_p * \nabla T = \nabla(K \nabla T) + q$$

Where the transient term for the solid is neglected.

The transient process will show the evolution of the water temperature exiting the upper cross section. It is moreover interesting to calculate the velocity profile of the water inside the pipe, in order to use in the solution of thermal problem (the advective contribution of the energy balance).

In that way we solve the fluid dynamic problem at the beginning, and then we are considering the full process as steady state (the fluid flow occurs in steady state conditions, without variations of the velocity profile occurring in time), the thermal profile instead is solved in transient state.

The figure 14 shows that the fully developed profile of the velocity in the tube has a parabolic dependence from the radius, reached after a certain length where there is a flow development from the one at the inlet.

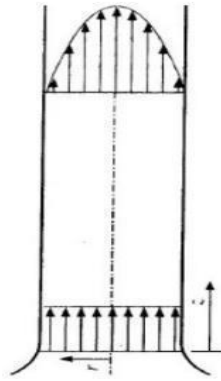


Figure 14 - Velocity profile of water in a pipe

Because of no slip condition, the water particles closer to the axis enter with a certain velocity ($u=0,1$ m/s, but the ones closer to the boundaries are subjected to viscosity, so their velocity tends to reduce to 0.

This effect propagates from the boundaries towards the center, as soon as the water is proceeding along the pipe, so from a certain section on the effect of viscosity will be perceived by the entire domain, producing the expected parabolic velocity profile.

More in general we will consider a laminar flow because the water velocity is supposed to be small. The conductivity is the one we found at the end of the previous step, for the ensemble PCM and fins (10.2 W/mK).

At the boundary is considered the non-slip assumption, and also the axial-symmetry geometry. At the inlet the velocities we considered are respectively $u=0.1$ m/s and $u=0.3$ m/s, in order give a look at the T profile variation with water velocity.

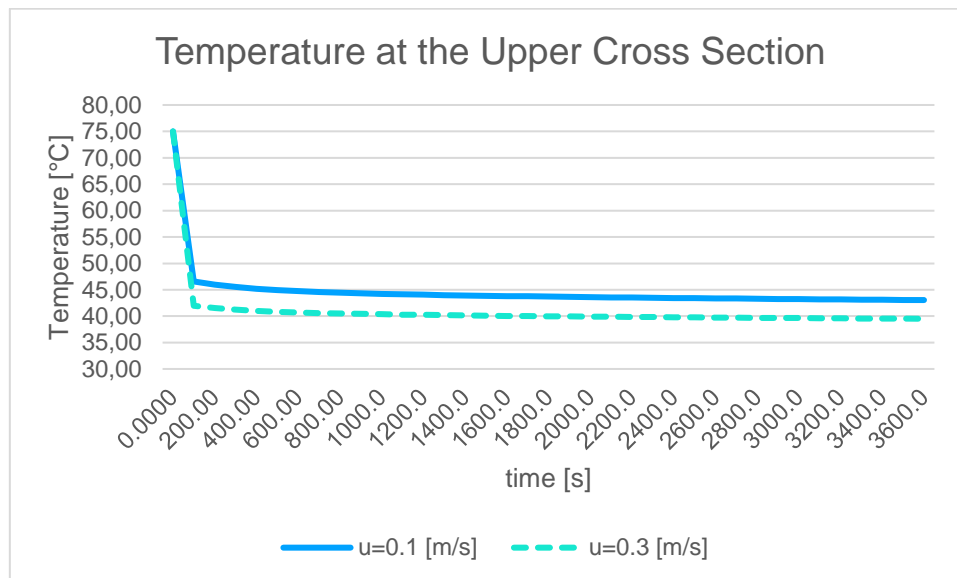


Figure 15 - Temperature at the Upper Cross Section

As we can see from the figure 15, an increase in terms of the inlet velocity produces a reduction of the temperature of the water flow at the outlet, because of the reduced time for the heat exchange occurring within the pipe.

Design improvement considering a 2D horizontal cross section

In this last section the first step is that of representing the entropy generation map for the optimized case in steady state, obtained at the end of point 1.

In our case we have only one contribution to heat generation, which is the heat transfer:

$$\Sigma_{irr} = k \frac{\nabla T^2}{T^2}$$

We implement this formula on Comsol computing the entropy generation map related to the optimized case represented in the figure 16.

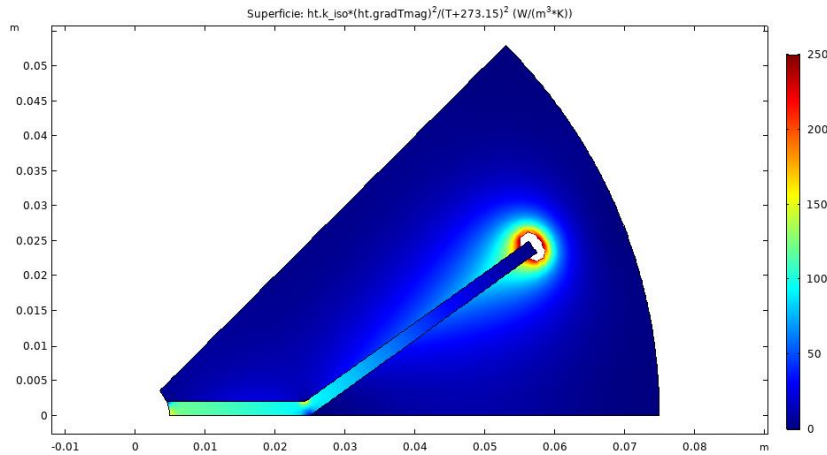


Figure 16 – Entropy generation map, optimized case 1st step

We have not considered the portion of white area closed to the fin's edges, because it is linked to a numerical issue, due to the presence of sharp angles, which would disappear if we choose for example to round them (while the global entropy generation would not change significantly).

So not considering the spots on the right we try to act on the fin shape in order to make the entropy distribution more homogeneous.

In particular we have that entropy generation increases in the portion of the fin close to the boundary (on the left), because considering the expression of the entropy generation reported previously, there is a dependence of the entropy generation on the square of the temperature gradient, which thinking to Fourier equation can be related to heat conduction mechanism.

Looking at Figure 16 we have a very large heat flux per unit surface that is exchanged in this portion, while in the right part of the fin the heat flux per unit surface is not particularly important.

What we could do is try to increase the fin surface close to the boundary, where we have the contact between the water and the pipe, keeping always constant the total volume of the fin.

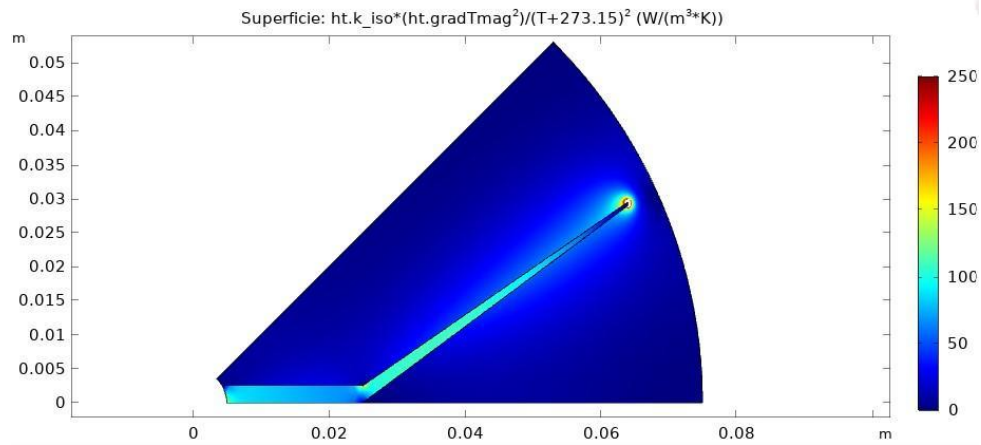


Figure 17 – Entropy generation map, first attempt

As the first step we try to increase the surface closed to the boundary (the height passes from 0.002 to 0.0025), while the surface of the portion of the fin on the right has been reduced by turning its shape from rectangular to trapezoidal.

In that way the entropy generation starts to become more homogeneous, but we still have some darker portions, as shown in figure 17.

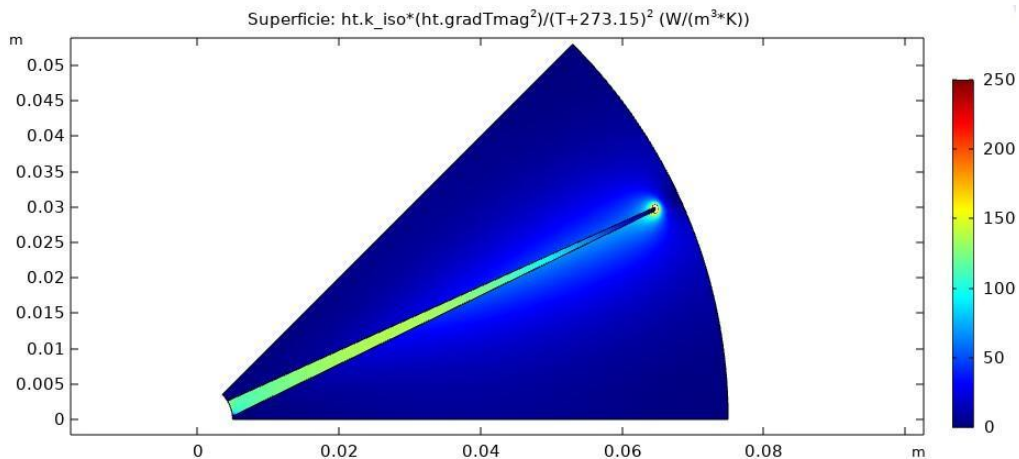


Figure 18 – Entropy generation map, second attempt

As a second attempt we try to increase the angle of the first portion (keeping constant the area) in order to have both the segments on the same line. In that case we obtain a configuration even worse than the previous one, as shown in the figure 18. So we try again to change the fin shape, keeping the inclination of the first segment in a new position, almost in the middle between the two previous cases.

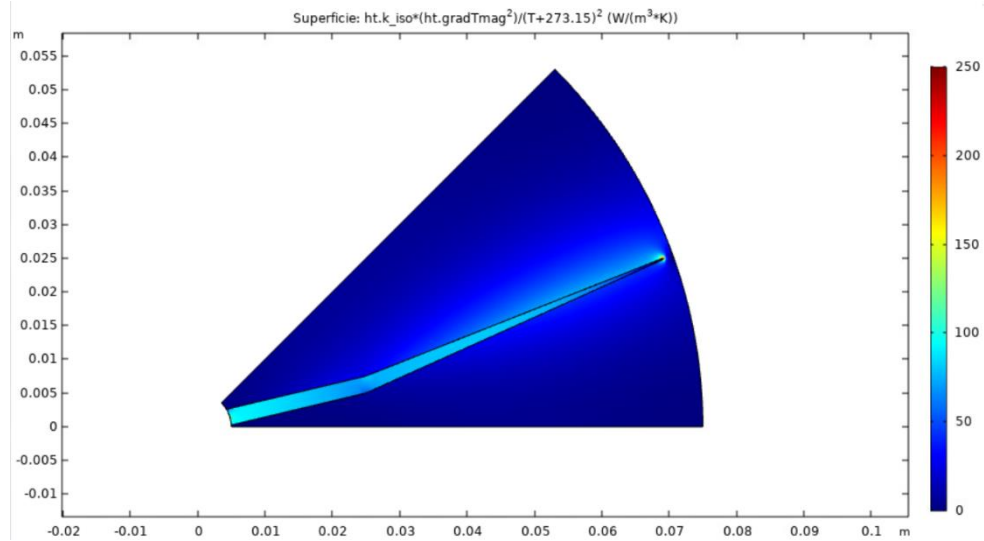


Figure 18 – Entropy generation map, final attempt

In this final configuration we have changed a bit the right portion of the fin (always trapezoidal) by reducing the smaller base and consequently increasing the length in order to keep the volume constant.

In that way we obtain an almost homogeneous entropy generation distribution all over the fin surface, while the average temperature of the PCM drops from the initial value of 372 K to 358 K in this final configuration, so through this optimization path we have also improved a bit the performance of the fin.