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Blade Element Momentum Analysis of Horizontal Axis Wind Turbine

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SUMMARY

INTRODUCTION

The aim of this project is to design and then analyze an horizontal axis wind turbine using the blade element momentum (BEM) theory.
At first the Glauert model is used in order to obtain the geometry of the blade for an ideal case in which the wind flow has no viscosity.
The second step consists again in finding the geometry of the blade, but this time the fluid flow has not been considered inviscid, therefore also the drag force has been taken into account and the model is more complicated.
The third step is then the analysis of the wind turbine designed in the second part, with the aim to find its produced power.

For the problem P44 the given data are:

Table 1 – Data for problem P44

Data		
U	12	m/s
D	150	m
Z	3	-
TSR	5	-
ρ air	1,2	kg/m^3

Where U is the undisturbed wind speed, D is the total diameter of rotor and blades, Z is the number of blades, TSR is the tip speed ratio and ρ air is the density of the fluid, in this case, air.

The analyzed section foil is the S818, a thick, laminar-flow airfoil developed by the National Renewable Energy Lab in Colorado (US), its geometry normalized by the cord is shown in the figure below:

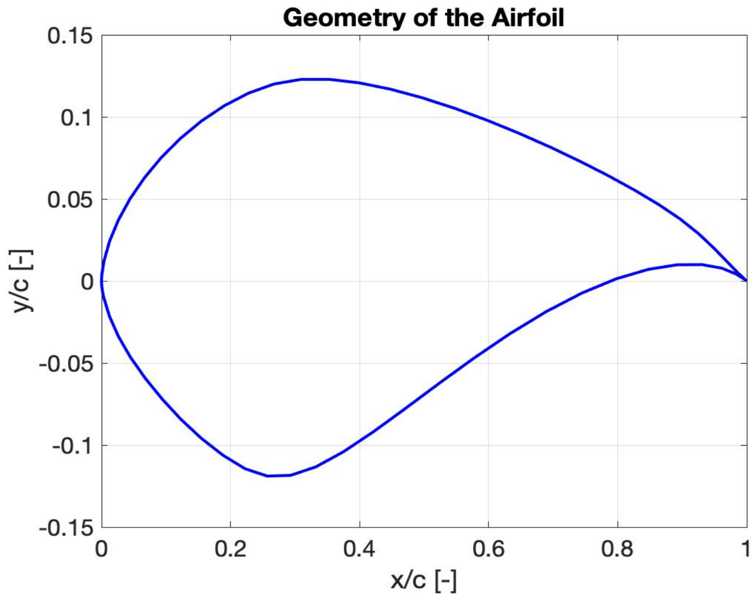

Figure 1 – Geometry of the airfoil

For this foil also the drag coefficient C_D and the lift coefficient C_L as a function of the angle of attack are provided, they are computed for a Reynolds number equal to $1x10^6$:

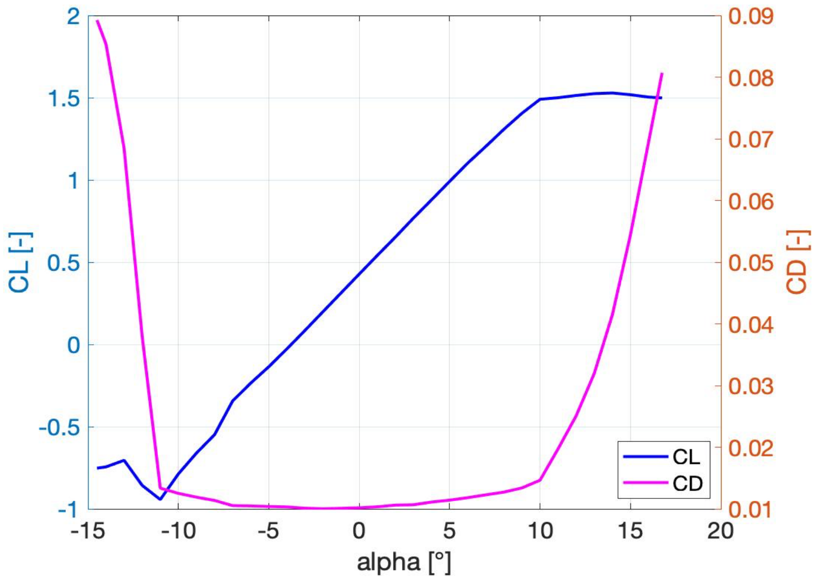

Figure 2 – Lift and drag coefficients of the foil

An important thing to point out is that we make the hypothesis that the airfoil geometry does not change with the radius of the blade. In fact, normally the airfoil is thicker nearer the rotor and thinner near the tip, but for simplicity we hypnotize that this geometry change does not occur.

PART 1: INVISCID FLOW DESIGN

As already said, in the first part the fluid is considered inviscid, this means that there are no viscous forces and, as a consequence, that there is no drag force.

In order to obtain the geometry of the wind turbine we have to compute the chord of the airfoil and the real pitch angle, they both vary with the radius of the blade.
So, the first step of the design of the wind turbine is to divide the blade in sections, in this case the blade is divided in 20 sections from the radius of the rotor to the total radius R = 75 m, which is half of the diameter D. The diameter of the rotor has been assumed equal to 30 m since typical diameter values for medium/large wind turbines range from 20 m to 50 m. Once the blade is divided into sections, the ratio r/R is calculated in order to work with non-dimensional values.

Then the dimensionless radial coordinate x is introduced, this value can be computed as:

$$x = \frac{\Omega\, r}{U}$$

In order to find the x we need to calculate the angular velocity first, it can easily be obtained from the tip speed ratio TSR:

$$TSR = \frac{\Omega\, R}{U}$$

$$\Omega = \frac{TSR\; U}{R}$$

With the dimensionless radial coordinate it is possible to find the axial induction factor a and the rotational induction factor a', these factors take into account the phenomena of the induction which is the decrease of the wind velocity (both axial and rotational) along the wind speed direction due to the interaction with the turbine.
The two factors can be computed as:

$$x = (4a - 1)\sqrt{\frac{1 - a}{1 - 3a}}$$

$$a' = \frac{1 - 3a}{4a - 1}$$

Since the first equation is nonlinear, an iterative solution has to be implemented. In this case the equation has been rearranged as the following:

$$a = \frac{1 - \left(\frac{x}{4a - 1}\right)^2}{1 - 3\left(\frac{x}{4a - 1}\right)^2}$$

In order to start the iterative cycle an initial value of a = 0.3262 has been decided, this value is near enough to the real value so that the method should converge in a fair number of iterations. With this guess a second value of this factor is computed and so on, the iterations stop when either a number of 100 cycles or a tolerance of 0.001 is reached.

At this point it is possible to compute also the aerodynamic undisturbed pitch angle β and the aerodynamic induced pitch angle β_i as:

$$\beta = \tan^{-1}\frac{1}{x}$$

$$\beta_i = \tan^{-1}\frac{1 - a}{x(1 + a')}$$

In order to calculate the chord and the pitch angle we still have to find the angle of attack α and lift coefficient C_L. As already said, we are provided with a plot of C_D and C_L as a function of α, so we can choose the design angle of attack as the one that maximizes the ratio C_L/C_D, this is a compromise between maximizing the numerator C_L and minimizing the denominator C_D:

$$\alpha_{Design} = \max\frac{C_L}{C_D}$$

For this purpose, some particular points in the graph of C_D and C_L plotted with α have been examinate (as shown in Table 2 and Table 3) and the design angle of attach has been chosen equal to 9° with the respective lift coefficient equal to 1.4052.

Table 2 – Design value of the angle of attack

α [°]	CL	CD	CL/CD
6,0000	1,1020	0,0118	93,2318
7,0000	1,2035	0,0123	98,0049
8,0000	1,3077	0,0127	102,8066
9,0000	1,4052	0,0134	104,8657
10,0000	1,4902	0,0147	101,7201
11,0000	1,4997	0,0197	75,9726
12,0000	1,5134	0,0251	60,2948

Table 3 – Chosen design values

Design Values		
α design	9,0000	°
CL	1,4052	-
CD	0,0134	-

Since in this first point the drag force is not present, the drag coefficient C_D should be zero and α should be set as the one that maximizes the lift, but in order to set an angle of attack equal for the case of a viscous flow (that has been examined in part 2), in this case also the drag coefficient has been considered.

At this point the pitch angle and the chord can be obtained from the following equations:

$$\psi = \beta_i - \alpha$$

$$\frac{c}{R} = \frac{8\pi x^2}{Z C_L \lambda}\frac{a'}{1-a}\sin\beta_i$$

The results are shown in the table below:

Table 4 – Inviscid flow design

r [m]	r/R	x	a guess	a	x check	a'	β [rad]	β [°]	βi [rad]	βi [°]	Ψ [°]	c/R	c [m]
15,000	0,200	1,000	0,31699	0,31699	1,000	0,18301	0,785	45,000	0,524	30,000	21,000	0,15975	11,98103
18,000	0,240	1,200	0,32074	0,32074	1,200	0,13348	0,695	39,806	0,463	26,537	17,537	0,15075	11,30591
21,000	0,280	1,400	0,32341	0,32341	1,400	0,10137	0,620	35,538	0,413	23,692	14,692	0,14069	10,55172
24,000	0,320	1,600	0,32535	0,32535	1,600	0,07943	0,559	32,005	0,372	21,337	12,337	0,13077	9,80743
27,000	0,360	1,800	0,32680	0,32680	1,800	0,06383	0,507	29,055	0,338	19,370	10,370	0,12148	9,11121
30,000	0,400	2,000	0,32790	0,32790	2,000	0,05235	0,464	26,565	0,309	17,710	8,710	0,11302	8,47627
33,000	0,440	2,200	0,32875	0,32875	2,200	0,04369	0,427	24,444	0,284	16,296	7,296	0,10539	7,90405
36,000	0,480	2,400	0,32942	0,32942	2,400	0,03698	0,395	22,620	0,263	15,080	6,080	0,09855	7,39089
39,000	0,520	2,600	0,32995	0,32995	2,600	0,03170	0,367	21,038	0,245	14,025	5,025	0,09242	6,93117
42,000	0,560	2,800	0,33039	0,33039	2,800	0,02746	0,343	19,654	0,229	13,103	4,103	0,08692	6,51887
45,000	0,600	3,000	0,33075	0,33075	3,000	0,02402	0,322	18,435	0,215	12,290	3,290	0,08198	6,14827
48,000	0,640	3,200	0,33104	0,33104	3,200	0,02118	0,303	17,354	0,202	11,569	2,569	0,07752	5,81418
51,000	0,680	3,400	0,33129	0,33129	3,400	0,01881	0,286	16,390	0,191	10,926	1,926	0,07349	5,51201
54,000	0,720	3,600	0,33151	0,33151	3,600	0,01682	0,271	15,524	0,181	10,349	1,349	0,06984	5,23780
57,000	0,760	3,800	0,33169	0,33169	3,800	0,01512	0,257	14,744	0,172	9,829	0,829	0,06651	4,98812
60,000	0,800	4,000	0,33184	0,33184	4,000	0,01367	0,245	14,036	0,163	9,357	0,357	0,06347	4,76004
63,000	0,840	4,200	0,33198	0,33198	4,200	0,01242	0,234	13,392	0,156	8,928	-0,072	0,06068	4,55101
66,000	0,880	4,400	0,33209	0,33209	4,400	0,01133	0,223	12,804	0,149	8,536	-0,464	0,05812	4,35886
69,000	0,920	4,600	0,33220	0,33220	4,600	0,01038	0,214	12,265	0,143	8,177	-0,823	0,05576	4,18171
72,000	0,960	4,800	0,33229	0,33229	4,800	0,00954	0,205	11,768	0,137	7,846	-1,154	0,05357	4,01794
75,000	1,000	5,000	0,33237	0,33237	5,000	0,00880	0,197	11,310	0,132	7,540	-1,460	0,05155	3,86614

Table 3 summarizes all the obtained results, the column “x check” calculates x with the values of a found thanks to the iterations, it is done in order to make sure that the loops have computed consistent values of the induction factors.

The plots below show the different computed values with respect to the section of the blade:

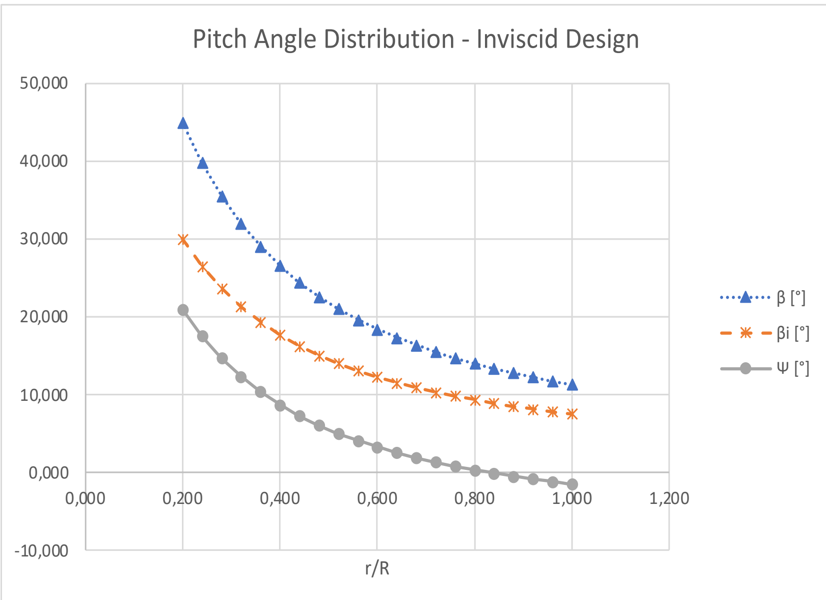

Figure 3 – Pitch angle distribution

As expected, the aerodynamic induced pitch angle is always lower than the undisturbed one, this is because the first angle takes into account the effect of induction while the second one does not. Some values of the pitch angle go below the value of zero degrees, this is because the angle of attach is higher than β_i for radius near the tip of the blade.

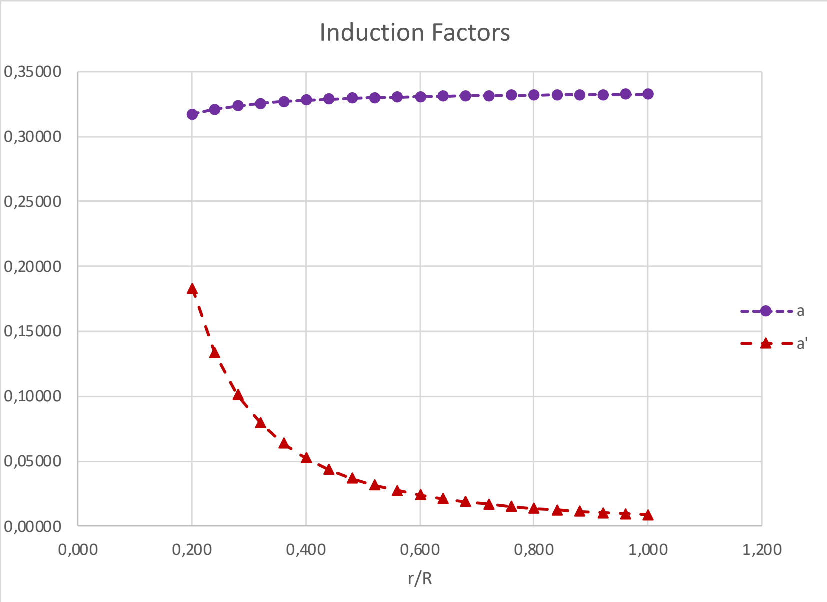

Figure 4 – Induction factors

The induction factor a ranges between 0 and 1, an induction factor equal to zero means that the turbine completely stops the wind (it is the ideal case, not possible in reality), while an induction factor equal to one means that the upwind velocity is not affected by the turbine. In this case the factor ranges between 0.317 to 0.332, it is slightly lower at the beginning of the blade, near the rotor, because the blades are bigger and can slow down more the wind with respect to the tip, where the blade is smaller.

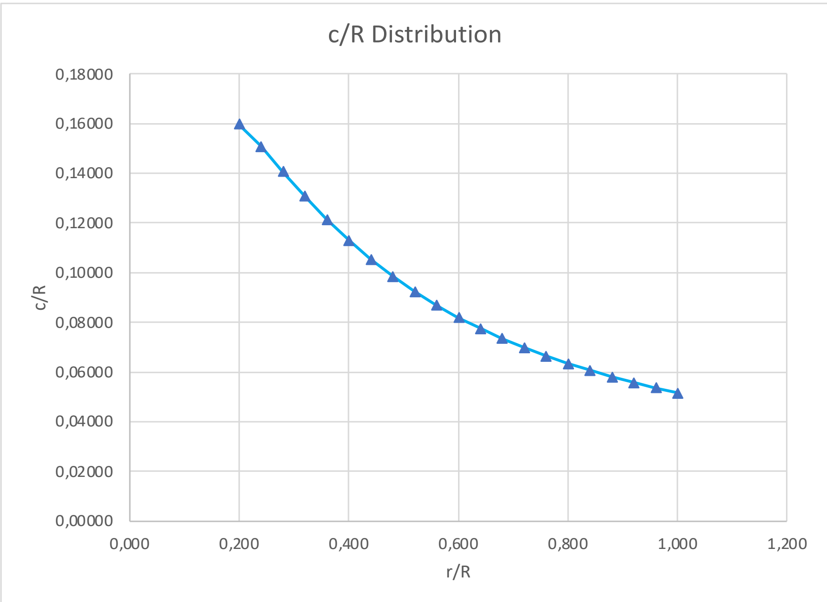

Figure 5 – c/R distribution

As expected, where the radius is lower the chord is higher because the blade is bigger near the rotor. Furthermore, where the rotational speed of the blade is smaller (near the rotor), the chord is larger and vice versa where the blade rotates faster (at the tip) the chord is smaller.

PART 2: REAL FLOW DESIGN

In this second part the aim is again to determine the geometry of the blade, so the pitch angle and the chord. However, in this case the wind has not been considered an ideal fluid with no viscosity, therefore the contribution of the drag force has to be considered.

The design angle of attack α and the derived C_D and C_L are the same of the first part, this is done in order to be able to compare the two results obtained. Also the radial sections of the blade do not change.

As before, the unknowns are the two induction factors, a and a', and the aerodynamic induced pitch angle β_i, while the aerodynamic undisturbed pitch angle β remains the same since it depends on the dimensionless coordinate x that does not change. The three unknowns are bound together by nonlinear equations, so an iterative procedure is needed: in this case the initial value of β_i has been guessed equal to 0.524 (which is the value obtained from the first part for r/R = 0.2). Then a and a' have been calculated and with those value a second β_i is computed and so on, until the maximum number of cycles or the tolerance is reached.

The equations that have been used are:

$$\frac{a}{1-a} = \frac{\mathcal{F}\cos\beta_i}{8\pi k\ sin^2\beta_i}(1+\varepsilon\ \tan\beta_i)$$

$$\frac{a'}{1+a'} = \frac{\mathcal{F}}{8\pi k\ cos\beta_i}(1+\varepsilon\ \cot\beta_i)$$

$$\tan\beta_i = \frac{1-a}{x(1+a')}$$

Where:

- $\varepsilon = \frac{C_D}{C_L}$
- $\mathcal{F} = \frac{ZcC_L}{r} = \mathcal{F}_{Glauert}k$
- $k = \frac{2}{\pi}\cos^{-1}\frac{\cosh(rf/R)}{\cosh f}$ and $f = \frac{ZR}{2r\tan\beta} - \frac{1}{2}$

As written, the factor $\mathcal{F}$ is equal to $\mathcal{F}_{Glauert}$ times the Prandtl factor k, where $\mathcal{F}_{Glauert}$ has the same expression of $\mathcal{F}$ but calculated for an inviscid flow, so in this case the values of c/r computed in the first part have been used.

After the iterative procedure, all the requested values have been obtained and they are summarized in the table below:

Table 5 – Real flow design

r/R	x	F glauert	β i guess [rad]	a	a'	β i [rad]	β i [°]	Ψ [°]	k	F	c/R	c [m]
0,2	1	3,367	0,524	0,31818	0,17945	0,524	30,000	21,000	0,99750	3,35874	0,15935	11,95111
0,24	1,2	2,648	0,464	0,32090	0,13067	0,464	26,589	17,589	0,99678	2,63931	0,15026	11,26947
0,28	1,4	2,118	0,414	0,32352	0,09899	0,414	23,734	14,734	0,99580	2,10928	0,14010	10,50737
0,32	1,6	1,723	0,373	0,32544	0,07737	0,373	21,372	12,372	0,99449	1,71317	0,13004	9,75334
0,36	1,8	1,423	0,339	0,32686	0,06200	0,339	19,399	10,399	0,99274	1,41223	0,12060	9,04505
0,4	2	1,191	0,310	0,32794	0,05072	0,310	17,735	8,735	0,99042	1,17967	0,11193	8,39505
0,44	2,2	1,010	0,285	0,32879	0,04221	0,285	16,317	7,317	0,98734	0,99693	0,10405	7,80400
0,48	2,4	0,865	0,264	0,32945	0,03563	0,264	15,098	6,098	0,98327	0,85099	0,09690	7,26720
0,52	2,6	0,749	0,245	0,32998	0,03046	0,245	14,041	5,041	0,97787	0,73262	0,09037	6,77776
0,56	2,8	0,654	0,229	0,33041	0,02631	0,229	13,116	4,116	0,97072	0,63515	0,08437	6,32800
0,6	3	0,576	0,215	0,33076	0,02294	0,215	12,302	3,302	0,96125	0,55365	0,07880	5,91006
0,64	3,2	0,511	0,202	0,33106	0,02017	0,202	11,580	2,580	0,94872	0,48444	0,07355	5,51600
0,68	3,4	0,456	0,191	0,33131	0,01787	0,191	10,936	1,936	0,93209	0,42468	0,06850	5,13770
0,72	3,6	0,409	0,181	0,33152	0,01593	0,181	10,358	1,358	0,91002	0,37211	0,06355	4,76652
0,76	3,8	0,369	0,172	0,33170	0,01428	0,172	9,837	0,837	0,88065	0,32488	0,05857	4,39279
0,8	4	0,334	0,163	0,33185	0,01287	0,163	9,365	0,365	0,84137	0,28139	0,05340	4,00497
0,84	4,2	0,305	0,156	0,33198	0,01166	0,156	8,935	-0,065	0,78841	0,24009	0,04784	3,58805
0,88	4,4	0,278	0,149	0,33210	0,01060	0,149	8,542	-0,458	0,71582	0,19929	0,04160	3,12015
0,92	4,6	0,255	0,143	0,33220	0,00968	0,143	8,182	-0,818	0,61295	0,15660	0,03418	2,56317
0,96	4,8	0,235	0,137	0,33229	0,00887	0,137	7,851	-1,149	0,45451	0,10692	0,02435	1,82620
1	5	0,217	0,132	0,33237	0,00816	0,132	7,545	-1,455	0,00000	0,00000	0,00000	0,00000

As shown in the table, the value of the Prandtl factor k is zero for r = R which is the tip of the blade. The physical explanation for this behavior is that this factor corrects the fact that, in reality, we do not have a uniform disk but a finite number of blades. Since k describes the probability of interaction of the air with the blades, at radius near to R it becomes small until the value of zero is reached, because the blade itself is smaller and interacts less with the wind.

The plots below show the new values of the unknowns:

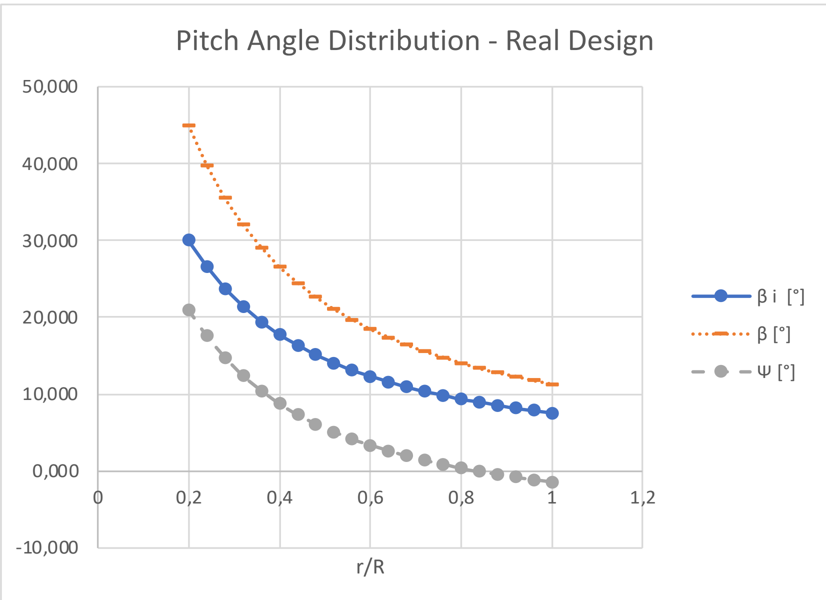

Figure 6 – Pitch angle distribution for real flow

The aerodynamic induced pitch angle β_i is now slightly higher than the previous case, this is because of the contribution of the drag force that in part one has been neglected. As a result, since the angle of attack is the same, also the real pitch angle ψ will be a little higher.

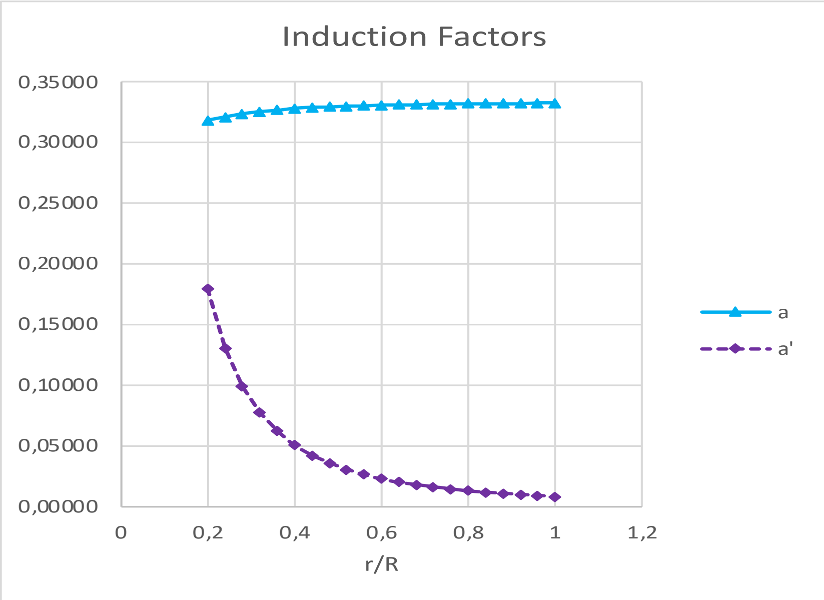

Figure 7 – Induction factors for real flow

The axial induction factor a remains almost equal to the inviscid case, while the rotational induction factor a' slightly decreases, this means that the rotational speed upwind the rotor also decreases with respect to the previous case.

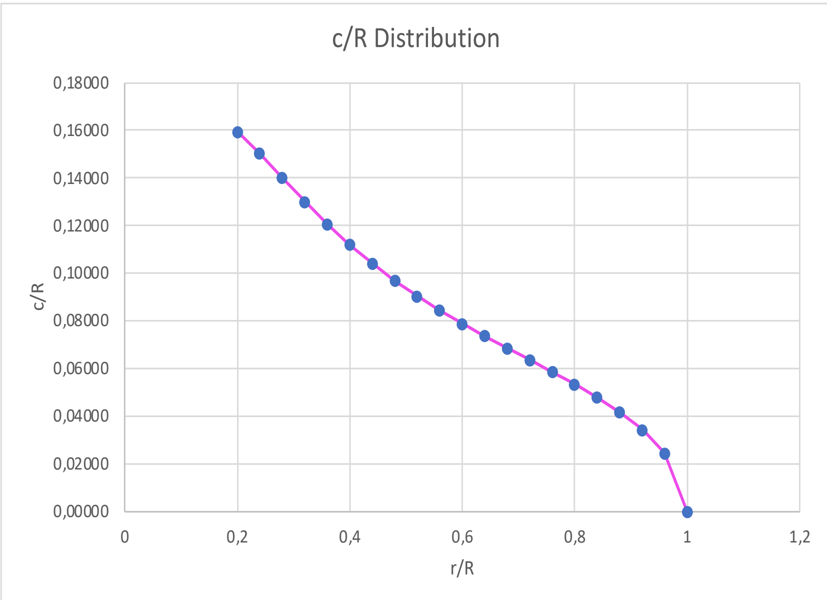

Figure 8 – c/R distribution for real flow

Also the c/R distribution slightly changes, in particular at the end of the blade we reach a value of the chord equal to zero meters, this has not occurred for the inviscid flow, so we can say that the real flow method is more precise that the ideal flow one.

With these values of chord and knowing the geometry of the airfoil, it is possible to visualize how the airfoil varies with respect to the radius of the blade:

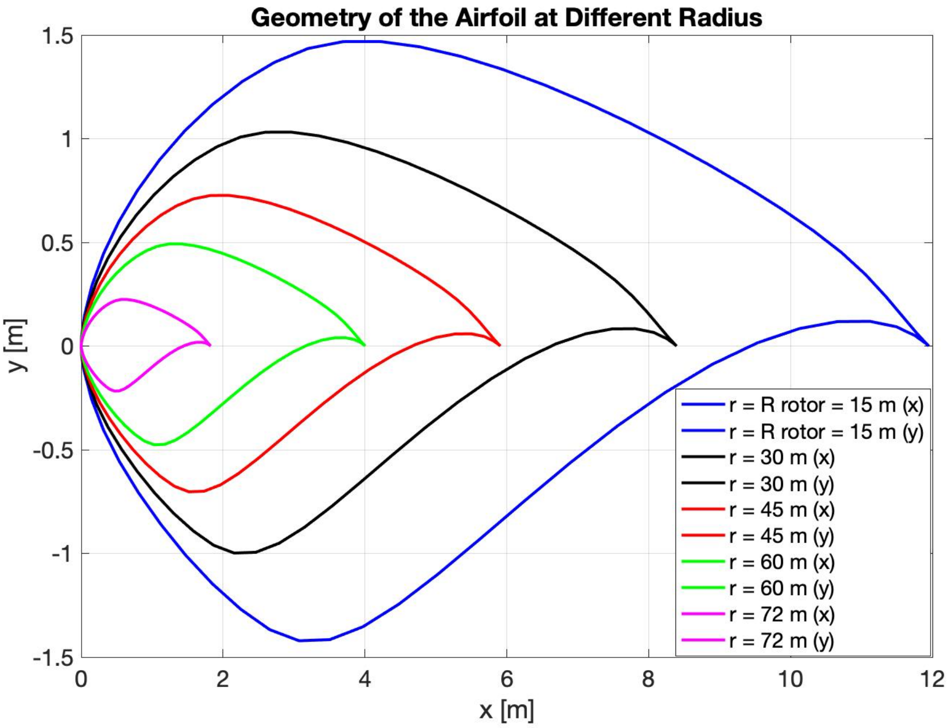

Figure 9 – Airfoil for different radius

Since the hypothesis that the thickness of the blade did not change with the radius has been made, the airfoils are similar, they are just smaller for different radius.

PART 3: REAL FLOW ANALYSIS

In the third point of the assignment the geometry is fixed to the one calculated for the real flow, the aim is now to analyze the designed blade in order to find out the turbine power.

The angle of attack α in not fixed anymore, this time it is calculated using the aerodynamic induced pitch angle β_i and the real pitch angle ψ obtained in the previous point:

$$\alpha = \beta_i - \psi$$

Since β_i and ψ vary with the radius of the blade, also the angle of attack will be a function of the radius. Furthermore, since α changes, also the lift and drag coefficients and ε will change.

In order to compute C_L and C_D as a function of the angle of attack, an analytical expression is needed. The analytical expression is obtained from the provided data, in particular the discrete values of the lift and drag coefficients as a function of α are plotted and a trendline is then obtained.

The results are shown in the figures below:

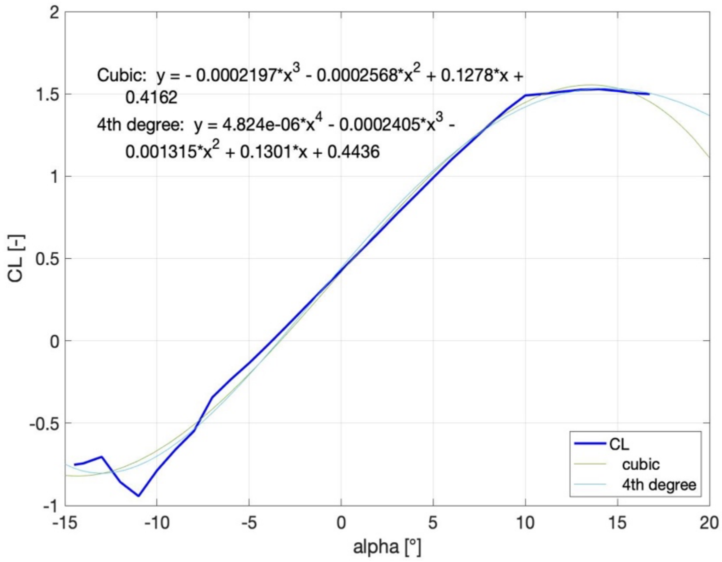

Figure 10 – Trendline for C_L

The variation of the lift coefficient with the angle of attack can be modelled with a cubic or with a fourth-degree polynomial, both curves are pretty accurate but in this case, for the sake of precision, the fourth-degree polynomial has been chosen, therefore the analytical expression is:

$$C_L = 4.824 \times 10^{-6}\alpha^4 - 0.0002405\alpha^3 - 0.001315\alpha^2 + 0.1301\alpha + 0.4436$$

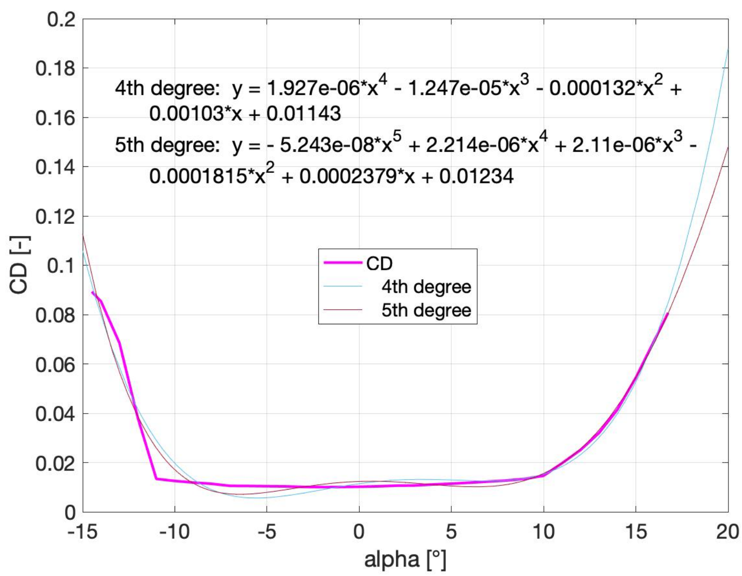

Figure 11 – Trendline for C_D

Also for the drag coefficient two different approximations could be used, a fourth-degree and a fifth-degree polynomial. Like the previous case the polynomial with the highest order has been chosen for the sake of precision and because the edges of the experimental distribution are approximated better with the fifth-order polynomial. So the analytical expression is:

$$C_D = -5.243 \times 10^{-8}\alpha^5 + 2.214 \times 10^{-6}\alpha^4 + 2.11 \times 10^{-6}\alpha^3 - 0.0001815\alpha^2 + 0.0002379\alpha + 0.01234$$

At this point, with the same equations used in the part 2, the induction factors a and a' and the aerodynamic induced pitch angle β_i are calculated with an iterative procedure that starts with a guess for β_i, also in this case the initial guess has been chosen equal to the value of β_i obtained in the previous part for r/R = 0.2.

Finally, the power coefficient C_P, the power P and the thrust coefficient C_T of the turbine can be calculated, their expressions are:

$$C_P = \frac{8}{TSR^2}\int_0^{TSR}(1-a)a'x^3\,dx$$

$$P = C_p\frac{1}{2}\rho U^3 A$$

$$T = \int_0^R 4\pi r\rho U^2 ka(1-a)dr$$

$$C_T = \frac{T}{\frac{1}{2}\rho U^2 A}$$

Where:

- ρ is the density of the fluid, in this case air, and is equal to 1.2 kg/m^3
- $A = \pi R^2$ is the surface of the circumference composed by the rotor and the blades, it is equal to 17671.46 m^2
- T is the thrust force, an axial force

In order to perform a numerical integration, the integrals for C_P and T have been discretized and written as a summation:

$$C_P = \frac{8}{TSR^2}\sum_i[(1-a_i)a'_i x_i^3 + (1-a_{i+1})a'_{i+1}x_{i+1}^3]\frac{\Delta x}{2}$$

$$T = 4\pi\rho U^2\sum_i[r_i k_i a_i(1-a_i) + r_{i+1}k_{i+1}a_{i+1}(1-a_{i+1})]\frac{\Delta r}{2}$$

The obtained results are summarized in the tables below:

Table 6 – Power and power coefficient

Cp	0,48220838	-
Power	8834910,17	W
	8834,91017	kW
	8,83491017	MW

Table 7 – Thrust and thrust coefficient

Thrust	1009286,32	W
	1009,28632	kW
	1,00928632	MW
Ct	0,66104077	-

An important comment on the power coefficient is that it is lower than the Betz limit of about 0.59, but it is not too low and the power extracted from the wind is significant, so we can say that the results are consistent.

The turbine has been designed for a specific tip speed ratio, this means that C_P and P have been maximized for a certain value of TSR, therefore in order to see if the design procedure has been done correctly the values of C_P and P have been plotted with different TSR along with the thrust coefficient C_T and without changing anything else:

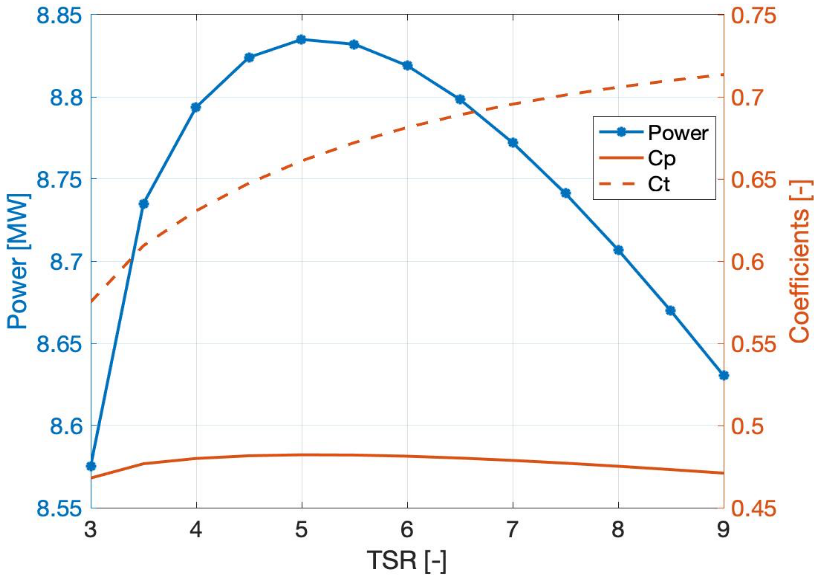

Figure 12 – P and C_P for different TSR

From the figure above it is clear that the maximum power is obtained for TSR = 5 which is the value for which the turbine has been designed and analyzed, as a consequence, also the power coefficient C_P is the highest for the given tip speed ratio.

The figure 12 is also important because it shows that changing the tip speed ratio could be a form of regulation of the turbine, in fact when the power request is less than the maximum power that the turbine can provide, it is possible to change the TSR and therefore the power output, on the contrary when the demand is high the TSR can be set back to the design value, in order to obtain the maximum power.

CONCLUSION

In the first part of this assignment the geometry of the blade for an inviscid flow has been found, as a consequence the drag force is not present since it is a viscous force. On the contrary, in the second part a real flow has been considered with $C_D \neq 0$.
The angle of attack α has been kept constant in order to be able to compare the two parts, it has been chosen as the one that maximizes the fraction C_L/C_D even if in the first part C_D should have not been taken into account as the flow is inviscid.
The aerodynamic induced angle of attack β_i slightly increases in the second point with respect to the first one because of the contribution of the drag force in the force triangle.
Regarding instead the chord, we can say that near the rotor it slightly decreases with respect to the first case, while at the tip it goes to zero, this is reasonable from a physical point of view because the blade is "finished" at the tip and the chord is reduced to a single point.
Overall, it is possible to say that the real flow model is more accurate than the inviscid one even if it is more complicated to implement.
In the third point there is the analysis of the turbine designed in the second point, now the angle of attack is not fixed anymore because we do not need to compare these results with the previous points.
The angle of attack is calculated with the pitch angle obtained in the second part and so it varies with the radius, consequently also C_D and C_L are varying with the radius.
The power produced is about 8.83 MW and the power coefficient is equal to 0.48, this indicates a fair conversion between the power absorbed from the wind and the power available at the end, this value of C_P is of course lower than the Betz limit.
The last thing to point out is that the turbine has been designed for a value of TSR = 5, therefore if the tip speed ratio changes, the power produced by the turbine decreases because the blades have been optimized for the given tip speed ratio, this can be useful when it is better not to operate at maximum power because the demand is low.

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