Laboratory Report

Wave generation, stability of floating bodies and test of an offshore wind platform

Offshore Wind Energy

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1. Introduction

The scope of the laboratory activity is to obtain the values of different Response Attitude Operator (RAO) by means of three different methods. The RAO represents the ratio of the response of the amplitude normalised by the wave amplitude.

The first approach is the WaMIT method, created by the Massachusetts Institute of Technology, based on the Panel Method. Here the effects of viscosity are neglected because of potential flow.

We will perform the other two with a scaled model 1:160th of a semisubmersible platform for a wind turbine under the action of waves. The geometry chosen is the one of the semisubmersible platforms developed for the NREL 5MW wind turbine, shown in figure 1.

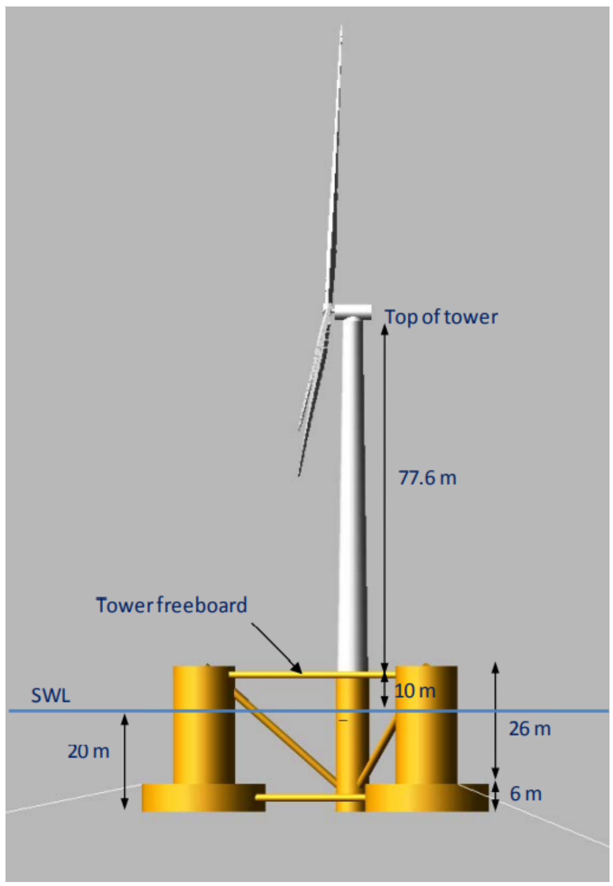

Figure 1 - Semisubmersible floating offshore wind system

The tests are carried out at the IST wave flume that is 20 m long, 0.7 wide and a water depth of 0.5 m, and in order to reduce the reflection of the waves there will be a wavemaker on one side of the tank and a wave absorption beach on the other side.

In figure 2 it is possible to acquire the geometry data for the platform at full-scale.

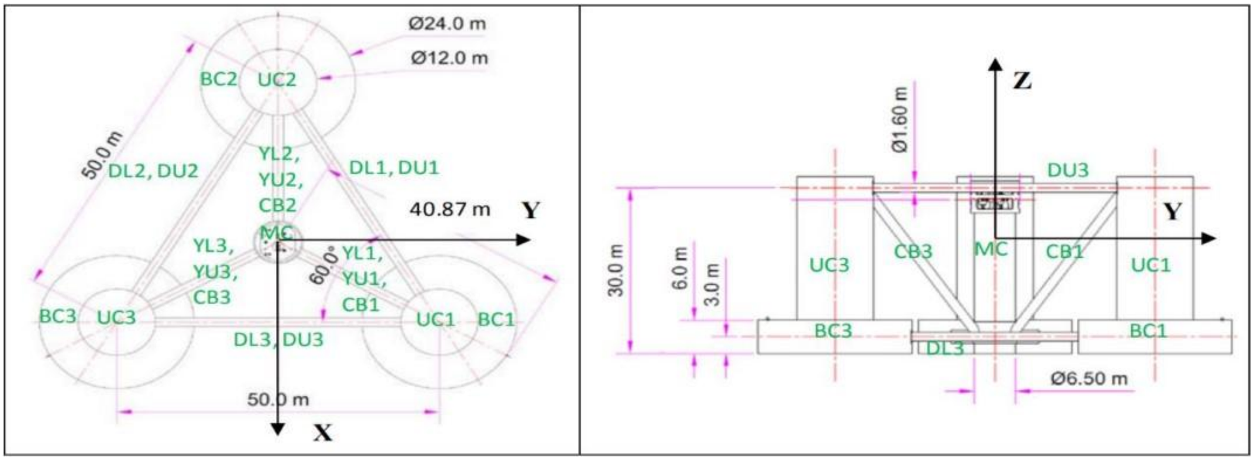

Figure 2 - Geometry and dimensions

The model will be subjected to decay test and model test in order to study the platform motion under real flow conditions. Thanks to the decay test it is possible to obtain the natural characteristics of the oscillatory behaviour of the floating body. With the model tests it is possible to study the dynamics of the scaled platform under regular waves excitation. To determine the hydrodynamic coefficients, it is necessary to also perform the dimensional analysis and also to better describe and predict the prototype motion.

The Table 1 resumes all the structural dimensions of the tower and the floater.

Table 1 - Initial structural dimensions of the full scale platform.

Platform Dimensions		
D1	24	m
D2	12	m
D3	6.5	m
L1	50	m
L2	40.87	m
H1	6	m
H2	14	m
H3	20	m
T	20	m

It is possible to assume that the platform has solid ballasts. Using dimensional analysis it is possible to find the draft, KG, and I_{yy} is the moment of inertia in Pitch at the centre of gravity. (Table 2)

Table 2 - Additional model information

Weights	Configuration	Mass [kg]	KG[m]	Draft [m]	Jyy [kg m^2]
-	A	2.725	0.05248	0.087	0.06079
M10+0	B	3.196	0.05524	0.115	0.06925
M10+1	C	3.475	0.05582	0.133	0.07384
M10+2	D	3.755	0.05706	0.153	0.07849
M10+3	E	4.034	0.05882	0.173	0.08323

The scale factor λ it's important to build the model, the density of the model and the prototype are considerate the same so we can assume that the draft of the prototype is equal to the draft of the model times the scale factor and also *KG* of the prototype is consider as the *KG* of the model times the scale factor. Lastly the I_{yy} of the prototyper is the I_{yy} of the model multiplied by the scale factor raised to 5.

2. Dimensional Analysis

The dimensional analysis is necessary to achieve accurate models to study the dynamics of an offshore wind platform. The use of scaled physical models is important in order to replicate the physical phenomena that appears in a full-scale prototype.

It is necessary to verify geometric similarity, that is verified if all the ratios between the dimensions of the model and prototype are complied. Then it is necessary to verify the kinematic similarity is guaranteed if the ratios between displacements, velocities and accelerations are respected and finally the dynamic similarity is verified if the ratio between forces involved in the process are constant. With this analysis we can guarantee that the model is properly scaled.

In figure 3 , it is given the expressions to properly scale several parameters and dimensions.

Property	Dimensions	Scaling factor
Length	L	ϵ
Wave height	L	ϵ
Water depth	L	ϵ
Displacement	L	ϵ
Velocity	LT^{-1}	$\epsilon^{1/2}$
Acceleration	LT^{-2}	1
Time	T	$\epsilon^{1/2}$
Wave period	T	$\epsilon^{1/2}$
Wave frequency	T^{-1}	$\epsilon^{-1/2}$
Angle	–	1
Mass	M	$(\varrho_p/\varrho_m)\epsilon^3$
Moment of inertia	ML^2	$(\varrho_p/\varrho_m)\epsilon^5$
Moment of area	L^4	ϵ^4
Force	MLT^{-2}	$(\varrho_p/\varrho_m)\epsilon^3$
Moment	ML^2T^{-2}	$(\varrho_p/\varrho_m)\epsilon^4$
Pressure	$ML^{-1}T^{-2}$	$(\varrho_p/\varrho_m)\epsilon$
Power	ML^2T^{-3}	$(\varrho_p/\varrho_m)\epsilon^{7/2}$

Figure 3 - Froude scaling factors used for scaled model testing

3. Metacentric Height

The metacentric height can be computed both theoretically and experimentally, in this case the calculation will be performed in both ways.

3.1. Theoretical Metacentric Height

We know that:

GM = KB + BM - KG

More in detail:

- KB is the distance between the keel of the platform and the center of buoyancy, calculated as:

$$KB = \frac{\Sigma z_i V_i}{\nabla} = \frac{3*\nabla_{thick}*KG_{thick}+3*\nabla_{thin}*KG_{thin}+\nabla_{main}*KG_{main}}{\nabla}$$

- BM is the distance between the center of buoyancy and the metacenter of the platform, it can be computed as:

$$BM = \frac{I}{\nabla}$$

KG is the distance between the keel and the center of gravity, computed from the one of the model using dimensional analysis.

Where I is the second moment of inertia of the waterplane area, ∇ is the volume of water displaced by the platform, z_i and V_i are respectively the center of buoyancy and the volume displaced by each section of the platform.

The displaced volume ∇ (and consequently the displaced mass m) can be computed thanks to the dimensions of the platform defined in the NREL report, after the computation of the volume ∇ the mass is calculated thanks to the Arquimede's principle:

$$\nabla = \pi \cdot 12^2 \cdot 6 \cdot 3 + \pi \cdot 6^2 \cdot 15,28 \cdot 3 + \pi \cdot 3,25^2 \cdot 21,28 = 14033,526\, m^3$$

$$m_{prototype} = \nabla \cdot \rho = 14033,526 \cdot 1025 = 14,384 \cdot 10^6\, kg$$

Two more things to point out before carrying on with the estimation of GM are:

1. The draft, KG and moment of inertia have been calculated using dimensional analysis with the draft of the model
2. For simplicity, the cross components of the platform have been neglected, therefore the waterplane area and the second moment of area of the waterplane will be slightly lower than in reality

The data summarised in table 3 were obtained for the full scale prototype:

Table 3- Full Scale Prototype Data

Draft	21,28	m
Displaced Volume	13886,43	m^3
∑(zi*Vi)	102657,28	m^4
Iyy	144537,28	m^4
Jyy	7,74E+09	kg*m^2
KG	8,931	m
KB	7,315	m
BM	10,299	m
GM	**8,683**	**m**

Since the metacentric height is positive, the platform is stable.

3.2. Experimental Metacentric Height

The objective of the experimental procedure is to change the centre of mass of the floating body and observe its influence on the stability of the barge, for this purpose a barge equipped with a vertically movable weight was used in the lab.

The experimental metacentric height can therefore be computed as:

$$GM = \frac{m_j}{m_F + m_j} \frac{dx}{d\theta} \text{ [m]}$$

Where:

- m_j is the jockey's mass
- m_F is the model's mass

The data for the computation of the GM of the model were provided from the experimental procedure and are summarised in table 4:

Table 4 - Full Scale Model Data

GM Model (Experimental)		
Jockey mass	0,0919	kg
mass	3,475	kg
dθ	5,10	deg
dx	0,2	m
GM (experimental)	0,05784	m

From the experimental metacentric height of the model we can obtain the one of the prototype with dimensional analysis:

$$GM_{prototype} = GM_{model} \cdot \epsilon$$

The obtained metacentric height is:

Table 5 - GM prototype

GM Full-scale prototype		
Scale Factor	160	-
GM (experimental)	**9,254**	**m**

4. RAO Estimation from WaMIT

The WaMIT analysis is used to study the dynamics of the platform under wave excitation, it is based on the Panel Method which is basically a numerical approximation that relies on using discrete elements on the surface of an object. For simplicity in this approach it is assumed potential flow i.e., the viscous effect on the platform motion is neglected. It is important to say that even if in the WaMIT analysis the viscous effects are neglected, the waves will still absorb part of the energy from the platform thus causing an additional damping effect. For this reason the damping effect in this theoretical analysis will be much smaller than the one of the real case.

The data necessary to compute the RAO with this method was acquired from the report NREL/TP-5000-60601, in particular it has been used to obtain the added mass, damping coefficient and the hydrodynamic wave excitation as function of the frequency.

So, the Response Amplitude Operator (RAO), can be computed thanks to the following expressions:

$$RAO_{33}(\omega) = \frac{X}{\zeta_a} = \frac{\frac{F_0(\omega)}{\zeta_a}}{\sqrt{\left(C_{33}-\left(M+A_{33}(\omega)\right)\omega^2\right)^2+\left(B_{33}(\omega)\omega\right)^2}}\ [-]$$

$$RAO_{55}(\omega) = \frac{\theta}{\zeta_a} = \frac{\frac{M_0(\omega)}{\zeta_a}}{\sqrt{\left(C_{55}-\left(J_{yy}+A_{55}(\omega)\right)\omega^2\right)^2+\left(B_{55}(\omega)\omega\right)^2}}\left[\frac{\circ}{m}\right]$$

Where the subscript "33" is the one for heave and the subscript "55" is the one for pitch.

The second step is then to evaluate the spring constants C_{33} and C_{55} for heave and pitch, from the theory we know that they can be computed as:

$$C_{33} = \rho \cdot g \cdot A_w \left[\frac{N}{m}\right]$$

$$C_{55} = \rho \cdot g \cdot \nabla \cdot GM\ [N \cdot m]$$

Where:

- ρ is the density of salt water, assumed equal to 1025 kg/m^3
- g is the gravitational acceleration, equal to 9.81 m/s^2
- A_w is the waterplane area, computed as: $A_w = 3\pi R_{thin}^2 + \pi R_{main}^2$
- ∇ is the volume of water displaced by the platform
- GM is the theoretical metacentric height

The results are summarised in table 6:

Table 6- Spring Constants

Spring Constants		
c33	3745330,038	N/m
c55	1225315964	N*m

The parameters that vary with the frequency like the added mass A33, the added inertia A55, the dampings B33 and B55 and the magnitudes F0 and M0 have been acquired from the given excel file named "OWE_Semisubmersible_hydrodynamic_coefs".

It is finally possible to compute the RAOs for heave and pitch as a function of the angular frequency as shown in the graphs below:

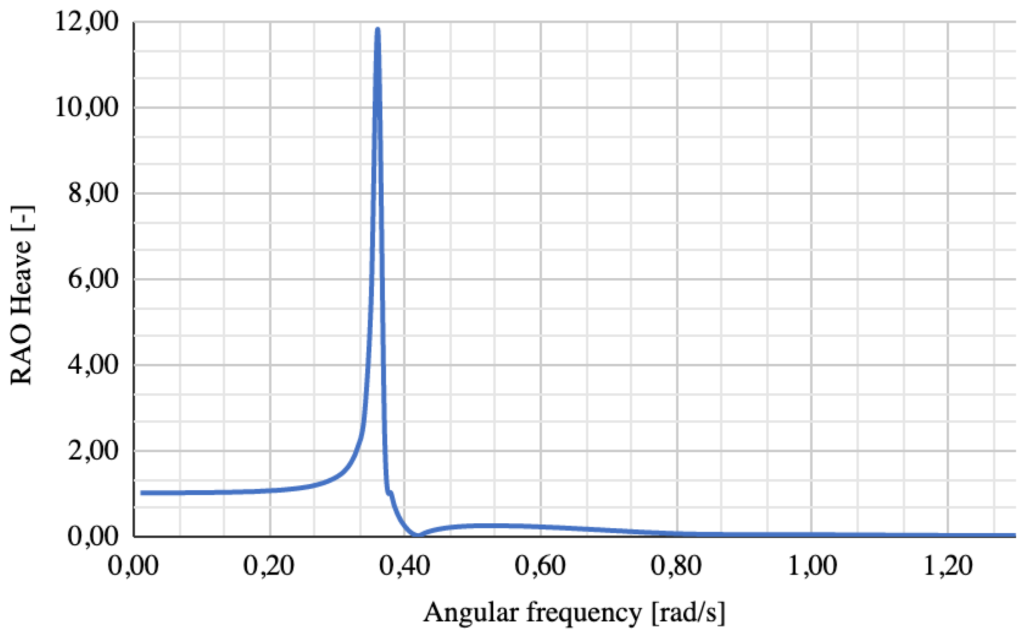

Figure 4 - RAO for heave motion (WaMIT)

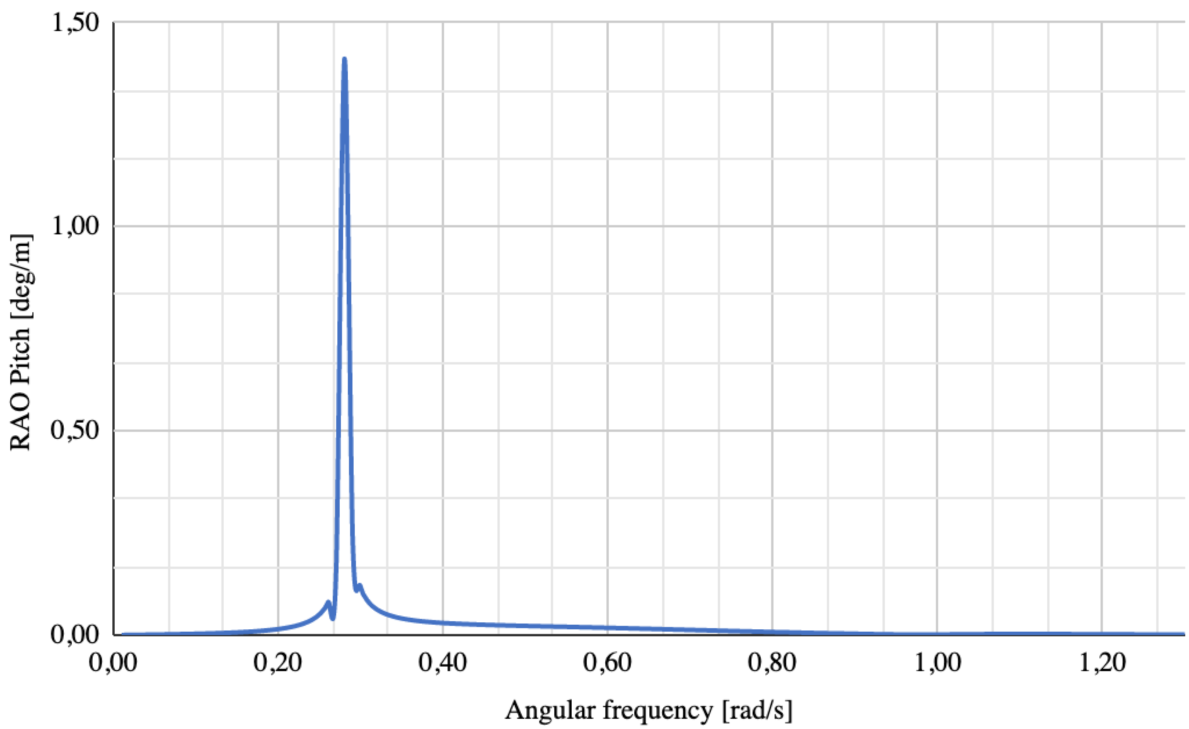

Figure 5 - RAO for pitch motion (WaMIT)

From figure 4 and figure 5 it is possible to see that when the wave frequency tends to zero, the RAO for heave tends to one and the one for pitch tends to zero, this is a confirmation of the value that we obtain when performing the limit for ω that tends to zero to the theoretical expression of the RAO for both heave and pitch.

Another important thing to point out is that when the frequency of the waves increases, the wavelength decreases (and so the period increases). Therefore, the platform will experience at the same time more than one wave excitation, this causes the heave and pitch RAOs to tend to zero with the increase of the wave frequency, so the waves excitation will cancel.

5. RAO Estimation - Decay tests

In order to determine the natural characteristics of the oscillatory behaviour of the platform, such as: added mass (heave), added inertia (pitch), damping constant and spring constant, free-decay tests have been conducted. The tests were done for heave motion and pitch motion separately in still-water conditions by creating an initial displacement of the model from the equilibrium position. After the model was released, the infrared cameras were monitoring its motion to create a timeseries motion trends for both analysed degrees of freedom. The frequency of cameras' recordings was 100 [Hz]. This means that the timestep of every record obtained in the motion recording is equal to 0.01 [s].

Registered motion obtained from the infrared cameras for the heave motion decay test are presented in Figure 6.

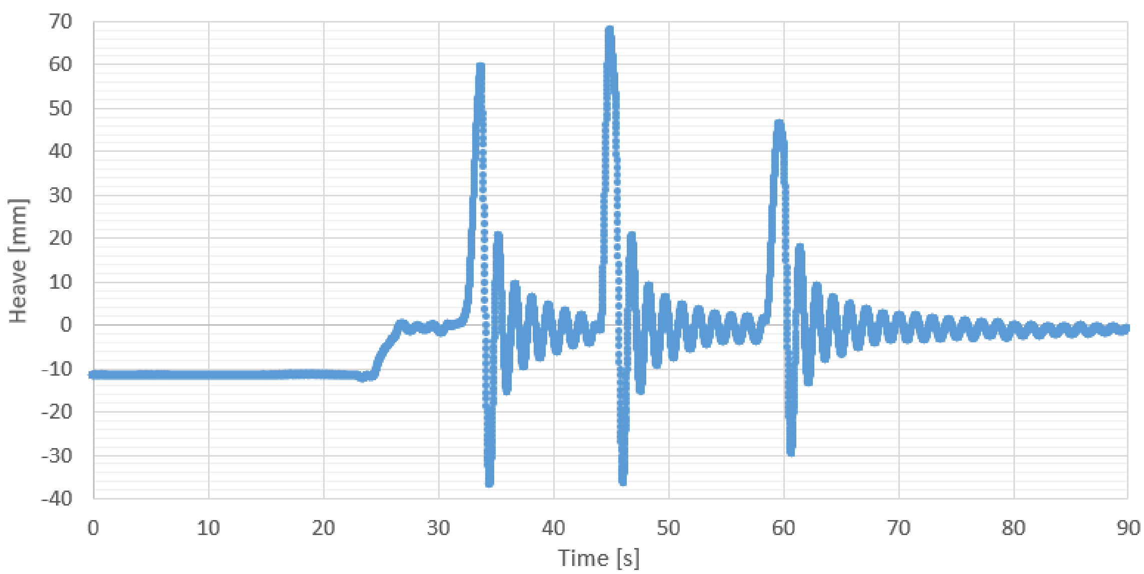

Figure 6 - Heave motion decay test results.

As one can notice from the figure above, three heave motion decay tests were conducted during the analysis. In order to conduct further calculations, the third test was chosen, as it is maintained for the longest. The time of this test ranges from 58s to 90s. This particular test's results are presented on Figure 7.

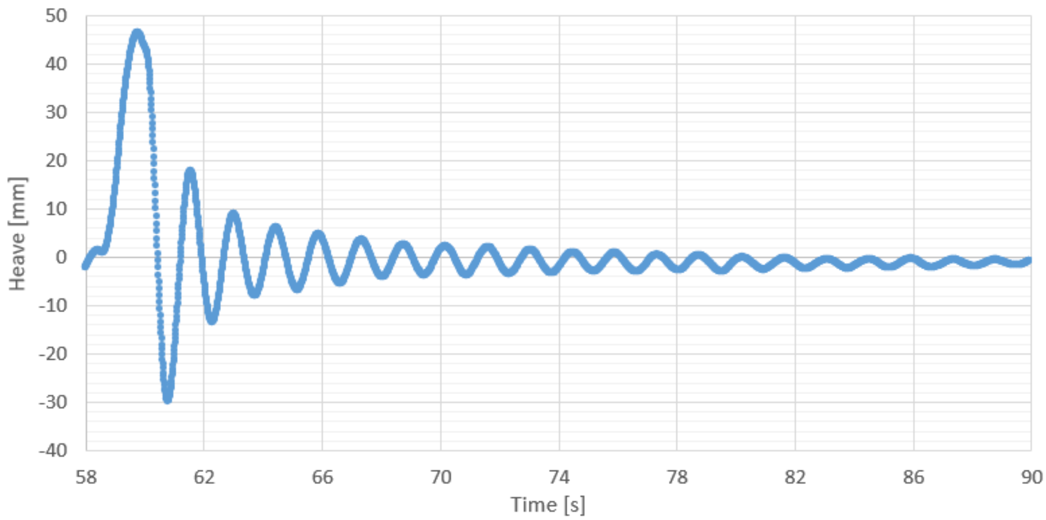

Figure 7 - Heave motion decay test results - test 3.

Registered motion obtained from the infrared cameras for the pitch motion decay test are presented in Figure 8.

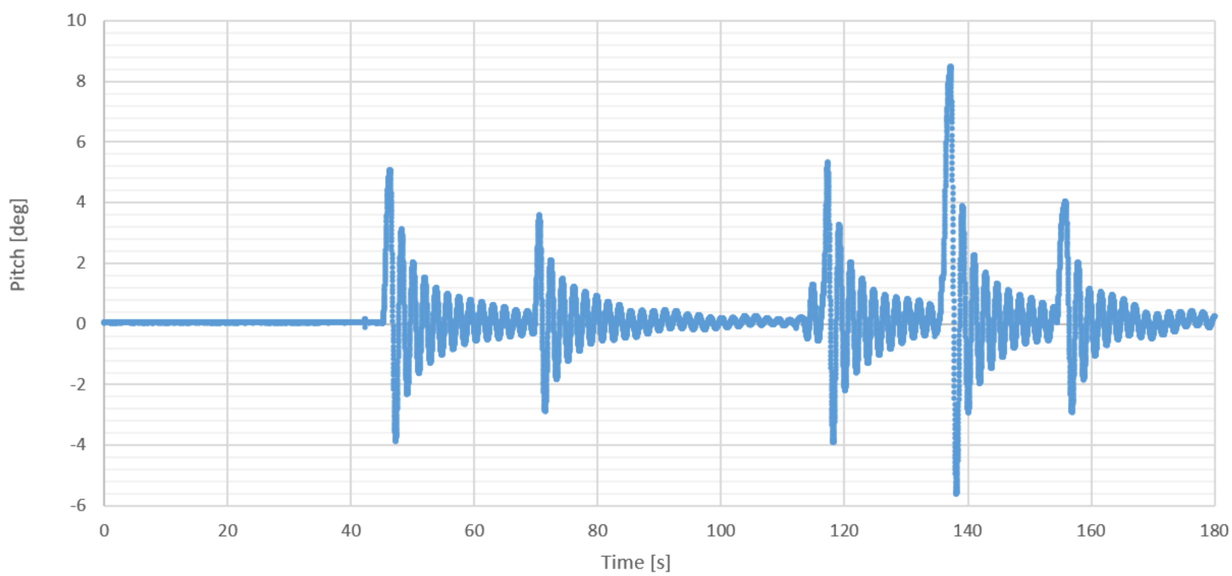

Figure 8 - Pitch motion decay test results

As one can notice from the figure above, five pitch motion decay tests were conducted during the analysis. In order to conduct further calculations, the second test was chosen, as it is maintained for the longest. The time of this test ranges from 68s to 110s. This particular test's results are presented on Figure 9.

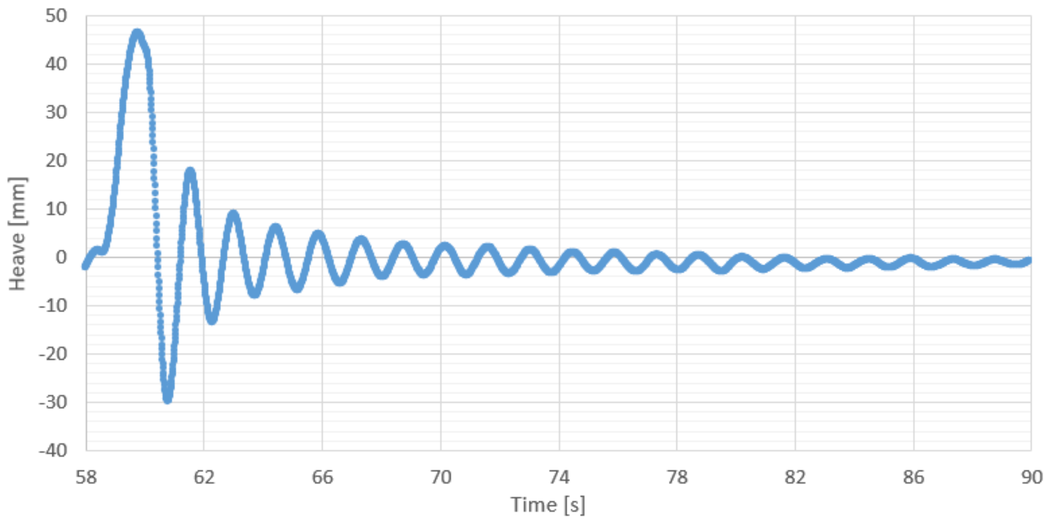

Figure 9 - Pitch motion decay test results - test 2

Firstly, the motion period, damping coefficient and natural period of the motion need to be calculated for both decay tests. In order to compute them, eight peak points of the motion were chosen for every decay test. The first peak was ignored because it represents the initial displacement imposed on the model to conduct the tests.

Using the time values and heave and pitch displacement values in time, damping coefficient and motion period for every step were calculated for both of the tests using the equations:

$$\xi_i = \frac{ln(\frac{x_i(t)}{x_i(t+T)})}{2\pi} \; [-]$$

$$T = t_f - t_i \; [s]$$

Chosen peak points with calculated values of the damping coefficients and motion periods are presented in Table 7 and Table 8 for the heave and pitch motion respectively.

Table 7 - Heave decay test motion peaks

Heave Decay Test - Motion Peaks								
Time [s]	61,57	63,03	64,45	65,91	67,35	68,78	70,19	71,64
Heave [mm]	17,708	8,829	6,038	4,638	3,482	2,503	2,146	1,990
Damping ξ	-	0,11076	0,06047	0,04198	0,04563	0,05254	0,02449	0,01200
Period [s]	-	1,46	1,42	1,46	1,44	1,43	1,41	1,45

Table 8 - Pitch decay test motion peaks

Heave Decay Test - Motion Peaks								
Time [s]	72,37	74,21	76,05	77,91	79,73	81,52	83,38	85,2
Pitch [deg]	3,513	2,248	1,707	1,356	1,152	0,947	0,727	0,561
Damping ξ	-	0,07107	0,04383	0,03665	0,02593	0,03119	0,04210	0,04114
Period [s]	-	1,84	1,84	1,86	1,82	1,79	1,86	1,82

After that, the average period and damping coefficients were calculated for both degrees of freedom. Using the averaged values for the laboratory model, average damping frequency and average natural frequency were calculated for the tested model using equations presented below.

$$\omega_d = \frac{2\pi}{T} \left[\frac{rad}{s}\right]$$

$$\omega_n = \frac{\omega_d}{\sqrt{1-\xi^2}} \left[\frac{rad}{s}\right]$$

The results of the calculated average period, damping coefficient, damping and natural frequency are presented in Table 9.

Table 9 - Heave and pitch decay tests averaged values for the tested model

	Heave Decay Test		**Pitch Decay Test**	
Experimental GM	0,0578	[m]	0,0578	[m]
Average damping coefficient	0,04969	[-]	0,04170	[-]
Average natural period	1,439	[s]	1,833	[s]
Average damping frequency	4,368	[rad/s]	3,428	[rad/s]
Average natural frequency	4,373	[rad/s]	3,431	[rad/s]

Next, the natural period, damping coefficient and natural frequency were scaled to represent the values for the full-scale prototype using equations:

$$\xi_{full-scale} = \xi_{model} \, [-]$$

$$T_{full-scale} = T_{model} \cdot \epsilon^{\frac{1}{2}} \, [s]$$

$$\omega_{full-scale} = \omega_{model} \cdot \epsilon^{-\frac{1}{2}} \left[\frac{rad}{s}\right]$$

After that, the spring constants C_{33} and C_{55} for heave and pitch motion were calculated using the equations below. The metacentric height used in the calculations for pitch motion spring constant was the experimentally derived one.

$$C_{33} = \rho \cdot g \cdot A_w \left[\frac{N}{m}\right]$$

$$C_{55} = \rho \cdot g \cdot \nabla \cdot GM \, [N \cdot m]$$

After that, added mass A_{33} and added inertia A_{55} were computed for both degrees of freedom using the relation below:

$$C_i = \omega_n^2 \cdot (M + A_i)$$

Lastly, the damping B_{33} and B_{55} were calculated for both degrees of freedom using the relation below:

$$B_i = 2 \cdot \xi_i \cdot \sqrt{(M + A_i) \cdot C_i}$$

The calculated values for both degrees of freedom are presented in Table 10.

Table 10 - Natural characteristics of the oscillatory behaviour of the full-scale platform

	Heave Decay Test		Pitch Decay test	
Damping Coefficient	0,04969	[-]	0,04170	[-]
Natural Period	18,197	[s]	23,184	[s]
Natural Frequency	0,346	[rad/s]	0,271	[rad/s]
Spring constant Ci	3,75E+06	[N/m]	1,31E+09	[N*m]
Added mass A33 / Added Inertia A55	1,70E+07	[kg]	1,00E+10	[kg*m^2]
Damping Bi	1,08E+06	[kg/s]	4,02E+08	[kg*m^2 / s]
Equivalent mass m_eq	3,13E+07	[kg]	-	-
Equivalent inertia Jyy_eq	-	-	1,77E+10	[kg*m^2]

In this analysis, the added mass/inertia , damping and spring constants are the same and independent of the wave frequency. The only values dependent on wave frequency are the excitation forces, which can be taken from the NREL report. Using the above values, the RAOs for both degrees of freedom in relation to the wave frequency can be calculated using the following equations:

$$RAO_{33}(\omega) = \frac{X}{\zeta_a} = \frac{\frac{F_0(\omega)}{\zeta_a}}{\sqrt{\left(C_{33}-\left(M+A_{33}\right)\omega^2\right)^2+\left(B_{33}\cdot\omega\right)^2}}\,[-]$$

$$RAO_{55}(\omega) = \frac{\theta}{\zeta_a} = \frac{\frac{M_0(\omega)}{\zeta_a}}{\sqrt{\left(C_{55}-\left(J_{yy}+A_{55}\right)\omega^2\right)^2+\left(B_{55}\cdot\omega\right)^2}}\left[\frac{\circ}{m}\right]$$

The results of the calculated RAO values are presented on Figure 10 and Figure 11, respectively for the heave and pitch motion.

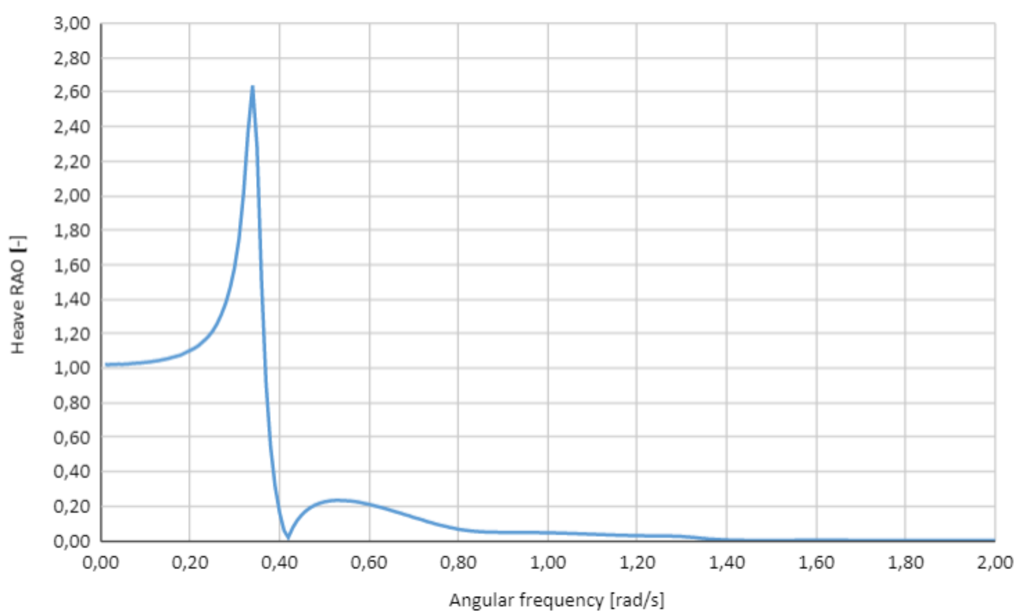

Figure 10 - RAO from wave frequency relation for heave motion for the analysed full-scale model.

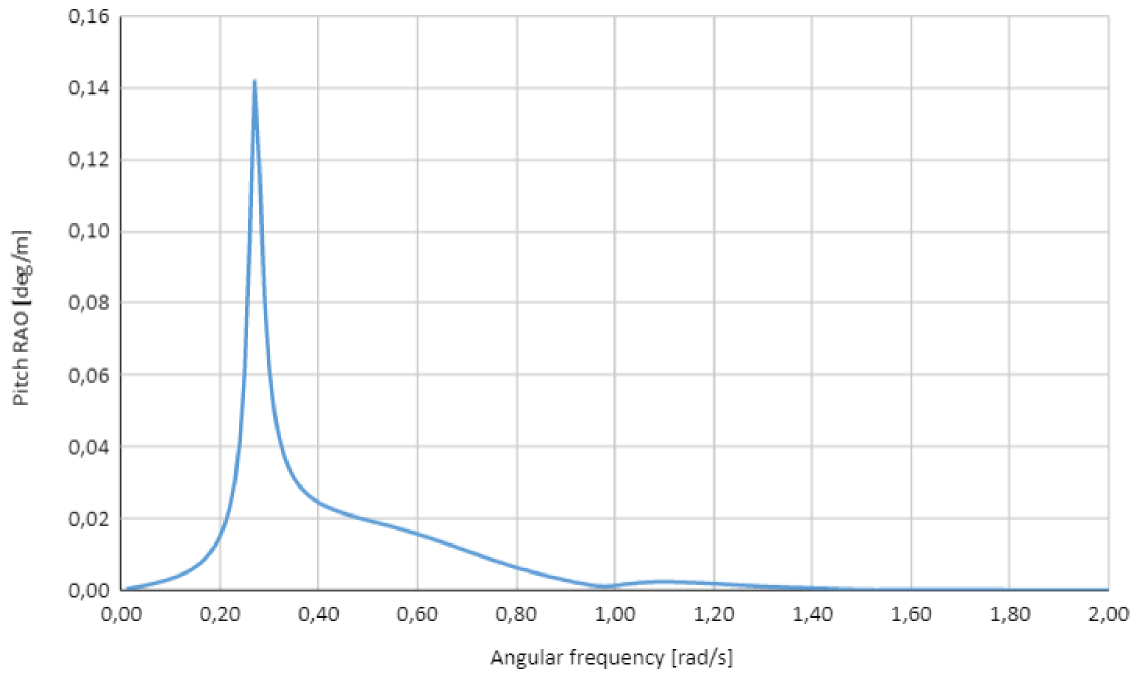

Figure 11 - RAO from wave frequency relation for pitch motion for the analysed full-scale model.

6. Model test

We now consider the semisubmersible platform model dynamics under regular wave excitation. An infrared camera motion tracking system measures the Heave and Pitch motions, while the free surface elevation is measured by a resistive gauge installed in the channel. The Heave measurements are reported in millimeters, whereas the Pitch measurements are reported in degrees. The data acquisition equipment is installed inside the wave flume control room. The duration time of each test is 120 s; each measurement is performed after 0.01 s, for a total frame rate of 100 Hz.

The file Global_results.exe already provides the numerical values of RAOs for Heave and Pitch as function of frequencies. The frequency chosen is 0.42 Hz. The Heave RAO is provided as non-dimensional value; the Pitch RAO must be scaled considering the following formula

$$RAO_{pitch}\left[\frac{deg}{m}\right] * \frac{1}{\varepsilon} = RAO_{pitch}\,[-]$$

The Figure 12 and Figure 13 show the behaviour of the Heave and Pitch RAOs as function of frequency.

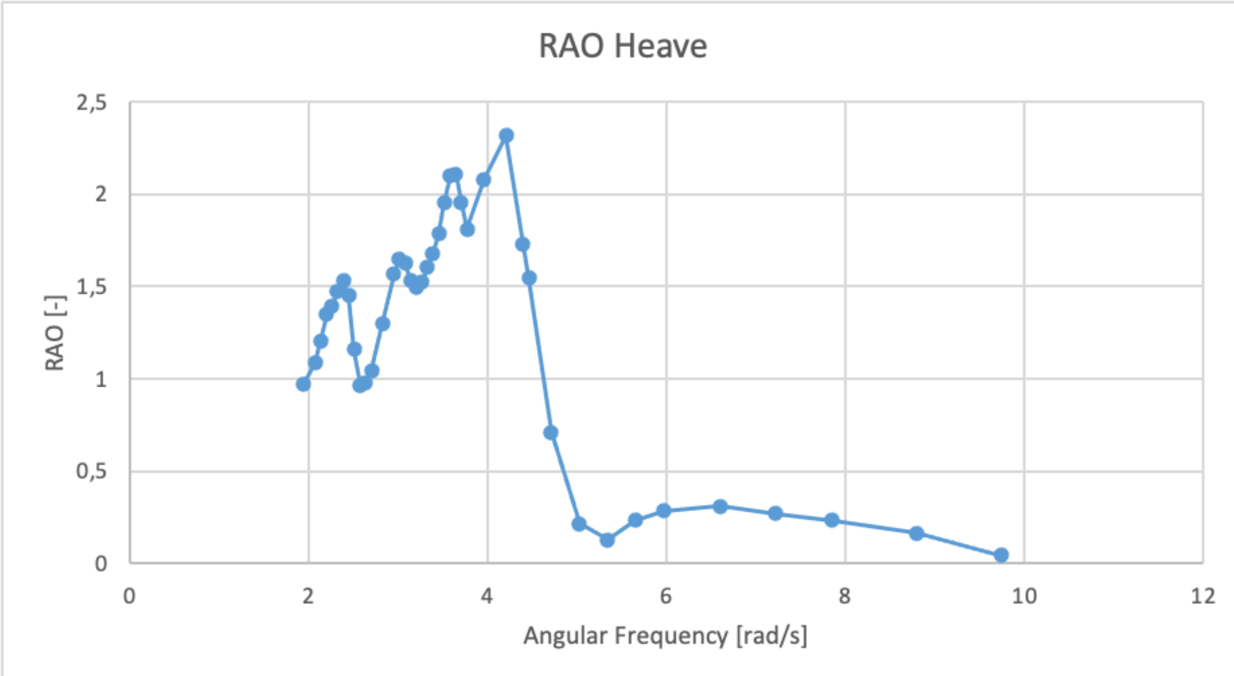

Figure 12 - RAO for Heave motion

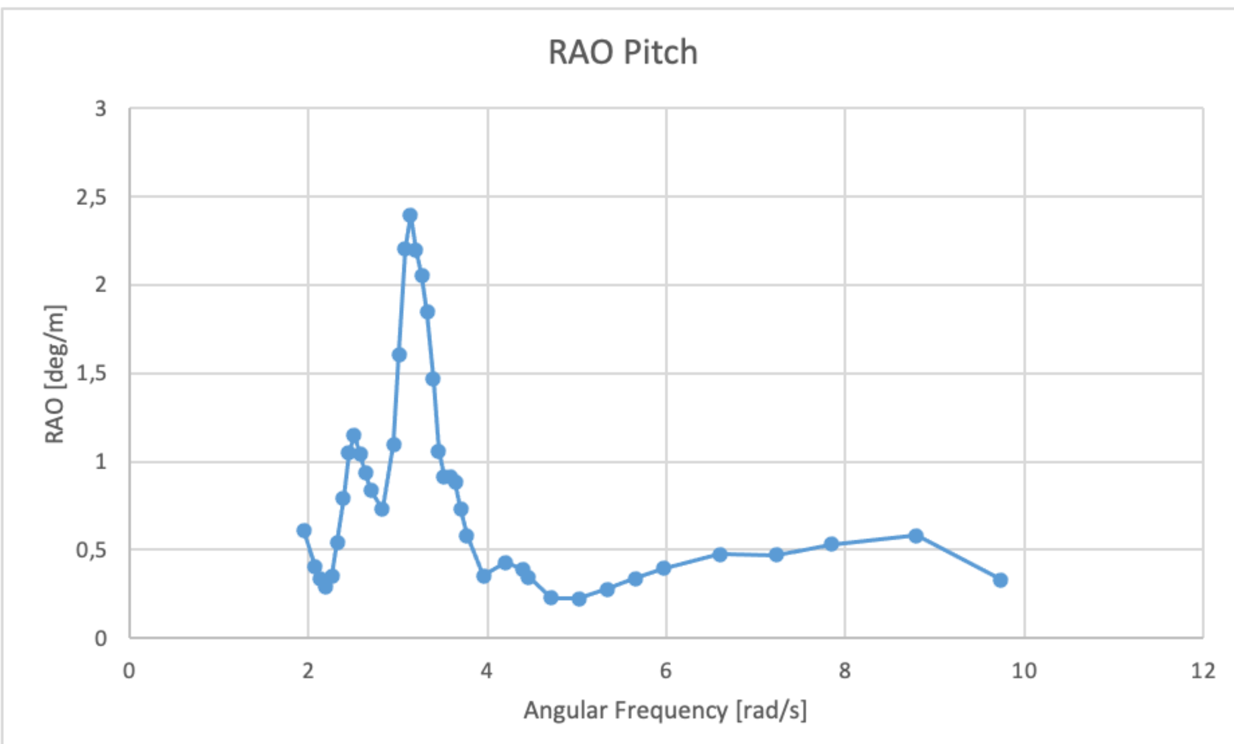

Figure 13 - RAO for PItch motion

6.1. Theoretical estimation of RAOs

In order to evaluate theoretically the Pitch and Heave RAOs, we consider the wave amplitude data for a specific frequency. The wave amplitude measurement is performed by choosing substantial number of values in an interval of time, when the transitory is already developed. The wave amplitude (WA) is half of the summation between absolute value of consecutive Free Surface Elevation (FSE) peaks.

$$WaveAmplitude \ = \frac{|FSEpeak_{i+1}|+|FSEpeak_{i}|}{2}$$

The period is calculated by the formula of half of the difference between the time of two consecutive peaks.

$$T \ = \frac{time\ peak_{i+2} - time\ peak_{i}}{2}$$

The Average Wave Amplitude and the Average Period is obtained by the mean value of those obtained.

The Angular Velocity is obtained by the Average Period

$$\omega_{ave} \ = \frac{2*\pi}{T_{ave}}$$

All the results obtained above are related to the model. The measurements must be converted by the scaling factor for the prototype.

The wavelength and the wave number are obtained by the formulas

$$\omega^2 \ = g\ *\ k\ *\ \tanh tanh\,(kh)$$

$$\lambda \ = \frac{2*\pi}{k}$$

The Heave and Pitch amplitude are computed similarly to wave amplitude. Once it is identified an interval of time distant from the transitory, a substantial number of values are chosen. The Heave Amplitude are calculated as half of the summation between absolute value of consecutive Heave peaks

$$\zeta_H = \frac{|Heave\ peak_{i+1}|+|Heave\ peak_{i}|}{2}$$

The Pitch Amplitude are calculated half of the summation between absolute value of consecutive Pitch peaks

$$\zeta_P \ = \frac{|Pitch\ peak_{i+1}|+|Pitch\ peak_{i}|}{2}$$

The Average values for heave and pitch are afterwards obtained

$$\zeta_{Have} = \frac{\sum_{n} \zeta_{Hi}}{n}$$

$$\zeta_{Pave} = \frac{\sum_{n} \zeta_{Pi}}{n}$$

The Heave and Pitch RAOs are evaluated by means of the formulas

$$RAO_{33}(\omega) = \frac{Z}{\zeta_{Have}} \; [m]$$

$$RAO_{55}(\omega) = \frac{Z}{\zeta_{Pave}} \; [m]$$

In order to deal with data regarding the prototype, we multiply the RAOs above for the scaling factor and convert them into meters.

$$RAO_{33}(\omega)_{prototype} = RAO_{33}(\omega) * \frac{160}{1000} \; [m]$$

$$RAO_{55}(\omega)_{prototype} = RAO_{55}(\omega) * \frac{160}{1000} \; [m]$$

7. Conclusions

The results obtained thanks to the WaMIT analysis, decay test and model test are plotted together in the following graphs in order to make a comparison between the different methods:

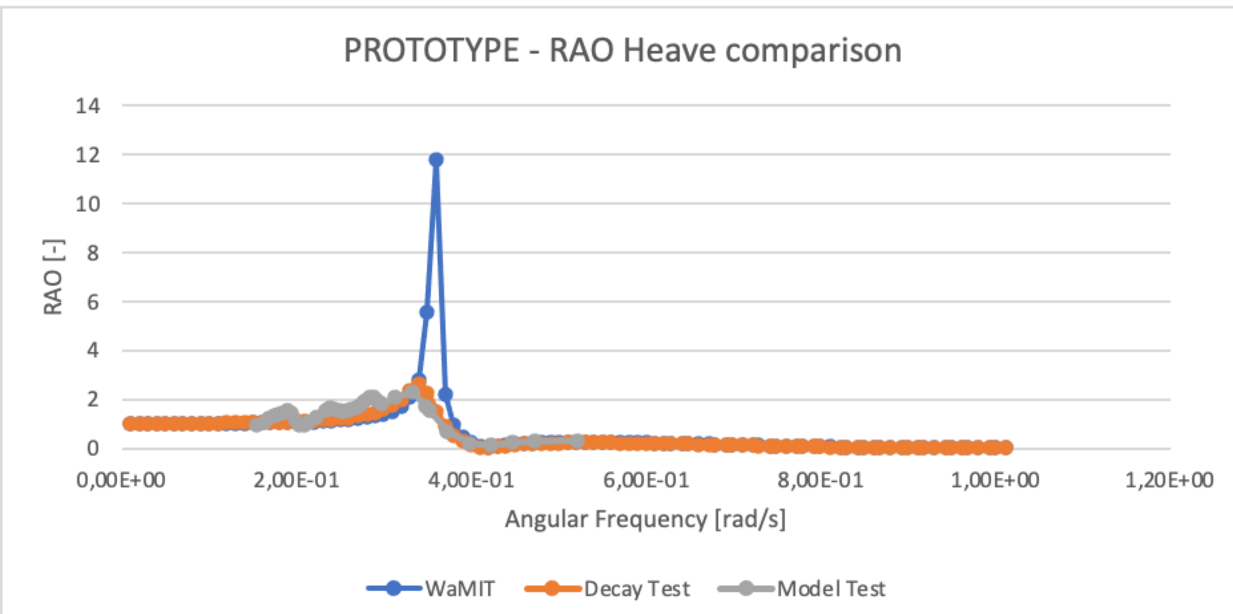

Figure 14 - RAO Heave comparison

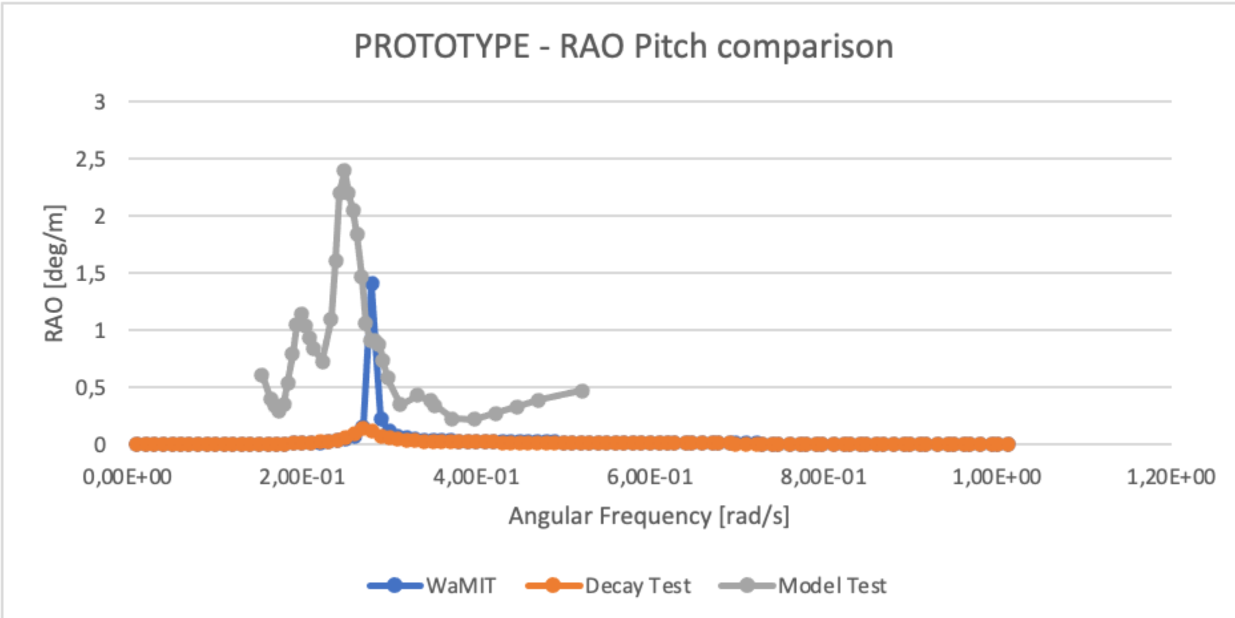

Figure 15 - RAO Pitch comparison

It is possible to notice that the damping coefficient for heave and pitch is smaller in the WaMIT analysis with respect to the other two analysis, this is because in the WaMIT analysis the effects of the viscous forces are neglected i.e. potential flow is assumed. As a consequence the RAO for this analysis is expected to be bigger than the others, and this is confirmed by the obtained results. Even if potential flow is assumed, the damping effect is non null because the radiated waves will behave as a dissipative medium absorbing part of the energy of the platform.

Regarding the Decay Test, in order to determine the characteristics of the platform, it has been displaced and its oscillation has been analyzed. The Model test, on the other hand, is different because the platform is subjected to a wave regime created by a wave maker. We know that the RAOs computed with the Decay Test will depend on the number of measurements taken in the lab, because the damping coefficient usually decreases with the number of measurements. This is not a problem because a lower damping coefficient leads to a more conservative analysis (because we obtain an higher RAO).

For what concerns the Model Test, it is possible to notice that the RAO for PItch is much higher than the RAO computed with the other two methods, in fact while WaMIT Analysis and Decay Test show a comparable trend, the Model Test's RAO is different, it does not tend to zero for the frequency that tends to infinite. A possible explanation for this incongruity is that, since the Model Test is an experimental procedure, there will always be an experimental error in the measurements.

As a conclusion it can be pointed out that the most accurate method is the WaMIT analysis, it is also the most conservative, although its limitation is that it is only valid for potential flow, while the other two methods are better in describing real flow conditions.

The dimensional analysis is an important tool because we are able to study a small model in the lab and then the obtained results, properly scaled, can be applied to the full scale prototype which would be difficult and expensive to study directly in the open sea without the small model.