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Artificial Potential Field Based Path Planning for Mobile Robots Using a Virtual Obstacle Concept

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Abstract
The artificial potential field (APF) based path planning methods have a local minimum problem, which can trap mobile robots before reaching it's goal. In this study, a new concept using a virtual obstacle is proposed to escape local minimums occurred in local path planning. A virtual obstacle is located around local minimums to repel a mobile robot from local minimums. A sensor based discrete modeling method is also proposed for modeling of the mobile robot with range sensors. This modeling method is adaptable for a real-time path planning because it provides lower complexity.

Keywords
Mobile robot, path planning, artificial potential field, local minimum problem, virtual obstacle, sensor based discrete modeling.

1. INTRODUCTION
The APF method provides simple and effective motion planners for practical purpose [1]. The applications of APF for obstacle avoidance was first developed by Khatib [2]. However, there is a major problem in the local path planning using the APF approaches, which is that the local minimums can trap a robot before reaching its goal. The local minimum problem is sometimes inevitable in the local path planning because the robot only can detect local informations of obstacles. In other words, the robot cannot predict local minimums before experiencing the environments. The avoidance of local minimum has been an active research topic in the APF based path planning [3]-[8]. However, the previous solutions are limited to simple formations of obstacles or available for known environments.

In this research, a virtual obstacle concept is proposed as an idea to escape a local minimum. The virtual obstacle is located around local minimum point to repel the robot from the point. This technique is useful for the local path planning in unknown environments. The sensor based discrete modeling method is also proposed for the simple modeling of a mobile robot with range sensors. This modeling method is simple and reliable because it is designed for a real-time path planning.

2. POTENTIAL THEORY AND LOCAL MINIMUM PROBLEM
The APF approaches are basically operated by a gradient descent search method which is directed toward minimizing the potential function. Obstacles which have to be avoided are surrounded by repulsive potential fields and the goal point is surrounded by an attractive potential field. The attractive potential is generally a bowl shaped energy well which drives a robot to its center if the environment is unobstructed. In an obstructed environment, a repulsive potential energy hills to repel the robot are added to the attractive potential field at the locations of the obstacles. The robot experiences the force which equals to the negative gradient of the potential. This force drives the robot downhill until the robot reaches the position with the minimum energy.

The attractive potential function used in this study is the conical well proposed by Andrews [9]. The conical well U_{att} is described by

$$U_{att}(\mathbf{x}) = \begin{cases} k_a|\mathbf{x}-\mathbf{x}_d|^2 & \text{if } |\mathbf{x}-\mathbf{x}_d| \le d_a \\ k_a(2d_a|\mathbf{x}-\mathbf{x}_d| - d_a^2) & \text{if } |\mathbf{x}-\mathbf{x}_d| > d_a \end{cases} \quad (1)$$

where $\mathbf{x}$ represents the position vector of a robot, $\mathbf{x}_d$ represents the position vector of a goal, d_a is the radius of a quadratic range, and k_a is a proportional gain of the function. The attractive force $\mathbf{F}_{att}$ may be obtained by the negative gradient of this attractive potential :

$$\mathbf{F}_{att}(\mathbf{x}) = -\nabla U_{att} = \begin{cases} -2k_a(\mathbf{x}-\mathbf{x}_d) & \text{if } |\mathbf{x}-\mathbf{x}_d| \le d_a \\ -2\,d_a k_a \dfrac{\mathbf{x}-\mathbf{x}_d}{|\mathbf{x}-\mathbf{x}_d|} & \text{if } |\mathbf{x}-\mathbf{x}_d| > d_a\,. \end{cases} \quad (2)$$

The second category of potentials, repulsive potential, is necessary to repel the robot away from obstacles which obstruct robot's path of motion in the global attractive potential field. The following repulsive potential function, FIRAS function proposed by Khatib, is

$$U_{rep}(\mathbf{x}) = \begin{cases} \frac{1}{2} k_r \left(\frac{1}{\rho} - \frac{1}{\rho_0} \right)^2 & \text{if} \quad \rho \leq \rho_0 \\ 0 & \text{if} \quad \rho > \rho_0 \end{cases} \tag{3}$$

where ρ_0 represents the limit distance of potential field influence and ρ is the shortest distance to the obstacle. The selection of the distance ρ_0 depends on the maximum speed of the robot and the control period [2]. The repulsive force is driven as

$$\mathbf{F}_{rep}(\mathbf{x}) = -\nabla U_{rep}$$
$$= \begin{cases} k_r \left(\frac{1}{\rho} - \frac{1}{\rho_0} \right) \frac{1}{\rho^2} \frac{\partial \rho}{\partial \mathbf{x}} & \text{if} \quad \rho \leq \rho_0 \\ 0 & \text{if} \quad \rho > \rho_0 \end{cases} \tag{4}$$

where $\partial\rho/\partial\mathbf{x}$ can be represented as

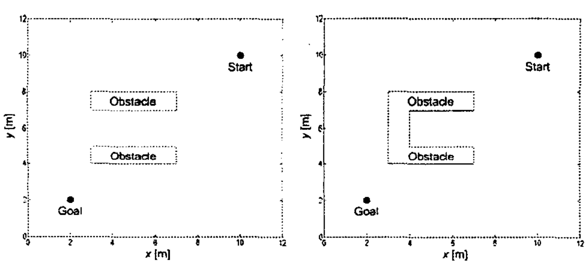
$$\frac{\partial \rho}{\partial \mathbf{x}} = \left(\frac{\partial \rho}{\partial x} \;\; \frac{\partial \rho}{\partial y} \right)^T = \frac{\mathbf{x} - \mathbf{x}_o}{\rho} \tag{5}$$

where $\mathbf{x}_o$ is the position vector of shortest obstacle in an xy-coordinate system [2].

The global potential can be obtained by sum of the attractive potential and repulsive potential. The principle of superposition can be applied to get $U(\mathbf{x})$ as

$$\mathbf{F}(\mathbf{x}) = -\nabla U(\mathbf{x})$$
$$= -\nabla U_{att}(\mathbf{x}) - \nabla U_{rep}(\mathbf{x}) = \mathbf{F}_{att}(\mathbf{x}) + \mathbf{F}_{rep}(\mathbf{x}) \,. \tag{6}$$

In unknown environments, the robot initially has not any informations for the environments and it has a limited sensing range to detect obstacle. In this study, it is assumed that the robot can detect obstacles up to 1m far from itself. The environment of Figure 1(a) has not local minimum, thus robots may successfully reach goal using the APF approach. In the environment of Figure 1(b), however, robots may be trapped by a local minimum. In this case, the global potential has the formation of Figure 2. The robot cannot move to anywhere in the local minimum, because the local minimum is a point where the gradient of the potential becomes zero. In Figure 1, the robot only can detect the obstacle located in 1m far from itself. Therefore the robot can not judge wether the deep aisle-shaped obstacle has a dead-end (Figure 1(a)) or not (Figure 1(b)) before going into the aisle and may be trapped in the local minimum.



(a) Opened aisle (b) Closed aisle

Figure 1. Two types of obstacles with aisle shapes

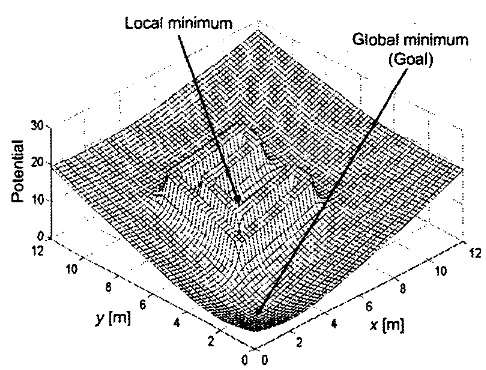


Figure 2. The potential field of the closed aisle

3. SENSOR BASED DISCRETE MODELING OF A MOBILE ROBOT

The sensor based discrete modeling method is useful for modeling of robots with range sensors such as ultrasonic sensors or laser sensors. In this modeling method, the robot skeleton point $\mathbf{p}_i$ is located at position of the range sensor. This technique lets us easily get artificial forces acted on a robot. Figure 3 shows the example of an arbitrary shaped robot with range sensors. The robot has the local coordinate system fixed on it. The point C is the center of mass and the origin of the local coordinate system. Robot skeleton point $\mathbf{p}_i$ is position of a range sensor. To analyze the dynamics of the robot, we should get the position vector of the robot skeleton points relative to O which is the origin of the global coordinate system. The position vector of the robot skeleton points can be gotten by

$$\mathbf{p}_{i/O} = \mathbf{p}_{i/C} + \mathbf{x}_{C/O} \tag{7}$$

where $\mathbf{x}_{C/O}$ is position vector of C relative to O and the set of $\mathbf{p}_{i/C}$ should be initially defined by user.

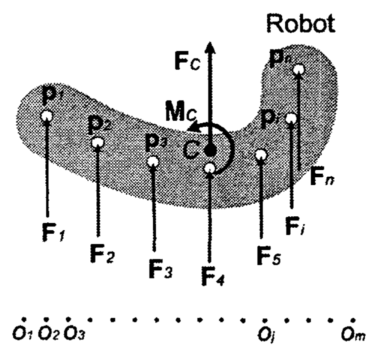


Figure 3. Sensor based discrete modeling and forces acting on robot skeleton points

In the Figure 3, each robot skeleton point is under the action of the force generated in a potential field. These forces can be represented as the total attractive force $\mathbf{F}'_{C,att}$, the total repulsive force $\mathbf{F}'_{C,rep}$, and the total moment $\mathbf{M}'_C$ acted on the center of mass. These forces are simply obtained by

$$\mathbf{F}'_{C,att} = \sum_{i=1}^{n} \mathbf{F}_{att}(\mathbf{p}_{i/O}) \tag{8}$$

$$\mathbf{F}'_{C,rep} = \sum_{i=1}^{n} \mathbf{F}_{rep}(\mathbf{p}_{i/O}) \tag{9}$$

$$\mathbf{M}_C = \sum_{i=1}^{n} \{ \mathbf{p}_{i/C} \times (\mathbf{F}_{att}(\mathbf{p}_{i/O}) + \mathbf{F}_{rep}(\mathbf{p}_{i/O})) \} \tag{10}$$

The robot used in this research has the two drive wheels and a caster and the robot can only make x -directional translational motion and rotational motion. Therefore y-directional forces is converted to moment as

$$\mathbf{M}_C = \mathbf{M}_C + \{R(\mathbf{F}'_{C,att} + \mathbf{F}'_{C,rep}) \cdot \hat{\mathbf{j}}\} \left(\frac{\mathbf{F}_C \times \hat{\mathbf{j}}}{|\mathbf{F}_C|} \right) \tag{11}$$

where R is a radius of a robot and $(\mathbf{F}_C \times \hat{\mathbf{j}})/|\mathbf{F}_C|$ is the unit vector to determine the direction of the rotational motion. The attractive and repulsive force in x-direction are derived by

$$\mathbf{F}_{C,att} = F_{C,att,x}\, \hat{\mathbf{i}} \tag{12}$$

where $F_{C,att,x} = \mathbf{F}'_{C,att} \cdot \hat{\mathbf{i}}$.

$$\mathbf{F}_{C,rep} = F_{C,rep,x}\, \hat{\mathbf{i}} \tag{13}$$

where

$$F_{C,rep,x} = \begin{cases} \mathbf{F}'_{C,rep} \cdot \hat{\mathbf{i}} & \text{if } \mathbf{F}'_{C,rep} \cdot \hat{\mathbf{i}} \le F_{C,att,x} \\ F_{C,att,x} & \text{if } \mathbf{F}'_{C,rep} \cdot \hat{\mathbf{i}} > F_{C,att,x} \end{cases} \tag{14}$$

In Eq. (14), $F_{C,rep,x}$ has the limited value because $F_{C,rep,x}$ sometimes has very high value when the sensors detect very close obstacles and this makes the robot unstable. The maximum value of $F_{C,rep,x}$ is set as $F_{C,att,x}$. The total force $\mathbf{F}_C$ is obtained as.

$$\mathbf{F}_C = \mathbf{F}_{C,att} + \mathbf{F}_{C,rep} = (F_{C,att,x} + F_{C,rep,x})\, \hat{\mathbf{i}} \ . \tag{15}$$

Consequently, we can get the linear acceleration and the angular acceleration of the robot by Newton's 2nd Law as follows :

$$\dot{\mathbf{v}}_C = \frac{\mathbf{F}_C}{m} \tag{16}$$

$$\dot{w}_C = \frac{\mathbf{M}_C}{I} \tag{17}$$

where m is mass of a robot and I is inertia.

The linear velocity of the robot with a control period T can be driven as

$$\begin{aligned} \mathbf{v}'_C(t+T) &= \mathbf{v}_C(t) + \Delta \mathbf{v}_C(t) \\ &= \mathbf{v}_C(t) + T\dot{\mathbf{v}}_C(t) \end{aligned} \tag{18}$$

$$\mathbf{v}_C = \begin{cases} \mathbf{v}'_C & \text{if } |\mathbf{v}'_C| \le v_{\max} \\ v_{\max} \dfrac{\mathbf{v}'_C}{|\mathbf{v}'_C|} & \text{if } |\mathbf{v}'_C| > v_{\max} \ . \end{cases} \tag{19}$$

Because the velocity of the robot has upper limit in real world, the maximum magnitude of the linear velocity is set as $v_{\max}$ when $|\mathbf{v}'_C|$ is greater than $v_{\max}$. In the same way, the angular velocity with the maximum magnitude of $w_{\max}$ is got as

$$\begin{aligned} w'_C(t+T) &= w_C(t) + \Delta w_C(t) \\ &= w_C(t) + T\dot{w}_C(t) \ , \end{aligned} \tag{20}$$

$$w_C = \begin{cases} w'_C & \text{if } |w'_C| \le w_{\max} \\ w_{\max} \dfrac{w'_C}{|w'_C|} & \text{if } |w'_C| > w_{\max} \ . \end{cases} \tag{21}$$

In the path planning of a real robot, these velocities are used as control inputs.

4. VIRTUAL OBSTACLE CONCEPT AND EXTRA POTENTIAL FUNCTION

A virtual obstacle is a new concept to escape local minimum when a robot is trapped by a local minimum. In conventional APF method, the local minimum is formed when an attractive force is equal to a repulsive force. A virtual obstacle has the role of repelling a robot from a local minimum. The virtual obstacle is generated when a robot is trapped by a local minimum and then it makes extra force which repels a robot from a local minimum point. To judge whether the robot is trapped by local minimum or not, we defined the following criterion.

Local-minimum-criterion

When $t \ge T_a$, if $|\mathbf{x}_C(t) - \mathbf{x}_C(t - T_a)| \le S_a$ then the robot is trapped by local minimum,

where $\mathbf{x}_C$ presents position vector of a robot, T_a is time interval, and S_a is set as minimum distance that the robot moves for T_a in non-local minimum condition. S_a should be set as a very small value because the distance between $\mathbf{x}_C(t)$ and $\mathbf{x}_C(t - T_a)$ has very small value when the robot is trapped by local minimum.

When a robot is trapped by a local minimum, a virtual obstacle is located at a trapping point to repel the robot from the local minimum point. The position vector of the trapping point is defined as $\mathbf{x}_{TP}$ which satisfies the following identical equation :

$$\begin{aligned} &\mathbf{F}_{att}(\mathbf{x}_{TP}) \bullet (-\mathbf{F}_{rep}(\mathbf{x}_{TP})) \\ &= MAX\{ \mathbf{F}_{att}(\mathbf{p}_1) \bullet (-\mathbf{F}_{rep}(\mathbf{p}_1)), \\ &\qquad \mathbf{F}_{att}(\mathbf{p}_2) \bullet (-\mathbf{F}_{rep}(\mathbf{p}_2)), \\ &\qquad \cdots, \\ &\qquad \mathbf{F}_{att}(\mathbf{p}_n) \bullet (-\mathbf{F}_{rep}(\mathbf{p}_n)) \} \ . \end{aligned} \tag{22}$$

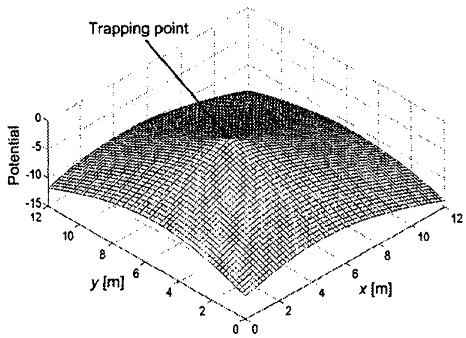


Figure 4. Extra potential by a virtual obstacle

The trapping point is selected among the robot skeleton points, and the inner product of the attractive force and the repulsive force has the maximum value at this point. The repulsive force at the trapping point has the maximum component force about the opposit direction of attractive force among all the skeleton points of the robot. Therefore, that point may has major influence on trapping the robot, and the robot should be far away from this point.

The virtual obstacle has an extra potential to repel the robot from the trapping point. The extra potential U_{ext} (Figure 4) is defined as

$$U_{ext}(\mathbf{x}) = \begin{cases} -\dfrac{k_e}{2d_e}|\mathbf{x}-\mathbf{x}_{TP}|^2 & \text{if } |\mathbf{x}-\mathbf{x}_{TP}| \le d_e \\ -k_e(|\mathbf{x}-\mathbf{x}_{TP}| - \dfrac{d_e}{2}) & \text{if } |\mathbf{x}-\mathbf{x}_{TP}| > d_e \end{cases} \quad (23)$$

where d_e is the range of the quadratic part in the extra potential. This quadratic part is required for differential at trapping point. The extra potential has maximum value at the trapping point and it decreases as the robot moves from trapping point. Therefore, this potential can repel the robot from the virtual obstacle. The extra force $\mathbf{F}_{ext}$ is obtained by the negative gradient of an extra potential as follows :

$$\mathbf{F}_{ext}(\mathbf{x}) = -\nabla U_{ext} = \begin{cases} \dfrac{k_e}{d_e}(\mathbf{x}-\mathbf{x}_{TP}) & \text{if } |\mathbf{x}-\mathbf{x}_{TP}| \le d_e \\ k_e\dfrac{\mathbf{x}-\mathbf{x}_{TP}}{|\mathbf{x}-\mathbf{x}_{TP}|} & \text{if } |\mathbf{x}-\mathbf{x}_{TP}| > d_e. \end{cases} \quad (24)$$

If d_e is set as a very small value, the extra force can be redefined as

$$\mathbf{F}_{ext}(\mathbf{x}) = -\nabla U_{ext} \approx \begin{cases} 0 & \text{if } |\mathbf{x}-\mathbf{x}_{TP}| = 0 \\ k_e\dfrac{\mathbf{x}-\mathbf{x}_{TP}}{|\mathbf{x}-\mathbf{x}_{TP}|} & \text{if } |\mathbf{x}-\mathbf{x}_{TP}| > 0. \end{cases} \quad (25)$$

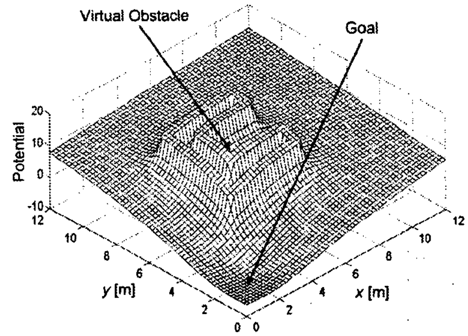


Figure 5. Global potential with virtual obstacle

$U_1(\mathbf{x})$ of a global potential without an extra potential and $U_2(\mathbf{x})$ containing an extra potential are defined as follows :

$$U_1(\mathbf{x}) = U_{att}(\mathbf{x}) + U_{rep}(\mathbf{x}) \quad (26)$$

$$U_2(\mathbf{x}) = U_1(\mathbf{x}) + U_{ext}(\mathbf{x}) = U_{att}(\mathbf{x}) + U_{rep}(\mathbf{x}) + U_{ext}(\mathbf{x}) . \quad (27)$$

The total extra force is obtained as

$$\mathbf{F}'_{C,ext} = \sum_{i=1}^{n} \mathbf{F}_{ext}(\mathbf{p}_{i/O}). \quad (28)$$

The extra force acted on the real robot in y-direction is derived as

$$\mathbf{F}_{C,ext} = F_{C,ext,x}\,\hat{\mathbf{i}} \quad (29)$$

where $F_{C,ext,x} = \mathbf{F}'_{C,ext} \cdot \hat{\mathbf{i}}$

Then, the distributed forces are expressed as

$$\mathbf{F}_{C,1} = \mathbf{F}_{C,att} + \mathbf{F}_{C,rep} \quad (30)$$

$$\mathbf{F}_{C,2} = \mathbf{F}_{C,att} + \mathbf{F}_{C,rep} + \mathbf{F}_{C,ext} \quad (31)$$

Figure 5 shows the extra potential added to the original global potential. In this formation of potential, the robot can escape from a local minimum because the trapping point has higher potential than its around. The extra potential is applied while a robot is in a local minimum area. The local minimum area means the region that the robot may return to local minimum when the virtual obstacle is cleared.

When the robot is in a local minimum area, it may move away from the local minimum by a virtual obstacle, and also it may move away from goal because the direction of the goal is similar to the direction of the trapping point when the robot is located at the local minimum area. The robot should move to goal only after it escapes local minimum area. Therefore, we define the following criterion to judge whether the robot escapes the local minimum area or not :

<u>*Escape-local-minimum-area-criterion*</u>

When $t - t_{TP} \ge T_b$,

if $|\mathbf{x}_C(t)-\mathbf{x}_d| \leq |\mathbf{x}_C(t-T_b)-\mathbf{x}_d|$ then it is assumed that the robot escaped the local minimum

where, t_{TP} is the time when a robot is located at a trapping point, T_b is the time interval between current time and previous time to get the change of the distance to goal, and the conditional expression means that the robot moves to the goal direction.
The whole path planning algorithm is as follows:

Path-planning-algorithm

Step 1
$t=0$
$\mathbf{x}_C(0)=\mathbf{x}_{START}$
$\mathbf{v}_C(0)=0$
$w_C(0)=0$
$LocalMinimumFlag = 0.$

Step 2
Sensing obstacles
Detect robot position by encoders.
If $LocalMinimumFlag = 0,$
  $\mathbf{F}_C=\mathbf{F}_{C,1}$
Else if $LocalMinimumFlag = 1,$
  $\mathbf{F}_C=\mathbf{F}_{C,2}$.
Calculate $\dot{\mathbf{v}}_C(t)$, $\dot{w}_C(t)$ by Eqs. (16) and (17)
Calculate $\mathbf{v}_C(t)$, $w_C(t)$ by Eqs. (19) and (21)
Control the robot by $\mathbf{v}_C(t)$, $w_C(t)$
$t=t+T$

Step 3
Local-minimum-criterion
if the robot is trapped by local minimum
  then $LocalMinimumFlag = 1$
Search the trapping point $\mathbf{x}_{TP}$ satisfying Eq. (22).

Step 4
Escape-local-minimum-area-criterion.
if the robot escapes local minimum area
  then $LocalMinimumFlag = 0.$

Step 5
if $|\mathbf{x}_C-\mathbf{x}_d| \leq Tolerance$
  then path planning is completed,
else return to Step 2.

5. EXPERIMENTS

We performed some experiments to evaluate the sensor based discrete modeling method and the virtual obstacle approach. Figure 6 shows the arrangement of ultrasonic sensors attached on the robot. The position vectors of sensors are defined in Table 1. The robot initially has not any informations for the environments and it can detect obstacles up to 1m far from itself. The experiments of Figure 7 and Figure 8 have not any local minimum, therefore the robot can successfully reach goal by only the APF approach. In Figure 9, the obstacle has the shape of a closed aisle. Therefore, the robot was trapped by a local minimum when the APF approach is applied. In the same environment, Figure 10 shows that the robot can escape the local minimum by the virtual obstacle approach and it can successfully reach goal. These results of the experiments show that this technique is useful for concave obstacles and for deep aisle-shaped obstacles.

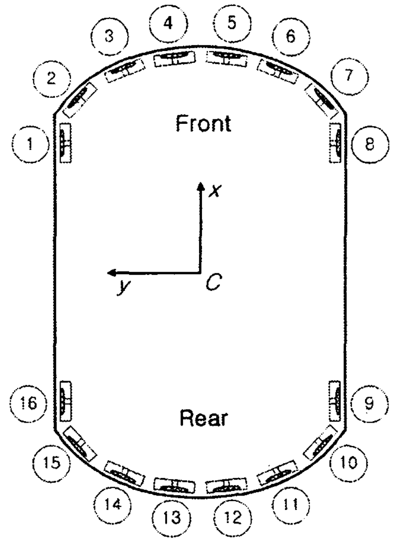


Figure 6. Arrangement of ultrasonic sensors

Table 1. The position of the sensors

Sensor number (i)	Position vector ($\mathbf{p}_{i/C}$ $[m]$)	Sensor Angle (α_i)
1	$0.115\,\hat{i}+0.130\,\hat{j}$	90°
2	$0.155\,\hat{i}+0.115\,\hat{j}$	50°
3	$0.190\,\hat{i}+0.080\,\hat{j}$	30°
4	$0.210\,\hat{i}+0.025\,\hat{j}$	10°
5	$0.210\,\hat{i}-0.025\,\hat{j}$	−10°
6	$0.190\,\hat{i}-0.080\,\hat{j}$	−30°
7	$0.155\,\hat{i}-0.115\,\hat{j}$	−50°
8	$0.115\,\hat{i}-0.130\,\hat{j}$	−90°
9	$-0.115\,\hat{i}-0.130\,\hat{j}$	−90°
10	$-0.155\,\hat{i}-0.115\,\hat{j}$	−130°
11	$-0.190\,\hat{i}-0.080\,\hat{j}$	−150°
12	$-0.210\,\hat{i}-0.025\,\hat{j}$	−170°
13	$-0.210\,\hat{i}+0.025\,\hat{j}$	170°
14	$-0.190\,\hat{i}+0.080\,\hat{j}$	150°
15	$-0.155\,\hat{i}+0.115\,\hat{j}$	130°
16	$-0.115\,\hat{i}+0.130\,\hat{j}$	90°

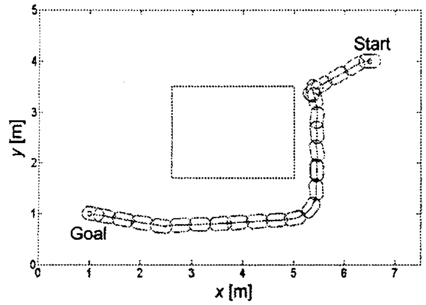


Figure 7. Rectangular obstacle and path planning by APF method

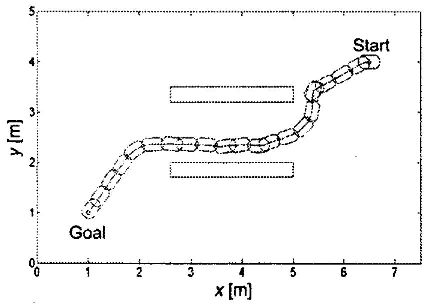


Figure 8. Oped aisle and path planning by APF method

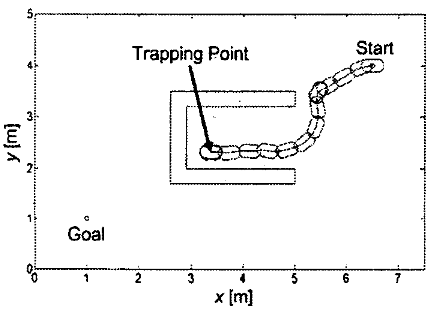


Figure 9. Closed aisle and failed path planning by APF method

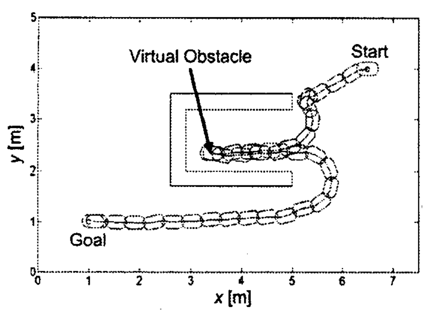


Figure 10. Closed aisle and path planning by APF method with virtual obstacle

6. CONCLUSIONS

In this study, we proposed the virtual obstacle concept to escape local minimums in local path planning based on APF approach. The virtual obstacle with the extra potential is located at a trapping point when robots are trapped by local minimums. The extra potential is added to global potential and it can repel the robot from the local minimum. The major advantage of the proposed path planner is simple and efficient to solve the local minimum problem. The results of the experiments showed that this technique is useful for the path planning in various environments. The sensor based discrete modeling method was also proposed to get a simple model of the robots with range sensors. This method is useful for real-time path planning because it lets us easily get artificial forces

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