CENTRO DE INVESTIGACIÓN Y DE ESTUDIOS AVANZADOS DEL IPN
UNIDAD TAMAULIPAS

Assignment #2

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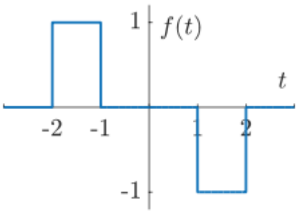
Course: *Introducción al Análisis de Fourier (Sep - Dec 2022)*
Professor: *Dr. Wilfrido Gómez-Flores*
November 15, 2022

Función 1



$$f(t) = \begin{cases} +1, & -2 < t < -1 \\ -1, & 1 < t < 2 \\ 0, & otro\ caso \end{cases}$$

Transformada de Fourier

$$\int_{-\infty}^{\infty} f(t)\, e^{-jwt}\, dt$$

$$\int_{-2}^{-1} 1 * e^{-jwt} dt + \cancel{\int_{-1}^{1} 0 * e^{-jwt}} dt + \int_{1}^{2} -1 * e^{-jwt} dt = \int_{-2}^{-1} e^{-jwt} dt - \int_{1}^{2} e^{-jwt} dt$$

Recordando la integral de una exponencial:

$$\int e^{u} du = e^{u} + C \Rightarrow \frac{-e^{-jwt}}{jw}\Big|_{-2}^{-1} + \frac{-e^{-jwt}}{jw}\Big|_{1}^{2}$$

$$= \left[\frac{e^{jw}(e^{jw}-1) - e^{-2jw}(e^{jw}-1)}{jw}\right] = \left[\frac{e^{2jw} - e^{jw} - e^{-jw} + e^{-2jw}}{jw}\right]$$

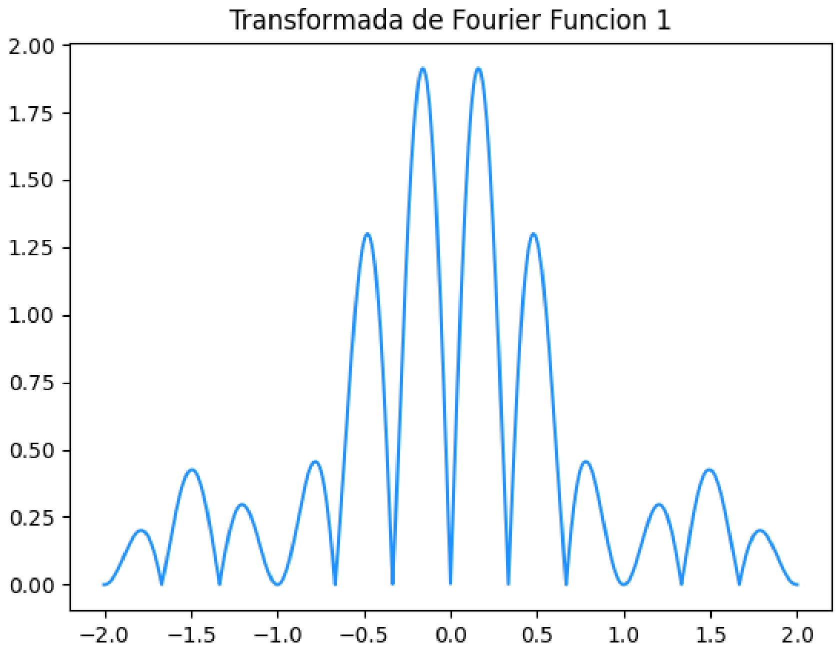
$$= \left[\frac{e^{2jw} + e^{-2jw}}{jw} - \frac{e^{jw} + e^{-jw}}{jw}\right] * \frac{2}{2} = \frac{2}{w} * \left[\left(\frac{e^{2jw} + e^{-2jw}}{2j}\right) - \left(\frac{e^{jw} + e^{-jw}}{2j}\right)\right]$$

Aplicando identidad:

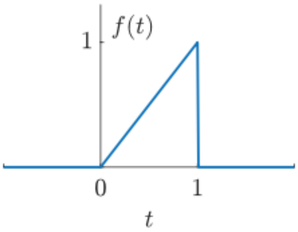
$$\frac{2}{w} * \left[\left(\frac{e^{2jw} + e^{-2jw}}{2j}\right) - \left(\frac{e^{jw} + e^{-jw}}{2j}\right)\right] \Rightarrow cos(\theta) = \frac{e^{i\theta} + e^{-i\theta}}{2j}$$

$$\boxed{\frac{2}{w} * (cos(2w) - cos(w))}$$

Función 1


Transformada de Fourier Funcion 1
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0

Función 2



$$f(t) = \begin{cases} t, & 0 < t < 1 \\ 0, & otro\ caso \end{cases}$$

Transformada de Fourier

$$\int_{-\infty}^{\infty} f(t)\, e^{-jwt}\, dt$$

$$\int_0^1 t * e^{-jwt} dt$$

Resolviendo la integral por partes
sea $u = t; du = dt;$
$dv = e^{-jwt}dt;$
$v = \frac{-e^{-jwt}}{jw}$

$$= \left(\frac{-t * e^{-jwt}}{jw}\right)\Big|_0^1 + \frac{1}{jw}\int_0^1 e^{-jwt}dt$$

$$\frac{-1 * e^{-jw}}{jw} + \frac{1}{jw}\int_0^1 e^{-jwt}dt = \frac{-e^{-jw}}{jw} + \frac{1}{jw}\left(\frac{-e^{-jw}+1}{jw}\right)$$

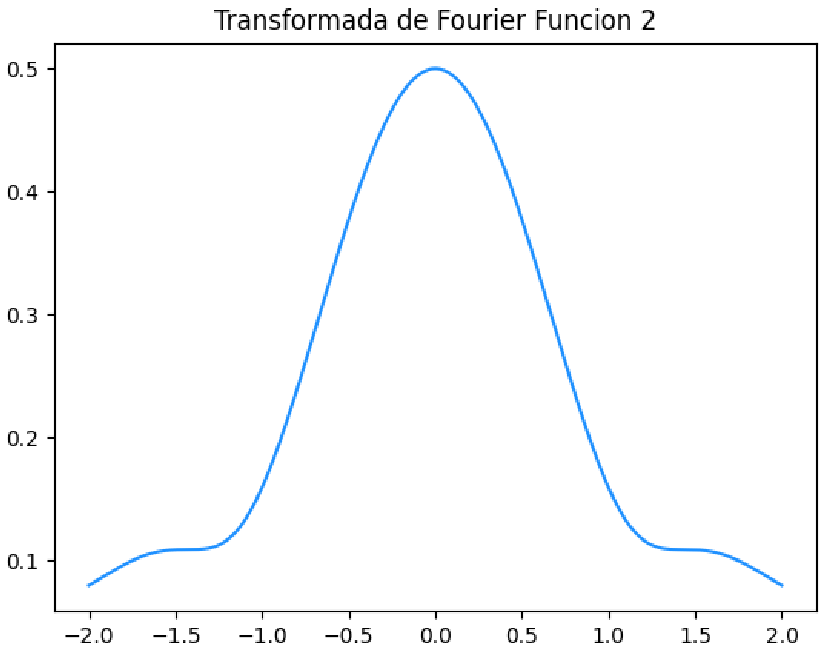
$$\frac{-e^{-jw}}{jw} - \left(\frac{e^{-jw}-1}{(jw)^2}\right)$$

$$= \left[\frac{-e^{-jw} * jw - e^{-jw} + 1}{(jw)^2}\right]$$

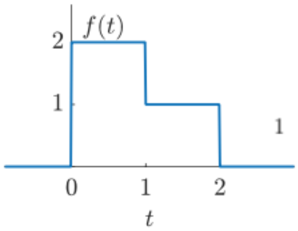
Factorizando

$$\boxed{\frac{e^{-jw}(-jw + e^{jw} - 1)}{(jw)^2}}$$

Función 2


Transformada de Fourier Funcion 2
0.5
0.4
0.3
0.2
0.1
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0

Función 3



$$f(t) = \begin{cases} 2, & 0 < t < 1 \\ 1, & 1 < t < 2 \\ 0, & otro\ caso \end{cases}$$

Transformada de Fourier

$$\int_{-\infty}^{\infty} f(t)\, e^{-jwt}\, dt$$

$$\int_0^1 2 * e^{-jwt} dt + \int_1^2 1 * e^{-jwt} dt + 0 = 2\int_0^1 e^{-jwt} dt + \int_1^2 e^{-jwt} dt + 0$$

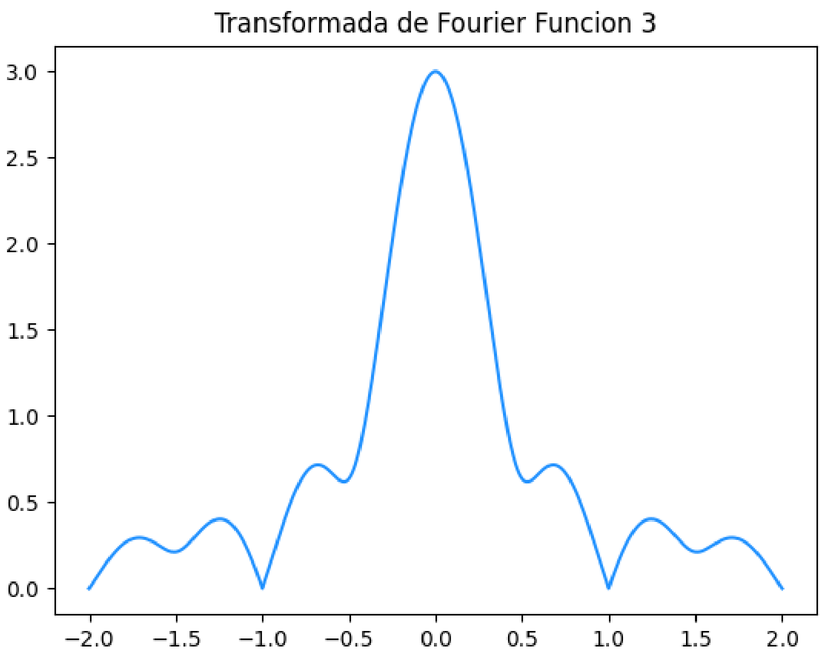
$$2\left(\frac{-e^{-jwt}}{jw}\right)\Big|_0^1 + \frac{-e^{-jwt}}{jw}\Big|_1^2$$

$$= 2\left(\frac{1 - e^{-jw}}{jw}\right) + \frac{e^{-2jw} * (e^{jw} - 1)}{jw}$$

$$= \frac{2 - 2 * e^{-jw}}{jw} + \frac{e^{-jw} - e^{-2jw}}{jw} = \frac{2 - e^{-jw} - e^{-2jw}}{jw}$$

$$\boxed{\frac{2 - e^{-jw}(1 + e^{-jw})}{jw}}$$

Función 3



Código python

Con uso de las librerias

```
import numpy as np
import math
import matplotlib.pyplot as plt

_x_ = np.linspace(-2,2,10000)
f = []

#iterar para los valores de t
for i in _x_:
    w = 2*np.pi*i
    #f.append(abs((2/w)*(np.cos(2*w)-np.cos(w)))) #Funcion1
    #f.append(abs((np.exp(-1j*w)*(-1j*w+np.exp(1j*w)-1))/((1j*w)*(1j*w)))) #Funcion2
    f.append(abs((2-np.exp(-1j*w)-np.exp(-2j*w))/(1j*w))) #Funcion3

plt.plot(_x_,f,color="dodgerblue")
plt.title("Transformada de Fourier Funcion 3")
plt.savefig("trans_fourier_3.png")
```