Centro de Investigación y de Estudios Avanzados del IPN
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Assignment #3

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Section 1.1 (page 12)

Question 1

Knowing that a proposition is a declarative sentence (that is, a sentence that declares a fact) that is either true or false, but both we can answer the following question.

1. Which of these sentences are propositions? What are the truth values of those are propositions?

   (a) Boston is the capital of Massachusetts
   It is a proposition because it is True and Valid
   (YES, TRUE)

   (b) Miami is the capital of Florida
   It is a proposition but it is not True, according to Google the capital of Florida is Tallahassee
   (YES, FALSE)

   (c) $2 + 3 = 5$
   It is a proposition and the math is correct.
   (YES, TRUE)

   (d) $5 + 7 = 10$
   It is a proposition but the math is not correct.
   (YES, FALSE)

   (e) $x + 2 = 11$
   It is **not** a proposition because does not have all the data.
   (NO PROPOSITION)

   (f) Answer this question
   It is **not** a proposition because it is ambiguous.
   (NO PROPOSITION)

Question 3

3. What is the negation of each of these propositions?

   (a) Mei has an MP3 player
   Mei does not have an MP3 Player

(b) There is no pollution in New Jersey
There is pollution in New Jersey

(c) $2 + 1 = 3$
$\mathbf{2 + 1 \neq 3}$

(d) The summer in Maine is hot and sunny
The summer in Maine is not hot and not sunny

Question 5

5. What is the negation of each of these propositions?

(a) Steve has more than 100 GB free disk space on his laptop
Steve does not have more than 100 GB freedisk

(b) Zach blocks e-mails and texts from Jennifer
Zach does not block emails from Jennifer

(c) $7 * 11 * 13 = 990$
$\mathbf{7 * 11 * 13 \neq 990}$

(d) Diane rode her bicycle 100 miles on Sunday
Diane did not ride her bicycle 100 miles on Sunday

Question 9

9. Let $\boldsymbol{p}$ "Swimming at the New Jersey shore is allowed" and $\boldsymbol{q}$ "Sharks have been spotted near the shore," Express each of these compound propositions as an English sentence

(a) $\neg q$
Sharks have not been spotted near the Shore

(b) $p \wedge q$
Swimming at the New Jersey shore is allowed and Sharks have been spotted near the shore

(c) $\neg p \vee q$
Swimming at the New Jersey shore is not allowed or sharks have been spotted near the shore

(d) $p \implies \neg q$
If swimming at the New Jersey shore is allowed, then sharks have not been spotted near the shore

(e) $\neg q \implies p$
If sharks have not been spotted near the shore, then swimming at the New Jersey shore is allowed

(f) $\neg p \implies \neg q$
If swimming at the New Jersey shore is not allowed, then sharks have not been spotted near the shore

(g) $p \iff \neg q$
Swimming at the New Jersey shore is allowed if and only if sharks have not been spotted near the shore

(h) ✯$\neg p \wedge (p \vee \neg q)$
Looking for a equivalence from the original expresion we have:

$$(\neg p \wedge p) \vee (\neg p \vee \neg q) \textit{ Appliying distributive}$$
$$F \vee (\neg p \vee \neg q) \textit{ Dominance}$$
$$\neg p \vee \neg q \textit{ Equivalent expression}$$

Swimming at the New Jersey shore is not allowed or sharks have not been spotted near the shore

Question 13

13. Let **p** and **q** be the propositions
p: You drive over 65 miles per hour.
q: You get a speeding ticket.

Write these propositions using p and q and logical connectives (including negations)

(a) You do not drive over 65 miles per hour
$\boldsymbol{\neg p}$

(b) You drive over 65 miles per hour, but you do not get a speeding ticket
$\boldsymbol{p \wedge \neg q}$
***Note that in logic the word **but** sometimes is used insted of **and** in conjunction*

(c) You will get a speeding ticket if you drive over 65 miles per hour
$\boldsymbol{p \implies q}$

(d) If you do not drive over 65 miles per hour, then you will not get a speeding ticket
$\boldsymbol{\neg p \implies \neg q}$

(e) Driving over 65 miles per hour is sufficient for getting a speeding ticket
$\boldsymbol{p \iff q}$

(f) You get a speeding ticket, but you do not drive over 65 miles per hour
$\boldsymbol{q \wedge \neg p}$

(g) Whenever you get a speeding ticket, you are driving over 65 miles per hour
$\boldsymbol{q \implies p}$

Question 15

15. Let p, q, and r be the propositions
p: Grizzly bears have been seen in the area.
q: Hiking is safe on the trail.
r: Berries are ripe along the trail.

Write these propositions using p, q and r and logical connectives (including negations)

(a) Berries are ripe along the trail, but grizzly bears have not been seen in the area
$\boldsymbol{r \land \neg p}$

(b) Grizzly bears have not been seen in the area and hiking on the trail is safe, but berries are ripe along the trail
$\boldsymbol{\neg p \land q \land r}$

(c) If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area.
$\boldsymbol{r \implies (q \iff \neg p)}$

(d) It is not safe to hike on the trail, but grizzly bears have not been seen in the area and the berries along the trail are ripe
$\boldsymbol{\neg q \land \neg p \land r}$

(e) For hiking on the trail to be safe, it is necessary but not sufficient that berries not be ripe along the trail and for grizzly bears not to have been seen in the area
$\boldsymbol{(q \implies (\neg r \land \neg p))}$ ✯

(f) Hiking is not safe on the trail whenever grizzly bears have been seen in the area and berries are ripe along the trail
$\boldsymbol{(p \land r) \implies \neg q}$

Question 17

17. Determine whether each of these conditional statements is true or false

(a) If $1 + 1 = 2$, then $2 + 2 = 5$
The first part is true, but the second not.
By the table of truth $\boldsymbol{True \implies False \cong False}$

(b) If $1 + 1 = 3$, then $2 + 2 = 4$
The first part is false, but the second is true.
By the implication table of truth $\boldsymbol{False \implies True \cong True}$

(c) If $1 + 1 = 3$, then $2 + 2 = 5$
The first part is false also the second.
By the implication table of truth $\boldsymbol{False \implies False \cong True}$

(d) If monkey can fly, then $1 + 1 = 3$
The first part is not true also the second.
By the implication table of truth $\boldsymbol{False \implies False \cong True}$

Question 27

27. State the converse, contrapositice, and inverse of each of these conditional statements.
Let us remmember that
$p \implies q$ the converse is $q \implies p$, the inverse is $\neg p \implies \neg q$ and the contrapositive is $\neg q \implies \neg p$ knowing that ..

   (a) If it snows today, I will ski tomorrow

      i. **Converse:** I will ski tomorrow **only if** it snows today.
      ii. **Contrapositive:** If I do **not** ski tomorrow, then it will **not** have snowed today.
      iii. **Inverse:** If it does **not** snow today, then I will **not** ski tomorrow.

   (b) I come to class whenever there is going to be a quiz

      i. **Converse:** I come to class **only if** there will be a quiz.
      ii. **Contrapositive:** If I do **not** come to class, then there will **not** be a quiz.
      iii. **Inverse:** If there is **not** going to be a quiz, then I do **not** come to class

   (c) A positive integer is a prime only if it has no divisors other than 1 and itself

      i. **Converse:** A positive integer is a prime **only if** it has no divisors other than 1 and itself
      ii. **Contrapositive:** If a positive integer has a divisor other than 1 and itself, then it is not prime
      iii. **Inverse:** If a positive integer is **not** prime, then it has a divisor other than 1 and itself

Question 29

29. How many rows appear in a truth table for each of these compound propositions? There we can take advantage of what we leart in the past module by the product rule 2^n

   (a) $p \implies \neg p$
   $\mathbf{2^1 = 2}$

   (b) $(p \vee \neg r) \wedge (q \vee \neg s)$
   $\mathbf{2^4 = 16}$

   (c) $q \vee p \vee \neg s \neg r \vee \neg t \vee u$
   $\mathbf{2^6 = 64}$

   (d) $(p \wedge r \wedge t) \iff (q \wedge t)$
   $\mathbf{2^4 = 16}$

Question 35

35. Construct a truth table for each of these compound propositions

(a)

p	q	$\neg q$	$(p \to \neg q)$
0	0	1	1
0	1	0	1
1	0	1	1
1	1	0	0

(b)

p	q	$\neg p$	$(\neg p \leftrightarrow q)$
0	0	1	0
0	1	1	1
1	0	0	1
1	1	0	0

(c)

p	q	$\neg p$	$(p \to q)$	$(\neg p \to q)$	$(p \to q) \vee (\neg p \to q)$
0	0	1	1	0	1
0	1	1	1	1	1
1	0	0	0	1	1
1	1	0	1	1	1

(d)

p	q	$\neg p$	$(p \to q)$	$(\neg p \to q)$	$(p \to q) \wedge (\neg p \to q)$
0	0	1	1	0	0
0	1	1	1	1	1
1	0	0	0	1	0
1	1	0	1	1	1

(e)

p	q	$\neg p$	$(p \leftrightarrow q)$	$(\neg p \leftrightarrow q)$	$(p \leftrightarrow q) \vee (\neg p \leftrightarrow q)$
0	0	1	1	0	1
0	1	1	0	1	1
1	0	0	0	1	1
1	1	0	1	0	1

(f)

p	q	$\neg p$	$\neg q$	$(\neg p \leftrightarrow \neg q)$	$(p \leftrightarrow q)$	$(\neg p \leftrightarrow \neg q) \leftrightarrow (p \leftrightarrow q)$
0	0	1	1	1	1	1
0	1	1	0	0	0	1
1	0	0	1	0	0	1
1	1	0	0	1	1	1

Section 1.3

Question 7

7. Use De Morgan's laws to find the negation of each of the following statements.

   (a) Jan is rich and happy
   Jan is not rich or Jan is not happy

   (b) Carlos will bicycle or run tomorrow
   Carlos will not bicycle, and Carlos will not run tomorrow

   (c) Mei walks or takes the bus to class
   Mei does not walks, and Mei does not takes the bus to class

   (d) Ibrahim is smart and hard working
   Ibrahim is not smart or Ibrahim is not hard working

Question 9

9. Show that each of these conditional statements is a tautology by using truth tables.

(a)

p	q	$(p \wedge q)$	$(p \wedge q) \rightarrow p$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	1

(b)

p	q	$(p \vee q)$	$p \implies (p \vee q)$
0	0	0	1
0	1	1	1
1	0	1	1
1	1	1	1

(c)

p	q	$\neg p$	$(p \implies q)$	$(\neg p \implies (p \implies q)$
0	0	1	1	1
0	1	1	1	1
1	0	0	0	1
1	1	0	1	1

(d)

p	q	$p \wedge q$	$p \implies q$	$(p \wedge q) \implies (p \implies q)$
0	0	0	1	1
0	1	0	1	1
1	0	0	0	1
1	1	1	1	1

(e) We can find the equivalence of the expresion
**remembering that
$\neg(p \implies q) \equiv \neg(\neg p \vee q) \equiv (p \wedge \neg q)$

p	q	$\neg q$	$p \wedge \neg q$	p	$\neg(p \implies q) \implies p$
0	0	1	0	0	1
0	1	0	0	0	1
1	0	1	1	1	1
1	1	0	0	1	1

(f) $\neg(p \implies q) \implies \neg q$
Using De Morgans Law $\neg(p \implies q) \equiv p \wedge \neg q$

p	q	$\neg q$	$p \wedge \neg q$	$\neg q$	$\neg(p \implies q) \implies \neg q$
0	0	1	0	1	1
0	1	0	0	0	1
1	0	1	1	1	1
1	1	0	0	0	1

Question 11

11. Show that each conditional statement in Question 9 is a tautology without using truth tables.

(a) $(p \wedge q) \implies p$
It has an "and" connection, that give us some information, to make the condition happens p and q must be **True**. If that happens p is also **True**. In other hand also works when both p and q are False by the implication table of truth that tell us if p and q does not happens the conditional is **True**.

(b) $p \implies (p \vee q)$
The p variable has the main function because in the part of $p \vee q$ for every positive value is **True** and the conditional give us the reason, in the other hand if it is False, by the conditional table of truth $False \implies False$ give us **True**

(c) $\neg p \implies (p \implies q)$
Evaluatin p as True in the conditional statement give us $False \implies (True \implies MaybeFalse)$, what le us think in the conditional table of truth $False \implies False$ is True in the other hand evaluating p as False. $True \implies True$ that is True and we can certantly say that is a **tautology**

(d) $(p \wedge q) \implies (p \implies q)$
Taking $p \wedge q$ as a positive value $True \implies True$ give us as result **True**, because the conditional $True \implies Maybe_True_or_False$ give us **True**

(e) $\neg(p \implies q) \implies p$
The possibilities to give us **True** are when $\neg(p \implies q)$ is **False** and p is **True**, in the other hand when $\neg(p \implies q)$ is **True**, p should be also **True**

(f) $\neg(p \implies q) \implies \neg q$
Evaluating the expresion $\neg(p \implies q)$ as **True** and $\neg q$ as True .. to the condi-

tion works, and in the other hand if $\neg(p \implies q)$ as False, but the negation of False is **True** the condition works.

Question 15

15. Determine whether $(\neg q \wedge (p \implies q)) \implies \neg p$ is a tautology Making the truth table to know it

p	q	$\neg p$	$\neg q$	$p \implies q$	$\neg q \wedge (p \implies q)$	$\neg p$	$(\neg q \wedge (p \implies q)) \implies \neg p$
0	0	1	1	1	1	1	1
0	1	1	0	1	0	1	1
1	0	0	1	0	0	0	1
1	1	0	0	1	0	0	1

Question 17

17. Show that $\neg(p \iff q)$ and $p \iff \neg q$ are logically equivalent.

p	q	$\neg q$	$\neg p$	$p \iff \neg q$	$\neg p \wedge \neg q$	$p \vee q$	$\neg(p \iff q)$
0	0	1	1	0	1	0	0
0	1	0	1	1	1	1	1
1	0	1	0	1	1	1	1
1	1	0	0	0	0	1	0

Question 31

31. Show that $(p \implies q) \implies r$ and $p \implies (q \implies r)$ are not logically equivalent.

p	q	r	$p \implies q$	r	$(p \implies q) \implies r$	$(p \implies r)$	$q \implies r$	$(p \implies r) \wedge (q \implies r)$
0	0	0	1	0	0	1	1	1
0	0	1	1	1	1	1	1	1
0	1	0	1	0	0	1	0	0
0	1	1	1	1	1	1	1	1
1	0	0	0	0	1	0	1	0
1	0	1	0	1	1	1	0	0
1	1	0	1	0	0	0	0	0
1	1	1	1	1	1	1	1	1

Question 35

35. Find the dual of each of these compound propositions.
A proposition is said to be dual if and only if it is dual is equivalent to the given function.
Chnaging the and for or, and vice-versa. A constant 1 (or true) of a given function is changed to a constant 0 (or false) and vice-versa

(a) $p \wedge \neg q \wedge \neg r$ DUAL: $\boldsymbol{p \vee \neg q \vee \neg r}$

(b) $(p \wedge q \wedge r) \vee s$ DUAL: $\boldsymbol{(p \vee q \vee r) \wedge s}$

(c) $(p \vee F) \wedge (q \vee T)$ DUAL: $\boldsymbol{(p \wedge T) \vee (q \wedge F)}$