

The Effects of Computer-Augmented Geometry Instruction on Student Performance and Attitudes

Charles Funkhouser
California State University, Fullerton

Abstract

This study reports on the effects of computer software on mathematics achievement and attitudes toward mathematics of secondary school students. Members of an experimental group were involved in geometry instruction using a constructivist approach to computer-augmented activities, while members of a control group were involved in more traditional geometry instruction. Members of the experimental group demonstrated significantly better performance on a standardized test of geometry concepts than the control group at the 0.05 level of significance. Results of assessments of student attitudes toward mathematics were mixed. Implications of results for mathematics instruction are discussed. (Keywords: cognitive science, computer-assisted instruction, constructivism, geometry, mathematics.)

The Mathematical Association of Great Britain offered that the mathematics of today may not be remembered as much as the newer methods we might integrate into our mathematics and our schools (Mann & Tall, 1992). A review of the literature on methods for instruction in mathematics education suggests two useful approaches: constructivism and computer-augmented instruction. Professional publications such as the *Journal of Research on Technology in Education*, the *Mathematics Teacher*, and *School Science and Mathematics* often report on research or curriculum proposals based wholly or in part on these two approaches. Methods such as constructivism and computer-augmented instruction are among the possible innovative instructional procedures that may affect how our mathematics instruction is remembered.

Geometry is advanced as a topic of increased importance in the mathematics curriculum (National Council of Teachers of Mathematics [NCTM], 1985), but at the same time, traditional instructional approaches to geometry instruction are seen as not responsive to the potential of today's learners (Del Grande & Morrow, 1993; Meiring, 1992; NCTM, 1985). The NCTM (2000) strongly advocates a new focus for geometry instruction in the modern mathematics curriculum.

How can the methods of constructivism and computer-augmented instruction be used to more effectively develop student geometry performance and student attitudes toward mathematics?

This study and the related research grew out of efforts to work with high school mathematics teachers to respond to a changing mathematics curriculum and new instructional procedures to affect that curriculum. Much of the theoretical basis for the instructional procedures was derived from cognitive science. These procedures centered on providing an environment in which students might construct a coherent system of geometry using software specifically designed for geometric exploration.

BACKGROUND

Constructivism

Cognitive psychologists such as Bruner (1960), Dienes (1964, 1967), and Piaget (1960) have long suggested the importance of active student involvement in learning. These theoreticians believed that learning is a very personal matter involving internalization of the concepts by the learner. In contrast to behaviorists (see Gagne, 1985; Skinner, 1938; Thorndike, 1911), cognitive psychologists emphasize not only the parts of knowledge but also the interrelationship among the parts and the relationship between the parts and the whole.

An instructional methodology derived from cognitive theory is constructivism. Its fundamental premise is the idea that all knowledge is constructed by the learner using the learner's past experiences and existing knowledge structures (schema). The role of the teacher, from a constructivist point of view, is to create an environment in which learners can construct, develop, and extend their mathematical view of the world.

This study reports on a computer-based environment created to allow students to "construct" the geometry commonly taught at the secondary level. This computer-augmented environment allowed the students to explore, discover, conjecture, and confirm mathematics in ways consistent with the theory of cognitive psychology.

Computer-Augmented Geometry Instruction

Niemiec and Walberg (1987) and Dennis and Kansky (1984) noted the anticipated effects of computers on mathematics instruction. They further noted that since the emergence of the microcomputer in the late 1970s, computers have been more widely available at every educational level. They reported that 90% of schools use computers in some mode of instruction. While microcomputers may be widely available, Cuban (1989) reported that computer-assisted instruction (CAI) accounts for only 5% of all instruction in schools. Nevertheless, a meta-analysis of studies of secondary school mathematics students by Kulik, Bangert, and Williams (1983) found that CAI methods tended to moderately raise mathematics achievement and foster more positive attitudes toward mathematics.

Papert (1980, 1993) maintained that, through Logo, active participation in mathematical explorations would allow students to master geometric concepts in an informal manner. Battista and Clements (1988), Clements (1998), Kenney (1987), and Martin and Bearden (1985) supported Papert's claims by suggesting that Logo environments promote conjecturing, discovery learning, and problem-solving skills. Their research further suggested that Logo helps learners move through van Heile's (1987) hierarchy of cognitive development in geometry.

Outcomes from other research on computer-augmented geometry instruction is mixed. Turner (1988) reported that, in a controlled study, students who received a Logo-based treatment in geometry showed no significant increase in conceptualization of geometric shapes, coordinate systems, negative numbers, or variables. (Results from this study were even less promising for low-achieving students.) Gallini (1987) found that students who used Logo and other

computer-augmented instruction were better at following directions and formulating hypotheses than students receiving programmed instruction. In studies similar in design to the current study, Clements and Battista (1990) reported that students who used computer-augmented methods as a supplement to traditional instruction were better able than a control group to visualize and describe angles and polygons. More recently, King and Schattschneider (1997) and Barron and Hynes (1996) found similar results.

In curriculum development projects directly related to the procedures of this study, Schwartz (1989); Schwartz, Yerushalmy, and Wilson (1993); Yerushalmy (1986, 1990); and Yerushalmy, Chazan, and Gordon (1987) suggested that The Geometric Supposer (1993) promoted the development of geometry concepts and positive attitudes toward mathematics. This constructivist approach to computer-assisted instruction is documented and anecdotally supported by Chazan and Houde (1989).

To summarize, a review of the literature and informal work with high school teachers suggested that a constructivist instructional approach using computer software might enhance student learning of geometry concepts and promote more positive student attitudes toward mathematics. What follows are the procedures undertaken to enhance mathematical performance and promote more positive attitudes, the results of that undertaking, and the implications for geometry instruction.

METHOD

Subjects

The 49 participants in the study were enrolled in geometry courses at a large public high school in the western United States. The subjects were in either grade 10 or 11. The distribution of participants according to grade level and gender is given in Table 1.

Table 1. Distribution of Participants by Grade Level and Gender

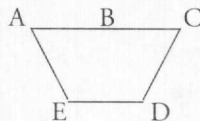
		Grade	
Group		10	11
Male	Control	14	4
	Treatment	8	1
Female	Control	8	1
	Treatment	12	1

The study was conducted over the course of two 18-week semesters. A control group consisting of 27 students was given a standardized test of geometry after instruction in plane geometry using a noncomputer-based approach. They were also given an attitude assessment both before and after the course. The treatment group consisting of 22 students was given a standardized test of geometry performance after instruction in plane geometry augmented by activities related

to The Geometric Supposer (1993). Sample geometry performance test questions are given in Figure 1. Students were also given an attitude assessment both before and after the course. The items in the mathematics attitude assessment are given in Figure 2.

1. For quadrilateral ACDE, what is $m\angle DBC$?

a. 50° b. 60° c. 70° d. 65°



2. If $\sin A = 3/5$, which of the following is true?

I. $\sin B = 3/5$ II. $\cos C = 3/5$ III. $\tan A = 3/4$

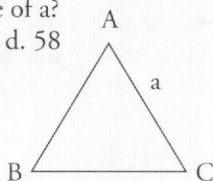
a. I b. II c. I and II d. II and III

3. The triangle inequality theorem states that the sum of the lengths of two sides of a triangle is _____ the length of the third side.

a. less than b. greater than c. equal to d. twice

4. In $\triangle ABC$, what is the value of a ?

a. 103 b. 32 c. 148 d. 58



5. l_1 and l_2 are parallel if they:

a. have no point in common. b. are not skewed.
c. do not intersect. d. are coplanar and do not intersect.

6. Which of the following is *not* characteristic of *all* parallelograms?

a. diagonals are congruent b. diagonals bisect each other
c. a diagonal separates the area into two congruent parts
d. consecutive angles are supplementary

7. In parallelogram ABCD, $AB = 3x - 4$, $BC = x + 5$, and $CD = 2x + 10$. What is the measure of AD?

a. 14 b. 19 c. 38 d. not enough information

8. A building casts a 90-foot shadow. A 6-foot man nearby casts a 9-foot shadow. How tall is the building?

a. 135 ft. b. 54 ft. c. 60 ft. d. 5,760 ft.

9. The diagonal of a square measure 8 units. What is the measure of a side of the square?

a. 4.5 b. 3.3 c. 6.3 d. 16

10. A right triangle has a leg of 9 cm and a hypotenuse measuring 15 cm. What is the measure of the other leg?

a. 17 b. 7 c. 9 d. 12

Figure 1. Sample Geometry Performance Test Questions

Both groups included a wide range of abilities. The only prerequisite at the high school for enrollment in geometry was a passing grade of at least a D in Algebra I. The comparable mathematical ability and attitudes toward mathematics of the two groups were demonstrated in two ways: (a) an analysis of student mean final grades for Algebra I and (b) a statistical analysis of the initial assessment of student attitudes. The mean final grades for the control and treatment groups were 3.17 and 3.27 (out of 4.00), respectively. A *t*-test of these mean scores showed that the differences were not statistically significant at the 0.05 level of significance. As shown in Table 2, results of the initial assessment of student attitudes toward mathematics suggested comparable attitudes for the two groups, except for one item. Students in the treatment group were more likely to agree with the statement "I look forward to coming to school." A *t*-test of differences showed a significant difference at the 0.05 level of significance.

Students were assigned to either group by computer scheduling. Though this is not strictly random assignment, Campbell and Stanley (1968) stated that this method, when used in a large school setting, comes close to random assignment.

The results of the analyses of the mean final Algebra I grades and the initial administration of the attitude assessment and the "functional" random assignment to groups diminish the likelihood that selection based on mathematical ability or disposition toward mathematics was a threat to the internal validity of this study.

Activities

Course activities undertaken by the control group centered on those suggested in the textbook *Geometry*, a textbook by Jurgensen, Brown, and Jurgensen (1992). Topics included points, lines, angles, polygons, deductive proof, and other topics traditionally considered in a high school geometry

1. One of my best subjects is mathematics.
2. I feel comfortable in mathematics class.
3. I am satisfied with the work I do in mathematics.
4. I look forward to coming to mathematics class.
5. One of my favorite subjects is mathematics.
6. If I cannot solve a problem on the first try, I keep trying.
7. I will raise my hand to ask a question in mathematics class.
8. I feel confident when I take a mathematics test.
9. Mathematics is valuable in the "real world."
10. When my teacher explains a mathematics problem, I understand it as well as others in my class.

Note: Ratings on the Mathematics Attitude Assessment Likert scale had integral values ranging from 1 to 5; 1 indicated the student strongly disagreed with the statement and 5 indicated the student strongly agreed with the statement.

Figure 2. Mathematics Attitude Assessment Items

Table 2. *t*-test for Preassessment on Mathematics Attitude Assessment

Item	Control Group		Treatment Group		<i>t</i>
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	
1	3.50	1.30	3.60	0.97	0.32
2	3.66	1.02	4.00	0.85	1.32
3	3.41	1.09	3.60	1.09	0.64
4	2.63	1.11	3.44	1.02	2.78*
5	3.03	1.33	3.40	1.26	0.70
6	3.59	0.96	3.84	1.01	0.94
7	3.34	1.31	3.80	0.98	1.09
8	3.47	1.17	3.60	0.74	0.47
9	4.09	1.10	4.16	1.01	0.24
10	3.88	1.11	3.92	0.89	0.25

*There were 27 subjects in the control group and 22 subjects in the treatment group. Mean values for ratings on the Mathematics Attitude Assessment scale had minimum and maximum values of 1 and 5, respectively. * $p < .05$.*

course. Teachers and students used materials such as a compass, straightedge, protractor, and transparencies and overhead projector.

Course activities undertaken by the treatment group centered on those suggested in the Jurgensen et al. (1992) geometry text and The Geometric Supposer (1993). Topics were the same as those considered by the control group, but the treatment group used the computer software rather than the materials used by the control group. A sample Geometric Supposer lab sheet is given in Figure 3.

Computer Software

As noted earlier, the Geometric Supposer (1993) was suggested in the literature to be useful in enabling students to construct an understanding of geometry concepts. As also noted earlier, the literature suggested that The Geometric Supposer might improve student attitudes toward mathematics. This software was selected based on these suggestions from the review of related literature.

The Geometric Supposer (1993) allows students to make measurements, perform geometric constructions commonly accomplished by a straightedge and compass, and make conjectures and generalizations about classes of figures. By providing data without interpretation, the authors of The Geometric Supposer suggest that students are able to discover and construct geometric concepts through inductive reasoning.

Instrumentation

As described earlier, after 36 weeks of instruction, a geometry performance test and the mathematics attitude assessment were administered to both groups. Differences were noted between groups. A statistical analysis was run on these differences.

Task: To investigate and define the midsegments of a triangle.

Procedures:

1. Draw an acute triangle.
2. Draw a midsegment for the triangle.
3. Measure all segments and angles.

Define: Midsegment

Drawings and Data:

Conjectures:

Additional Procedures:

1. Draw the other midsegments of the same triangle.
2. Measure the new segments and angles formed.

Additional Conjectures:

Figure 3. Sample Geometric Supposer (1993) lab sheet.

RESULTS

Geometry Performance

Both groups were given a standardized test of geometry performance after receiving instruction in plane geometry using either a noncomputer-based or a computer-augmented approach. A geometry performance score was obtained for each student by totaling the number of correct items on the standardized test. Means and standard deviations were computed for both groups. Differences between the means were analyzed by use of a *t*-test.

As shown in Table 3, the group receiving computer-augmented instruction demonstrated a significantly better performance on the geometry test than the control group at the 0.05 level of significance.

Table 3. *t*-test for Geometry Performance Scores

	<i>M</i>	<i>SD</i>	<i>df</i>	<i>t</i>
Control group	34.26	7.65	47	2.06*
Treatment group	37.00	5.15		

*There were 27 subjects in the control group and 22 subjects in the treatment group. Mean values for Geometry Performance Test scores had minimum and maximum values of 0 and 50, respectively. * $p < .05$.*

Student Attitude toward Mathematics

Both groups were given a 10-item mathematics attitude assessment both before and after receiving instruction in plane geometry using either a noncomputer-based or a computer-augmented approach. Means and standard deviations were found for each item on the pre- and postassessments for both groups. Differences between the pre- and postassessment for both groups and differences between the postassessments for both groups were analyzed by means of a *t*-test.

As shown in Tables 4 and 5, except for item 1 for the control group, there were no statistically significant differences in student attitudes toward mathematics from pre- to postassessments within the groups, or on the postassessments between the groups. The one exception given for the control group on item 1 ("One of my best subjects is mathematics.") was at the 0.05 level of significance. Unlike the preassessment of student attitudes given earlier, an analysis of the differences between the means of the control and treatment groups on the postassessment of attitudes by use of a *t*-test showed no statistically significant differences.

DISCUSSION AND CONCLUSION

The hypotheses suggested by the literature were as follows:

1. Students who receive a constructivist instructional approach to geometry using computer-augmented activities would achieve stronger gains in knowl-

Table 4. *t*-test for Pre- and Postassessment Measures of Control Group Mathematics Attitude Assessment

Item	Pre		Post		<i>t</i>
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	
1	3.50	1.30	4.15	0.97	2.11*
2	3.66	1.02	4.24	1.07	1.72
3	3.41	1.09	3.33	1.15	0.27
4	2.63	1.11	2.85	1.21	0.72
5	3.03	1.33	3.33	1.31	0.85
6	3.59	0.96	3.77	1.20	0.63
7	3.34	1.31	3.41	1.34	0.16
8	3.47	1.17	3.74	1.24	0.85
9	4.09	1.10	4.07	1.30	0.06
10	3.88	1.11	4.04	0.88	1.01

*There were 27 subjects in the control group. Mean values for ratings on the Mathematics Attitude Assessment scale had minimum and maximum values of 1 and 5, respectively. *p < .05.*

Table 5. *t*-test for Pre- and Postassessment Measures of Treatment Group Mathematics Attitude Assessment

Item	Pre		Post		<i>t</i>
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	
1	3.60	0.97	3.50	1.37	0.28
2	4.00	0.85	4.04	0.98	0.14
3	3.60	1.09	3.59	1.12	0.03
4	3.44	1.02	3.14	1.10	0.95
5	3.40	1.26	3.22	1.31	0.47
6	3.84	1.01	3.68	1.10	0.51
7	3.80	0.98	3.95	0.98	0.51
8	3.60	0.74	3.86	0.97	1.01
9	4.16	1.01	4.00	1.41	0.50
10	3.92	0.89	4.14	0.97	0.95

There were 22 subjects in the treatment group. Mean values for ratings on the Mathematics Attitude Assessment scale had minimum and maximum values of 1 and 5, respectively.

- edge of geometry than would students who receive a more traditional approach.
- Students who receive a constructivist instructional approach to geometry using computer-augmented activities would develop a more positive attitude toward mathematics than would students who receive a more traditional approach.

Based on the results of this study, it appears students who receive geometry instruction using a constructivist approach by means of computer-augmented activities do achieve stronger gains in knowledge of geometry concepts than students who receive more traditional geometry instruction. This result is consistent with several related studies on the effects of constructivist and computer-augmented approaches to mathematics instruction, including previous studies undertaken by me (Funkhouser, 1990; Funkhouser & Dennis, 1992).

Based on the results of this study, it appears that students who receive geometry instruction using a constructivist approach by means of computer-augmented activities do not develop more positive attitudes toward mathematics than students who receive a more traditional approach. In fact, as suggested by the analysis of the differences in the postassessments of the two groups, there appears to be a trend toward students in the control group viewing mathematics as one of their best subjects. This result is inconsistent with much of the research on the effects of constructivist and computer-augmented methods of mathematics instruction, including my previous studies (Funkhouser, 1993).

In summary, students who received a constructivist approach to geometry instruction using computer-augmented methods tended to make significantly stronger gains in acquiring geometry concepts, but did not tend to develop more positive attitudes toward mathematics than did a control group. In fact, on at least one measure ("One of my best subjects is mathematics"), nonconstructivist, noncomputer-augmented instruction produced significantly more positive attitudes toward mathematics over the course of the study.

Implications for Mathematics Instruction

This study sought to further investigate the effects of a constructivist instructional approach and computer-augmented methods on student mathematics performance and attitudes toward mathematics. Based on the results of this investigation, the following recommendations for mathematics education should be considered:

1. The significantly better performance of the treatment group in a test of geometry ability suggests that mathematics instruction using computer-augmented activities (especially for geometry) should be more widely used. Integrated augmentations of a traditional geometry course with computer-based methods, such as those used in this study, appear to enhance student mathematics performance without an "opportunity cost" for mathematics content. The replacement of a traditional geometry course with a totally computer-based approach may not yield similar results. (The use of The Geometer's Sketchpad [1995] is a promising example of integrative software.)
2. Factors other than use of computer-based methods may be just as important in developing positive attitudes toward mathematics. Stated another way, the use of a computer in a mathematics class *in and of itself* does not appear to produce more positive attitudes toward mathematics. It could be that a longer instructional timeline, such as that used in this study, "washes out" any Hawthorne effect that may be caused by use of novel technology.

3. The relationship between initial student attitudes toward mathematics class and geometry performance should be noted. In response to item 4, "I look forward to coming to mathematics class," students in the treatment group were more likely to respond favorably than the control group at the 0.05 level of significance. In fact, it could be that initial student disposition toward mathematics has a critical effect on mathematics performance, regardless of the instructional methodology. ■

Contributor

Charles Funkhouser is an associate professor of mathematics education at the California State University, Fullerton. His research interests include the application of instructional technology and cognitive science to mathematics education. (Address: Charles Funkhouser, California State University, Fullerton, Department of Mathematics, 800 N. State College, Fullerton, CA 92834-9480; cpf7951@aol.com.)

References

- Barron, A. E., & Hynes, M. C. (1996). Using technology to enhance communication in mathematics. In P. C. Elliott & M. J. Kenney (Eds.), *Communication in mathematics, K-12 and beyond* (pp. 95-100). Reston, VA: National Council of Teachers of Mathematics.
- Battista, M. T., & Clements, D. H. (1988). Using Logo pseudoprimitives for geometric investigations. *Mathematics Teacher*, 81, 166-174.
- Bruner, J. S. (1960). *The process of education*. Cambridge, MA: Harvard University Press.
- Campbell, D. T., & Stanley, J. C. (1968). *Experimental and quasi-experimental designs for research*. Chicago: Rand McNally and Company.
- Chazan, D., & Houde, R. (1989). *How to use conjecturing and microcomputers to teach geometry*. Reston, VA: National Council of Teachers of Mathematics.
- Clements, D. H. (1998). *Turtle paths: 2-D geometry*. Menlo Park, CA: Dale Seymour Publications.
- Clements, D. H., & Battista, M. T. (1990). The effects of Logo on children's conceptualizations of angle and polygons. *Journal of Research in Mathematics Education*, 21, 356-371.
- Cuban, L. (1989). Neoprogessive visions and organizational realities. *Harvard Educational Review*, 59, 217-222.
- Del Grande, J. J., & Morrow, L. (1993). *Geometry and spatial sense*. Reston, VA: National Council of Teachers of Mathematics.
- Dennis, J. R., & Kansky, R. J. (1984). *Instructional computing: An action guide for educators*. Glenview, IL: Scott, Foresman.
- Dienes, Z. P. (1964). *The power of mathematics: A study of the transition from the constructive to the analytical phase of mathematical thinking in children*. London: Hutchinson Educational Ltd.
- Dienes, Z. P. (1967). *Building up mathematics*. London: Hutchinson Educational Ltd.

Funkhouser, C. P. (1990). *The effects of problem solving software on the problem solving ability of secondary school students*. (Doctoral dissertation, University of Illinois, Urbana). Dissertation Abstracts International, 51, 1186A.

Funkhouser, C. P. (1993). The influence of problem-solving software on student attitudes. *Journal of Research on Computing in Education*, 25, 339–346.

Funkhouser, C. P., & Dennis, J. R. (1992). The effects of problem-solving software on problem-solving ability. *Journal of Research on Computing in Education*, 24, 338–347.

Gagne, R. M. (1985). *The conditions of learning and instruction*. New York: Holt, Rinehart & Winston.

Gallini, J. K. (1987). A comparison of the effects of Logo and a CAI learning environment on skills acquisition. *Journal of Educational Computing Research*, 3, 461–477.

The Geometer's Sketchpad: Dynamic geometry for the 21st century [Computer program]. (1995). Berkeley, CA: Key Curriculum Press.

The Geometric Supposer [Computer software]. (1993). Pleasantville, NY: Sunburst Communications.

Jurgensen, R. C., Brown, R. G., & Jurgensen, J. W. (1992). *Geometry*. Boston: Houghton Mifflin Company.

Kenney, M. J. (1987). Logo adds a new dimension to geometry programs at the secondary level. In M. M. Lindquist (Ed.), *Learning and teaching geometry K–12: 1987 Yearbook* (pp.85–100). Reston, VA: National Council of Teachers of Mathematics.

King, J. R., & Schattschneider, D. (Eds.) (1997). *Geometry turned on!: Dynamic software in learning, teaching and research*. Washington, D.C.: Mathematical Association of America.

Kulik, J. A., Bangert, R. L., & Williams, G. W. (1983). Effects of computer-based teaching on secondary school students. *Journal of Education Psychology*, 75, 19–26.

Mann, W. J., & Tall, D. (1992). *Computers in the mathematics curriculum*. Leicester, United Kingdom: The Mathematical Association.

Martin, K., & Bearden, D. (1985). *Mathematics and Logo: A turtle trip through geometry*. Reston, VA: Reston Publishing Company.

Meiring, S. P. (1992). *A core curriculum: Making mathematics count for everyone*. Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (1985). *The secondary school mathematics curriculum*. Reston, VA: Author.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Niemiec, R., & Walberg, H. J. (1987). Comparative effects of computer-assisted instruction: A synthesis of reviews. *Journal of Education Computing Research*, 3, 19–37.

Papert, S. (1980). *Mindstorms: Children, computers, and powerful ideas*. New York: Basic Books.

Papert, S. (1993). *The children's machine: Rethinking schools in the age of the computer*. New York, NY: Basic Books.

- Piaget, J. (1960). *The psychology of intelligence*. Littlefield, NJ: Adams.
- Schwartz, J. L. (1989). Intellectual mirrors: A step in the direction of making schools knowledge-making places. *Harvard Educational Review*, 59, 51–61.
- Schwartz, J. L., Yerushalmy, M., & Wilson, B. (1993). *The Geometric Supposer: What is it a case of?* Hillsdale, NJ: Erlbaum Associates.
- Skinner, B. F. (1938). *The behavior of organisms*. New York: Appleton.
- Thorndike, E. L. (1911). *Animal intelligence*. New York, NY: Macmillan.
- Turner, S. V. (1988). Cognitive effects of a Logo-enriched mathematics program for middle school students. *Journal of Educational Computing Research*, 4, 443–452.
- van Heile, P. M. (1987). In M. M. Lindquist (Ed.), *Learning and teaching geometry K–12: 1987 Yearbook* (pp. 1–15). Reston, VA: National Council of Teachers of Mathematics.
- Yerushalmy, M. (1986). The Geometric Supposer: Promoting thinking and learning. *Mathematics Teacher*, 79, 418–422.
- Yerushalmy, M. (1990). Using empirical information in geometry: Students' and designers' expectations. *Journal of Computers in Mathematics and Science Teaching*, 9, 23–41.
- Yerushalmy, M., Chazan, D., & Gordon, M. (1987). *Guided inquiry and technology: A year long study of children and teachers using the Geometric Supposer*. (ERIC No. ED 294711)