Primal & Dual Problems

$$\max \quad \zeta(x) = 4x_1 + x_2 + 3x_3 \qquad x_i \geq 0$$

$$\text{s.t} \quad x_1 + 4x_2 \leq 1$$

$$3x_1 - x_2 + x_3 \leq 3$$

lin comb
of constraints
$$\begin{cases} y_1(x_1 + 4x_2 \qquad) \leq y_1 \cdot 1 \\ y_2(3x_1 - x_2 + x_3) \leq y_2 \cdot 3 \end{cases}$$

$$(y_1 + 3y_2)x_1 + (4y_1 - y_2)x_2 + (y_2)x_3 \leq y_1 + 3y_2$$

$$\begin{matrix} \vee\mkern-9mu\vert \; \in bc & \vee\mkern-9mu\vert & \vee\mkern-9mu\vert \\ 4 \quad \text{of max} & 1 & 3 \end{matrix}$$

$$\zeta(x) = 4x_1 + 1x_2 + 3x_3$$

$$\leq (y_1 + 3y_2)x_1 + (4y_1 - y_2)x_2 + (y_2)x_3$$

$$\leq 1y_2 + 3y_2 = \zeta(y) \qquad \boxed{\zeta(x) \leq \zeta(y)}$$

Dual: $\min \; \zeta(y) = y_1 + 3y_2$ \qquad For any feasible x, y

$$\text{s.t} \quad y_1 + 3y_2 \geq 4$$

$$4y_1 - y_2 \geq 1$$

$$y_3 \geq 3$$

(P) $\max \ f(x) = 4x_1 + x_2 + 3x_2 \rightarrow (4 \ 1 \ 3) \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$\text{s.t} \quad x_1 + 4x_2 \qquad \leq 1$

$\qquad 3x_1 - x_2 + x_3 \leq 3 \quad \leadsto \quad \begin{bmatrix} 1 & 4 & 0 \\ 3 & -1 & 1 \end{bmatrix}$

$\qquad x_i \geq 0$

(D) $\min \ f(y) = 1y_1 + 3y_2 \rightarrow (1 \ 3) \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$

$\text{s.t} \quad y_1 + 3y_2 \geq 4$

$\qquad 4y_1 - y_2 \geq 1 \quad \leadsto \quad \begin{bmatrix} 1 & 3 \\ 4 & -1 \\ 0 & 1 \end{bmatrix}$

$\qquad y_2 \geq 3$

$\qquad y_i \geq 0$

If $\ f(x^*) = f(y^*) \implies$ strong duality