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"2do. Parcial: Ecuaciones Diferenciales"

1- Resolver aplicando Coeficientes Ideterminados

a) $y'' + y' + 1/4\, y = e^{x}[\operatorname{Sen}3x - \operatorname{Cos}3x)$

$(D^2 + D + \frac{1}{4})y = e^{x}(\operatorname{sen}x - \cos 3x)$

Lo ponemos como...

$m^2 + m + \frac{1}{4} = 0$

$4m^2 + 4m + 1 = 0$

$4m^2 + 2m + 2m + 1 = 0$

$2m(2m+1) + 1(2m+1) = 0$

$(2m+1)(2m+1)2 = 0$

$m = -\frac{1}{2}, -\frac{1}{2}$

$y_c(t) = (C_1 + C_2 C)e^{-\frac{1}{2}x}$

Ahora... Gauss para

$(A\cos x + B\operatorname{sen}x)$

Gauss...

$[C\cos 3x + D\operatorname{sen}3x)$

Gauss para exponencial

ke^{x}

(A)

(A)

$y_p(t) = ke^{x}[(A\cos x + B\operatorname{sen}x) - (C\cos 3x + D\operatorname{sen}3x)]$

$y = y_c(t) + y_p(t)$

$y = (C_1 + C_2 x)e^{-1/2x} + ke^{x}[(A\cos x + B\operatorname{sen}x) - [C\cos 3x + D\operatorname{sen}3x)]$

b) $y'' + 2y' + y = x^2 e^{-x}$

Tenemos como solución general $y(x) = C.F + P.I$

Para encontrar C.F resolveremos

$$(D^2 + 2D + 1) = 0$$

$$(D+1)^2 = 0$$

$$D = -1$$

Compl. la función

$$C.F = C_1 e^{x} + C_2 e^{-x} x$$

Ahora para encontrar la sol. particular (P.I). aplicamos coeficientes indeterminados.

$$y_p'' + 2y_p + y_p = e^{-x} x^2$$

$$y_p(x) = x^2 (a_1 e^{-x} + a_2 e^{-x} x + a_3 e^{-x} x^2)$$

$$y_p' = -a_1 e^{-x} x^2 + 2a_1 e^{-x} x - a_2 e^{-x} x^3 + 3a_2 e^{-x} x^2 - a_3 e^{-x} x^4 + 4a_3 e^{-x} x^3$$

$$y''_p = a_1 (2e^{-x} + e^{-x} x^2 - 4e^{-x} x) + a_2 (e^{-x} x^3 - 6e^{-x} x^2 + 6e^{-x} x) + a_3 (e^{-x} x^4 - 8e^{-x} x^3 + 12 e^{-x} x^2)$$

Ponemos el valor...

$$a_1 (2e^{-x} + e^{-x} x^2 - 4e^{-x} x) + a_2 (e^{-x} x^3 - 6e^{-x} x^2 + 6e^{-x} x) + a^3 (e^{-x} x^4 - 8e^{-x} x^3 + 12 e^{-x} x^2) + 2(-a_1 e^{-x} x^2 + 2a_1 e^{-x} x + a_2 e^{-x} x^3 + 3a_2 e^{-x} x^2 - a_3 e^{-x} x^4 + 4a_3 e^{-x} x^3) + a_1 e^{-x} x^2 + a_2 e^{-x} x^3 + a_3 e^{-x} x^4 + 4a_3 e^{-x} x^3$$

$$\therefore 2a_1 e^{-x} + 6a_2 e^{-x} x + 12 a_3 e^{-x} x^2 = e^{-x} x^2$$

Por ec.

$$2a_1 = 0$$
$$6a_2 = 0$$
$$12a_3 = 1$$

$$\boxed{\begin{array}{l} \therefore a_1 = 0, \quad a_2 = 0, \quad a_3 = 1/12 \\ y(x) = C.F + P.I \\ y(x) = C_1 e^{-x} + C_2 e^{-x} x + \frac{1}{12} e^{-x} x^4 \end{array}}$$

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2- Resolver aplicando variación de parametros

a) $y'' - 2y' + y = \dfrac{e^x}{1+x^2}$

Reacomodamos como...

$$y'' - 2y' + y = \frac{e^x}{x^2+1}$$

Dejamos $y = y_p + y_c$

$$y_c'' - 2y_c' + y_c = 0$$

$$m^2 - 2m + 1 = 0$$

$$(m-1)^2 = 0, \quad m = 1, 1$$

$$y_c = (A + Bx)e^x \quad \text{—— (1)}$$

Ponemos $y_p = u y_1 + v y_2$

$y_1 = e^x$, $y_2 = x e^x$

$y_1' = e^x$, $y_2' = e^x + x e^x$

$\therefore y_1 y_2' - y_2 y_1' = e^{2x}$

$$u = \int \frac{\frac{-e^x}{x^2+1} \times x e^x}{e^{2x}}\, dx = -\int \frac{x\,dx}{x^2+1}$$

$$u = -\frac{1}{2}\ln(x^2+1)$$

$$v = \int \frac{\frac{e^x}{x^2+1} \times e^x}{e^{2x}}\, dx = \int \frac{dx}{x^2+1} = \tan^{-1}(x)$$

$$y_p = -\frac{e^x}{2}\ln(x^2+1) + x e^x \tan^{-1}(x)$$

$$y = (A + Bx)e^x - \frac{e^x}{2}\ln(x^2+1) + x e^x \tan^{-1} x$$

(A)

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b) $y'' - 6y' + 30y = e^x \operatorname{Tan} 3x$

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3- Resolver las siguientes ec. dif.

a) $x^2 y'' - 2xy' + 2y = x^3 \ln x$

Tenemos la forma

$$y'' + Py' + Qy = R$$

Ahora...

$$P = -\frac{2}{x} ; Q = \frac{2}{x^2} , R = x \log x$$

∴ x es parte de la funcion comp.

∴ $y = vx$ es transformación

∴ La ecuación se transforma a...

$$v'' + \left[P + \frac{2}{u}\frac{du}{dx}\right]v' = \frac{R}{u} \quad \text{donde} = x$$

$$v'' + \left(-\frac{2}{x} + \frac{2}{x}\right)v' = \log x$$

$$v'' = \log x$$

$$v' = x(\log x - 1) + C_1$$

$$v = \frac{x^2(\log x - 1)}{2} - \frac{x^2}{4} + C_1 x + C_2$$

$$y = vx$$

$$= C_1 x^2 + C_2 x + \frac{x^3(2\log x - 3)}{4}$$

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b) $(1+y)y'' = (y')^2$

$$\frac{(1+y)d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$$

Ponemos $\frac{dy}{dx} = v$

$$\frac{d^2y}{dx^2} = \frac{dv}{dy} \cdot \frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{dv}{dy} v$$

$$(1+y)\, y'' = (y')^2$$

$$(1+y)\frac{dv}{dy} v = (v)^2$$

$$(1+y)\frac{dv}{dy} = v$$

$$\frac{dv}{v} = \frac{dy}{(1+y)}$$

Integramos ambos lados

$$\int \frac{dv}{v} = \int \frac{dy}{(1+y)}$$

$$\ln(v) = \ln(1+y) + c$$

$$v = e^{\ln(1+y)+c}$$

$$v = e^{\ln(1+y)}\, e^{c}$$

$$v = C(1+y)$$

$$\frac{dy}{dx} = C(1+y)$$

$$\frac{dy}{(1+y)} = C\,dx \quad ---- \quad A$$

A

$$\frac{dy}{(1+y)} = C\,dx$$

Integramos ambos lados

$$\int \frac{dy}{1+y} = \int C\,dx$$

$$\ln(1+y) = Cx + D$$

$$(1+y) = e^{Cx+D}$$

$$(1+y) = De^{Cx}$$

$$y = De^{Cx} - 1$$ donde C, D son constantes

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4-Problema

En el cálculo diferencial, la curvatura de una curva representada por $y = f(x)$, se define como sigue:

$$k = \frac{y''}{[1+(y')^2]^{3/2}}$$, si $k=1$, ¿Cuánto vale $y = f(x)$?

$k = 1$

$y'' = [1+(y')^2]^{3/2}$ ⟶ Sust $v = y'$

$v' = (1+v^2)^{3/2}$ ⟶ Tomando $v = \tan\theta \Rightarrow v' = \sec^2\theta \, d\theta$

Entonces

$$\sec^2\theta \frac{d\theta}{dt} = (1+\tan^2\theta)^{3/2} = \sec^3\theta$$

Entonces $\cos\theta \, d\theta = dt$

$$\int \cos\theta \, d\theta = \int dt$$

$\operatorname{sen}\theta = t$

$\Rightarrow \theta = \operatorname{sen}^{-1}(t)$

$\cos\theta = \sqrt{1-t^2}$

$$y' = \tan\theta = \frac{t}{\sqrt{1-t^2}}$$

$$y(t) = \int \frac{t}{\sqrt{1-t^2}} \, dt = -\sqrt{1-t^2}$$

$$\boxed{y(x) = -\sqrt{1-x^2}}$$

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