Ramirez Cotonieto Luis Fernando — 5-nov-2020

"Parcial 1: Ec. Diferenciales" — 1CM1

1. a) $$\frac{dy}{dx}=\frac{\sqrt{x+y}+\sqrt{x-y}}{\sqrt{x+y}-\sqrt{x-y}}$$

Pongamoslo en $f\left(\frac{y}{x}\right)$

$$=\frac{\sqrt{x}\sqrt{1+\frac{y}{x}}+\sqrt{x}\sqrt{1-\frac{y}{x}}}{\sqrt{x}\sqrt{1+\frac{y}{x}}-\sqrt{x}\sqrt{1-\frac{y}{x}}}$$

$$=\frac{\sqrt{1+\frac{y}{x}}+\sqrt{1-\frac{y}{x}}}{\sqrt{1+\frac{y}{x}}-\sqrt{1-\frac{y}{x}}}$$

ahora lo tenemos la forma $\frac{y}{x}$

$$\frac{dy}{dx}=\frac{\sqrt{1+\frac{y}{x}}+\sqrt{1-\frac{y}{x}}}{\sqrt{1+\frac{y}{x}}-\sqrt{1-\frac{y}{x}}}$$

$y=vx$

$$\frac{dy}{dx}=v+x\frac{dv}{dx}=\frac{\sqrt{1+v}+\sqrt{1-v}}{\sqrt{1+v}-\sqrt{1-v}} \quad \text{--- } (1)$$

$$\frac{(\sqrt{1+v}+\sqrt{1-v})^2}{(\sqrt{1+v}-\sqrt{1-v})(\sqrt{1+v}+\sqrt{1-v})}=\frac{(1+v)+(1-v)+2\sqrt{1+v}\sqrt{1-v}}{(\sqrt{1+v})^2-(\sqrt{1-v})^2}$$

$$=\frac{2+2\sqrt{1-v^2}}{2v}=\frac{1+\sqrt{1-v^2}}{v}$$

De la ecuación (1)

(A)

(A)

$$v+x\frac{dv}{dx}=\frac{1+\sqrt{1-v^2}}{v}$$

$$\frac{dv}{dx}=\frac{1+\sqrt{1-v^2}-v^2}{v}$$

$$\frac{dx}{x}=\frac{v}{1+\sqrt{1-v^2}-v^2}\,dv \quad \text{--- } (2)$$

Integramos ambos lados

$$\int\frac{1}{x}\,dx=\int\frac{v}{\sqrt{1-v^2}-v^2+1}\,dv \quad \text{--- } (2)$$

Consideramos.

$$\int\frac{v}{\sqrt{1-v^2}-v^2+1}\,dv$$

$$u=\sqrt{1-v^2} \longrightarrow \frac{du}{dv}=-\frac{v}{\sqrt{1-v^2}}$$

$$dv=-\frac{\sqrt{1-v^2}}{v}$$

$$v^2=1-u^2$$

$$\int\frac{v}{\sqrt{1-v^2}-v^2+1}=-\int\frac{1}{u+1}\,du$$

$$=-\ln(u+1)$$

$$=-\ln(\sqrt{1-v^2}+1)$$

(B)

(0)

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De 2

$$\ln(x)+c=-\ln(\sqrt{1-v^2}+1)$$

$$c=\ln(k)$$

$$\ln(x)+\ln(k)=-\ln(\sqrt{1-v^2}+1)$$

$$kn=(\sqrt{1-v^2}+1)^{-1}$$

$$\sqrt{1-v^2}+1=\frac{1}{kn}$$

$$v=\frac{y}{x}$$

$$\sqrt{1-\left(\frac{y}{x}\right)^2}+1=\frac{1}{kx}$$

$$\sqrt{1-\frac{y^2}{x^2}}+1=\frac{1}{kx}$$

$$\frac{\sqrt{x^2-y^2}}{x}+1=\frac{1}{kx}$$

$$\sqrt{x^2-y^2}+x=\frac{1}{k}$$

$$\sqrt{x^2-y^2}+x=c$$

$$\sqrt{x^2-y^2}=c-x$$

$$x^2-y^2=(c-x)^2$$

$$y=\sqrt{x^2-(c-x)^2}$$

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"Ecuaciones diferenciales"

1- Resolver las siguientes Ecuaciones Diferenciales.

a) $\frac{dy}{dx} = \frac{\sqrt{x+y} + \sqrt{x-y}}{\sqrt{x+y} - \sqrt{x-y}}$

se resuelve en la primera parte por falta de espacio :)

b) $\frac{dy}{dx} = \tan(x+y)$

$\frac{dy}{dx} = \tan(x+y)$ $u = x+y$

$\frac{du}{dx} = 1 + \frac{dy}{dx}$

$\frac{du}{dx} = 1 + \tan(u)$

$\frac{1}{1+\tan(u)} du = dx$

*Integrando ambos lados

$\int \frac{1}{1+\tan(u)} du = \int dx$

$\frac{1}{2}\left(u + \ln(\text{sen}(u) + \cos(u))\right) = x + C_0$

$x+y + \ln(\text{sen}(x+y) + \cos(x+y)) = 2x + 2C_0$

$2C_0 = C_1$

$\ln(\text{sen}(x+y) + \cos(x+y)) = x - y + C_1$

$e^{\ln(\text{sen}(x+y) + \cos(x+y))} = e^{(x-y+C_1)}$

$\text{sen}(x+y) + \cos(x+y) = e^{x-y} \cdot e^{C_1}$

$e^{C_1} = C$

$\text{Sen}(x+y) + \cos(x+y) = Ce^{x-y}$

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a) Resolver aplicando exactitud $(2xy^2+2xy)dx+(x^2y+x^2)dy=0$

$(2xy^2+2xy)dx+(x^2y+x^2)dy=0$ --- (1)

A

Entonces...

$M=2xy^2+2xy$ $N=x^2y+x^2$

Ahora considerando $M=2xy^2+2xy$

$\frac{\partial M}{\partial y}=4xy+2x$

Ahora con $N=x^2y+x^2$

$\frac{\partial N}{\partial x}=2xy+2x$

$$\frac{\frac{\partial N}{\partial x}-\frac{\partial m}{\partial y}}{M}=-\frac{1}{y+1}=P(y)$$

Fact. Int.

$$e^{\int P(y)dy}=e^{\int -\frac{1}{y+1}dy}$$

$$=e^{-\ln(y+1)}$$

$$=(y+1)^{-1}=\frac{1}{y+1}$$

Mult. (1) . Fact. Int.

$$\frac{2xy^2+2xy}{y+1}dx=\frac{x^2y+x^2}{y+1}dy=0$$

$$\frac{2xy\cancel{(y+1)}}{\cancel{(y+1)}}dx+\frac{x^2\cancel{(y+1)}}{\cancel{(y+1)}}dy=0$$

$2xy\,dx+x^2dy=0$ ------ (2)

Es exacta en $\frac{\partial M'}{\partial y}=\frac{\partial}{\partial y}2xy=2x$

$$\frac{\partial N'}{\partial x}=\frac{\partial x^2}{\partial x}=2x$$

A

Por lo tanto la sol en (2) es

$\int 2xy\,dx+\int$ Terminos N' sin x dy = y constante

$$2y\frac{x^2}{2}+C=0$$

$$\boxed{x^2y+C=0}$$

b) Resolver la siguiente ecuación $\frac{dy}{dx} = \frac{(y-1)(x-2)(y+3)}{(x-1)(y-2)(x+3)}$

$$\frac{dy}{dx} = \frac{(y-1)(x-2)(y+3)}{(x-1)(y-2)(x+3)}$$

$$\frac{(y-2)\,dy}{(y-1)(y-3)} = \frac{(x-2)}{(x-1)(x+3)}\,dx$$

$$\left(\frac{-1}{(y-1)\cdot 4} + \frac{(-3-2)}{(-3-1)(y+3)}\right)dy = \left(\frac{(1-2)}{(x-1)(1+3)} + \frac{(-3-2)}{(-3-1)(x+3)}\right)dx$$

$$\left(\frac{-1}{4(y-1)} + \frac{5}{4(y+3)}\right)dy = \left(\frac{-1}{4(x-1)} + \frac{5}{4(x+3)}\right)dx$$

$$\cancel{\frac{1}{4}}\left(\frac{5}{y+3} - \frac{1}{y-1}\right)dy = \cancel{\frac{1}{4}}\left(\frac{5}{x+3} - \frac{1}{x-1}\right)dx$$

Integramos entonces...

$$5\log(y+3) - \log(y-1) + C = 5\log(x+3) - \log(x-1) + \log A$$

$$\cancel{\log}\frac{(y+3)^5}{y-1} = \cancel{\log}\left(\frac{A(x+3)^5}{x-1}\right)$$

$$\frac{(y+3)^5}{y-1} = A\,\frac{(x+3)^5}{x-1}$$

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3. Resolver la siguiente ecuación de Ricatti:

$$\frac{dy}{dx} = e^{2x} + (1+2e^{x})y + y^2 ; \ \varnothing(x)_p = -e^{x}$$

Si $\varnothing(x)$ es una solución particular de la ec. de Ricatti, la solución general es $y = \varnothing(x) + u$

Sust. $y = \varnothing(x) + u$ en la ec.

$$y = -e^{x} + u$$

$$\frac{dy}{dx} = e^{2x} + (1+2e^{x})y + y^2$$

$$\frac{d}{dx}(-e^{x} + u) = e^{2x} + (1+2e^{x})(-e^{x} + u) + (-e^{x} + u)^2$$

$$\cancel{-e^{x}} + \frac{du}{dx} = e^{2x} + \cancel{-e^{x}} - 2e^{2x} + u(1+2e^{x}) + e^{+2x} - 2ue^{x} + u^2$$

$$\frac{du}{dx} = \cancel{2e^{2x}} - \cancel{2e^{2x}} + (1+2e^{x})u - 2e^{x}u + u^2$$

$$\frac{du}{dx} = u + \cancel{2ue^{x}} - \cancel{2ue^{x}} + u^2$$

$$\frac{du}{dx} = u + u^2$$

$$\frac{1}{u+u^2} du = dx$$

$$\frac{1}{u(u+1)} du = dx$$

$$\int \frac{1}{u(u+1)} du = dx$$

$$\int \left(\frac{1}{u} - \frac{1}{u+1} \right) du = dx \quad \ldots \quad \textcircled{A}$$

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$\ln|u| - \ln|u+1| = x + c$

(A) $\ln\left(\frac{u}{u+1}\right) = x + c$

Para ambos lados...

$$\frac{u}{u+1} = e^{(x+c)}$$

$$\frac{u}{u+1} = e^{x} \cdot e^{c}$$

$$\frac{u}{u+1} = ke^{x}$$

$$u = (u+1)ke^{2} = ke^{2}u + ke^{2}$$

$$u - uke^{x} = ke^{2}$$

$$u(1 - ke^{x}) = ke^{2}$$

$$u(1 - ke^{x}) = ke^{2}$$

$$u = \frac{ke^{x}}{1 - ke^{x}} = \frac{k(e^{x} \cdot e^{-x})}{e^{-x} - k(e^{-x} \cdot e^{x})}$$

$$u = \frac{k}{e^{-x} - k} = \frac{1}{\frac{1}{k}e^{-x} - \frac{k}{k}}$$

$\frac{1}{k} = C$ constate

$$u = \frac{1}{ce^{-x} - 1}$$

$$y = -e^{x} + \frac{1}{ce^{-x} - 1}$$

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