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1CM5

"Ec. D. F. Tercer Parcial"

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a) Aplicando la definición de la transformada de Laplace, calcular $L\{e^{-3t^2+4t}\}$

Sabemos que

$$L\{f(t)\} = \int_0^\infty e^{-st} \cdot f(t)\, dt$$

Para $f(t) = e^{-3t^2+4t}$

$$= \int_0^\infty e^{-st} \cdot \left[e^{-3t^2+4t}\right] dt$$

$$= \int_0^\infty e^{-3t^2+(4-s)t}\, dt$$

En $-3t^2 - st + 4t = \frac{(4-s)^2}{12} - \left(\sqrt{3}t - \frac{4-s}{2\sqrt{3}}\right)^2$

Completando el cuadrado

Sust. $u = \frac{6t+s-4}{2\sqrt{3}} \rightarrow \frac{du}{dt} = \sqrt{3}$

$$\rightarrow dt = \frac{1}{\sqrt{3}} du$$

$$= \int \frac{e^{\frac{(s-4)^2}{12} - u^2}}{\sqrt{3}} du$$

Sig. pag

$$= \frac{\sqrt{\pi}\, e^{\frac{(s-4)^2}{12}}}{2\sqrt{3}} \int \frac{2e^{-u^2} du}{\sqrt{\pi}}$$

Resolviendo

$$\int \frac{2e^{-u^2} du}{\sqrt{\pi}}$$

esta es una integral especial erf(u)

Agregando la integral resuelta.

$$\frac{\sqrt{\pi}\, e^{\frac{(s-4)^2}{12}}}{2\sqrt{3}} \int \frac{2e^{-u^2}}{\sqrt{\pi}}\,du$$

$$= \frac{\sqrt{\pi}\, e^{\frac{(s-4)^2}{12}}\, erf(u)}{2\sqrt{3}}$$

$$u = \frac{6t+s-4}{2\sqrt{3}}$$

$$= \sqrt{\pi}\, e^{\frac{(s-4)^2}{12}}\, erf\left(\frac{6t+s-4}{2\sqrt{3}}\right)$$

Ahora...

$$\int_0^{\infty} f(t)\,dt = \frac{\sqrt{\pi}\, e^{\frac{s^2}{12}-\frac{2s}{3}+\frac{4}{3}}}{2\sqrt{3}} - \frac{\sqrt{\pi}\, e^{\frac{s^2}{12}-\frac{2s}{3}+\frac{4}{3}}\, erf\left(\frac{\sqrt{3}s-4\sqrt{3}}{6}\right)}{2\sqrt{3}}$$

$$= \frac{\sqrt{\pi}\, e^{\frac{s^2}{12}-\frac{2s}{3}+\frac{4}{3}}\left(erf\left(\frac{\sqrt{3}s-4\sqrt{3}}{6}\right)-1\right)}{2\sqrt{3}}$$

b) Haciendo uso de las tablas para la transformada calcular $L\{(t^4+10t)^4\}$

$$L\{(t^4+10t)^4\}$$

$$=L\{(t^4)^4+4(t^4)^3 10t+6(t^4)^2(10t^2)+4t^4(10t)^3+(10t)^4\}$$

$$=L[t^{16}+4\cdot t^{12}\cdot 10t+6\cdot t^8\cdot 100\cdot t^2+4\cdot t^1\cdot 1000\cdot t^3+10000t^4\}$$

$$=L\{t^{16}+40t^{13}+600t^{10}+4000t^7+10000t^4\}$$

$$=L\{t^{16}\}+40L\{t^{13}\}+600L\{t^{10}\}+4000L\{t^7\}+10000L\{t^4\}$$

$$=\frac{16!}{s^{16+1}}+40\frac{13!}{s^{13+1}}+600\frac{10!}{s^{10+1}}+4000\frac{7!}{s^{7+1}}+10000\frac{4!}{s^{4+1}}$$

$$=\frac{16!}{s^{17}}+40\frac{13!}{s^{14}}+600\frac{10!}{s^{11}}+4000\frac{7!}{s^8}+10000\frac{4!}{s^5}$$

$$=\frac{16!}{s^{17}}+\frac{249080832000}{s^{14}}+\frac{2177280000}{s^{11}}+\frac{20160000}{s^8}$$

$$+\frac{240000}{s^5}$$

$$=\frac{48000}{s^{17}}\left(5s^{12}+420s^9+45360s^6+5189184s^3+435891456\right)$$

$$L(t^n)=\frac{n!}{s^{n+1}}$$

2.

a) Calcular la siguiente transformada de Laplace empleando el teorema de convolución $L\left\{\frac{e^{-3s}}{(s+1)(s^2+4)}\right\}$

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b) Calcular la siguiente transformada, aplicando teoremas de traslacion $L\left\{\frac{e^{-2s}}{(s+1)(s^2+7s+12)}\right\}$

$$= \mathcal{L}^{-1}\left\{\frac{1}{(s+1)(s+4)(s+3)}\right\} u(t-2)$$

$1 = A(s+4)(s+3) + B(s+1)(s+3) + C(s+1)(s+$

$$= \frac{A}{(s+1)} + \frac{D}{(s+4)} + \frac{C}{(s+3)}$$

A:

$$\lim_{s\to -1} (1) = A(s+4)(s+3) + B\cancel{(s+1)(s+3)} + C\cancel{(s+1)(s+4)}$$

$$A = \frac{1}{6}$$

B:

$$\lim_{s\to -4} (1) = A\cancel{(s+4)(s+3)} + B(s+1)(s+3) + C(s+1)\cancel{(s+4)}$$

$$B = \frac{1}{3}$$

C:

$$\lim_{s\to -3} (1) = A\cancel{(s+4)(s+3)} + B\cancel{(s+1)(s+3)} + C(s+1)(s+4)$$

$$C = -\frac{1}{2}$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{2(s+1)}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{3(s+4)}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{2(s+3)}\right\}$$

$$= \left(\frac{1}{2}e^{-t} + \frac{1}{3}e^{-4t} - \frac{1}{2}e^{-3t}\right) u(t-2)$$

3.

Resolver las siguientes ecuaciones diferenciales, aplicando la transformada de laplace

a) $y'' + 5y' + 4y = 0,\ y(0) = 1,\ y'(0) = 0$

$$L(y'' + 5y' + 4y) = 0$$

$$[s^2 y(s) - s y(0) - y'(0)] + 5(s y(s) - y(0)) + 4y(s) = 0$$

$$[s^2 y(s) - s - 0] + 5(s y(s) - 1) + 4y(s) = 0$$

$$(s^2 + 5s + 4)\, y(s) - s - 5 = 0$$

$$(s^2 + 5s + 4)\, y(s) = s + 5$$

$$y(s) = \frac{s+5}{s^2+5s+4}$$

$$\therefore \boxed{y(s) = \frac{s+5}{s^2+5s+4}}$$

Podemos escribir $s^2 + 5s + 4 = (s+1)(s+4)$

$$\frac{s+5}{(s+1)(s+4)} = \frac{A}{s+1} + \frac{B}{s+4}$$

$$\frac{s+5}{(s+1)(s+4)} = \frac{As + 4A + Bs + B}{(s+1)(s+4)}$$

$s + 5 = (A+B)s + 4A + B$ porque ambos lados tienen s

$A + B = 1 \quad , \quad 4A + B = 5$

$$\begin{array}{rcl} A + B &=& 1 \\ -4A + B &=& -5 \\ \hline -3A &=& -4 \end{array} \longrightarrow A = 4/3,\ B = -1/3$$

$$\therefore y(s)\ \frac{4}{3}\left(\frac{1}{s+1}\right) - \frac{1}{3}\left(\frac{1}{s+4}\right)$$

Inversa de Laplace

$$L^{-1}(y(s)) = \frac{4}{3}L^{-1}\left(\frac{1}{s+1}\right) - \frac{1}{3}L^{-1}\left(\frac{1}{s+4}\right)$$

Resultado

$$\boxed{y(t) = \frac{4}{3}e^{-t} - \frac{1}{3}e^{-4t}}$$

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b) $y'' - y' = e^{t} \cos t \,,\ y(0) = 0 \,,\ y'(0) = 0$

Aplicamos trans. Laplace en ambos lados

$$L(y'') - L(y') = L(e^{t} \cos t)$$

$$\Rightarrow (s^2 y(s) - s y(0) - y'(0)) - (s y(s) - y(0)) = \frac{1}{(s-1)^2 + 1^2}$$

$$s^2 y(s) - 0 - 0 - s y(s) - 0 = \frac{1}{(s-1)^2 + 1}$$

$$y(s)(s^2 - s) = \frac{1}{(s-1)^2 + 1}$$

$$y(s) = \frac{1}{((s-1)^2 + 1)(s^2 - s)}$$

$$y(s) = \frac{1}{[(s-1)^2 + 1]\, s(s-1)}$$

Tomamos t. par.

$$y(s) = \frac{-1}{2s} + \frac{1}{s-1} - \frac{s}{2[(s-1)^2 + 1)]}$$

$$y(s) = \frac{-1}{2s} + \frac{1}{s-1} - \frac{(s-1)+1}{2[(s-1)^2 + 1)]}$$

$$y(s) = \frac{-1}{2s} + \frac{1}{s-1} - \frac{(s-1)}{2[(s-1)^2 + 1]} - \frac{1}{2[(s-1)^2 + 1]}$$

Aplicando inversa de la trans. Laplace

$$L^{-1}(y(s)) = -\frac{1}{2} L^{-1}\left(\frac{1}{s}\right) + L^{-1}\left(\frac{1}{s-1}\right) - \frac{1}{2} L^{-1}\left(\frac{(s-1)}{(s-1)^2 + 1}\right)$$

$$- \frac{1}{2} L^{-1}\left(\frac{1}{(s-1)^2 + 1}\right)$$

$$y(t) = \frac{-u(t)}{2} - e^{t} - \frac{1}{2} e^{t} \cos t - \frac{1}{2} e^{t} \operatorname{sen} t$$

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