

# The RITAS algorithm

A constructive yield monitor data processing algorithm

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Luis Damiano

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Department of Statistics  
Iowa State University

# Road map

Introduction

The RITAS algorithm

Rectangles

Intersection

Tessellation

Aggregation

Smoothing

STRIPS

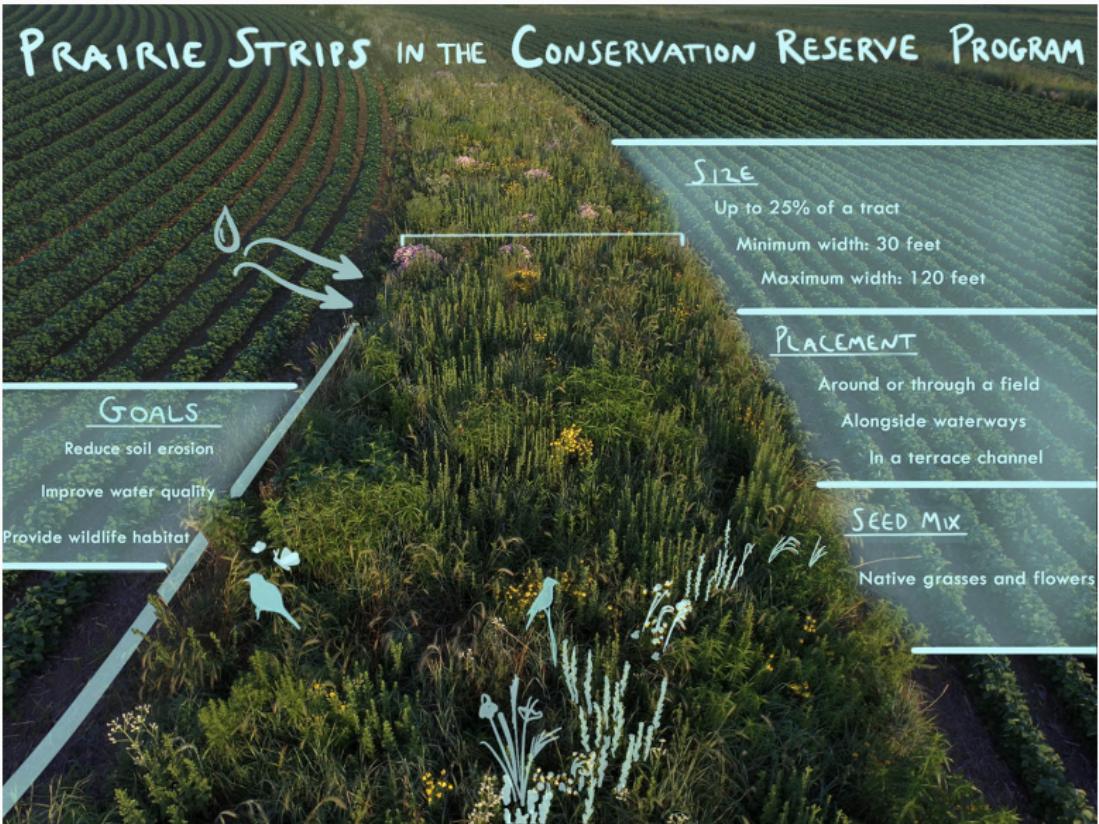
Discussion

Future work

# Introduction

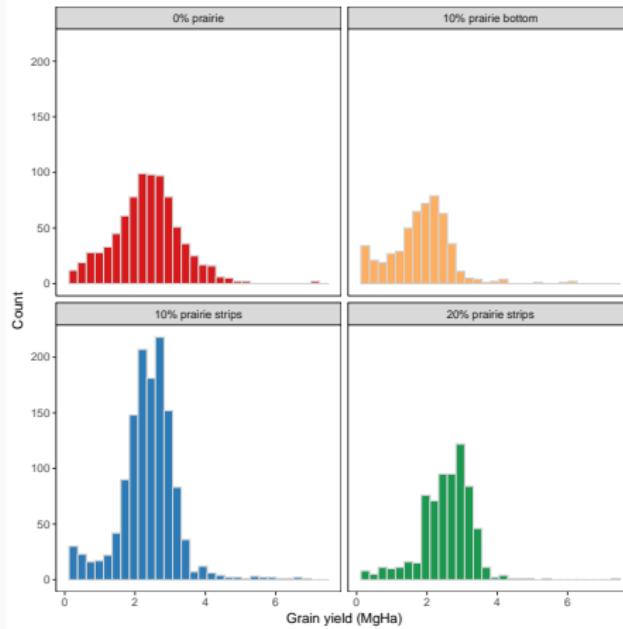
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# Context



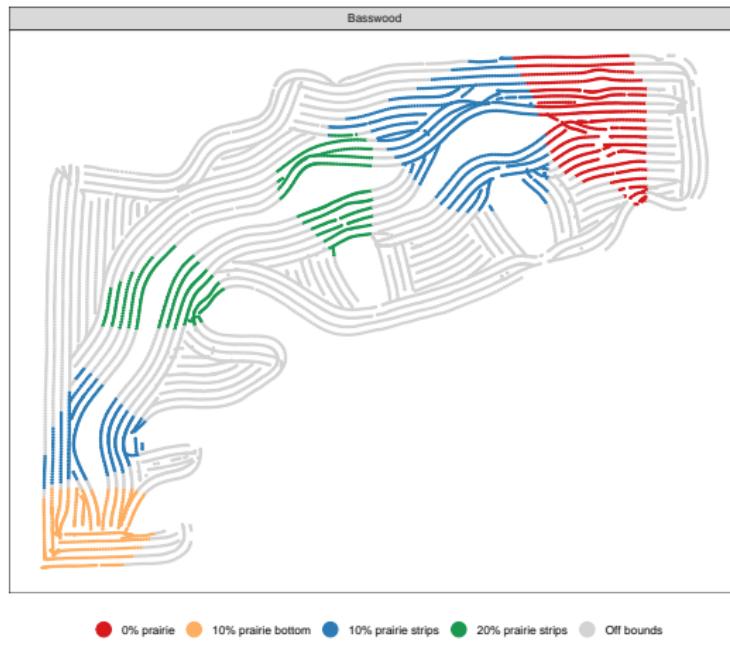
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# Data acquisition

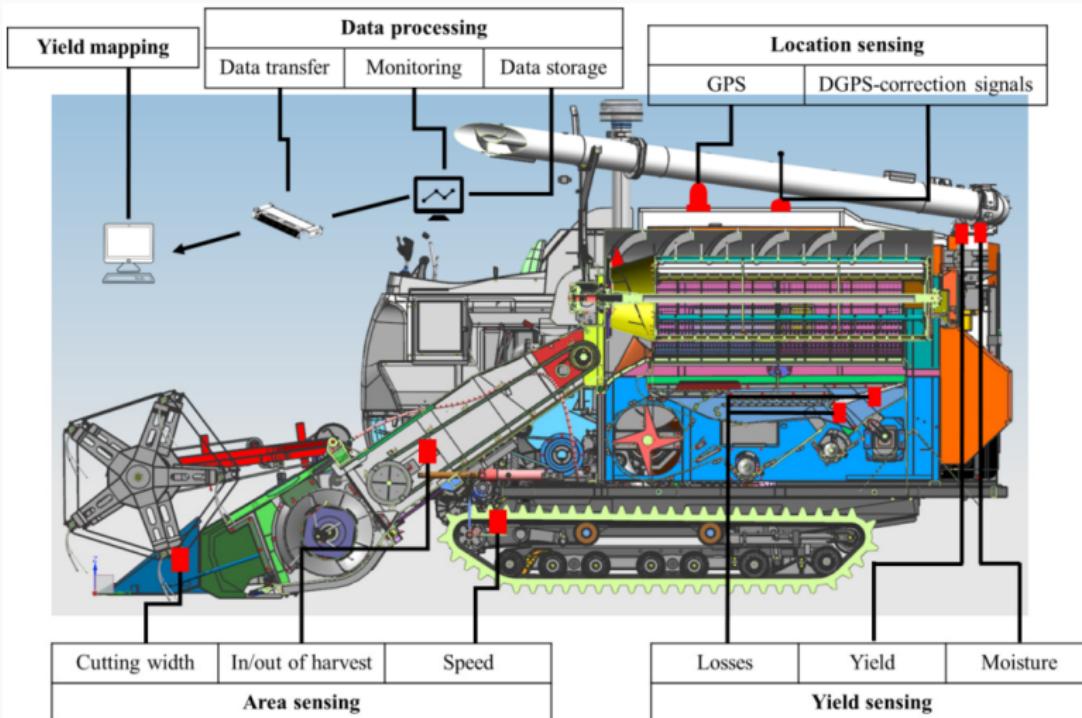
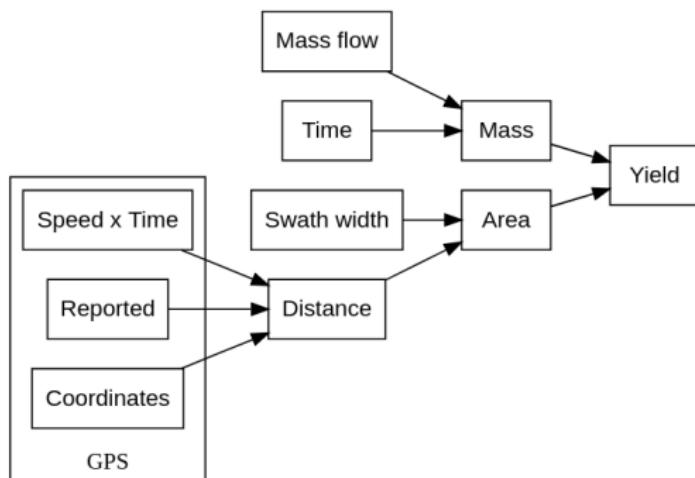


Figure 1: Yield monitoring component diagram from ??

# Uncertainty map



## Modeling challenges

- Harvesting dynamics (time lag, start-pass and end-pass delay)
- Continuous measurements (local harvest circumstances)
- Positioning information (GPS receiver accuracy, repeated locations)
- Operator-induced errors (short segments, sudden speed changes, narrow finishes and overlaps, sharp turns, calibration)

## Current analysis methodology

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- Spatial vs non-spatial analysis
- Heuristics

## The RITAS algorithm

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# RITAS

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Consider two subsequent locations

$s_t, s_{t-1}$

$$s_t = (x_t, y_t)$$

$$s_0 = (x_0, y_0)$$

## Rectangles

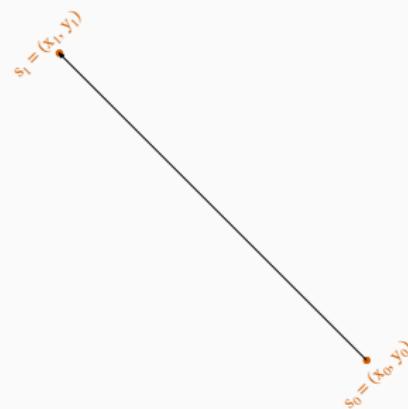
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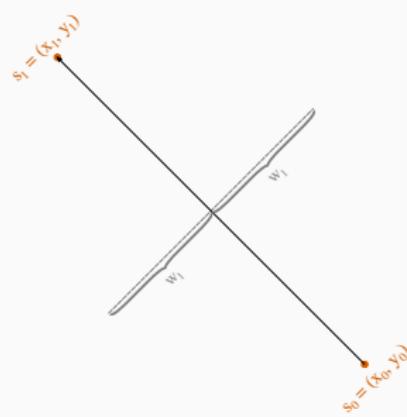
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$$dx_t = w_t (1 + b_t^{-2})^{-\frac{1}{2}}$$

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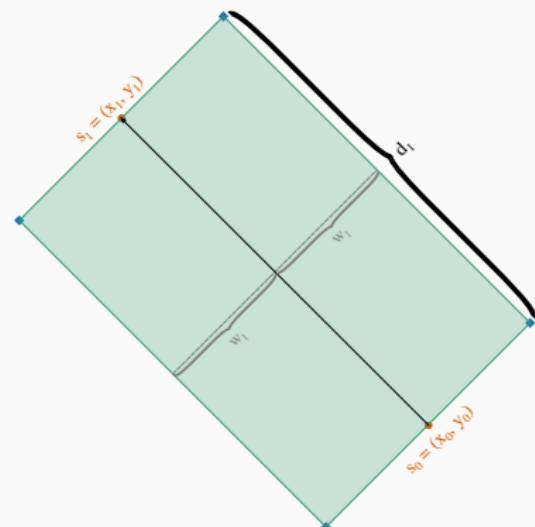
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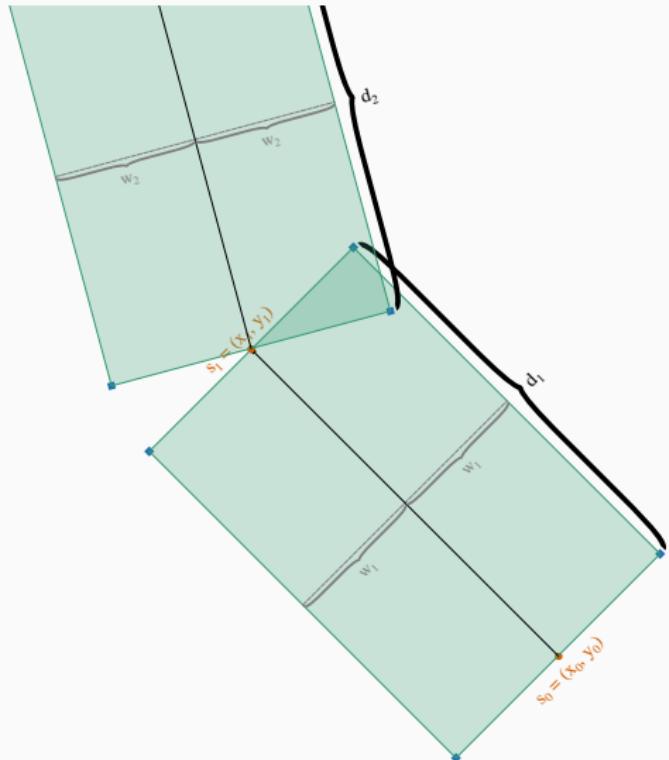
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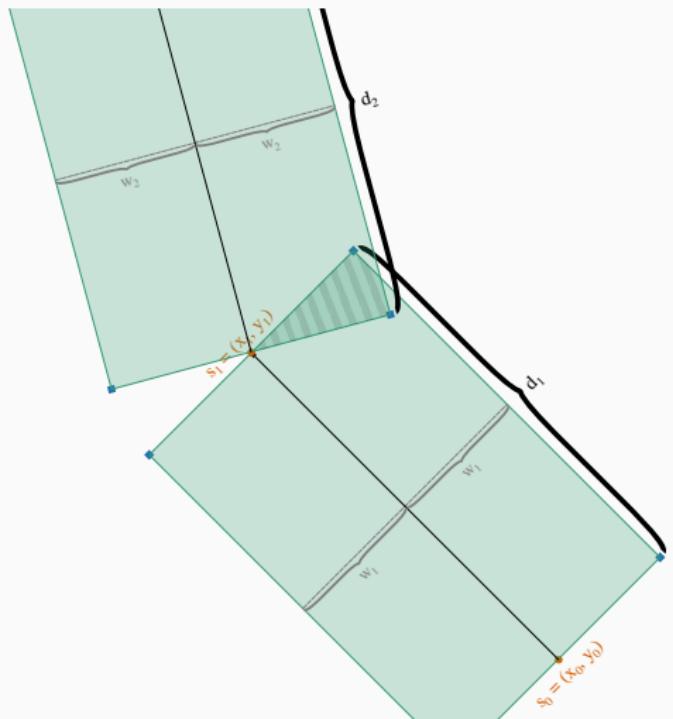
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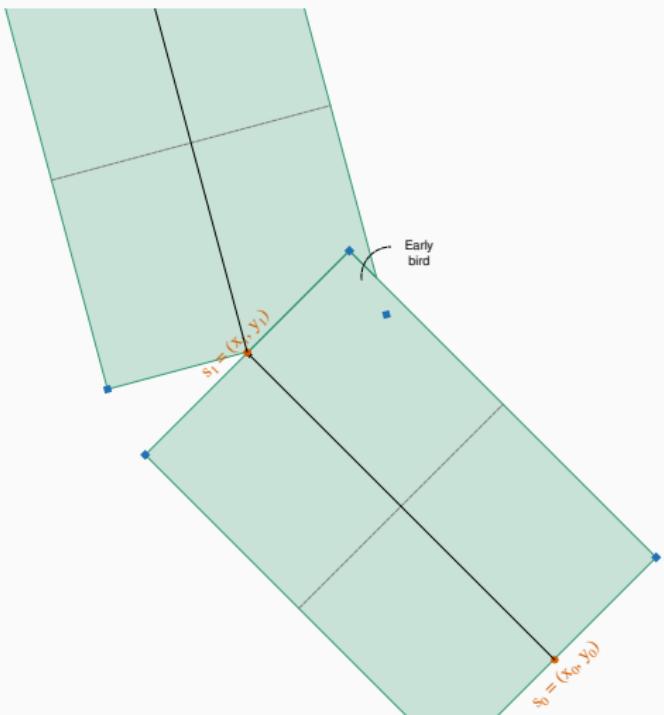


## Rectangle collection

$$\mathcal{P} = \{P_\tau : \tau \in \{1, \dots, T\}\}$$

**Intersection**

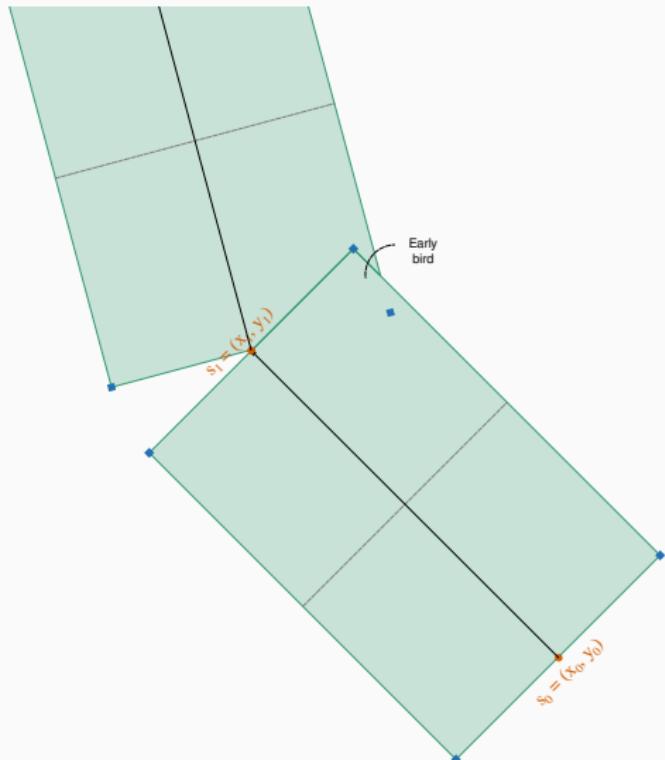
## Tessellation



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Define the time-ordered relative complement

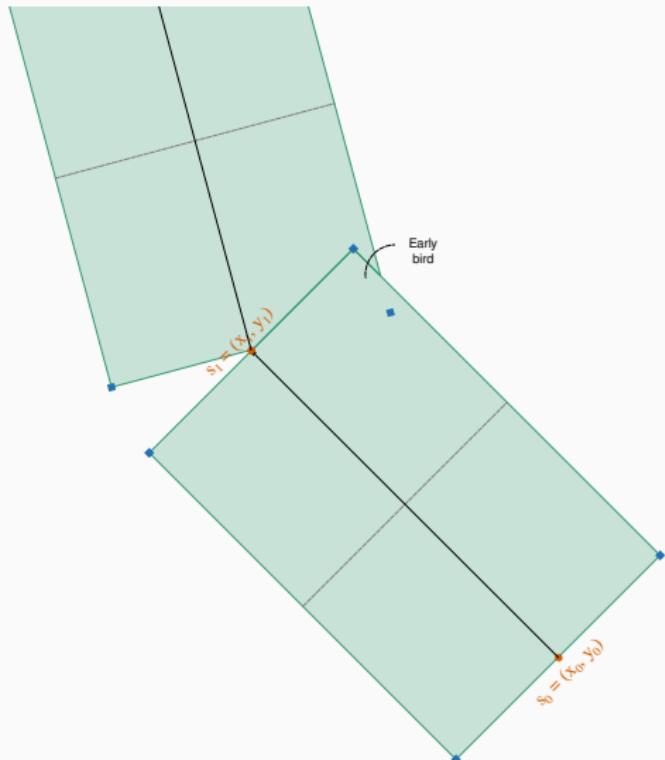
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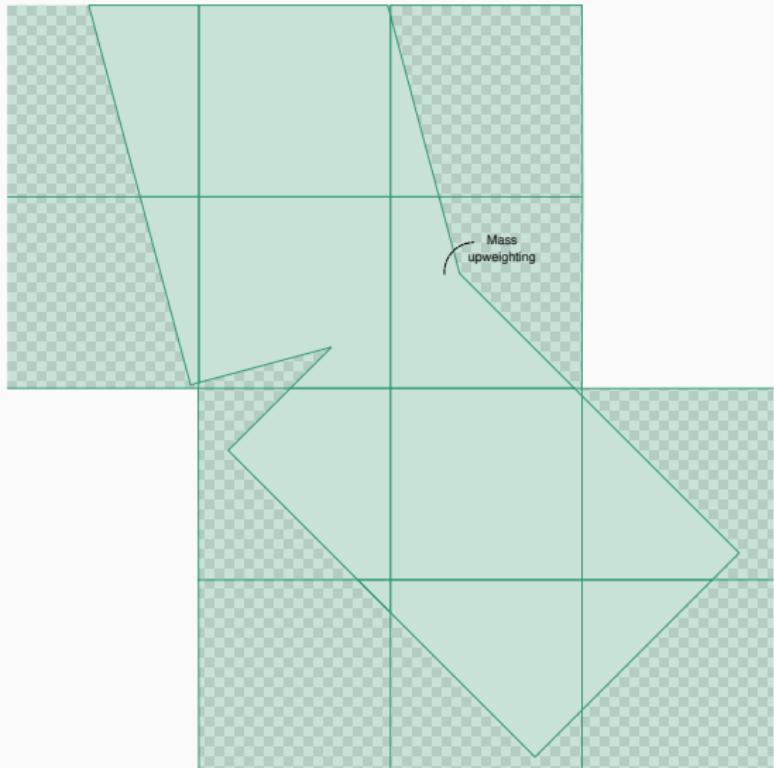
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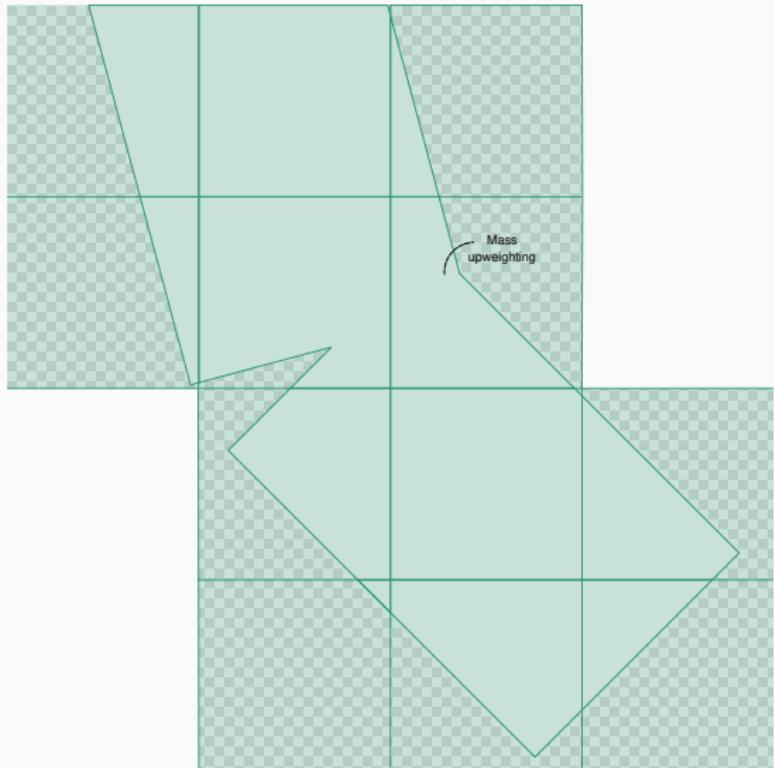
$$\tilde{\mathcal{P}} = \{\tilde{P}_\tau : \tau \in \{1, \dots, T\}\}$$

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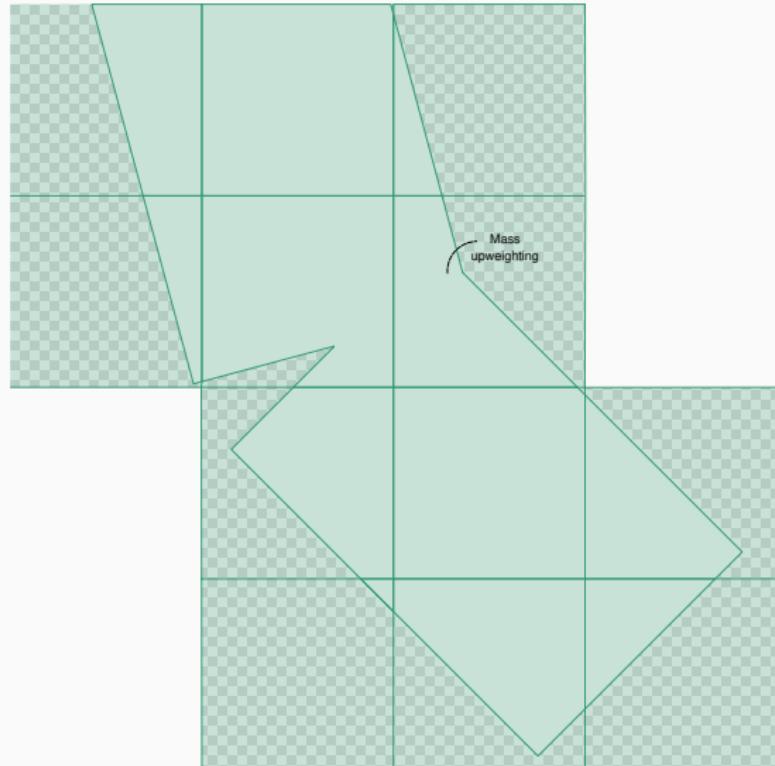
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## Apportioning

Fix  $N \in \mathbb{N}$

Create  $N$  equally-sized,  
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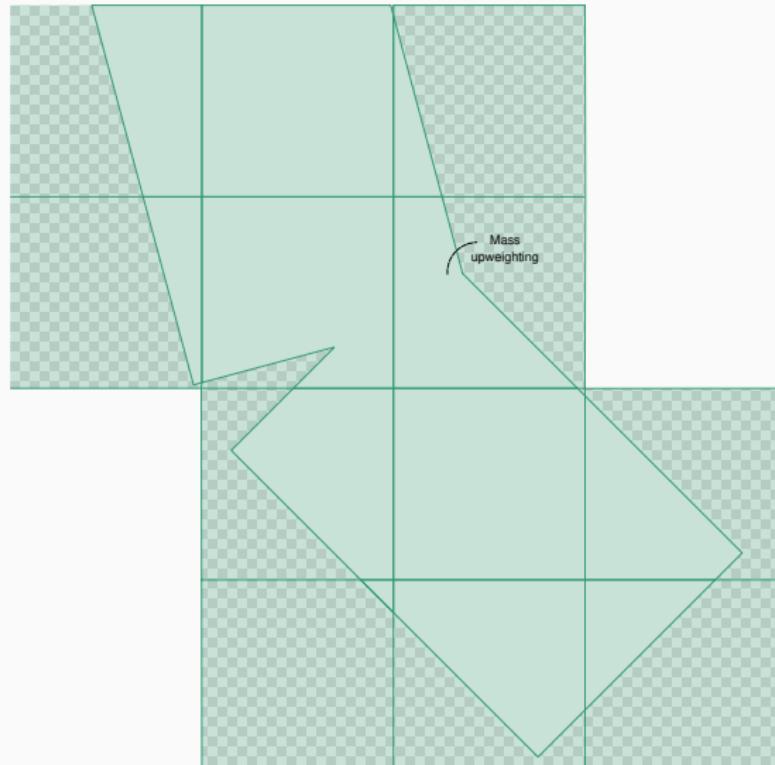


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polygon  $\tilde{P}_\tau$  and the  $n$ -th pixel  
for  $\tau \in \{1, \dots, T\}$  and  
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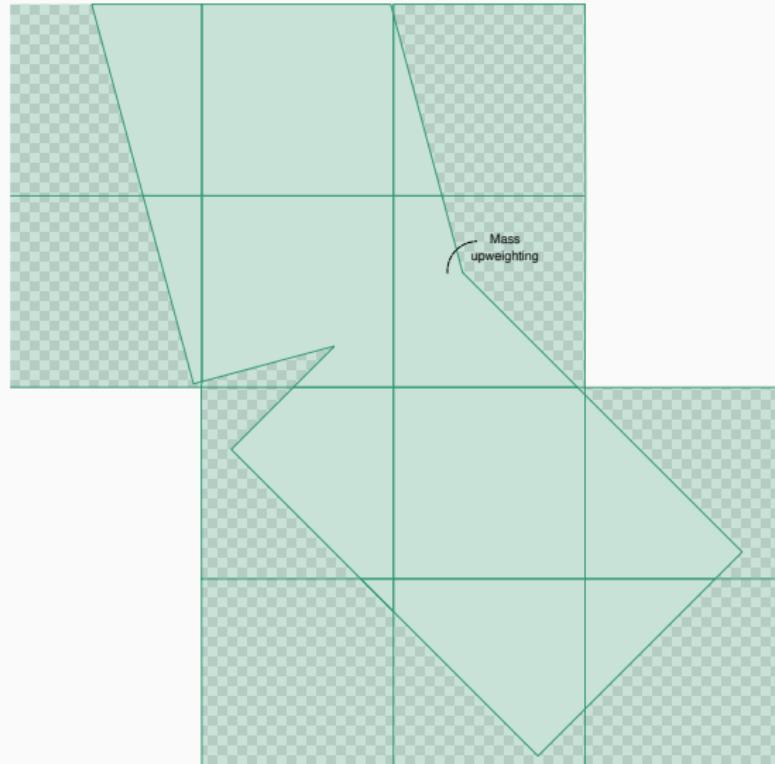
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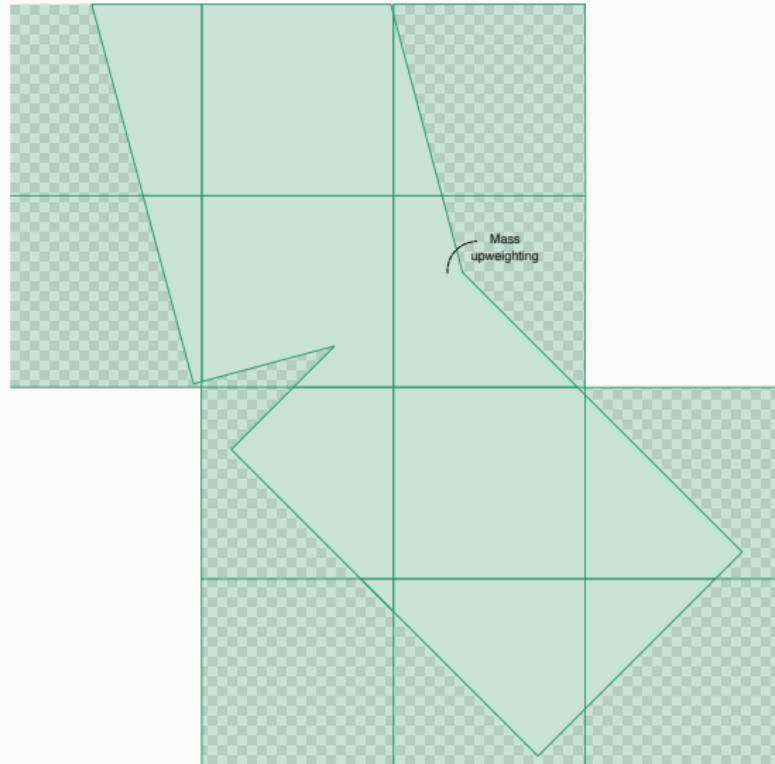
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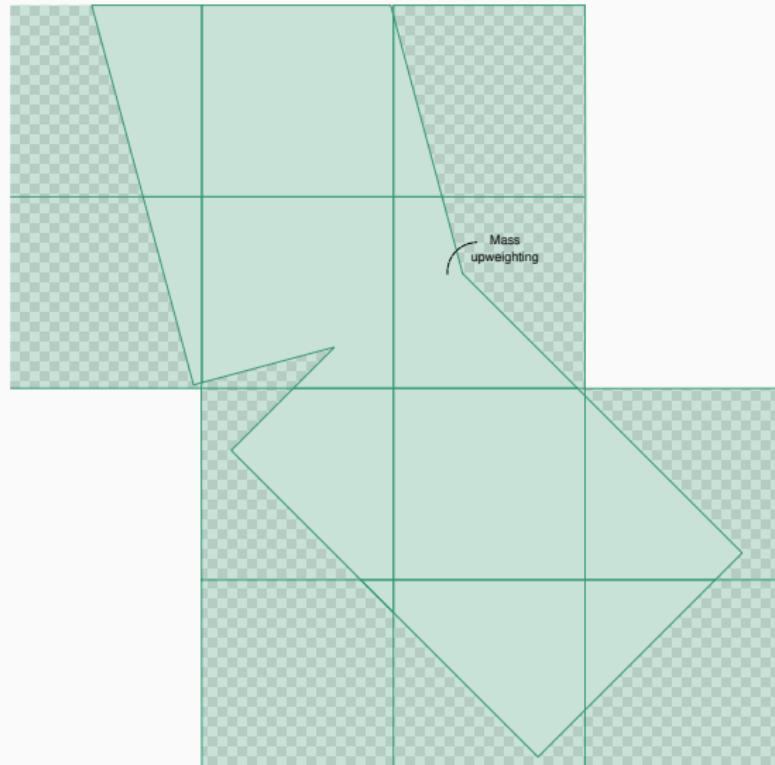
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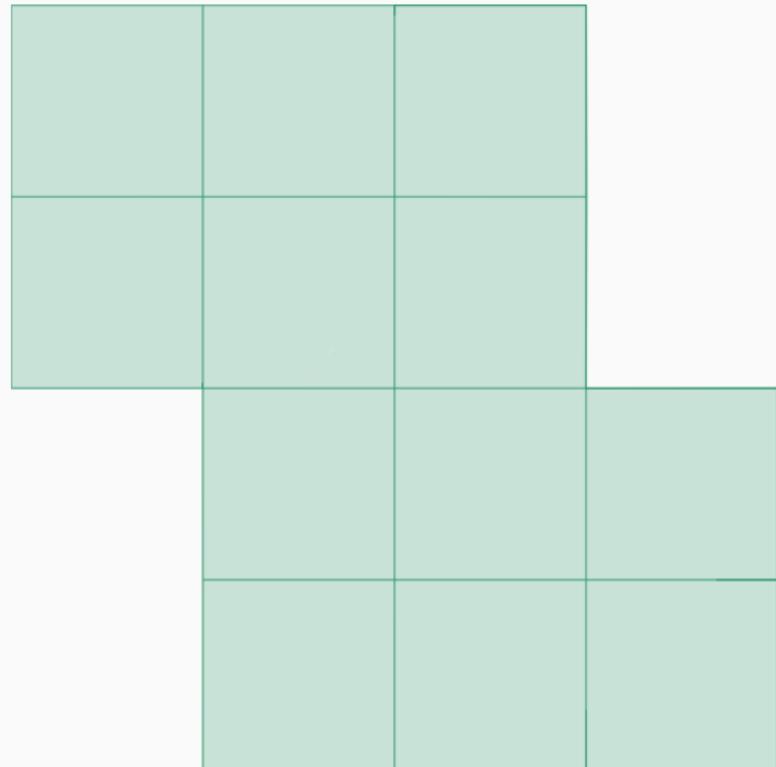
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## Grid collection

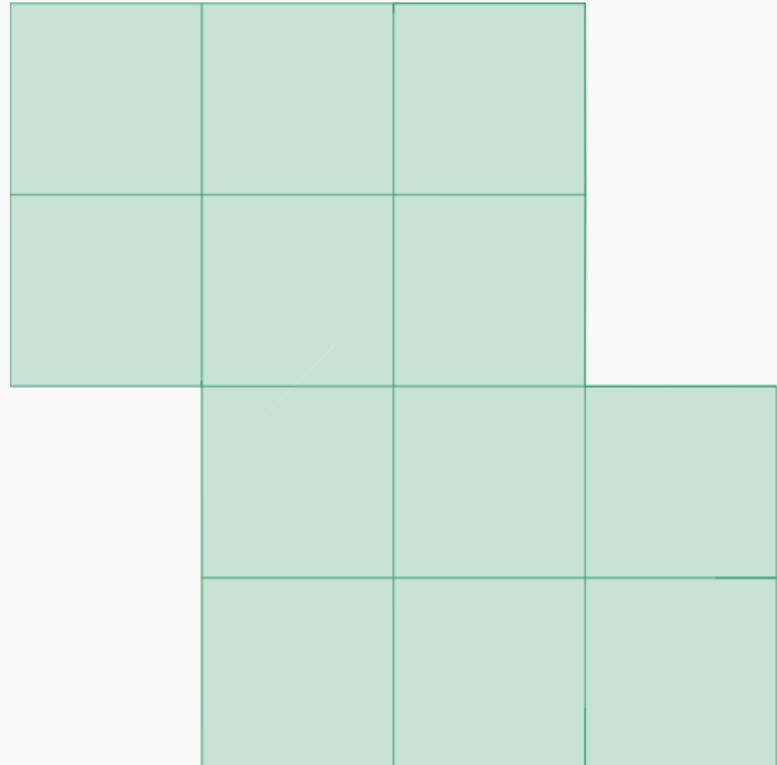
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## Smoothing



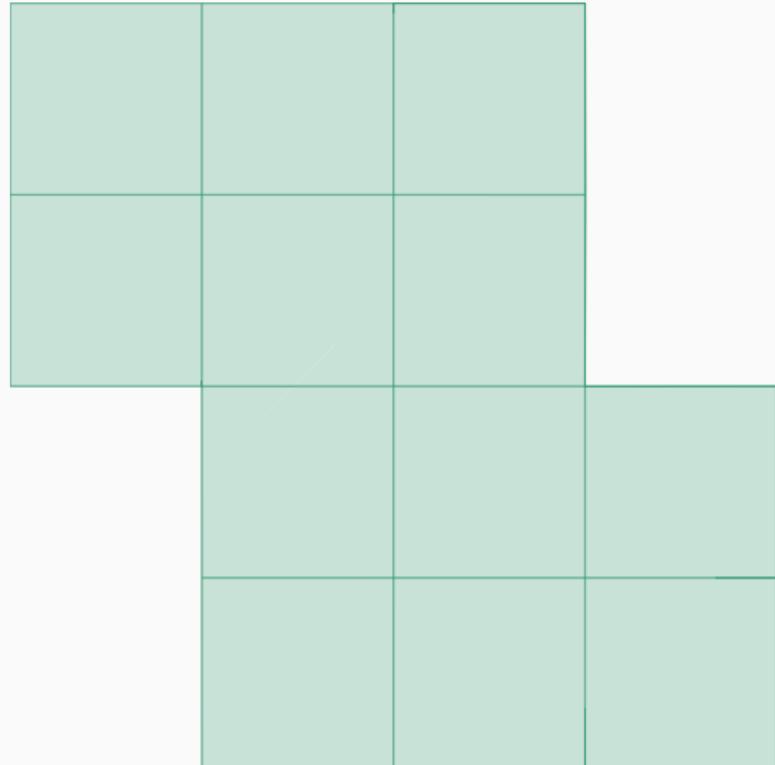
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- Model



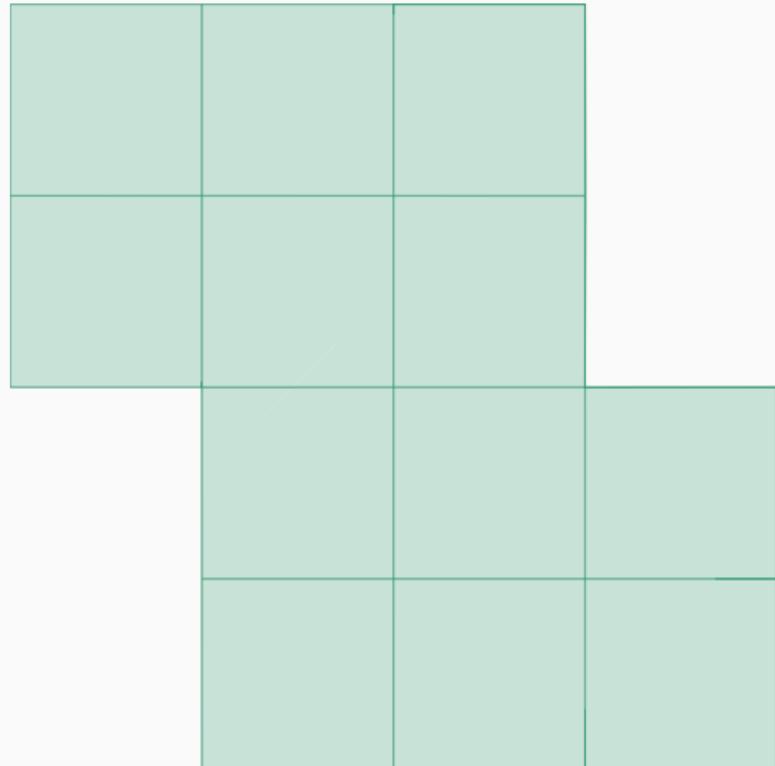
## Smoothing

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## Smoothing

- Model
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- Prediction



## **Smoothing**

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### Model

$$f(\mathbf{S}) \triangleq \log(\mathbf{m}) \sim \mathcal{GP} \left( m(\mathbf{S}), k \left( \mathbf{S}, \mathbf{S}' \right) \right)$$

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$$k(d) = \frac{2^{1-\nu}}{\Gamma(\nu)} \left( \frac{\sqrt{2\nu}d}{\ell} \right)^\nu K_\nu \left( \frac{\sqrt{2\nu}d}{\ell} \right)$$

for  $d \geq 0$  known and fixed,  $\ell, \nu > 0$  unknown

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### Log marginal likelihood

$$\log p(\mathbf{m} \mid \mathbf{S}) = -\frac{1}{2}\mathbf{y}^\top (K + \sigma_n^2 I)^{-1} \mathbf{y} - \frac{1}{2} \log |K + \sigma_n^2 I| - \frac{n}{2} \log 2\pi$$

for  $K = k(\mathbf{S}, \mathbf{S}')$

## Smoothing

Predictive distribution

$$\begin{bmatrix} \mathbf{m} \\ \mathbf{f}_* \end{bmatrix} \sim \mathcal{N}\left(\mathbf{0}, \begin{bmatrix} K(S, S) + \sigma_n^2 I & K(S, S_*) \\ K(S_*, S) & K(S_*, S_*) \end{bmatrix}\right)$$

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$$\mu_m = \exp(\mu_f + \sigma_f^2/2) \quad \text{and} \quad \sigma_m^2 = \exp(2\mu_f + \sigma_f^2) [\exp(\sigma_f^2) - 1]$$

# Implementation

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- R package, documentation at about 80%

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  - Has issues with small  $\nu < 0.3$

## Code snippet

```
proj4string <-
  "+proj=utm +zone=15 +datum=WGS84 +units=m +ellps=WGS84 +towgs84=0,0,0"

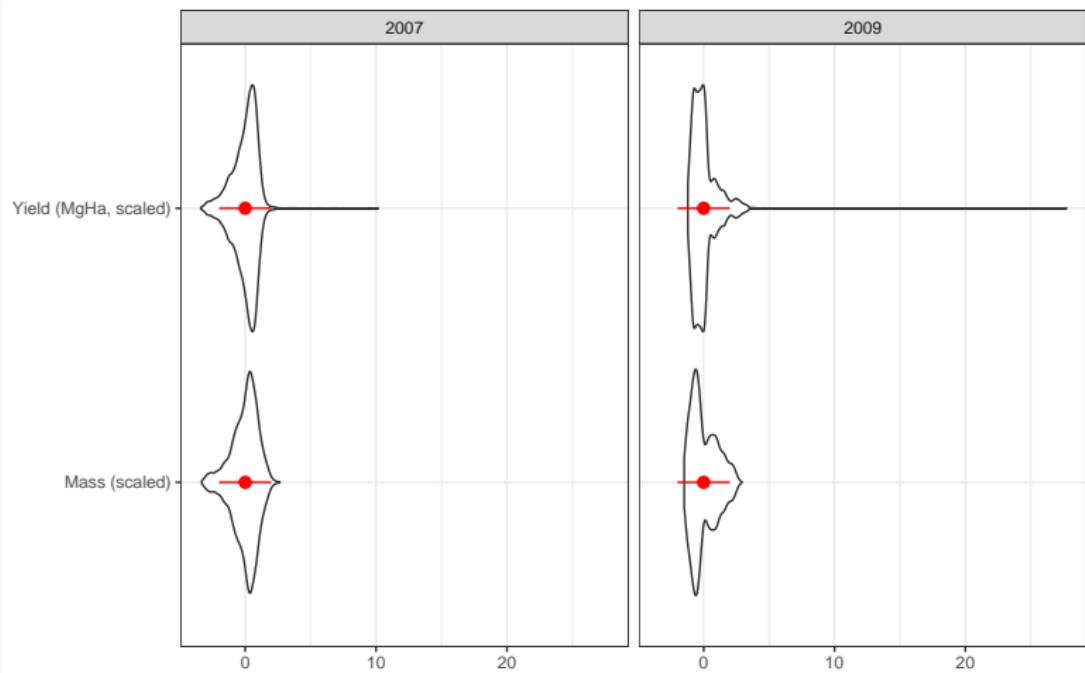
> head(DF)
    site year record      x      y mass swath     d
1 Basswood 2012        0 476877.0 4598424 3.67  6.09    NA
2 Basswood 2012        1 476879.5 4598424 7.87  6.09  1.87
3 Basswood 2012        2 476881.8 4598424 2.65  6.09  2.33
4 Basswood 2012        3 476882.6 4598424 9.26  6.09  0.76
5 Basswood 2012        4 476884.8 4598424 11.68 6.09  2.26
6 Basswood 2012        5 476887.0 4598424 18.14 6.09  2.23

ritas(DF, proj4string, "Basswood", 2012, res = 5, nmax = 0.4, nCores = 16)
```

# STRIPS

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# Data set



# Results



# Results



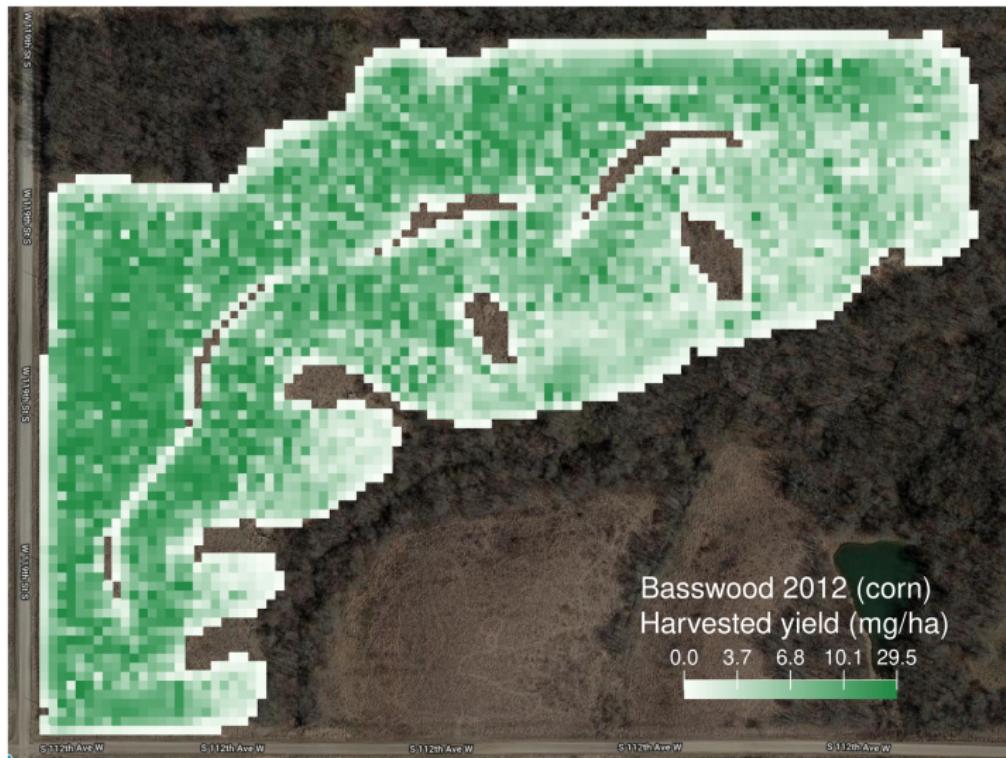
# Results



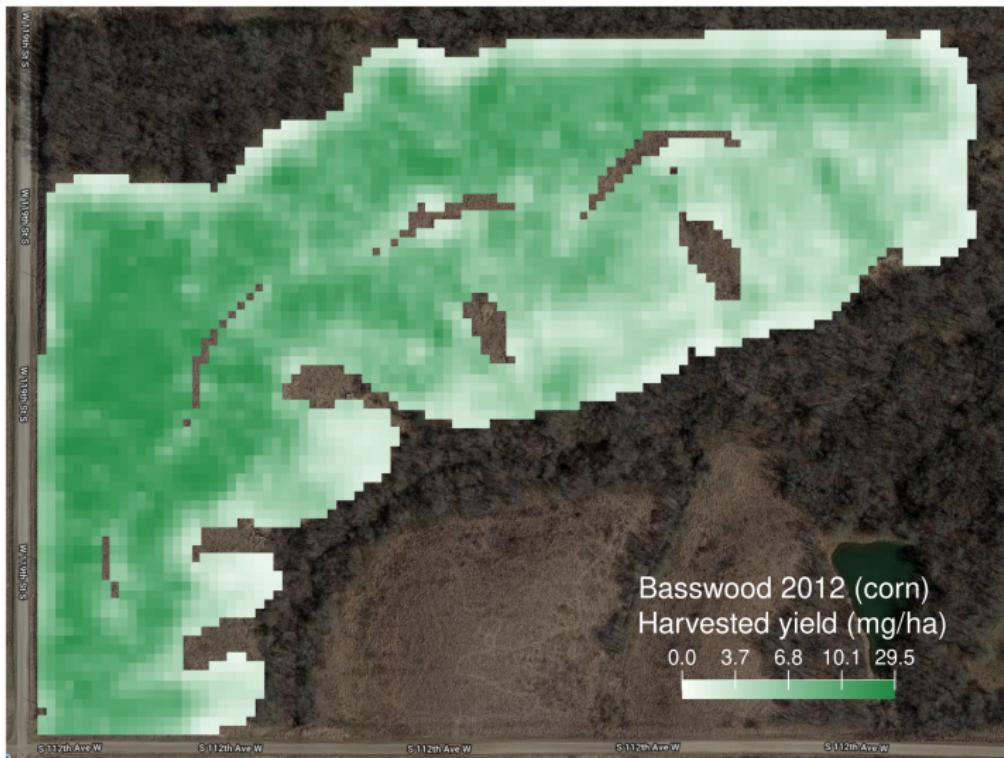
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## **Discussion**

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## The YAYs!

- Constructive approach motivated by data acquisition dynamics.
- Modeling mass to partially work around some of the uncertainty propagation channels.
- Autonomous: no user-defined thresholds for automatic processing and more consistent results across data sets.
- No observations were harmed during the making of this film.
- Apportioned observations are aligned for spatio-temporal analysis (spatial registration).

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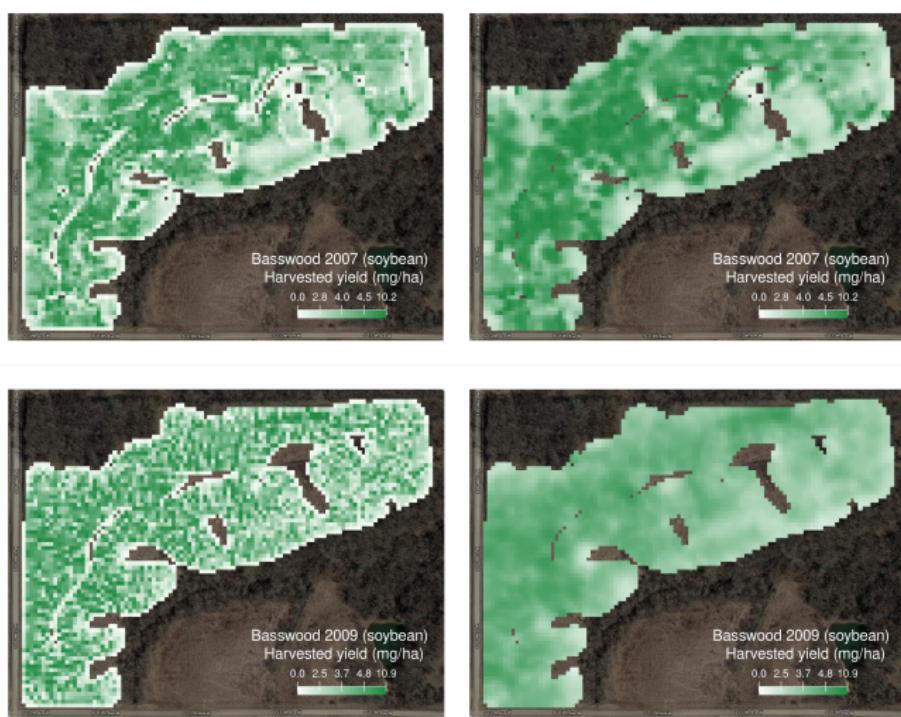
## The not-so-YAYs!

- Smoothing is  $O(N^3)$ , but many approximations are readily available.
- Tessellation, if improved naively, involves  $(N - 1)!$  operations.
- Time lag processing rules sold separately.

## **Future work**

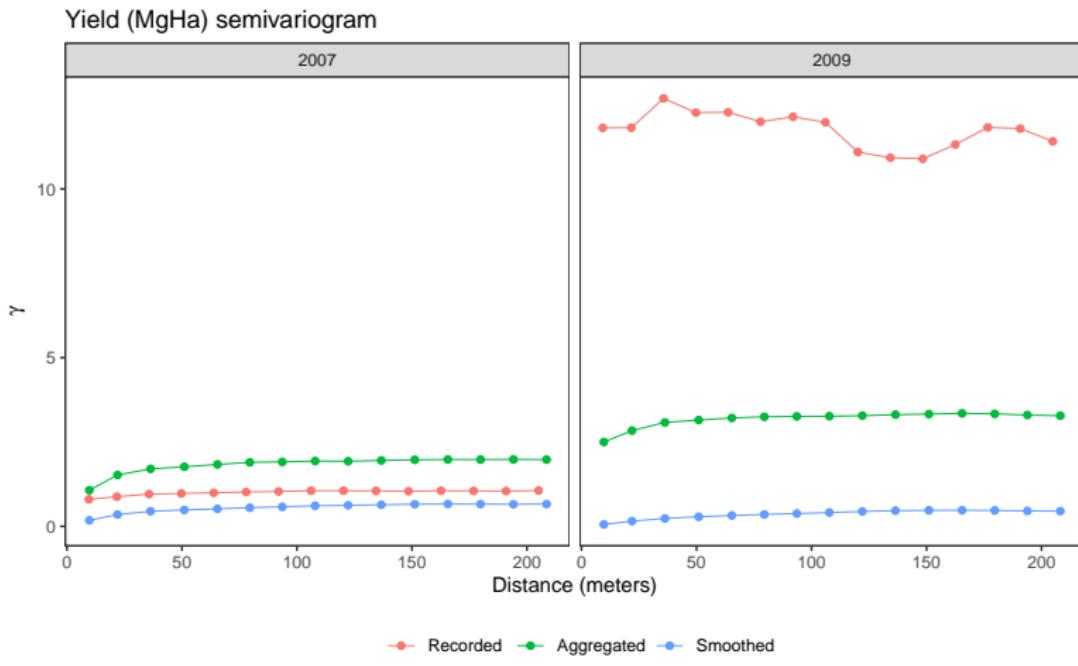
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# Variability quantification



**Figure 2:** Apportioned (left) and smoothed (right) maps for two datasets with an apparent difference in variability.

# Improvement quantification



**Figure 3:** Semivariogram of the yield monitor data

# Resolution selection

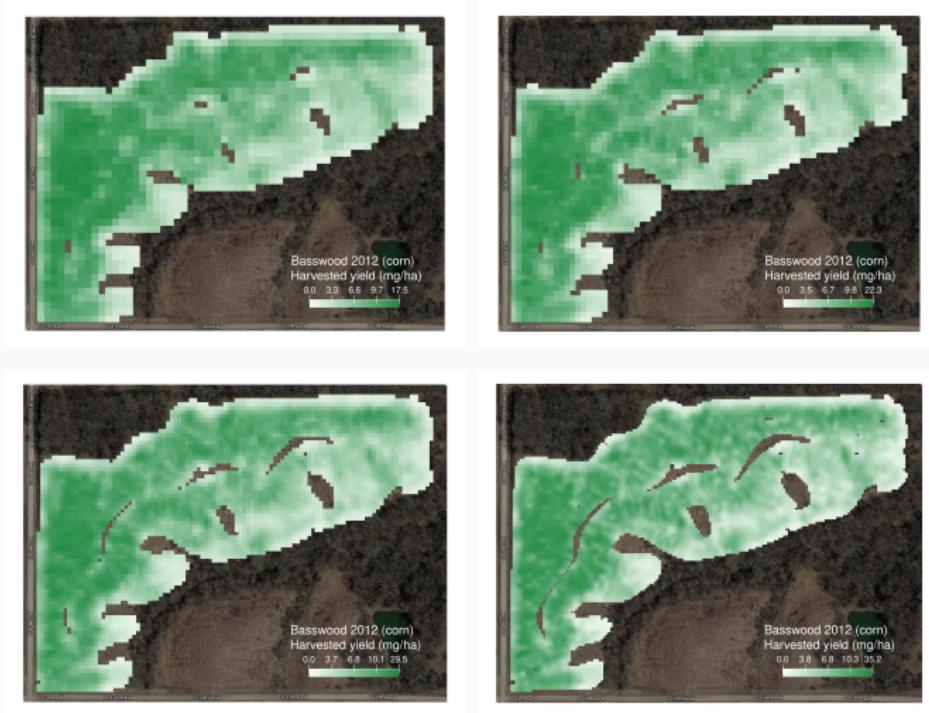


Figure 4: Smoothed map at four different resolutions. Top: 9m, 7m; bottom: 5m, 3m.

## References

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## References i

See Creative Component writing.