CS 5/7320
Artificial
Intelligence

Adversarial Search and Games

AIMA Chapter 5

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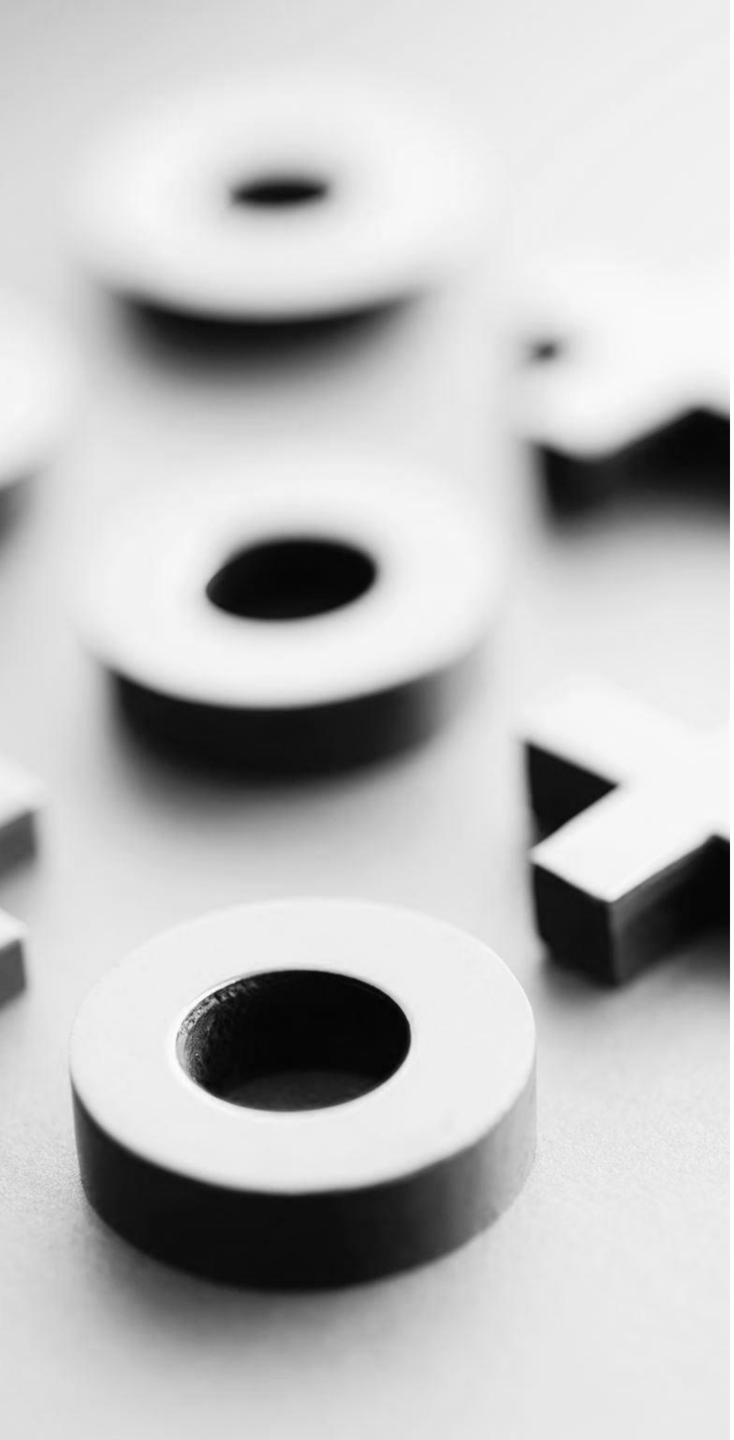


Games

- Games typically confront the agent with a competitive (adversarial) environment affected by an opponent.

- We will focus on deterministic two-player zero-sum games with perfect information.

- We call the two players:
  1) **Max** tries to maximize his utility.
  2) **Min** tries to minimize Max's utility since it is a zero-sum game.

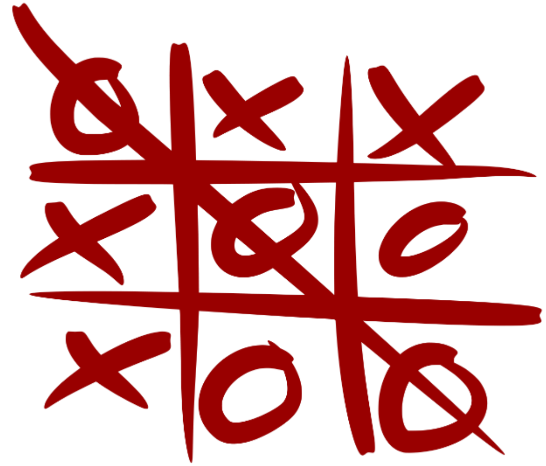
Definition of a Game

- **Definition:**

s_0	The initial state (position, board).
$Actions(s)$	Legal moves in state s.
$Result(s, a)$	Transition model.
$Terminal(s)$	Test for terminal states.
$Utility(s)$	Utility for player Max.

- **State space**: a graph defined by the initial state and the transition function containing all reachable states (e.g., chess positions).
- **Game tree**: a search tree superimposed on the state space. A complete game tree follows every sequence from the current state to the terminal state (the game ends).

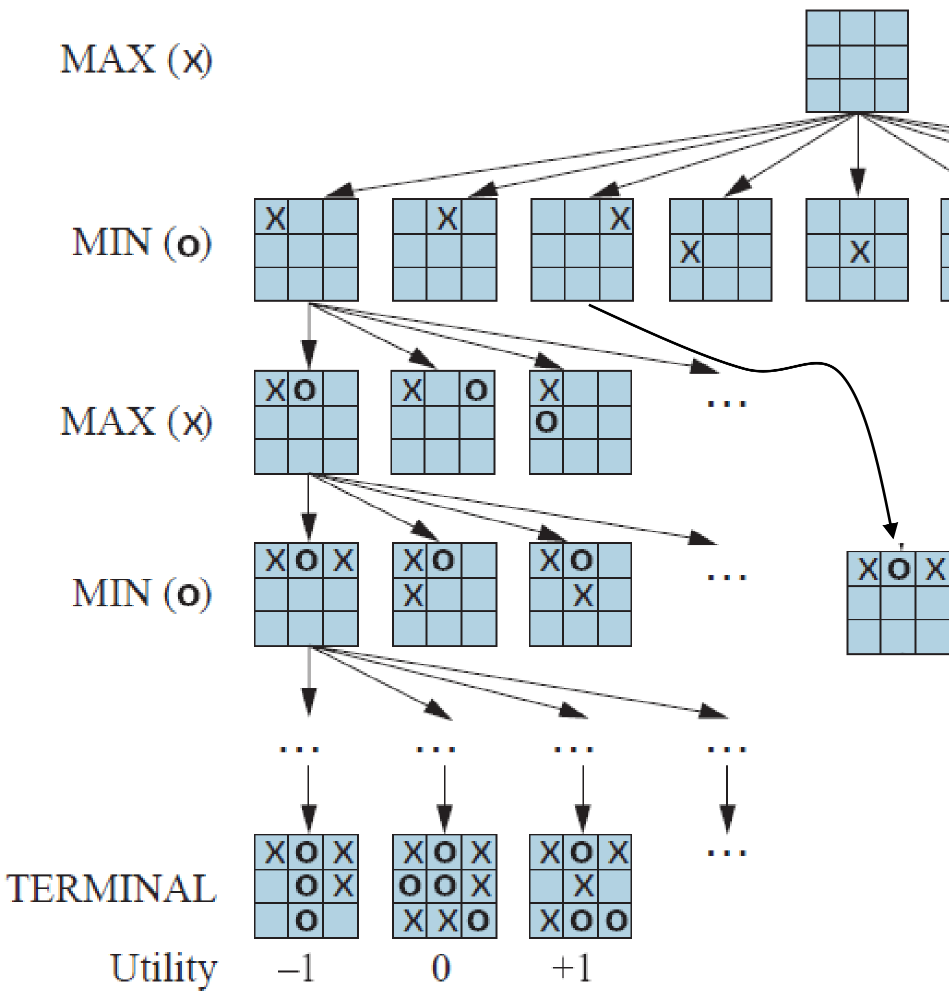
Example: Tic-tac-toe



s_0	Empty board.
$Actions(s)$	Empty squares.
$Result(s, a)$	Place symbol (x/o) on empty square.
$Terminal(s)$	Did a player win or is the game a draw?
$Utility(s)$	+1 if x wins, -1 if o wins and 0 for a draw.
	Utility is only defined for terminal states.

Here player x is Max and player o is Min.

Tic-tac-toe: Partial Game Tree



The state space is:

$$3^9 = 19{,}683$$

Why is the complete game tree size much larger? A little smaller than:

$$9! = 362{,}880$$

Methods for Adversarial Games

Exact Methods

- **Model as nondeterministic actions**: The opponent is seen as part of an environment with nondeterministic actions. Non-determinism is the result of the unknown moves by the opponent. We **consider all possible moves** by the opponent.
- **Find optimal decisions**: Minimax search and Alpha-Beta pruning where **each player plays optimal** to the end of the game.

Heuristic Methods

(state space is too large)

- **Heuristic Alpha-Beta Tree Search**:
  a. Cut off search tree and use heuristic for utility.
  b. Forward Pruning: ignore poor moves.
- **Monte Carlo Tree search**: Estimate utility of a state by simulating complete games and average the utility.

Nondeterministic Actions

- Recall And-Or Search from AIMA Chapter 4

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Nondeterministic Actions

Each action consists of the move by the player and all possible (i.e., nondeterministic) responses by the opponent.

Outcome of actions in the environment is nondeterministic = **transition model need to describe uncertainty about the opponent's behavior.**

Example transition:

$$Results(s_1, a) = \{s_2, s_4, s_5\}$$

i.e., action a in s_1 can lead to one of several states.

The set of possible states $\mathrm{b} = \{s_2, s_4, s_5\}$ is called a **belief state** of the agent.

AND-OR DFS Search Algorithm

= nested If-then-else statements

function AND-OR-SEARCH(*problem*) **returns** a conditional plan, or *failure*
  return OR-SEARCH(*problem*, *problem*.INITIAL, [ ])

function OR-SEARCH(*problem*, *state*, *path*) **returns** *a conditional plan, or failure*
  if *problem*.IS-GOAL(*state*) **then return** the empty plan
  if IS-CYCLE(*path*) **then return** *failure* // don't follow loops
  for each *action* **in** *problem*.ACTIONS(*state*) **do** // check all possible actions
    plan ← AND-SEARCH(*problem*, RESULTS(*state*, *action*), [*state*] + *path*])
    if *plan* ≠ *failure* **then return** [*action*] + *plan*]
  return *failure*

my moves

function AND-SEARCH(*problem*, *states*, *path*) **returns** *a conditional plan, or failure*
  for each s_i **in** *states* **do** // check all states in belief state
    $plan_i$ ← OR-SEARCH(*problem*, s_i, *path*)
    if $plan_i$ = *failure* **then return** *failure*
  return [**if** s_1 **then** $plan_1$ **else if** s_2 **then** $plan_2$ **else** ... **if** s_{n-1} **then** $plan_{n-1}$ **else** $plan_n$]

opponent moves

- And-Or Search searches the whole tree till it finds a subtree that leads only to goal nodes.
- BFS and A* search can also be used to search an AND-OR tree.

Optimal Decisions

Minimax Search and Alpha-Beta Pruning

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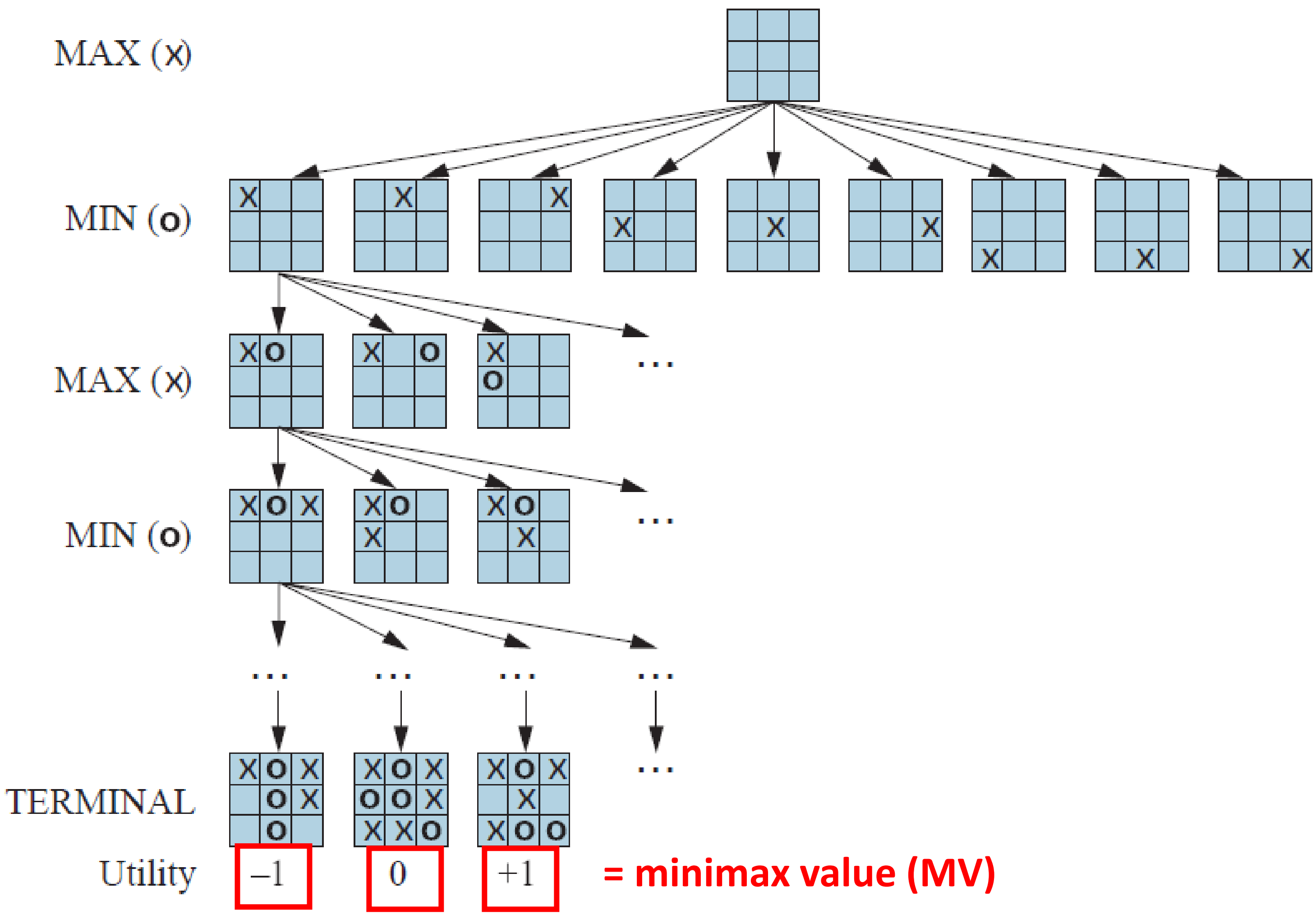
Idea: Minimax Decision

- Assign each state a **minimax value** that reflects how much Max prefers the state (= Min dislikes the state).

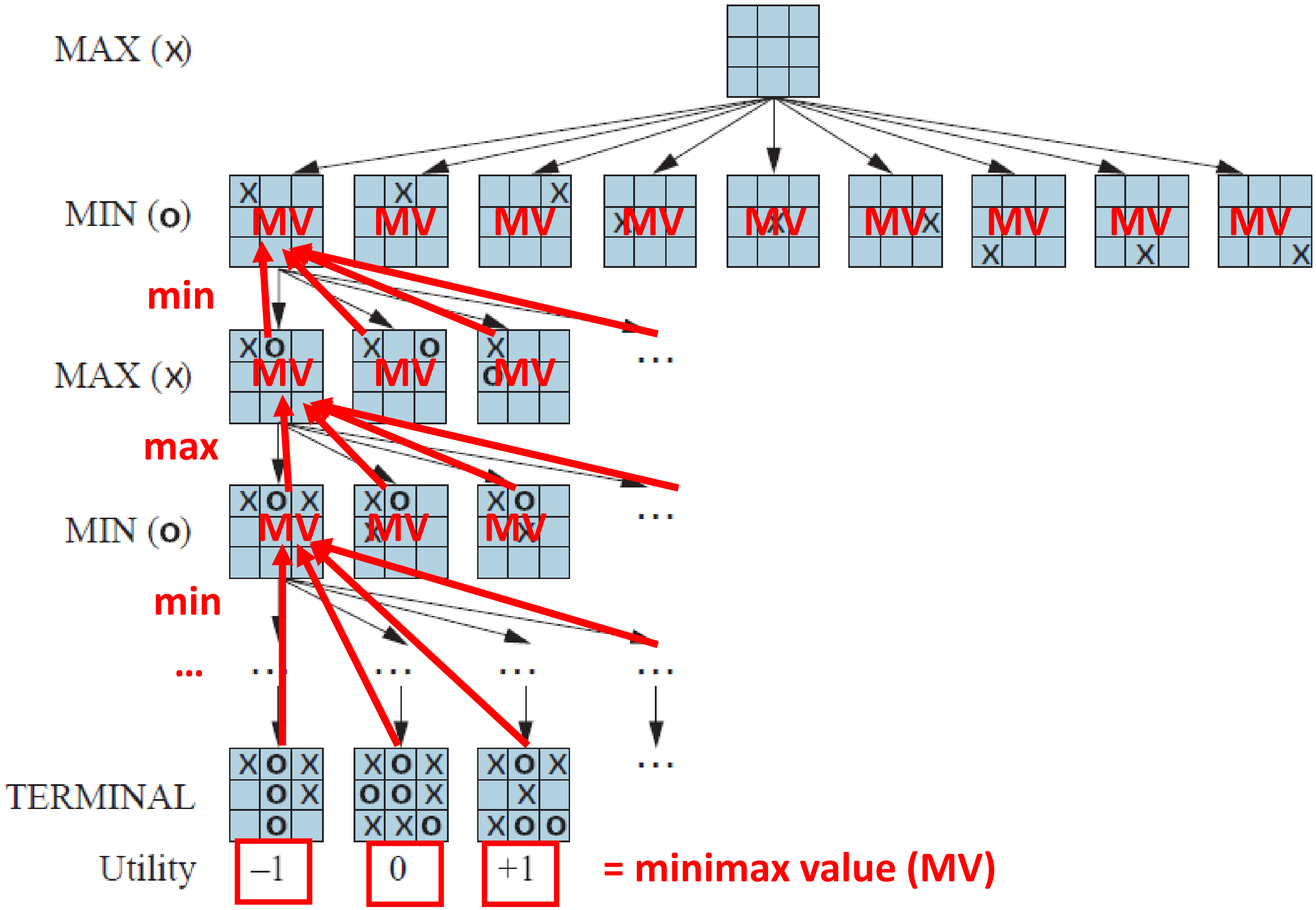
$$Minimax(s) = \begin{cases} Utility(s) & \text{if } terminal(s) \\ \max_{a \in Actions(s)} Minimax\big(Result(s,a)\big) & \text{if } move = Max \\ \min_{a \in Actions(s)} Minimax\big(Result(s,a)\big) & \text{if } move = Min \end{cases}$$

- The minimax value is the utility for Max in state s assuming that **both players play optimally** from s to the end of the game.
- The **optimal decision** for Max is the action that leads to the state with the largest minimax value.

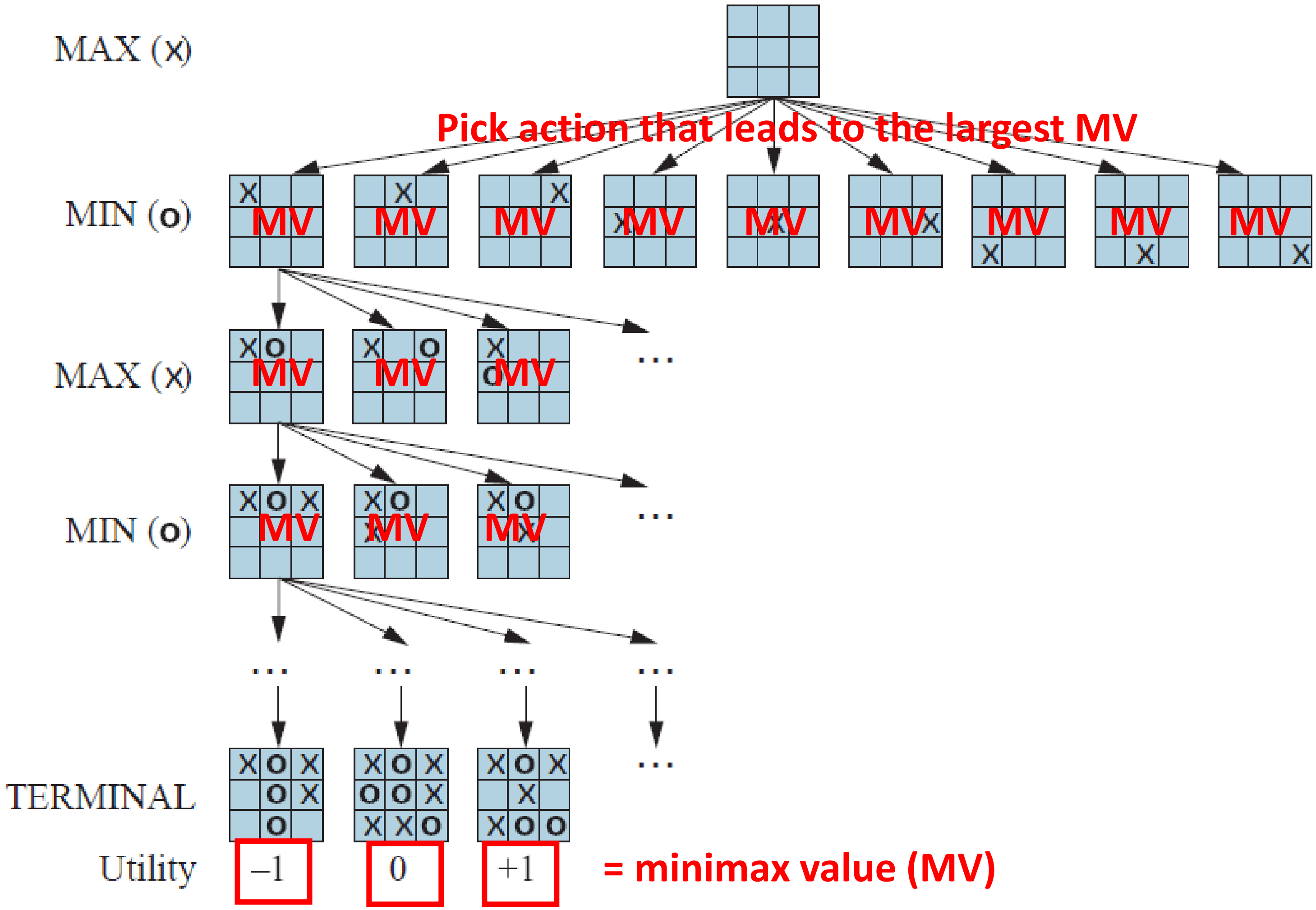
Minimax Search



Minimax Search : Back-up Minimax Values



Minimax Search: Decision



Approach: Follow tree to each terminal node and back up minimax value.

Note: This is just a generalization of the And-Or Tree Search and returns first action of the conditional plan.

function MINIMAX-SEARCH(*game*, *state*) **returns** *an action*
  player $\leftarrow$ *game*.TO-MOVE(*state*)
  value, *move* $\leftarrow$ MAX-VALUE(*game*, *state*)
  return *move*

function MAX-VALUE(*game*, *state*) **returns** a (*utility*, *move*) pair
  if *game*.IS-TERMINAL(*state*) **then return** *game*.UTILITY(*state*, *player*), *null*
  $v \leftarrow -\infty$
  for each a **in** *game*.ACTIONS(*state*) **do**
    v2, *a2* $\leftarrow$ MIN-VALUE(*game*, *game*.RESULT(*state*, *a*))
    if $v2 > v$ **then**
      v, *move* $\leftarrow$ *v2*, *a*
  return v, *move*

Represents OR Search

function MIN-VALUE(*game*, *state*) **returns** a (*utility*, *move*) pair
  if *game*.IS-TERMINAL(*state*) **then return** *game*.UTILITY(*state*, *player*), *null*
  $v \leftarrow +\infty$
  for each a **in** *game*.ACTIONS(*state*) **do**
    v2, *a2* $\leftarrow$ MAX-VALUE(*game*, *game*.RESULT(*state*, *a*))
    if $v2 < v$ **then**
      v, *move* $\leftarrow$ *v2*, *a*
  return v, *move*

Represents AND Search

Issue: Game Tree Size

- This traverses the complete game tree using DFS!

Time complexity: $O(b^m)$

- Only feasible for very simple games!
- Example: Tic-tac-toe

  $b = 9, m = 9 \rightarrow O(9^9) = O(387{,}420{,}489)$

  b decreases from 9 to 8, 7, …

  $\rightarrow$ actually less than $O(9!) = O(362{,}880)$

- We need to reduce the search space! -> Pruning

Alpha-Beta Pruning

- **Idea**: Do not search parts of the tree if they do not make a difference to the outcome.
- **Observations**:
  - $\min(3, x, y)$ can never be more than 3
  - $\max(5, \min(3, x, y, \dots))$ does not depend on the values of x or y.
  - Minimax search applies alternating min and max.
- **Approach**: maintain for each state bounds for the minimax value $[alpha, beta]$ and prune subtrees that cannot be part of the solution.
  - Alpha is used by Max and means "$Minimax(s)$ is at least."
  - Beta is used by Min and means "$Minimax(s)$ is at most."

```
function ALPHA-BETA-SEARCH(game, state) returns an action
  player ← game.TO-MOVE(state)
  value, move ← MAX-VALUE(game, state, −∞, +∞)
  return move

function MAX-VALUE(game, state, α, β) returns a (utility, move) pair
  if game.IS-TERMINAL(state) then return game.UTILITY(state, player), null
  v ← −∞
  for each a in game.ACTIONS(state) do
    v2, a2 ← MIN-VALUE(game, game.RESULT(state, a), α, β)
    if v2 > v then
      v, move ← v2, a
      α ← MAX(α, v)
    if v ≥ β then return v, move
  return v, move

function MIN-VALUE(game, state, α, β) returns a (utility, move) pair
  if game.IS-TERMINAL(state) then return game.UTILITY(state, player), null
  v ← +∞
  for each a in game.ACTIONS(state) do
    v2, a2 ← MAX-VALUE(game, game.RESULT(state, a), α, β)
    if v2 < v then
      v, move ← v2, a
      β ← MIN(β, v)
    if v ≤ α then return v, move
  return v, move
```

=minimax search + pruning.

Note: Pruning can be made more effective by **move ordering**: Check known good moves first to get a good bound early.
Optimal decision algorithms still scale poorly!

Heuristic Alpha-Beta Tree Search

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- **Find optimal decisions**: Minimax search and Alpha-Beta pruning where **each player plays optimal** to the end of the game.

Heuristic Methods
(state space is too large)

- **Heuristic Alpha-Beta Tree Search**:
  a. Cut off search tree and use heuristic for utility.
  b. Forward Pruning: ignore poor moves.
- **Monte Carlo Tree search**: Estimate utility of a state by simulating complete games and average the utility.

Idea: Cutting off search

Stop search in a state before the terminal node is reached. Use a heuristic evaluation function $Eval(s)$ to approximate the utility for that state.

Properties of the evaluation function:

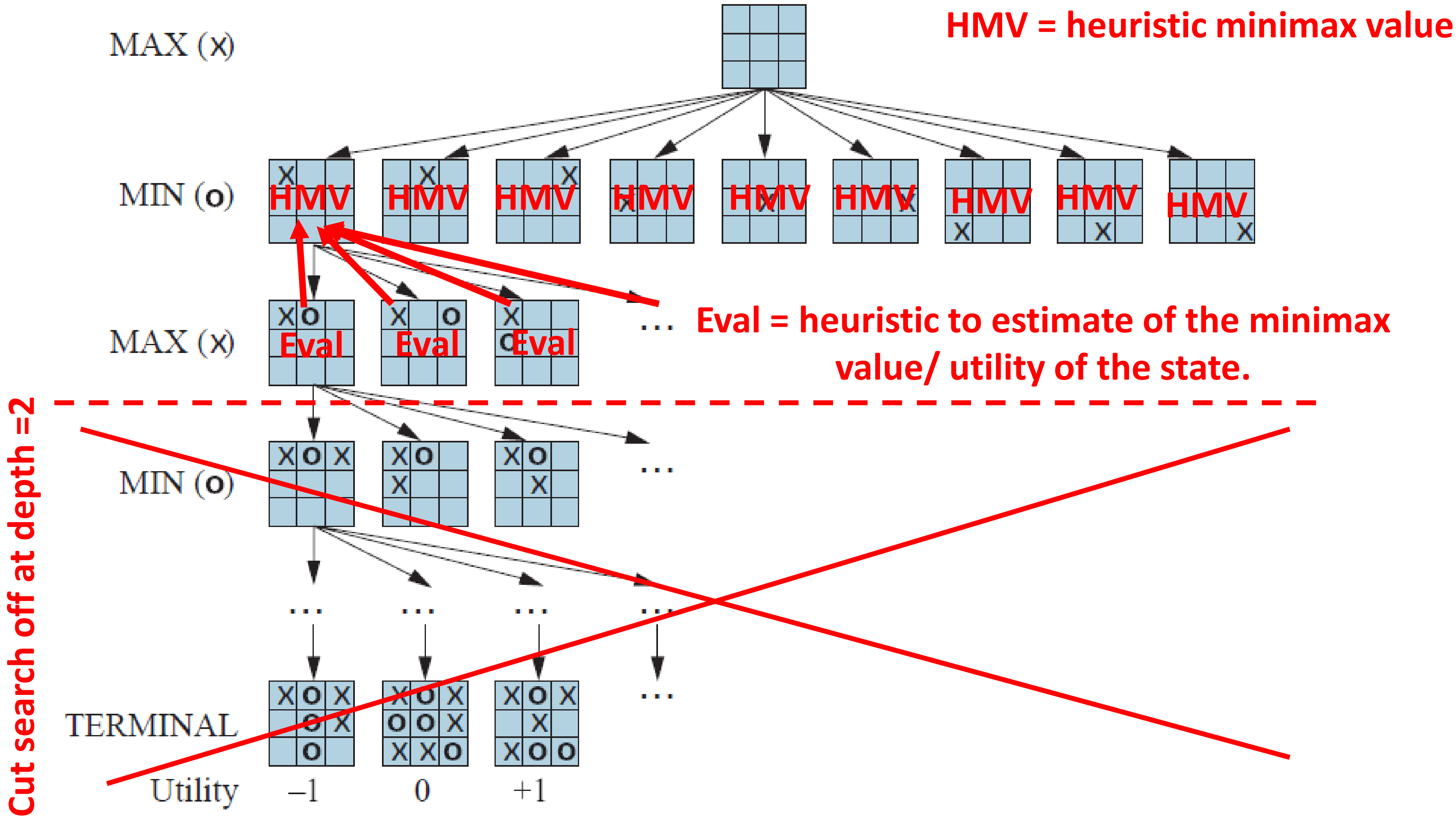
- Fast to compute.
- $Eval(s) \in [Utility(loss), Utility(win)]$
- Correlated with the actual chance of winning (e.g., using features of the state).

Example: A weighted linear function

$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \cdots + w_n f_n(s)$$

where f_i is a feature of the state.

Heuristic Alpha-Beta Tree Search: Cutting off search



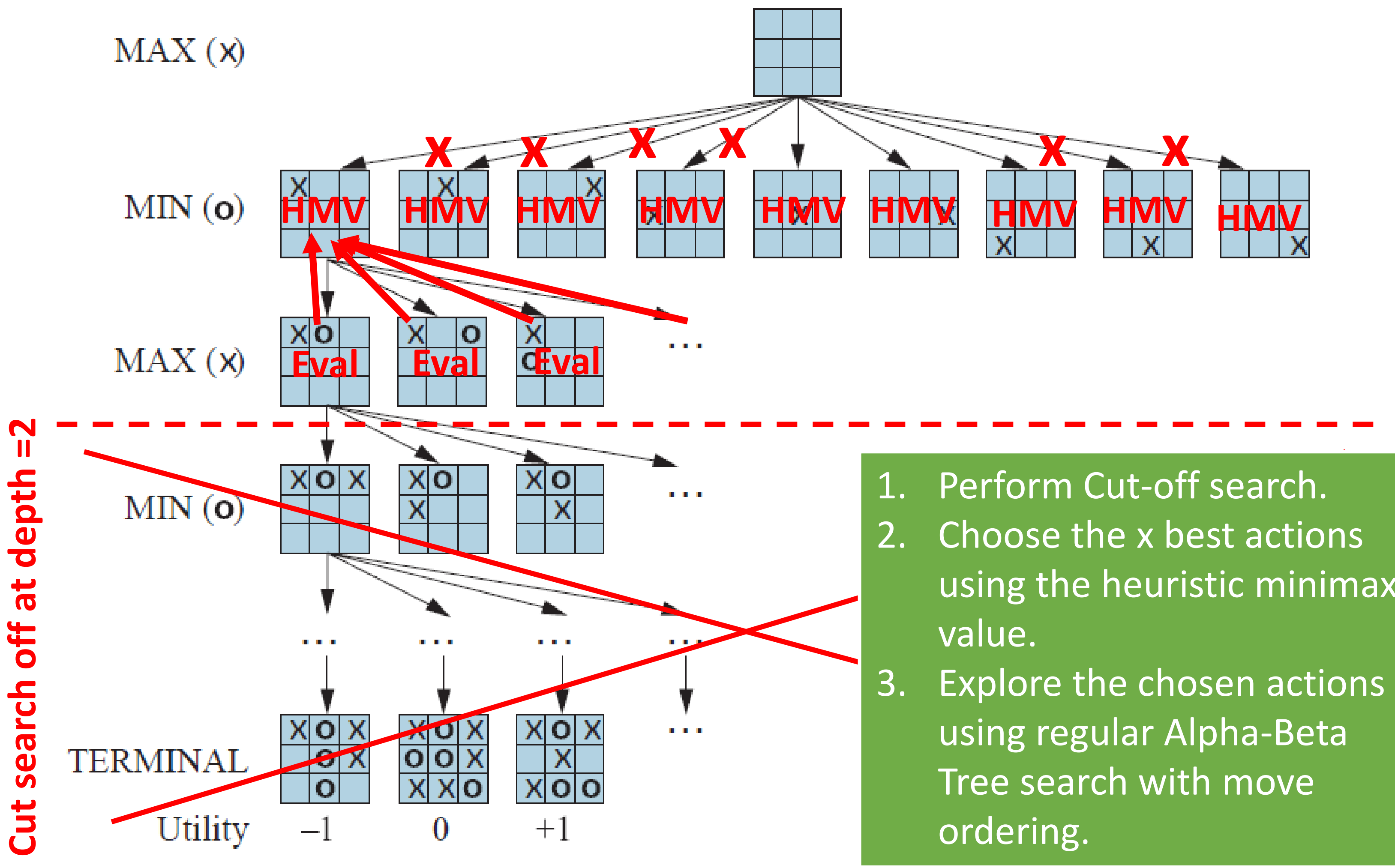
Idea: Forward pruning

Prune moves that appear poor. Poor can be evaluated in several ways:

- Low evaluation value after shallow search.
- Past experience.

Issue: May prune important moves.

Heuristic Alpha-Beta Tree Search: Forward Pruning Example



1. Perform Cut-off search.
2. Choose the x best actions using the heuristic minimax value.
3. Explore the chosen actions using regular Alpha-Beta Tree search with move ordering.

Monte Carlo Tree Search (MCTS)

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Idea

- **Approximate** $\boldsymbol{Eval(s)}$ as the average utility of a number of simulation runs to the terminal state (called playouts).
- **Playout policy**: How to choose moves during the simulation runs? Example policies:
  - Random.
  - Heuristics for good moves developed by experts.
  - Learn good moves from self-play (e.g., with deep neural networks).

- Typically used for problems with
  - High branching factor (many possible moves).
  - Unknown or hard to define good evaluation functions.

- **Questions**:
  - How many simulations/playouts should we run each time?
  - From what state should the playouts start?

Pure Monte Carlo Search

- Method
    1. Simulate N playouts from **current state**.
    2. Select the move with the highest win percentage.

- Converges to optimal play for stochastic games as N increases.
- **Do as many playouts as you can** given the available time.

Selection Policy

- Select the starting state for playouts to **focus** on important parts of the game tree. It is a tradeoff between:

  a) **Exploration**: search from states that have few playouts.

  b) **Exploitation**: more playouts for states that have done well to get more accurate estimates.

Selection using Upper Confidence Bounds applied to Trees (UCT)

Tradeoff constant ($\approx \sqrt{2}$) can be optimizes using experiments

$$UCB1(n) = \frac{U(n)}{N(n)} + C\sqrt{\frac{\log(N(Parent(n)))}{N(n)}}$$

Average utility (=exploitation)

High for nodes with few playouts (=exploration)

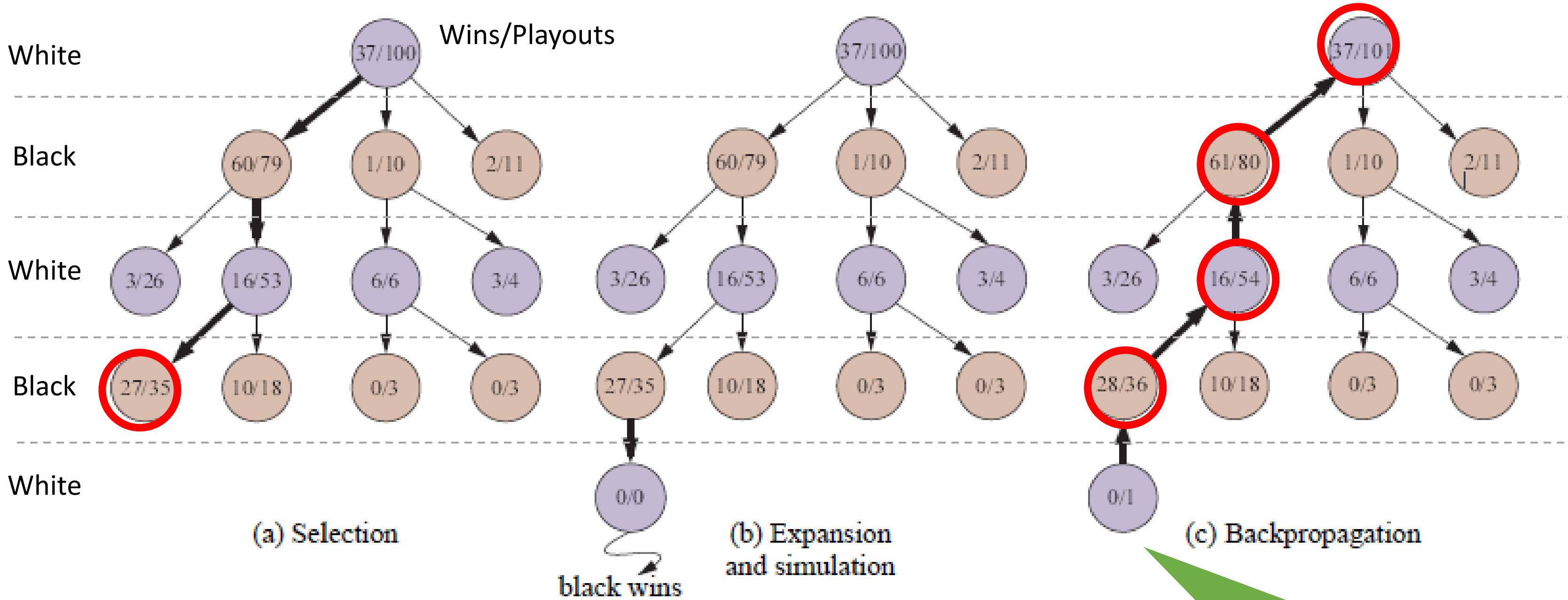
$U(n)$ ... total utility of all playouts going through node n
$N(n)$ ... number of playouts through n

Policy: Select leaf with highest UCB1 score.

function MONTE-CARLO-TREE-SEARCH(*state*) **returns** *an action*
  tree ← NODE(*state*)
  while IS-TIME-REMAINING() **do**
    leaf ← SELECT(*tree*)
    child ← EXPAND(*leaf*)
    result ← SIMULATE(*child*)
    BACK-PROPAGATE(*result*, *child*)
  return the move in ACTIONS(*state*) whose node has highest number of playouts

Highest UCB1 score

UCB1 selection favors win percentage more and more.



(a) Selection

(b) Expansion and simulation

(c) Backpropagation

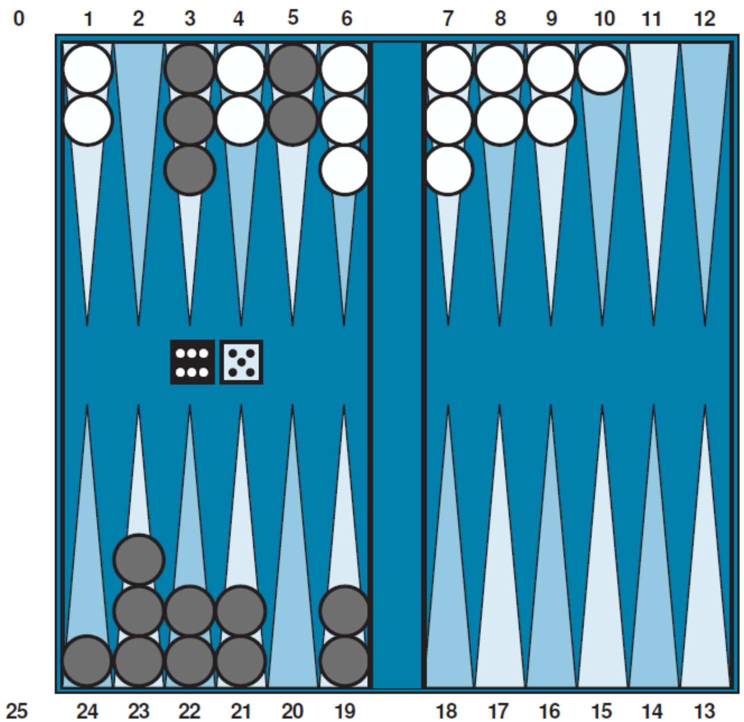
Note: the simulation path is not recorded!

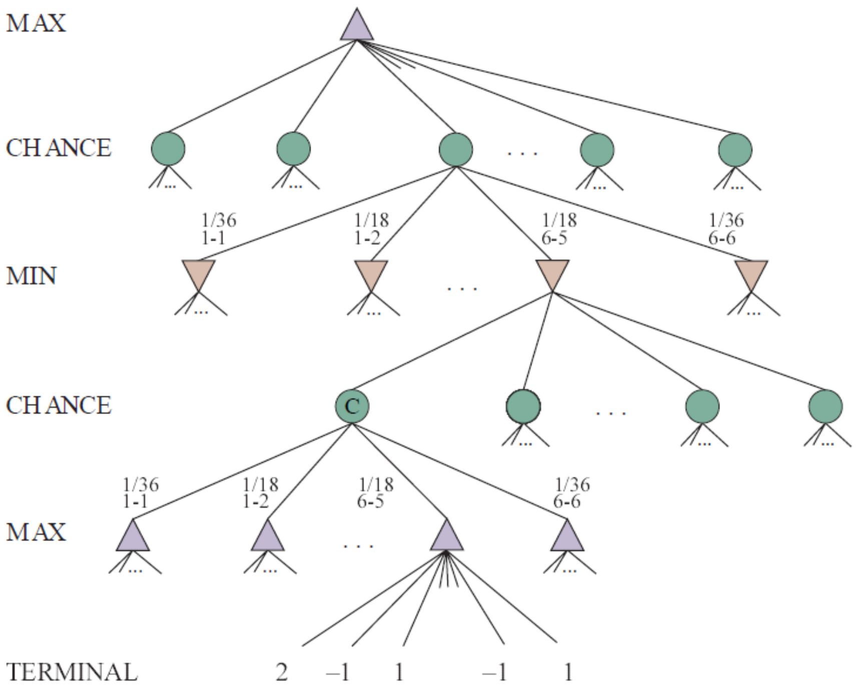
Stochastic Games

Games With Random Events

Stochastic Games

- Game includes a "random action" r (e.g., dice, dealt cards)
- Add **chance nodes** that calculate the expected value.





Backgammon

Expectiminimax

- Game includes a "random action" r (e.g., dice, dealt cards)
- Add **chance nodes** that calculate the expected value.

$$Expectiminimax(s) =$$

$$\begin{cases} Utility(s) & \text{if } terminal(s) \\ \max_{a \in Actions(s)} Expectiminimax\big(Result(s,a)\big) & \text{if } move = Max \\ \min_{a \in Actions(s)} Expectiminimax\big(Result(s,a)\big) & \text{if } move = Min \\ \boxed{\sum_r P(r) Expectiminimax\big(Result(s,r)\big) \qquad \text{if } move = Chance} \end{cases}$$

- Options:
  - Use Minimax algorithm. Issue: Search tree size explodes if the number of "random actions" is large. Think of drawing cards!
  - Approximate Expectiminimax with an evaluation function.
  - Perform Monte Carlo Tree Search.

Conclusion

Nondeterministic actions:

- The opponent is seen as part of an environment with nondeterministic actions. Non-determinism is the result of the unknown moves by the opponent.

Optimal decisions:

- Minimax search and Alpha-Beta pruning where each player plays optimal to the end of the game.
- Choice nodes and Expectiminimax for stochastic games.

Scale only for tiny problems!

Heuristic Alpha-Beta Tree Search:

- Cut off search tree and use heuristic evaluation function for utility (based on state features).
- Forward Pruning: ignore poor moves.

Monte Carlo Tree search:

- Simulate complete games and average the results.
- Learn playout policy using self-play and deep learning.
- Use modified UCB1 scores.

State of the Art