Population Prediction

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Dataset: International Database (IDB)

Question Of Interest: Predict the population of earth in 2122.

1 Data Understanding

1.1 Data Description

<class 'pandas.core.frame.DataFrame'>
RangeIndex: 34237 entries, 0 to 34236
Data columns (total 99 columns):

#	Column	Non-Null Count	Dtype
0	#YR	34237 non-null	int64
1	TFR	26163 non-null	float64
2	SRB	26163 non-null	float64
3	RNI	26172 non-null	float64
4	POP95_99	26171 non-null	float64
5	POP90_94	26171 non-null	float64
6	POP85_89	26171 non-null	float64
7	POP80_84	26171 non-null	float64
8	POP75_79	26171 non-null	float64
9	POP70_74	26171 non-null	float64
10	POP65_69	26171 non-null	float64
11	POP60_64	26171 non-null	float64
12	POP5_9	26171 non-null	float64
13	P0P55_59	26171 non-null	float64

```
14 POP50_54
               26171 non-null
                                float64
15
    POP45_49
               26171 non-null
                                float64
    POP40_44
               26171 non-null
                                float64
16
    POP35_39
               26171 non-null
17
                                float64
18
    POP30 34
               26171 non-null
                                float64
    P0P25_29
19
               26171 non-null
                                float64
20
    POP20 24
               26171 non-null
                                float64
21
    POP15_19
               26171 non-null
                                float64
22
    POP10_14
               26171 non-null
                                float64
23
    POP100_
               26171 non-null
                                float64
24
    POP0_4
               26171 non-null
                                float64
    POP
               34237 non-null
                                int64
25
26
               26172 non-null
                                float64
    NMR
27
    NAME
               34237 non-null
                                object
28
    MR1_4
               26163 non-null
                                float64
               26163 non-null
                                float64
29
    MRO_4
30
    MPOP95_99
               26171 non-null
                                float64
    MPOP90_94
               26171 non-null
                                float64
31
    MPOP85_89
               26171 non-null
                                float64
32
33
    MPOP80 84
               26171 non-null
                                float64
34
    MPOP75_79
               26171 non-null
                                float64
35
    MPOP70 74
               26171 non-null
                                float64
36
    MPOP65_69
               26171 non-null
                                float64
               26171 non-null
37
    MPOP60_64
                                float64
38
    MPOP5_9
               26171 non-null
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39
    MPOP55_59
               26171 non-null
                                float64
               26171 non-null
40
    MPOP50_54
                                float64
41
    MPOP45_49
               26171 non-null
                                float64
               26171 non-null
42
    MPOP40_44
                                float64
43
    MPOP35_39
               26171 non-null
                                float64
    MPOP30_34
               26171 non-null
                                float64
44
45
    MPOP25_29
               26171 non-null
                                float64
46
    MPOP20_24
               26171 non-null
                                float64
    MPOP15_19
               26171 non-null
47
                                float64
    MPOP10 14
               26171 non-null
                                float64
48
49
    MPOP100
               26171 non-null
                                float64
50
    MPOPO 4
               26171 non-null
                                float64
51
    MPOP
               26171 non-null
                                float64
                                float64
52
    MMR1_4
               26163 non-null
53
    MMRO 4
               26163 non-null
                                float64
54
    IMR_M
               26163 non-null
                                float64
               26163 non-null
                                float64
55
    IMR_F
56
    IMR
               26163 non-null
                                float64
57
    GRR
               26163 non-null
                                float64
58
               26172 non-null
                                float64
    GR
59
    FP0P95_99
               26171 non-null
                                float64
60
    FPOP90_94
               26171 non-null
                                float64
    FP0P85_89
               26171 non-null
                                float64
61
```

```
63 FPOP75_79
                    26171 non-null float64
     64
        FPOP70_74
                    26171 non-null float64
        FP0P65_69
                    26171 non-null float64
     65
        FPOP60 64
     66
                    26171 non-null float64
         FP0P5_9
                    26171 non-null float64
     67
         FP0P55 59
                    26171 non-null float64
                    26171 non-null float64
     69
        FP0P50_54
                    26171 non-null float64
     70 FP0P45_49
     71
        FP0P40_44
                    26171 non-null float64
     72 FPOP35_39
                    26171 non-null float64
     73 FPOP30_34
                    26171 non-null float64
     74
                    26171 non-null float64
        FP0P25_29
     75
        FP0P20_24
                    26171 non-null float64
                    26171 non-null float64
     76 FPOP15_19
        FPOP10_14
                    26171 non-null float64
     78
        FPOP100_
                    26171 non-null float64
     79
        FPOPO_4
                    26171 non-null float64
     80
        FP0P
                    26171 non-null float64
                    26163 non-null float64
     81
        FMR1 4
                    26163 non-null float64
     82
         FMRO 4
     83
         GENC
                    34086 non-null object
     84
        FIPS
                    34237 non-null object
        EO_M
                    26163 non-null float64
     85
     86
        EO_F
                    26163 non-null float64
     87
                    26163 non-null float64
         EΟ
         CDR
                    26172 non-null float64
     88
     89
         CBR
                    26172 non-null float64
                    26173 non-null float64
     90
         ASFR45_49
         ASFR40_44
                    26173 non-null float64
                    26173 non-null float64
     92
         ASFR35_39
     93
         ASFR30_34
                    26173 non-null float64
     94
         ASFR25_29
                    26173 non-null float64
     95
        ASFR20_24
                   26173 non-null float64
     96
         ASFR15 19
                    26173 non-null float64
     97
         AREA KM2
                    34237 non-null int64
     98 POP DENS
                    34237 non-null float64
    dtypes: float64(93), int64(3), object(3)
    memory usage: 25.9+ MB
[2]: import seaborn as sns
    import matplotlib.pyplot as plt
    %matplotlib inline
     # Make year column easier to understand
    df.rename(columns={'#YR':'YEAR'}, inplace=True)
```

62 FPOP80_84

26171 non-null float64

```
# Remove every column except for year & population
for col in df.columns.values:
   if col != 'YEAR' and col != 'POP':
        df.drop(col, axis=1, inplace=True)

df.describe()
```

```
[2]:
                   YEAR
                                  POP
    count 34237.000000 3.423700e+04
    mean
            2024.935158 3.306017e+07
    std
              43.571971 1.253471e+08
    min
            1950.000000 2.028000e+03
    25%
            1987.000000 4.304230e+05
    50%
            2025.000000 4.831844e+06
    75%
            2063.000000 2.070922e+07
            2100.000000 1.647894e+09
    max
```

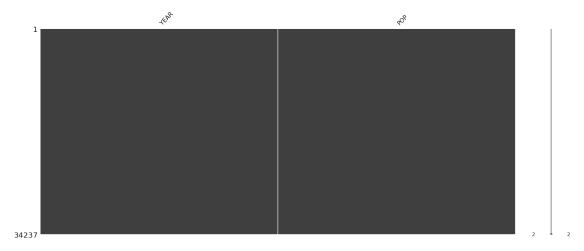
1.2 Data Quality

```
[3]: import missingno as mn

mn.matrix(df)

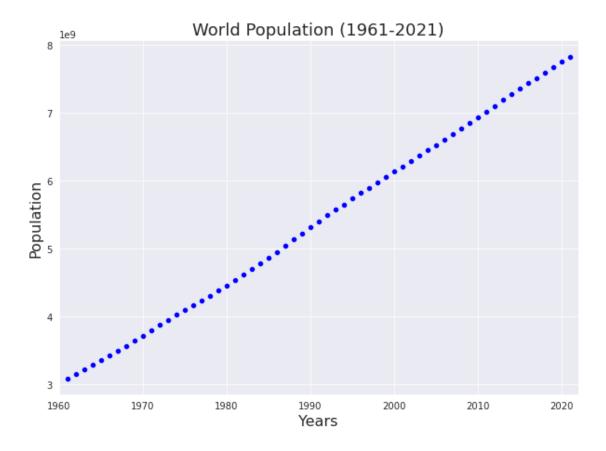
# Count unique values in column 'gameId' of the dataframe
print('Number of unique values in column "YEAR" : ', df['YEAR'].nunique())
```

Number of unique values in column "YEAR" : 151



1.3 Clearning the Dataset

```
[4]: # Group by year & get sum
     df_yr = df.groupby(by='YEAR')
     df_yr = df_yr['POP'].sum()
     # Create a new dataframe with new data (1961 - 2100)
     pop_sum = []
     for i in range(1961, 2022):
        pop_sum.append(df_yr[i])
     df_pop = pd.DataFrame({'YEAR': list(range(1961, 2022)), 'POP': pop_sum})
     df = df_pop
     print(f'\n--> Current Population (2021): {df["POP"][60]:,d}\n')
     df.tail(5)
    --> Current Population (2021): 7,831,718,605
[4]:
                     POP
        YEAR
     56 2017 7516769535
     57 2018 7597066210
    58 2019 7676686052
     59 2020 7756873419
     60 2021 7831718605
[5]: sns.set_style("darkgrid")
    plt.subplots(figsize=(10,7))
     ax = sns.scatterplot(data=df, x='YEAR', y='POP', color='blue')
     ax.set_xlabel('Years', fontsize=16)
     ax.set_ylabel('Population', fontsize=16)
     ax.set_title('World Population (1961-2021)', fontsize=18)
     plt.xlim(1960, 2022)
    plt.show()
```



```
[6]: # Define X & Y
if 'POP' in df_pop:
    y = df_pop['POP'].values
    del df_pop['POP']
    X = df_pop.to_numpy()
```

2 Modeling

Derived the formula for calculating the optimal values of the regression weights:

$$w = (X^T X)^{-1} X^T y$$

where X is the matrix of values with a bias column of ones appended onto it. For the population dataset one could construct this X matrix by stacking a column of ones onto the $\mathtt{df_pop.YEAR}$ matrix.

$$X = \begin{bmatrix} \vdots & 1 \\ \dots & \text{ds.data} & \dots & \vdots \\ \vdots & 1 \end{bmatrix}$$

```
# Calculate optimal values of the regression weights
w = np.linalg.inv(X.T @ X) @ X.T @ y
print("\n++++++++++ WEIGHTS ++++++++++++++++\n", pd.DataFrame(data=w))
diff = np.round(( (y - (abs(np.dot(X,w) - y))) / y ) * 100, 2)
print("\n========\n", pd.
 →DataFrame(data=diff))
+++++++++++ WEIGHTS ++++++++++++++
0 -1.550917e+11
1 8.060884e+07
======= TARGET PERCENT ACCURACY =========
0
   96.70
  97.54
1
  97.93
  98.25
   98.63
. .
56 99.73
57 99.74
58 99.75
59 99.76
60 99.84
[61 rows x 1 columns]
```

[7]: # Create a matrix full of ones & stack 2 matrices horizontally

X = np.hstack((np.ones((len(X), 1)), X))

To predict the output from our model, \hat{y} , from w and X we need to use $\hat{y} = w^T X^T$, for row vector \hat{y}

```
[8]: yHat_np = w.T @ X.T  # Shape : (1,61)
yHat_np = yHat_np.ravel() # Shape : (61,)

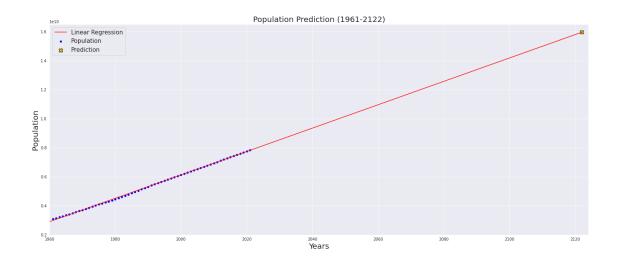
MSE_np = (np.square(y - yHat_np)).mean()
```

```
print(f'MSE: {round(MSE_np):,d}')
```

MSE: 1,150,330,787,912,923

```
[9]: X_new = np.array([[0], [2122]])
     X_test = np.c_[np.ones((len(X_new), 1)), X_new]
     y_test = X_test.dot(w)
     print(f'\n--> \sim Population (2122): \{round(y_test[1]):,d\}\n')
     sns.set style("darkgrid")
     plt.subplots(figsize=(25,10))
     plt.plot(X_new, y_test, "r-", color='red')
     sns.scatterplot(data=df, x='YEAR', y=y, color='blue')
     sns.scatterplot(x=X_new[1], y=y_test[1], s=100, marker="X", linewidth=1,__
     ⇔edgecolor='k', color='gold')
     plt.xlabel('Years', fontsize=20)
     plt.ylabel('Population', fontsize=20)
     plt.title('Population Prediction (1961-2122)', fontsize=20)
     plt.axis([1960, 2124, 2000000000, 16500000000])
     plt.legend(["Linear Regression", "Population", "Prediction"], prop={'size': 15})
     plt.show()
```

--> ~Population (2122): 15,960,289,855



3 Comparing Performance

```
[10]: from sklearn.metrics import mean squared error
      from sklearn.linear_model import LinearRegression
      reg = LinearRegression().fit(X, y)
      MSE_sk = mean_squared_error(y, reg.predict(X))
      y_testsk = reg.predict(X_test)
      print("******* Linear Equation *******")
      print("[Numpy] \th =", round(w[0],3), "* x + (" + str(round(w[1],5)) + ")")
      print("[Sklearn]\th =", round(reg.intercept_,3), "* x + (" + str(round(reg.
      \rightarrowcoef_[1],5)) + ")\n")
      print("****** Mean Squared Error ******")
      print(f'[Numpy] \tMSE: {round(MSE_np):,d}')
      print(f'[Sklearn]\tMSE: {round(MSE_sk):,d}\n')
      print("****** Population Prediction - 2122 *******")
      print(f'[Numpy] \t {round(y_test[1]):,d}')
      print(f'[Sklearn]\t {round(y_testsk[1]):,d}\n')
     ***** Linear Equation ******
     [Numpy]
                     h = -155091670931.498 * x + (80608841.08674)
     [Sklearn]
                     h = -155091670931.687 * x + (80608841.08683)
     ***** Mean Squared Error *****
                     MSE: 1,150,330,787,912,923
     [Numpy]
     [Sklearn]
                     MSE: 1,150,330,787,912,744
     ***** Population Prediction - 2122 ******
     [Numpy]
                      15,960,289,855
     [Sklearn]
                      15,960,289,855
```

 $\label{lem:constraint} {\bf References} \ \ {\bf Census.} \ \ \ International \ \ Database \ \ (IDB). \ \ https://www.census.gov/datatools/demo/idb/#/country?COUNTRY_YEAR=2022&COUNTRY_YR_ANIM=2022 \ \ \ (Accessed \ 01-22-2022)$