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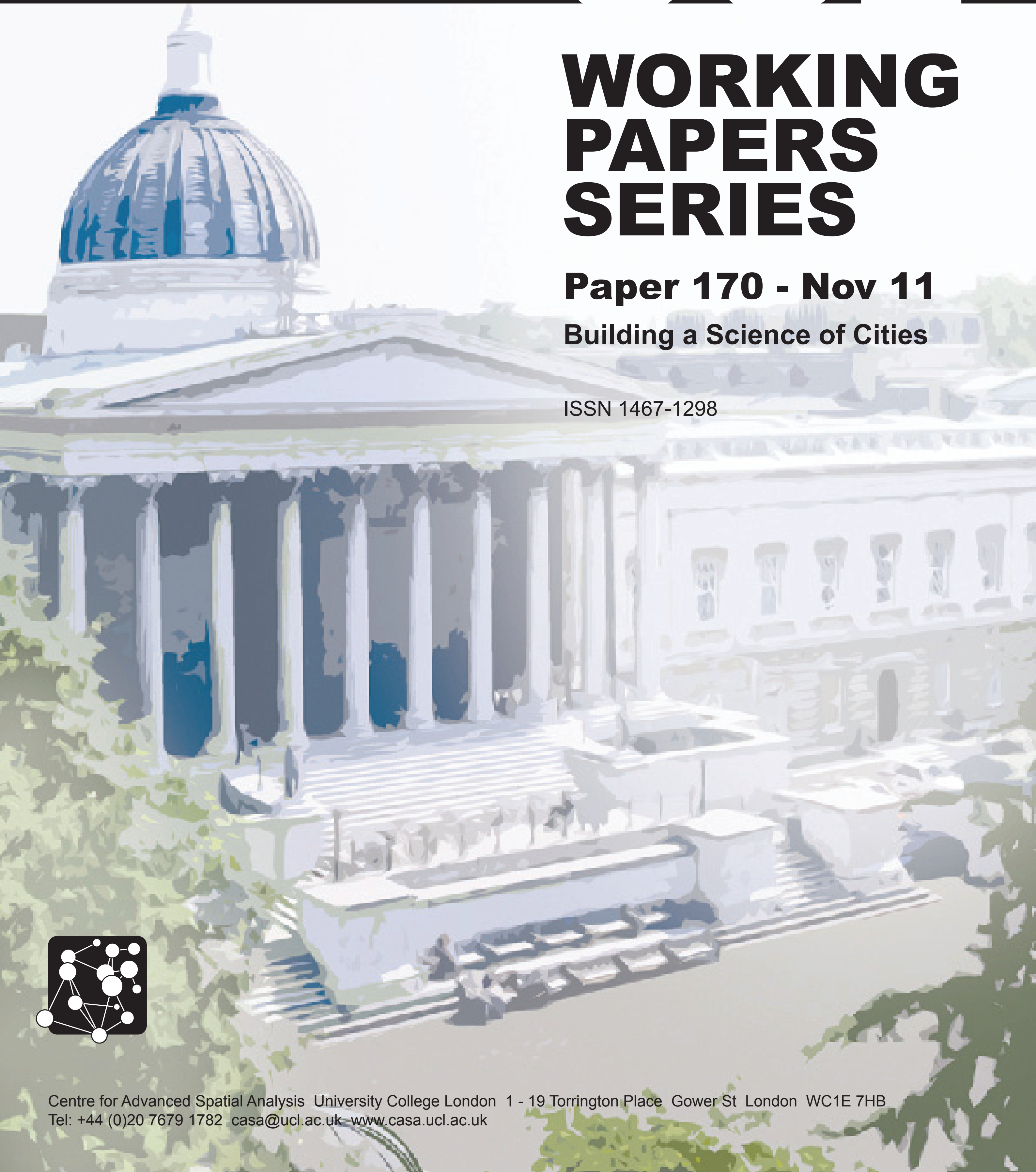

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Building a Science of Cities

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Building a Science of Cities

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Abstract

Our understanding of cities is being transformed by new approaches from the complexity sciences (Batty, 2005). Here we review progress, sketching the background beginning with the systems approach which treated systems as being organised from the top down to that which now dominates where systems are treated as evolving from the bottom up. The switch in thinking we describe is best pictured in the transition from thinking of 'cities as machines' to 'cities as organisms'. We first review developments in the dynamics of cities where the notion of equilibrium has been replaced by a veritable potpourri of different types such as chaos, catastrophes, and bifurcations. We then look at patterns and processes that give rise to morphologies that illustrate fractal patterns and self-similarity. We follow this with ideas about networks and interactions that sustain cities through their transport and then we show how such processes of movement and mobility can give rise to the diffusion and segregation of different spatial activities. In all these developments, ideas about scaling that relate size, shape and scale in space and time are evident and we thus sketch three scaling laws based on rank-size, allometry and gravitation that are central to our synthesis of how spatial processes give rise to physical morphologies. We conclude with notions about how these ideas are being embedded into models that have potential applications to inform policy.

Key Words

COMPLEXITY SCALING EMERGENCE FRACTALS NETWORKS EVOLUTION RANK-SIZE ALLOMETRY GRAVITATION

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The Focus

A Science of Cities has taken a long time coming but there is now considerable momentum in developing formal ideas about how cities are ordered and structured which are part of the rapidly expanding sciences of complexity (Batty, 2005, 2009). Half a century or more ago, cities were first formally considered as 'systems' which were defined as distinct collections of interacting entities, usually in equilibrium, but with explicit functions that could enable their control often in analogy to processes of their planning and management (Berry, 1964). These conceptions treated cities as organised from the top down, distinct from their wider environment which was assumed largely benign, with their functioning dependent on restoring their equilibrium through various negative feedbacks of which planning was central (Chadwick, 1971). As soon as this model was articulated, it was found wanting. Cities do not exist in benign environments and cannot be easily closed from the wider world, they do not automatically return to equilibrium for they are forever changing, indeed they are far-from-equilibrium. Nor are they centrally ordered but evolve mainly from the bottom up as the products of millions of individual and group decisions with only occasional top down centralised action. In short, cities are more like biological than mechanical systems and the rise of the sciences of complexity which has changed the direction of systems theory from top down to bottom up is one that treats such systems as open, based more on the product of evolutionary processes than one of grand design (Portugali, 2000). During the last half century, the image of a city as a 'machine' has been replaced by that of 'organism'.

This developing science has not abandoned more formal approaches to understanding cities which have come from urban economics, regional science, social physics and transportation planning for the new science builds on this edifice while changing its emphasis (Fujita, Krugman, and Venables, 1999). In essence, what is being forged is a much more comprehensive set of structures that allow us to understand the many perspectives on the city that reflect its diversity and plurality. Moreover, the complexity sciences are sufficiently open to embrace many different approaches for part of the very definition of complexity is the idea that no one approach is predominant (Miller and Page, 2007). The problem with a systems theory of cities is that it tends to view systems as being well-behaved in the sense that external shocks to the system tend to work themselves out restoring the previous equilibrium or at least evolving to something that is close to the pre-existing state. What has been realised in the last 50 years, is that this notion of systems freely adjusting to changed conditions is no longer valid, in fact it never was. Cities admit innovation, indeed they are the crucibles of innovation, they generate surprise, they display catastrophes.

To begin our discussion however, we will focus on this past imagery of equilibrium and dynamics for the processes that define how cities function and ultimately give rise to the spatial and physical patterns that we observe most superficially, belie a complexity of spatial behaviours that only now are we beginning to recognise and articulate. We follow this discussion with ideas about how complex systems (and cities) hold their elements together through interactions that flow on networks and through notions about how individual populations grow and change to generate the kinds of structures that compose the contemporary city. Our focus on spatial dynamics and behaviours leads to one of the key concepts in this new theory of cities, one which we refer to as 'emergence' and which underpins the idea that multiple decisions from the bottom up often give rise to unexpected, innovative and surprising behaviours.

Although our discussion will be quite general while being liberally illustrated with ways of articulating the various components of this science, we will attempt to stitch these ideas together through the key concept of ‘scaling’. Scaling pertains to how the elements as well as the entire system in question – the city – change in shape and size as their elements and their wholes grow and change, and we will fashion some basic relationships which are being used to reflect the way such a science hangs together. Scaling is the skeleton around which we can build such a science, and it provides us with the focus for new forms of simulation and visualisation (Batty, 2010), models consistent with such bottom up thinking. We will conclude with some brief speculations on what this science holds for policy making and for developing cities which enable a better quality of life than our current cities offer.

Equilibrium and Dynamics

We could all be forgiven for thinking that cities are in equilibrium for the built environment that conditions our immediate responses, changes only slowly in comparison to the functions that take place in such environments. Cities can be pictured as if they are in equilibrium, the focus being initially on representing and simulating static urban structures such as population density profiles, cross-sectional patterns of movement, and the configuration and location of different land use types often as concentric rings of use around the origin of settlement, invariably the central business district (Alonso, 1964). Insofar as change to these structures has been articulated, this is smooth change but as soon as scholars first became aware of the nature of actual change which was often discontinuous and lumpy, the need for widening the framework to embrace all kinds of non-smooth dynamics became obvious. In the 1970s, ideas about how cities developed in discrete jumps, catastrophes in development reminiscent of house price (and building) booms and crashes, and spatial behaviours that was revealed to be chaotic – qualitatively different from near identical initial conditions, came to dominate our thinking about the dynamics of change (Wilson, 1981).

Urban simulation models initially predicated on forecasting development at a cross section in time were the first to adjust to deal with such dynamics but these developments were painful for the wider theory of how change takes place in cities was hardly developed. Forrester’s (1969) exposition *Urban Dynamics* was an early statement that urban change had a quality which was often counterintuitive, built around notions of positive feedback giving rise to profiles leading to exponential growth and decline, capacitated logistic growth and oscillations as systems overshot and undershot their assumed equilibria (Batty, 1971). It was widely regarded that an acceptable science of cities must embrace dynamics directly enabling simulations to be made of widely differing growth scenarios. New developments in the mathematics of dynamics based on catastrophe, bifurcation, and deterministic chaos were quickly embraced by rudimentary complexity theory and have since become integral to the definition of every type of complex system (May 1976). We graph these kinds of dynamics for population change in Figure 1.

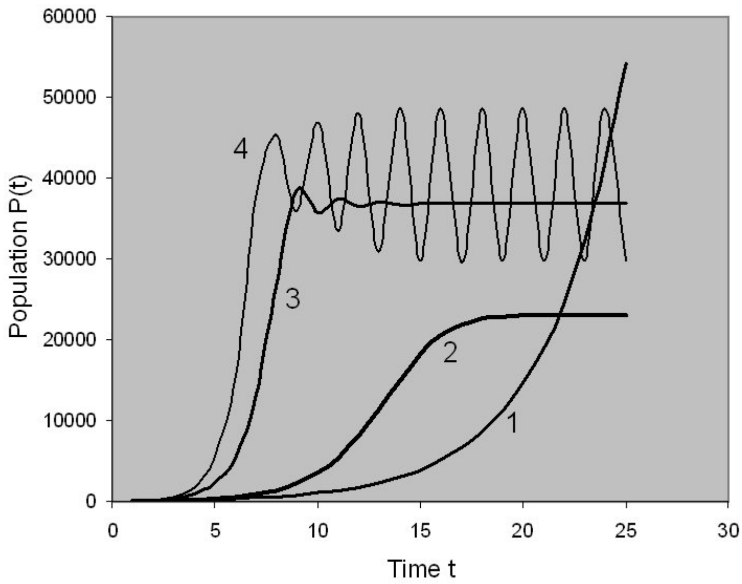

Figure 1: Varieties of Population Dynamics

Simulations from a generic model $P(t+1) = \varphi P(t) z^{\gamma} (\overline{P} - P(t))$ *of population growth. The parameters* φ , z *and* γ *control the positive and negative feedbacks that generate damped exponential growth, and* $\overline{P}$ *is the capacity of population growth. Curve 1 is the classic exponential growth model with* $\gamma = 0$, *Curves 2-4 pure logistic, then oscillating with damped exponential growth for different combinations of the parameter values.*

Patterns and Processes

The functioning of cities in space and time is based on multiple processes of spatial choice in which individuals and groups in the population locate with respect to one another and their wider activities in the form of land use types. These activities tend to be dominated by trade-offs between agglomeration economies and diseconomies which are often represented in terms of relative accessibilities between different locations. These trade-offs give rise to patterns of activity that reflect different levels of clustering and in turn these imply different density levels associated with different locations. For a long time, locational patterns were represented in rather coarse, abstract terms as density profiles around key hubs, as nested hierarchies of central places, and as patterns of accessibility reflecting more polycentric forms but with little sense of the morphological structures that they actually represented. This meant that urban researchers missed some significant signals that urban patterns manifest such as their self-similarity or spatial invariance across different scales, which in turn implies that the similar sorts of processes are operating across scales. Moreover patterns that repeat in modular-like form, such as those characteristic of central place theory (Christaller, 1933), generate hierarchies that can be handled using new types of geometry. In the 1980s, onto this canvas came ideas about how modular patterns in cities were structured and using ideas about self-similar processes of development, cities began to be interpreted as fractal structures (Batty and Longley, 1994). Figure 2 presents a potpourri of aggregate urban patterns that display such fractal structure with distinct hierarchical ordering that accords to the rank-size scaling that we introduce below.

This idea of using morphology as a signature to detect the different underlying processes at work in cities relates very strongly to notions about how individual spatial decisions determine how cities grow from the bottom up and how patterns repeat themselves at different spatial scales. Urban development rarely fills the entire space which defines the wider hinterland of a settlement or city and in this sense, it is regarded as space-filling in the same way that fractal forms fill space between the Euclidian (integer) dimensions. The whole paraphernalia of fractal geometry can thus be brought to bear on urban patterns and processes. A means for classifying cities using their relative densities and accessibilities, is through their fractal dimensions that determine the extent to which they fill the space that they occupy. Many of these ideas pertain to how cities develop in terms of compaction and sprawl and they find immediate expression in terms of the network structures that tie land uses together and provide the glue for achieving the sort of agglomeration economies and scaling effects that define cities in the first place.

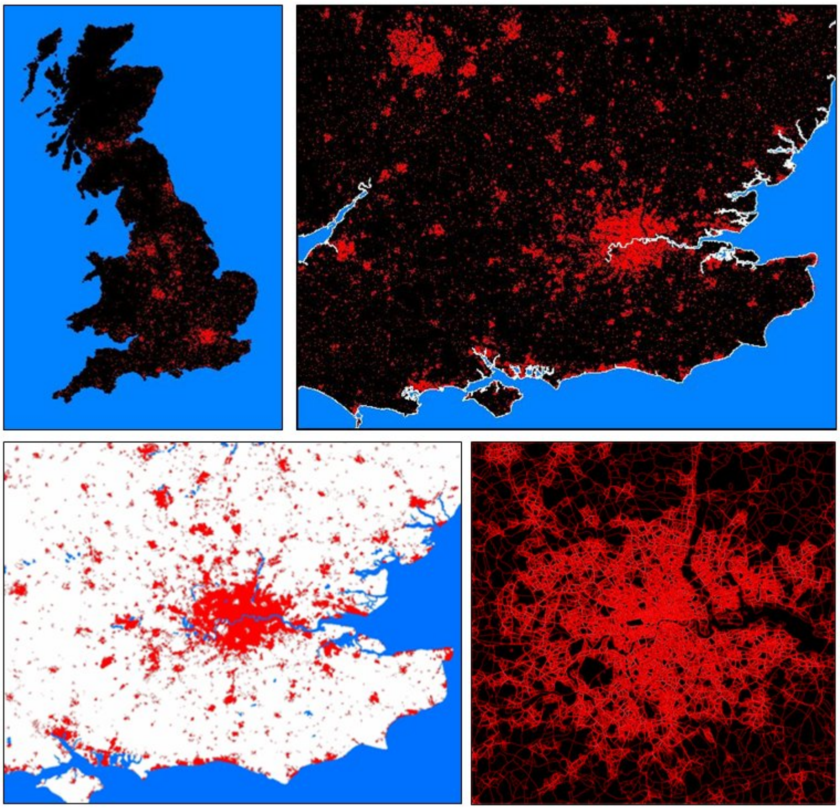

Figure 2: Self-Similar Urban Morphologies from Population, Remotely Sensed Imagery and Street Network Representations

Top left and right show the urban morphology of the UK and the South of England from 1991 gridded Population Census Data. Bottom left is an image taken from RS Data for 2000, and bottom right is from street network data for Greater London. Note the clusters on all scales that accord to the rank-size scaling that we discuss in a later section of the paper.

Interactions, Flows and Networks

The idea of interactions between different individuals rooted in time and space defines the nature of what a city is all about. Cities, as Glaeser (2011) and Jacobs (1961) before, have argued so persuasively, are about ‘connecting people’. The various processes that bring people together to produce and exchange goods and ideas that take place in cities define a multitude of networks that enable populations to deliver materials and information to support such endeavours. Physical and social networks tend to mutually reinforce one another as they develop. From an initial hub such as the origin of settlement, individuals are attracted usually in proportion to what already exists, that is, for an existing population P_i at location i, the growth of the population is proportional to this size λP_i where λ is the growth rate. Imagine that this is a hub or node in a network. Then the hub grows, assuming the rate of growth is greater than 1, by new links from other populations in the hinterland of the hub connecting to it, the number of such new links being in proportion to its size. Each of these other nodes which form the links attract links from other nodes in the same manner, with new nodes emerging randomly in the wider region. This is a model in which the rich get richer. It generates a cumulative causation and lies at the basis of how networks develop in many different domains. Barabasi (2003) calls it the ‘preferential attachment’ model and one of its striking features is that the size distribution of the nodes – which can be locations (cities), parts of cities, groups of individuals, institutions and so on for the model is generic – follows a scaling law. In short, the frequency of nodes of increasing size in terms of their links gets ever smaller in accordance with a power law, there being many small nodes and very few large ones. This is one of the basic signatures of this new science for it reflects the way competition determines size and the way resources are bid for in a competitive environment. It is not only applicable to networks but also to city sizes as well as the sizes of different locations within cities where it appears as Zipf’s Law (Zipf, 1949). We will return to this below when we pull together the key signatures of spatial complexity that define this new science.

How networks form in cities relates to the fractal patterns that we showed earlier with the physical imprint of these signatures forming the channels through which people and materials move. The flows on these networks also depend on the size of the locations between which links are established but usually these flows are mitigated by a deterrent effect of distance – the friction of distance as it is called – d_{ij} defined between any two locations i and j. A particularly long-standing form of this flow relation, first proposed to model movements in human space almost immediately after Newton published the laws of motion in his *Principia*, is the gravitational force defined as $T_{ij} \sim P_i P_j d_{ij}^{-2}$ where P_i, P_j are population masses and T_{ij} is the interaction. It is the so-called inverse square law used by Ravenstein to represent migration flows between cities and regions in Britain in the late 19^{th} century (Tobler, 1995). This is also a scaling law and from the 1950s, it has been used in many forms to model different kinds of traffic flow at a variety of scales. Indeed it lies at the basis of much operational land use and transportation modelling which still forms the basis of many applications of this science of cities (Batty, 1976).

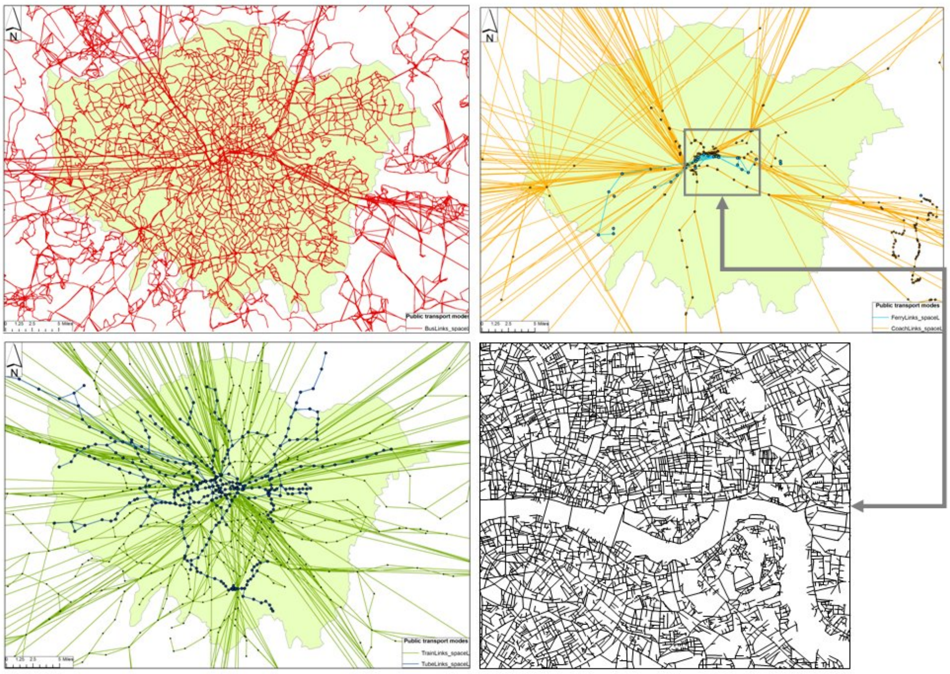

Figure 3: Coupled Transport Networks Generating a Convoluted Dynamics of Traffic

> *Top left shows bus routes (red) in London with the longer straight lines being the routes of long distance buses, top right shows intercity coaches (yellow) and ferries (blue), bottom left are long distance and overground rail (green) and tube (blue), and bottom right is a sample of the network in central London on which private car and taxi flows take place.*

Social networks in which space is merely implicit also characterise cities and the way their individuals and groups interact to trade, exchange, build friendships, exercise decisions, manage other groups and so on. Currently there is a great flurry of activity in modelling such networks and the emergence of a new form of network science where the focus is on detecting patterns in networks – clusters called small worlds, shortest links, weak ties, bridges between communities, and hierarchies – is becoming ever more central to our science. We show some of these networks in Figure 3 to give the reader a sense of what they portray. The big challenge is in coupling networks, in developing ways in which both material (energy) and ethereal (information) networks are coupled to one another, ways in which such networks are cascaded into each other, and the way processes spread and diffuse over such links (Newman, 2010). The cutting edge is figuring out how such networks interlink and interlock and how the patterns of morphology that are the physical manifestations of the social and economic processes that define the way cities work build on such network representations. The key to understanding how networks fracture and split, how economies of scale and innovations are realised through the way different networks relate, and the ways in which prosperity and the creation of wealth is linked to these network effects, is a central question that our science needs to address. Scaling ties these ideas together.

Evolution and Emergence

So far our descriptions of cities, apart from identifying the key role of feedback in conditioning how dynamic processes build on existing patterns to reinforce size and to generate economies of scale, have not formally treated how cities actually develop through time. The patterns that we showed earlier which are largely built from modules operating from the actions of individuals (or at least individuals acting for groups and institutions) from the bottom up at relatively small scales, evolve through time in such a manner that any snapshot at any cross section shows an emergent order that is the product of countless individual decisions. Patterns emerge from the operation of processes which spread the effects of these decisions spatially using various processes of segregation and diffusion.

Imagine that development proceeds in an orderly pattern in regular neighbourhoods around some seed that motivates the growth. If we define the urban landscape as a grid and a neighbourhood as the eight cells around any given cell, the first of which is the seed, then development takes place around a cell already developed, in a certain unvarying local pattern. If a cell is developed already, we might fix a rule that a cell that is then developed around this cell must be as far as possible from the source. This would then lead to the development of four radial lines around the seed cell that spanned the space as four lines of development. If we relaxed the rule and specified that at each time period, a cell is developed in any of the eight positions if one is already developed, then the process would lead to a diffusion of development around the seed cell with all cells being eventually developed but in random order with respect to the chronology. We show some of these patterns which are typical of such diffusion in Figure 4 where the patterns are constructed in a modular way, do not fill the whole space and are clearly fractal.

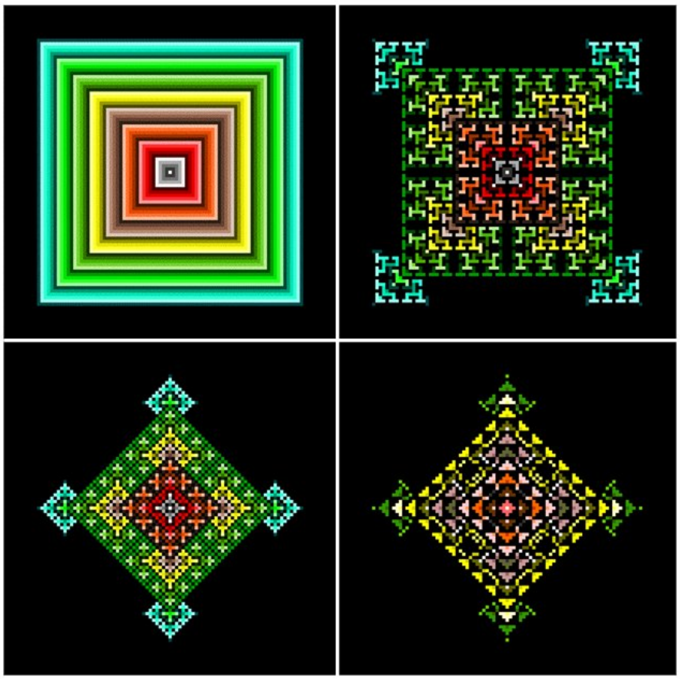

Figure 4: Idealised Urban Patterns Generated from the Bottom Up Using Modular Rules for Constructing Development Amongst Nearest Neighbours

In contrast, we might consider an already developed system composed of two kinds of individual – red and green – who are located randomly across the space. If the rule is that individuals are quite content to live side by side as long as their neighbours of a different kind are not in the majority, then we might assume that the landscape is one a checker board of reds and greens, say. However if one of these individuals switches colour arbitrarily so that one neighbourhood is now composed of 5 reds and 3 greens centred around a green which has switched its allegiance to red, the neighbourhood will become more segregated. In fact what happens is that the entire landscape unravels to produce highly segregated areas of red and green. This model was first proposed by Schelling (1969) and it produces a classic example of emergence in that although individuals are quite prepared to live side by side if there is an equality of view – reds balance greens – as soon as this difference shifts in favour of one other, the pattern begins to unravel and eventually what appears to be modest support for a balance of interests or views becomes extreme. Figure 5 shows how such segregation can take place from a set of initial conditions where reds and greens are distributed randomly.

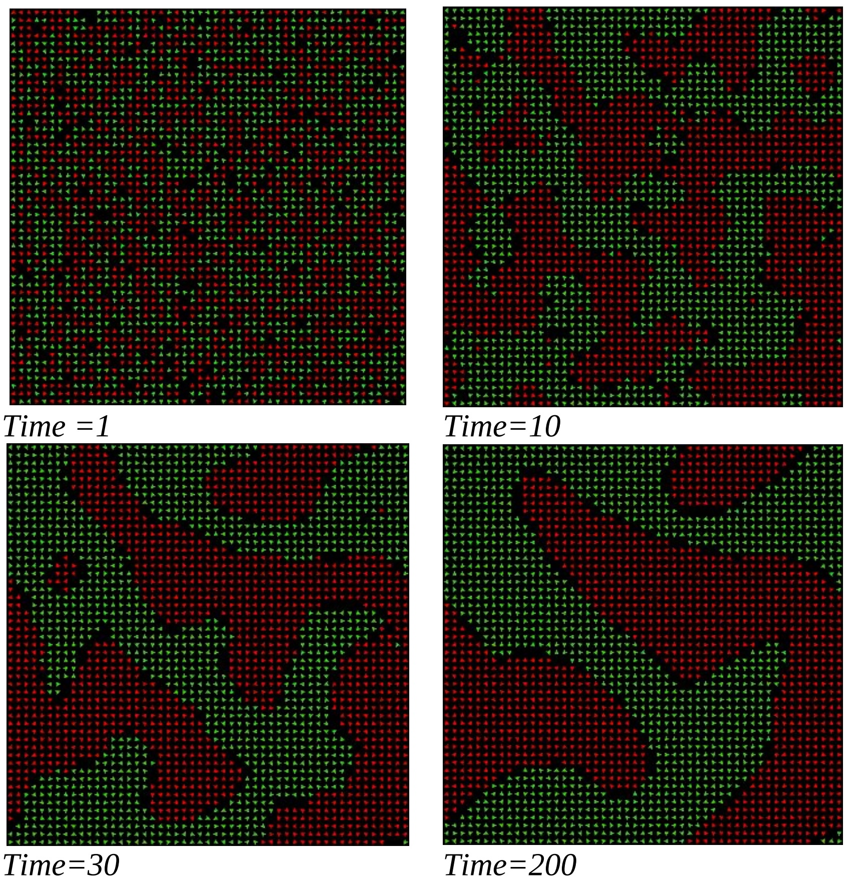

Figure 5: Segregation from a Random Spatial Distribution t=1 to a Highly Polarised but Stable Distribution by t=200

Size, Shape, Scale and Space: Three Laws of Scaling

What ties all these forms and processes together is the idea of scaling (Batty, 2008). When we say an object scales, we mean that the object resembles in some way a smaller or larger object of the same form although these forms might be different in some distinct and regular way. For example if an object scales with respect to its space, this means its spatial form might have the same proportions as a smaller or larger object (Bonner, 2006). More likely the object will have proportions that are distorted in some particular way due to the fact that as it changes in size, its proportions must adjust commensurately to conserve some critical functions of the object. For example, a small town is unlikely to have a well developed subway system because the physical dictates of movement by subway mean that certain stopping distances are required; if the town is small enough, it would be physically impossible to build such a structure. However in a small town, other forms of transport which scale accordingly such as trams might enable this function to be met.

The relationship between cities in a hierarchy of central places is one of scaling in that the frequency f_s of cities of size class s which have population P_s scales as a power law which in its strictest form is $f_s \sim P_s^{-2}$. In fact a much more convenient way to represent this frequency is to take the counter-cumulative distribution which is the rank r_s and to show that the typical rank-size distribution is represented as $P_s(r) \sim r_s^{-1}$. This is the pure Zipf (1949) relation, first popularised in his book *Human Behavior and the Principle of Least Effort*. In fact, it is our first scaling law – the so-called rank size rule – and it applies of course not only to city sizes but also to the evolution of the size of nodes in what Barabasi (2003) calls 'scale-free' networks. To see why such power laws are scale free, if we change the rank by doubling r to $2r$, our rank size rule becomes $P_{s'}(r) \sim (2r_s)^{-1} = 2^{-1} r_s{}^{-1} \sim 2^{-1} P_s(r)$. In short, the population size is a simple scaling. We can generate these rank size relations using various models: proportionate effect and preferential attachment of course, but also by subdividing a large hinterland into mutually exclusive subdivisions in a modular and regular manner, making various assumptions about population densities. This ties this kind of scaling back to both central place theory in the interurban context and to urban economics in the intraurban (Simon, 1955; Gabaix, 1999, Rozenfeld, Rybski, Andrade, Batty, Stanley, and Makse, 2008). In fact although rank-size scaling is highly stable through time, changes in the population of cities that make up such scaling can be highly volatile, and this remains a major puzzle in reconciling aggregate with disaggregate space-time correlations (Batty, 2010).

Our second scaling law relates not to the frequency of sizes but to the way attributes of cities change relative to their size and to one another. The easiest way to illustrate this is to consider how changes in the size of one geometric attribute of cities – the area A_i that they occupy or fill – relates to their population P_i. For example if cities expanded into the third dimension as well as the other 2, then we might consider cities increased in population as $P_i \sim d_i^3$ where d_i is a linear dimension. Area of course increases as the square of this dimension $A_i \sim d_i^2$ and thus we can relate population to area as follows $P_i \sim A_i^{3/2}$. This equation is a power law of a different kind. It is an example of positive allometry where the power is greater than 1. If population were to rise proportionately with area, we would consider this to be isometry and there is considerable evidence to think that the allometric coefficient – the power – is nearer to 1 than 3/2. If it is less than 1, then this is called negative allometry. Allometry is the study of changes in shape with size. Clearly population growing into its third dimension constitutes a change in shape with respect to what it is measured against which is the flat plane but it is more usual to consider changes in this power law to mirror economies or diseconomies of scale. There is a good deal of evidence for example to suggest that wages grow more than proportionately with population, that is $Y_i \sim P_i^{\alpha}$ where $\alpha > 1$ while physical infrastructures such as road space grow less than proportionately with $\alpha < 1$, reflecting economies and diseconomies of scale or positive and negative allometry respectively (Bettencourt, Lobo, Helbing, Kuchnert, and West, 2007).

Our third law of scaling pertains to interactions which we have already anticipated in a previous section. One reason why creative pursuits, innovations and even income scale more than proportionately with population is because of interaction effects. Consider the 'potential interactions' for a population P which we can write as P^2. Only a fraction of these interactions might be realised but as we have a population P where everyone interacts with him or herself than the total number of interactions T for this population will be $P \leq T \leq P^2$. Assume that self interactions are excluded and we only count symmetric interactions once, then the total is scaled down to $T = P(P-1)/2$. Of course as the population increases of a city, then the area over which the population has to interact also increases. The furthest distance to travel to engage in interaction is in fact linear with distance – if d is the radius of the area, then we might write $T = P(P-1)/2d$ but if this is constrained by area, then as area is the square of distance, the potential interaction collapses to $T \cong P/d$. We might also argue that there is a constant fraction ξ of any population that can interact and thus the potential interaction will still lie in the range from $\xi P/d \leq T \leq \xi P^2/d$. Now it is likely for many creative pursuits that a large potential interaction does force greater interaction than simply the size of the population would imply, and thus the findings that income and related attributes scale superlinearly (or as positive allometry) with population would be borne out.

This logic leads directly to our third form of scaling which pertains to modelling interaction itself. We have already stated that interaction between two different places is usually modelled using gravitational force, that is $T_{ij} \cong KP_iP_jd_{ij}^{-\phi}$ where K is a constant that contains various other scalars that determine the interaction and ϕ is a parameter that is usually greater than 1 and reflects the intensity of the friction that distance imposes on interaction (Wilson, 1970). Clearly this model is scaling for if we double the friction as $(2d_{ij})^{-\phi}$, we scale the interaction as $2^{-\phi}T_{ij}$. Our third law is much more generic in that models of interaction such as this have been (and continue to be) applied for many years in land use transportation modelling. Relating these three laws however is still somewhat of a challenge for all three come from different perspectives on city systems. They relate to many different developments whose proponents were and in some cases are still unaware of the way these scaling relationships are entangled with one another. It is still early days yet in the quest to map out a consistent theory which relates scale to shape to size in such a way that the spatial processes that determine city form are understandable as being consistent with one another.

Where Do We Go From Here?

Many of these ideas come from previous approaches to understanding and simulating city systems developed over the last half century or more (Berry, 1964). Social physics which was developed in analogy to gravitation, rank-size relationships, and notions about diffusion and segregation are deeply embedded in previous approaches to thinking about how cities are patterned. Many of these ideas have also been embedded into various generations of land use transport model that have been used to make predictions, to fashion 'what if' styles of scenario, and to inform policy making in general, notwithstanding a robust critique that has always dominated the field. New modelling approaches based on complexity theory are now in the ascendancy. The idea that we should build models that contain what we consider important to how cities function rather than seek the most parsimonious ways of distilling our knowledge into testable propositions that we match against data, is now significant. There is still a sense that we need to test models in various ways but the fact now that models are much more disaggregate down to the level often of individuals and households and that fact that such models assume processes that often cannot be tested, notwithstanding the fact that they might be eminently plausible, has changed the grounds rules of how we might judge and critique this new science. Agent-based, cellular automata, and micro-simulation models dominate applications and their use in policy is now tempered by notions that there needs to be dialogue between model builders and model users however they might be constituted (Heppenstall, Crooks, See and Batty, 2012). Models are being used increasingly to 'inform' rather than 'predict' as a new relativism sweeps the field.

In terms of theory, new data sets are coming on stream very rapidly and are enabling new theories to be tested. Much of this data is dynamic at the level of the individual and new techniques of model building, estimation, data mining, and pattern recognition not to say new ways of storing, retrieving and analysing massive data sets, are changing the context to the field. In one sense, cities are slowly beginning to be subject to the methods and approaches of 'big science' as data sets get ever larger and as teams of different experts are required to put together requisite models to engender this new science. There is little doubt that these developments are in their infancy as many disciplines begin to see that cities and their science have meaningful expression in terms of approaches developed elsewhere. This is but a mere beginning and in a decade, it is likely given the present rate of development, new and more powerful but also more pluralistic theories of how cities function and change will be with us.

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