Work as a criterion for determining *in situ* and yield stresses in clays

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A method of interpreting conventional oedometer test data using work per unit volume as a criterion for determining both *in situ* effective and yield stresses in clay is presented. This technique was applied to the results of oedometer tests carried out on samples of natural clay deposits and on specimens consolidated anisotropically from a slurry to a known effective stress state. The work per unit volume − effective stress relationship, using arithmetic scales, can be approximated or fitted using linear relationships. The intersections of these fitted lines are demonstrated to provide accurate values for *in situ* current and yield (preconsolidation) stresses. The yield stress is defined as the intersection of the initial fitted line and the linear relationship observed at higher stresses. The current effective stress is indicated by the first significant divergence of the data from the initial fitted line. These relationships apply to both conventionally (horizontally) trimmed specimens and to vertically trimmed oedometer samples. It is hypothesized that the *in situ* effective and yield stresses (in both the vertical and horizontal directions) in a natural clay can be determined by the work per unit volume interpretation of oedometer tests carried out on horizontally and vertically trimmed specimens.

Key words: *in situ*, stress, yield, oedometer, interpretation, clays, work, state, K_0, preconsolidation pressure.

Uné méthode d'interprétation des données d'essais oedométriques conventionnels basée sur le travail par unité de volume est présentée comme critère pour déterminer les contraintes effectives et les contraintes limites en place dans les argiles. Cette technique a été utilisée pour les résultats d'essais oedométriques réalisés sur des échantillons de dépôts d'argiles naturelles et sur des spécimens consolidés dans un état anisotrope en partant d'un coulis, jusqu'à un état connu de contrainte effective. La relation travail par unité de volume en fonction de la contrainte effective peut être évaluée sur des échelles arithmétiques ou tracée en utilisant des relations linéaires. Il est démontré que les intersections de ces courbes tracées fournissent des valeurs précises des contraintes actuelles ou des contraintes limites (de pré-consolidation). La contrainte limite est définie par l'intersection de la courbe initiale tracée, avec la relation linéaire observée à plus forte contrainte. La contrainte effective actuelle est donnée par la première divergence significative des données par rapport à la ligne initiale. Ces relations sont applicables tant aux spécimens conventionnels taillés horizontalement qu'aux échantillons oedométriques taillés verticalement. L'on pose comme hypothèse que les contraintes effectives en place et les contraintes limites (dans les deux directions horizontale et verticale) dans use argile naturelle peut être déterminée par l'interprétation basée sur le travail par unité de volume des essais oedométriques réalisés sur des spécimens taillés horizontalement et verticalement.

Mots clés : *in situ*, contrainte, limite élastique, oedomètre, interprétation, argiles, travail, état, K_0, pression de pré-consolidation.

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Introduction

The behaviour of a clay is governed by the conditions under which it exists. As is the case with other soils, the most significant conditions are void ratio and current geostatic stresses; these together describe the current state of a soil and can be represented as a point on the state diagram (i.e., in e − log stress space, Fig. 1). It is convenient to quantify current state in terms of distance from a reference state that can also be expressed as a unique relationship on the state diagram. For clays, the virgin consolidation line (VCL) has traditionally been used, either implicitly or explicitly, for this purpose. Current state has been quantified as the stress difference between the current state (σ'_0) and the state on the VCL following recompression (σ'_p), which is equivalent to log (σ'_p/σ'_0) on the state diagram. It is noted that σ'_p, the preconsolidation pressure, reflects the void ratio of the material. Thus, the current state of a clay can be expressed in terms of its overconsolidation ratio (OCR). It has been clearly demonstrated that the behavioural properties of clays correlate well with overconsolidation ratio (Roscoe and Burland 1968; Ladd and Foott 1974; Ladd *et al.* 1977; Wroth 1984). Further, the well-appreciated dependence of the ratio of undrained strength to preconsolidation pressure for various test types (see, e.g., Larsson 1980) is a manifestation of the same phenomenon. Finally, the effective stress path and yield behaviour of clays under field loading has been shown to be controlled by the *in situ* state and yield envelope; the latter is directly controlled by the maximum past pressure (Folkes and Crooks 1985; Crooks *et al.* 1984; Becker *et al.* 1984).

It is appreciated that factors other than void ratio and current stress regime (e.g., physical, chemical, and mineralogical conditions) affect clay behaviour. However, a clear appreciation of the first-order conditions is necessary to provide a rational framework within which the influence of other factors can be evaluated. Further, the current stresses existing within a clay are not adequately represented solely by the vertical stress

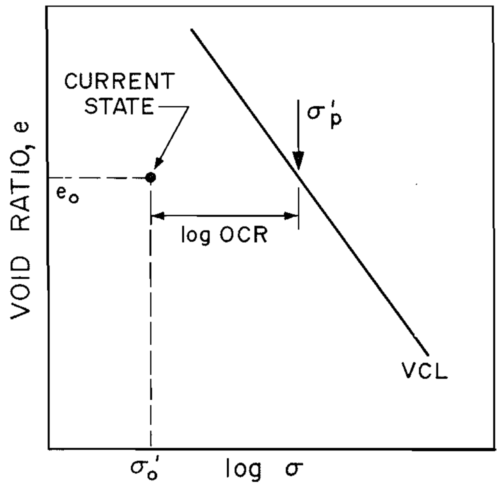

FIG. 1. Quantifying the current state of a clay.

(σ'_{v0}); the horizontal stress (σ'_{h0}) is also important. It is preferable that both the current and yield stresses in a clay are described in terms of the first stress invariant (I_1). Although it is a simplification of the general three-dimensional stress case, current effective stress can be described as $I'_0 = (\sigma'_{v0} + 2\,\sigma'_{h0})/3$ where $\sigma'_{h0} = K_0\sigma'_{v0}$ and mean yield stress I'_y can be described as $(\sigma'_{vy} + 2\sigma'_{hy})/3$. Becker *et al.* (1987) have demonstrated that field vane strength data can be well rationalized using I'_0 and I'_y. However, existing data and experience related to clay behaviour are almost universally expressed in terms of vertical stresses alone (i.e., OCR $= \sigma'_p/\sigma'_{v0}$).

Based on the above and assuming that the vertical and horizontal directions coincide with the principal stress directions, quantification of clay behaviour in terms of the state of the material requires knowledge of the vertical and horizontal effective *in situ* and yield stresses. *In situ* vertical effective stress can readily be determined; however, the determination of K_0 and yield stresses is more difficult. Several laboratory and field techniques to measure or infer K_0 have been proposed, such as instrumented oedometer cells to measure lateral stresses, triaxial consolidation under conditions of no lateral strain, hydraulic fracturing, installation of total pressure cells *in situ*, self-boring pressuremeters, and dilatometers (Brooker and Ireland 1965; Bishop and Henkel 1962; Bjerrum and Anderson 1972; Hughes 1973; Massarsch *et al.* 1975; Tavenas *et al.* 1975; Marchetti 1975, 1980). The advantages and limitations of these methods have been the subject of much discussion and controversy (American Society of Civil Engineers 1975; Ladd *et al.* 1977; Soos and Sallfors 1981; Mori 1981; Wroth 1984; Chan and Morgenstern 1986; Jefferies *et al.* 1987).

Traditionally, σ'_p has been determined from the results of the oedometer test. Various methods for interpretation of σ'_p from oedometer test data are available (Casagrande 1936; Schmertmann 1955; Burmister 1942 and 1951; Janbu *et al.* 1981). In general, these methods are satisfactory for clays that exhibit an e − log stress curve with a well-defined break in the vicinity of σ'_p. However, for soils that exhibit more "rounded" e − log stress curves, the above methods do not always provide an unambiguous definition of σ'_p. In these cases, σ'_p determined using conventional methods is usually reported in terms of a probable value with an associated range of possible values. Therefore, prediction of clay behaviour is also subject to a similar range of interpretation, which is not always adequate for analyses and design. It is often the case that the rounded nature of e − log stress curves is taken as evidence of soil disturbance during sampling and specimen preparation. This may not always be the case and rounded e − log stress curves may equally represent real soil behaviour. Regardless of which is the dominant effect, the interpretation of these test data still requires the accurate definition of yield stresses.

This paper presents an alternative approach for interpreting oedometer test data using work done per unit volume as a criterion to determine both *in situ* and yield stresses. Confirmation of this method is provided by the results of controlled laboratory testing of a reconstituted, slurried Beaufort Sea clay and samples of various natural clays. Comparisons of *in situ* stresses determined in the laboratory with computed *in situ* vertical stresses and horizontal stresses measured in self-bored pressuremeter tests are presented. The advantages of the work per unit volume approach for oedometer test interpretation over other conventional approaches are also discussed.

Oedometer test data interpretation in terms of work per unit volume

General approach

Regardless of how carefully sampling operations are performed and physical disturbance minimized, the stress regime acting on the clay is changed during sample retrieval. Consequently, the behaviour of the material in laboratory tests is also changed. The main issue, then, is to assess the *in situ* state of the material from the results of laboratory tests on specimens that have experienced a stress disturbance effect of unknown magnitude.

In the field, stresses start at σ'_{v0} but in the laboratory, applied stresses are initially less than σ'_{v0}. Because of sampling disturbance and changed stress conditions, a small deformation response occurs in the stress range below σ'_{v0}. Therefore, the laboratory e − log stress curve determination from an oedometer test on an overconsolidated clay should reflect three different deformation responses: one response for $\sigma'_v < \sigma'_{v0}$, a second response for $\sigma'_{v0} < \sigma'_v < \sigma'_p$, and a third response for $\sigma'_v > \sigma'_p$. Similarly, for a normally consolidated clay, two main deformation responses should be observed. Typical e − log stress curves indicate that changes in deformation response do occur at these reference stresses but the demarcation at σ'_{v0} is not well pronounced. Because σ'_{v0} can be readily calculated, the utility of a technique to determine σ'_{v0} from the results of oedometer tests is not immediately apparent. However, the significance of this capability will become obvious in later sections of this paper.

When a material under an existing state of stress is subjected to an incremental stress tensor and deforms by strains $d\epsilon_1$, $d\epsilon_2$, $d\epsilon_3$, the work done per unit volume (W) to the material can be expressed as

$$[1] \quad W = \int (\sigma_1 d\epsilon_1 + \sigma_2 d\epsilon_2 + \sigma_3 d\epsilon_3)$$

In soils, deformations are governed by changes in effective stresses and the stress−strain relationship is not linear. Therefore, for test interpretation, [1] must be written in terms of effective stresses, and work per unit volume computed incrementally.

The use of work per unit volume as a yield criterion to define the change from small strain response to large strain response to loading was introduced by Crooks and Graham (1976) to define yield envelopes for Belfast clays using drained triaxial stress probe tests. This approach has also been adopted for

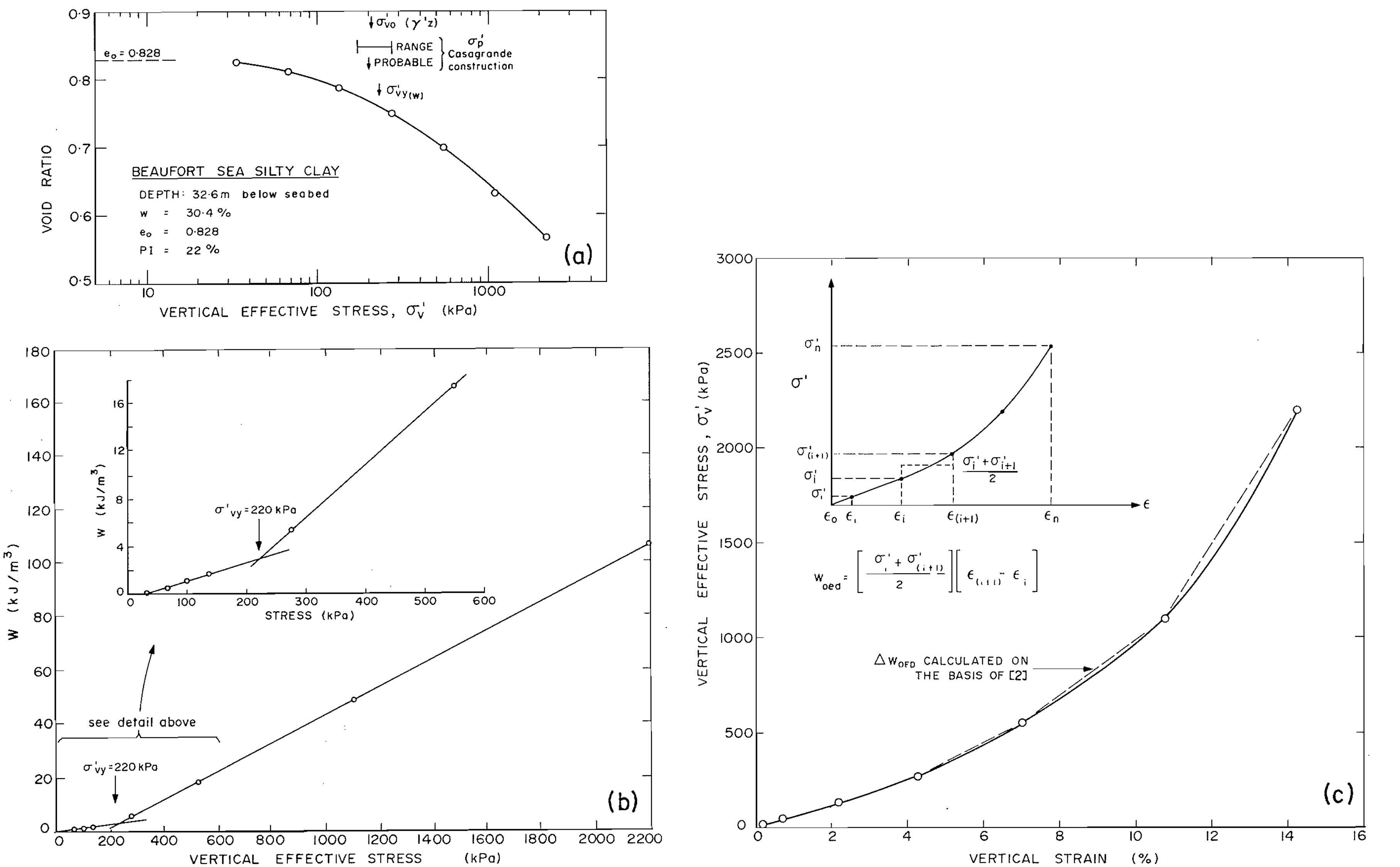

FIG. 2. Oedometer test on normally consolidated Beaufort Sea clay: (*a*) void ratio − log stress relationship; (*b*) work per unit volume interpretation of test data; (*c*) stress−strain relationship.

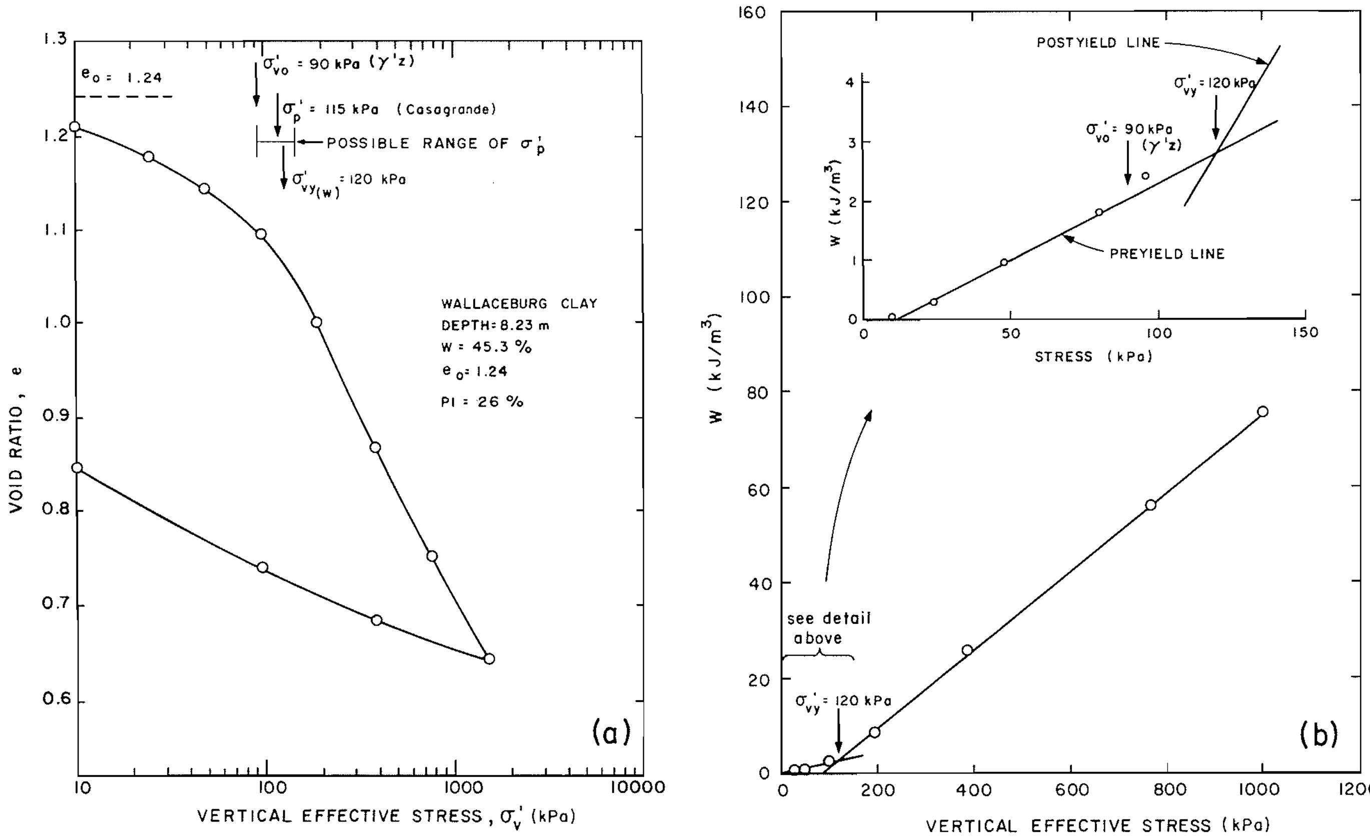

FIG. 3. Oedometer test on normally consolidated Wallaceburg clay: (*a*) void ratio — log stress relationship; (*b*) work per unit volume interpretation.

defining yield from triaxial test data for other clays (Tavenas *et al.* 1979; Graham *et al.* 1983). Although this criterion has been found to work well for interpreting triaxial test data, it has not been used to interpret oedometer test data. Since yield can be defined as a change from small strain to large strain response, it is postulated that yield in the oedometer test is synonymous with σ'_p. Lateral strains are prevented in oedometer tests and therefore the work per unit volume associated with a given load increment can be computed as follows:

$$[2]\quad \Delta W_{\text{oed}} = \left[\frac{\sigma'_i + \sigma'_{i+1}}{2}\right](\epsilon_{i+1} - \epsilon_i)$$

The strains used to define ΔW in [2] are incremented natural strains (i.e., the deformation within each increment is referenced to the sample height at the start of each increment). To interpret yield from an oedometer test, the cumulated work per unit volume is plotted against the applied vertical effective stress at the end of the relevant load increment using arithmetic scales.

A typical e — log stress curve obtained from a conventional oedometer test with a load increment ratio (LIR) of 1 on a natural, almost normally consolidated Beaufort Sea clay sample is presented in Fig. 2*a*. As is typical of these materials, the e — log stress curve appears to be rounded, which makes the accurate determination of σ'_p very uncertain. The probable value of σ'_p, using the Casagrande construction, is indicated together with a range of possible σ'_p values. Figure 2*b* shows the same oedometer test data interpreted using the work per unit volume approach as described above. The data at low stress levels are shown on an expanded scale (inset in Fig. 2*b*) and, as indicated, a linear approximation of these data seems to be reasonable. The linear relationship formed by the postyield data is clearly evident. The intersection of the preyield and postyield lines indicates the vertical yield stress (σ'_{vy}).

The oedometer stress—strain curve is shown in Fig. 2*c* to illustrate that the computed work per unit volume corresponds to the area beneath the stress—strain curve. The linear approximation between the individual loads inherent in [2] is quite adequate for calculating the work per unit volume during the oedometer test. The major discrepancy in calculating work per unit volume using [2] lies with the last two load increments. The ΔW_{oed} values computed for these last two increments is 31.1 and 56.6 kJ/m^3 respectively: the ΔW_{oed} beneath the actual σ' — ϵ curve for the same stress intervals are 30.3 and 55.0 kJ/m^3. Therefore, [2] overestimates work per unit volume by about 2%, which is negligible.

Normally consolidated clays

Although oedometer test data interpretation using the work per unit volume approach results in an unambiguous definition of σ'_{vy}, it remains to be demonstrated that the yield stress identified by this approach is the same as the preconsolidation pressure σ'_p interpreted using the more conventional approach. To examine this issue, e — log stress curves with a reasonably well-defined break in the vicinity of σ'_p were interpreted using the work per unit volume approach. Figure 3*a* presents the results of an oedometer test on a natural undisturbed specimen of soft, almost normally consolidated clay from Wallaceburg, Ontario (Becker 1981). The e — log stress curve has a reasonably well-defined break and the determination of σ'_p by the Casagrande construction can be carried out with some confidence. Work per unit volume interpretation of the same data

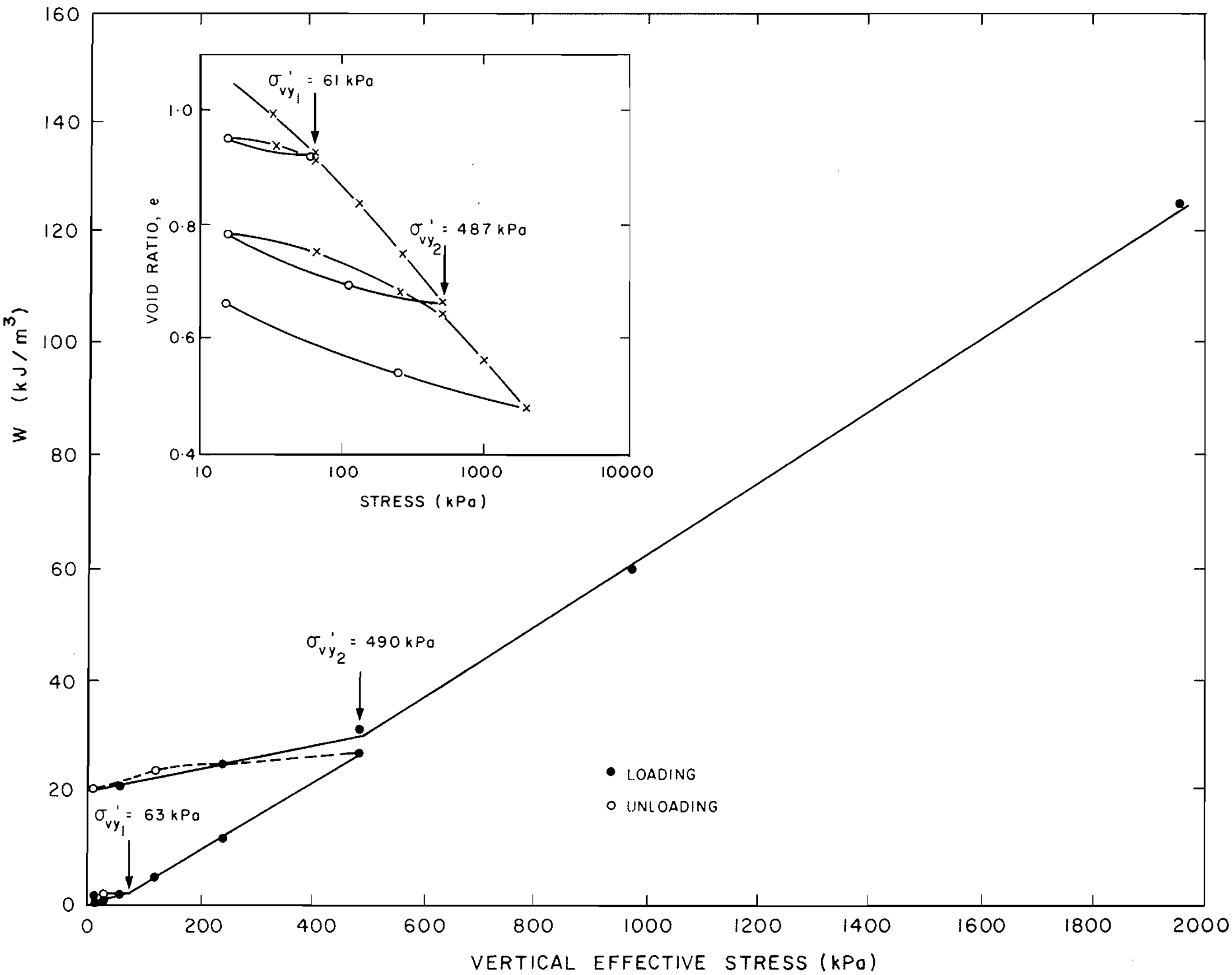

FIG. 4. Oedometer test on remoulded clay.

is shown in Fig. 3*b*. The postyield $W-$stress relationship is clearly linear, whereas the preyield data points can be reasonably represented by a straight line. The intersection of these fitted lines indicates a yield stress, σ'_{vy}, equal to 120 kPa. This value is in close agreement with the σ'_p value of 115 kPa determined from the e − log stress plot using Casagrande's construction.

The work per unit volume approach has also been applied to the results of an oedometer test on a sample of Beaufort Sea clay that was remoulded at its *in situ* water content prior to testing (Fig. 4). The sample was subjected to two full loading cycles with off-loading at 61 and 487 kPa. Each load increment was applied only for a sufficient time period to permit primary consolidation (i.e., negligible secondary compression was allowed). The test data were interpreted using the work per unit volume approach. For each loading cycle, pre- and post-yield lines can be readily defined. The intersections of these relationships define yield stresses of 63 and 490 kPa.

The above examples demonstrate that the yield stress determined by the work per unit volume approach corresponds to the preconsolidation pressure defined by the conventional Casagrande construction. Further, the distinct intersection of the fitted lines representing the pre- and post-yield $W-$stress data provides an unambiguous definition of σ'_{vy}.

Overconsolidated clays

The results of an oedometer test performed on a natural undisturbed specimen of an overconsolidated clay from Wallaceburg are shown in Fig. 5. The e − log stress curve (Fig. 5*a*) exhibits a well-defined break that enables σ'_p to be confidently determined by conventional methods. The work per unit volume interpretation of the same test data again permits approximation of the preyield data as a linear relationship; the postyield data form a distinctly linear relationship (Fig. 5*b*). The intersection of these lines (σ'_{vy}) occurs at a stress of 145 kPa, which agrees well with the σ'_p value of 150 kPa determined by the Casagrande construction.

Close examination of the W − stress plots presented in Figs. 2−5 together with the results of an oedometer test on a typical overconsolidated Beaufort Sea clay sample (Fig. 6) indicates a consistent difference between normally consolidated and overconsolidated samples. In the case of essentially normally consolidated clays (Figs. 2−4), the pre- and post-yield fitted lines pass through almost all data points. However, in the case of overconsolidated samples (Figs. 5 and 6), the W − stress data points in the stress range from σ'_{v0} to slightly beyond σ'_p lie above the preyield ($\sigma'_v < \sigma'_{v0}$) and postyield ($\sigma'_v > \sigma'_p$) lines. The intersection of the short dashed line connecting points slightly beyond σ'_{v0} and the preyield linear approximation of

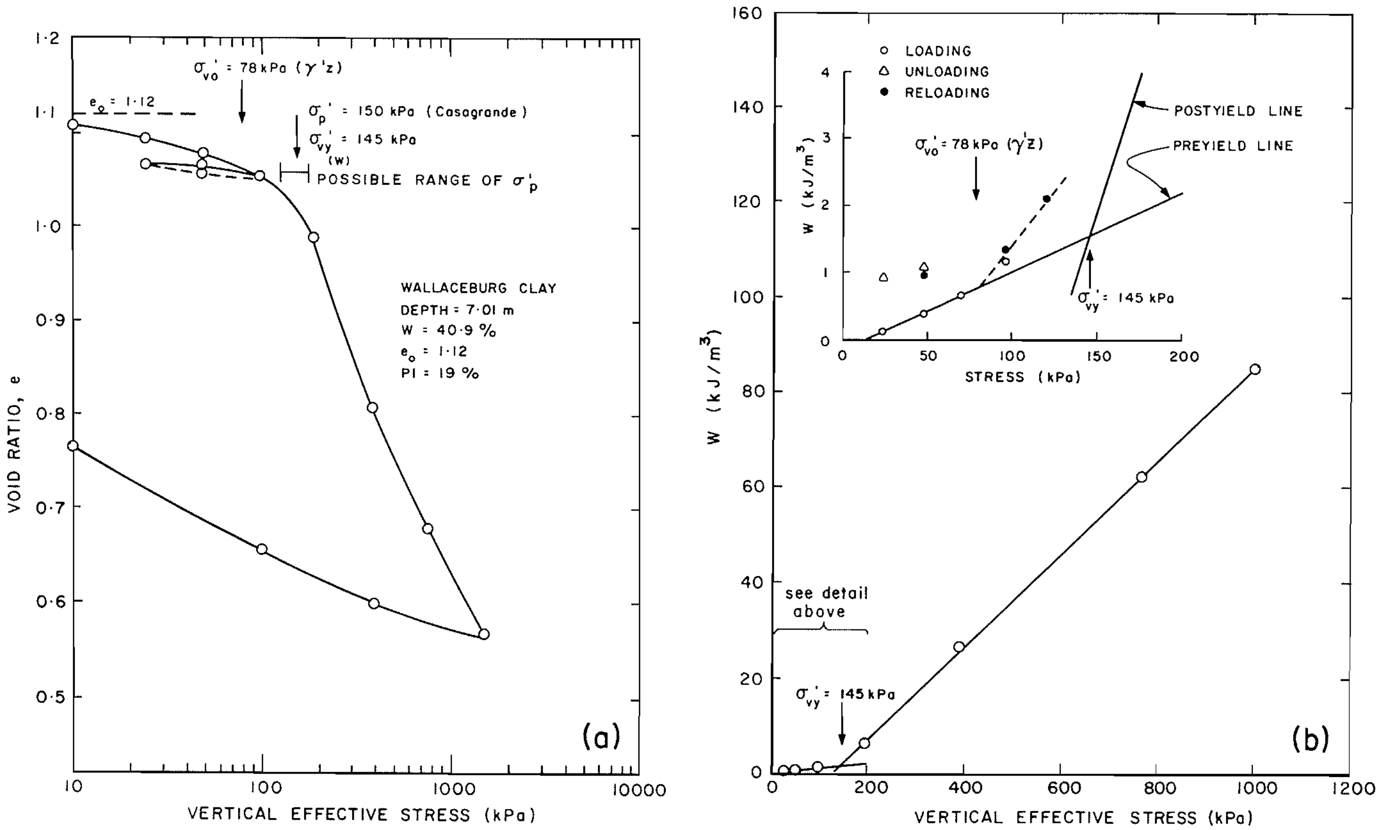

FIG. 5. Oedometer test on overconsolidated Wallaceburg clay: (*a*) void ratio − log stress relationship; (*b*) work per unit volume interpretation.

the data at stresses below σ'_{v0} coincides closely with the computed vertical effective stress. The intersection of these fitted lines allows for demarcation of the stress level at which a change in deformation response occurs. This observation is consistent with previous discussions and suggests that a change in deformation response in the oedometer test occurs at σ'_{v0}. The work per unit volume approach identifies this change.

To demonstrate the validity of defining σ'_{v0} as the stress level at which significant departure from the initial linear approximation of the test data is observed, σ'_{v0} values for the Tarsiut P45 site in the Beaufort Sea were interpreted using the work per unit volume interpretation of oedometer tests at various depths (Fig. 7). These compare favourably with the computed σ'_{v0} values at the same depths based on measured unit weights. Similar agreement between computed and measured σ'_{v0} values has been achieved at a large number of sites.

Horizontal stress (K_0)

The successful application of the work per unit volume approach for interpreting *in situ* vertical effective stresses from the results of oedometer tests on conventional "horizontally trimmed oedometer" (HTO) specimens encouraged the use of the same approach to interpret the results of tests on "vertically trimmed oedometer" (VTO) specimens (Fig. 8). Typical test results from a VTO test on an overconsolidated Beaufort Sea clay are shown in Fig. 9 and indicate the same response as was observed for the HTO specimens. The obvious inference is that the first intersection (point A) defines the current *in situ* horizontal effective stress (σ'_{h0}), whereas point B defines the horizontal yield stress (σ'_{hy}). This implies that work per unit volume interpretation of VTO tests provides a relatively simple method for determining K_0 as well as the horizontal yield stress. However, unlike σ'_{v0}, the value of σ'_{h0} *in situ* cannot be computed to allow direct validation.

Thus, it is necessary to rely on comparison with other measurements of K_0. In this respect, an adequate data base of K_0 values determined by *in situ* self-bored pressuremeter (SBP) tests is available for the Tarsiut P45 site offshore in the Beaufort Sea (Jefferies *et al.* 1987). The K_0 values obtained from the SBP tests are compared with those obtained from work per unit volume interpretations of oedometer tests on samples from the same depths at this site in Fig. 10. As indicated, there is reasonable agreement between the two approaches.

It is appreciated that the agreement indicated in Fig. 10 does not prove that the work per unit volume interpretation on oedometer data provides the correct value of K_0; both methods of measurement could be wrong. However, it is considered unlikely that both methods would be wrong by the same magnitude. Further, the fact that good agreement is achieved despite the very different natures of the two test types is compelling. Nevertheless, further validation of the work per unit volume approach is necessary as discussed below.

Laboratory verification of work per unit volume interpretation

To confirm the validity of the work per unit volume approach, HTO and VTO tests were carried out on clay samples with known stress histories. To "build in" a known stress history into a clay mass, a clay slurry was consolidated in a large triaxial cell to a specific stress state. It was necessary to consolidate from a slurry rather than remould a sample at its natural water content to ensure that the behaviour reflected by

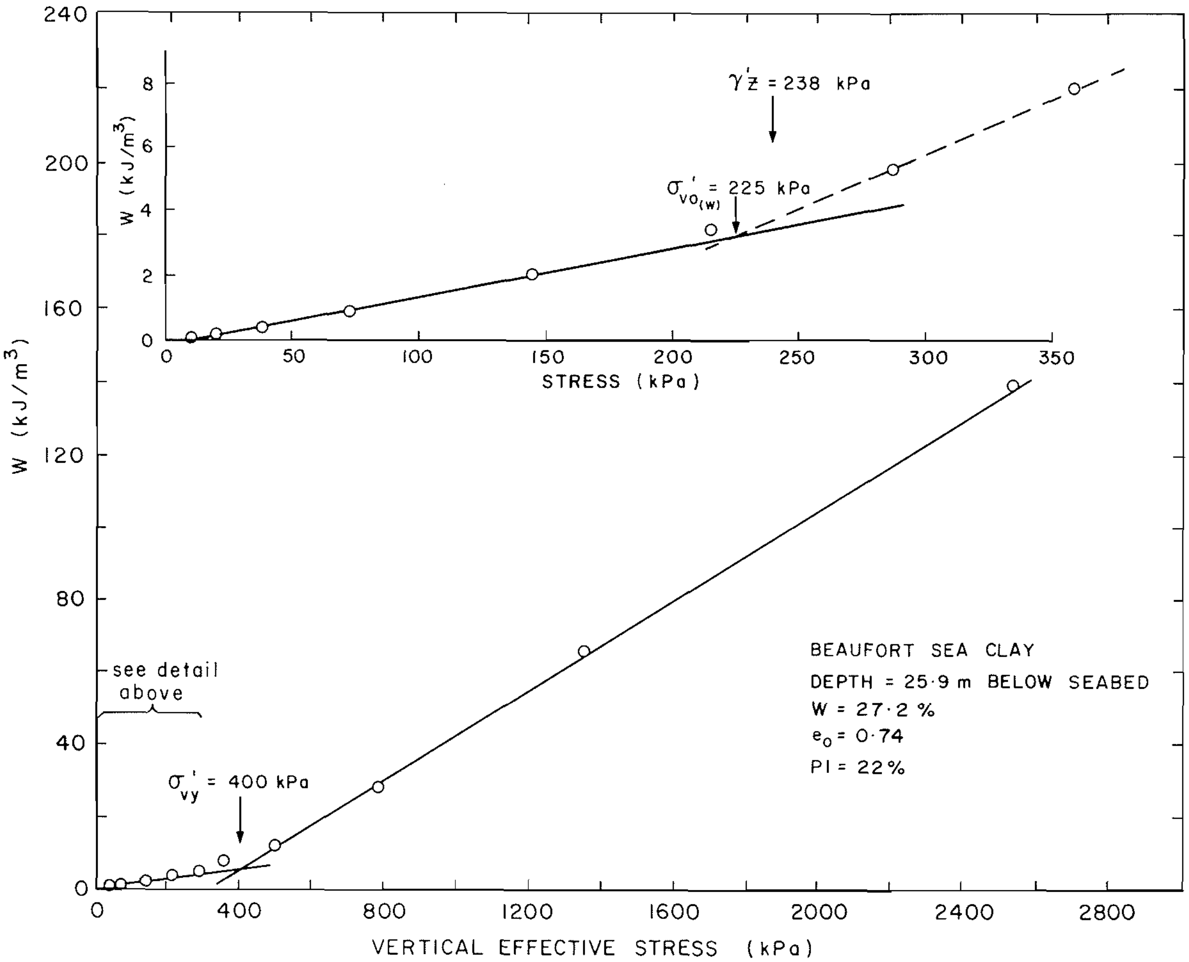

FIG. 6. Work per unit volume interpretation of oedometer test data for overconsolidated Beaufort Sea clay.

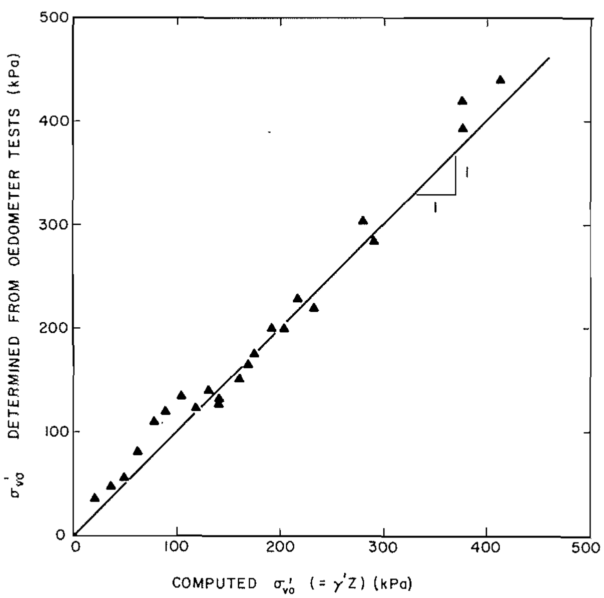

FIG. 7. Computed and measured *in situ* vertical stresses at Tarsiut P45.

the work per unit volume approach was the result of the known stress history and did not reflect any inherent relict stress or fabric effects. The removal of the consolidated clay mass from the triaxial chamber would essentially simulate release of total

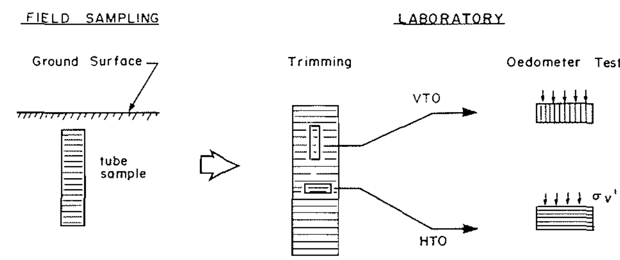

FIG. 8. Definition of horizontally trimmed (HTO) and vertically trimmed (VTO) oedometer tests.

stresses associated with sampling. In triaxial testing, it is normally assumed that axisymmetric conditions exist and that the stress conditions are uniform throughout the sample. However, an examination of the actual stress conditions in a triaxial specimen indicates that these assumptions are not necessarily true (Perloff and Baron 1976; Saada and Townsend 1980). To avoid the influence of possible stress concentrations and to ensure a reasonable approximation of the assumed triaxial stress conditions, small (50 mm diameter) oedometer specimens were trimmed from the central portions of 150 mm diameter triaxial samples that had a length-to-diameter ratio of two.

Sample preparation

The clay slurry was manufactured from samples of Beaufort Sea clay obtained from a depth range of 0–6.6 m below seabed. A grain size analysis performed on the prepared sample

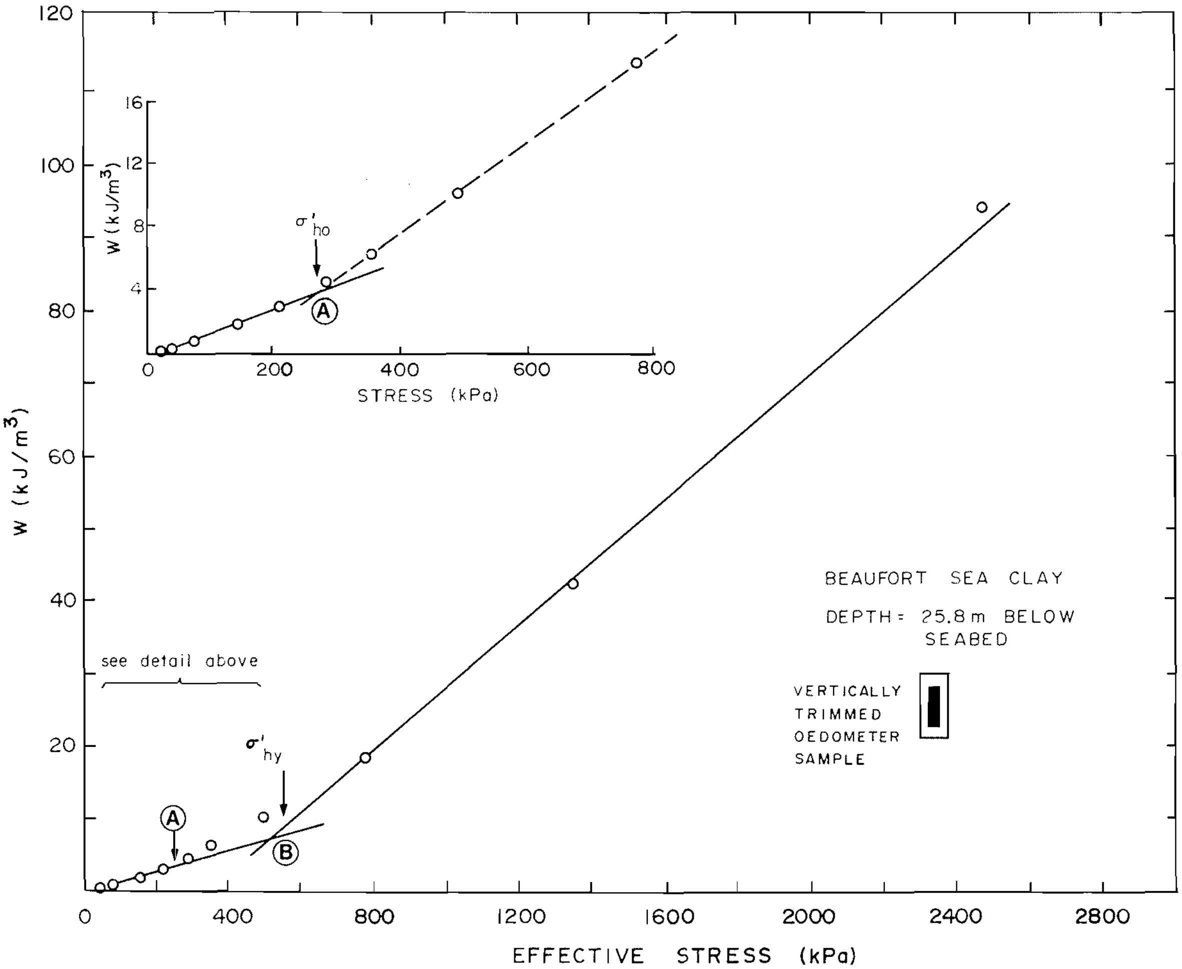

FIG. 9. Work per unit volume interpretation of VTO test on overconsolidated Beaufort Sea clay.

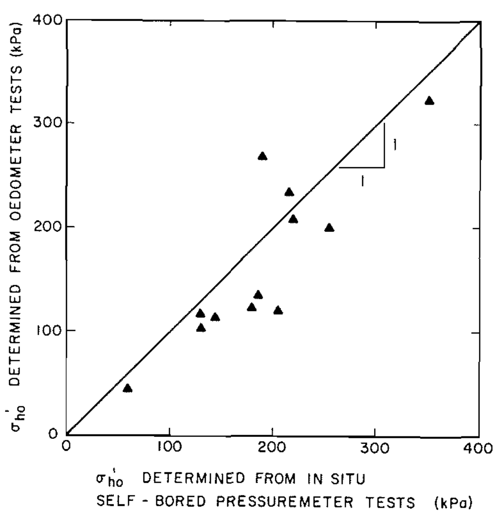

FIG. 10. Comparison of σ'_{h0} values from self-bored pressuremeter tests and work per unit volume interpretation of VTO tests at Tarsiut P45.

indicated that the material consisted of 37% silt sizes and 63% clay sizes by weight. The liquid and plastic limits were 57% and 27% respectively.

The natural clay samples were manually remoulded and cut into small pieces, which were then mixed with tap water in a blender to produce a thin smooth slurry. The slurry was allowed to cure for several days in a humid room to permit the sample to "bleed" water. The excess clear water was then decanted and the slurry transferred to a large mechanical mixing bowl. As mixing continued, small pieces of remoulded clay were added to thicken the slurry. Mechanical mixing operations and manual manipulation of lumps continued until a smooth paste-like consistency was achieved. The water content of the resultant slurry was 89%.

Because the slurry possessed minimal shear strength, the initial stage of consolidation required the confinement of an outer split mould. The purpose of this initial consolidation stage was to impart sufficient strength to the material so that it could support its self-weight and the top loading platen. After nominal strength gain occurred, consolidation in the triaxial chamber was completed using standard procedures.

A partial vacuum was employed in conjunction with peripheral filter paper strips to impose an isotropic consolidation pressure of 80 kPa. After consolidation under this stress was complete, as indicated by volume change measurement, the split mould was removed and the filter paper, porous stone, and loading platen were placed on the prepared top surface of the specimen. The 80 kPa vacuum was again applied (drainage valves open) and maintained until the triaxial chamber was in place and filled with water. Isotropic consolidation under 100 kPa was then effected followed by incremental anisotropic consolidation. The use of small consolidation increments was necessary to maintain the cylindrical shape of the consolidating

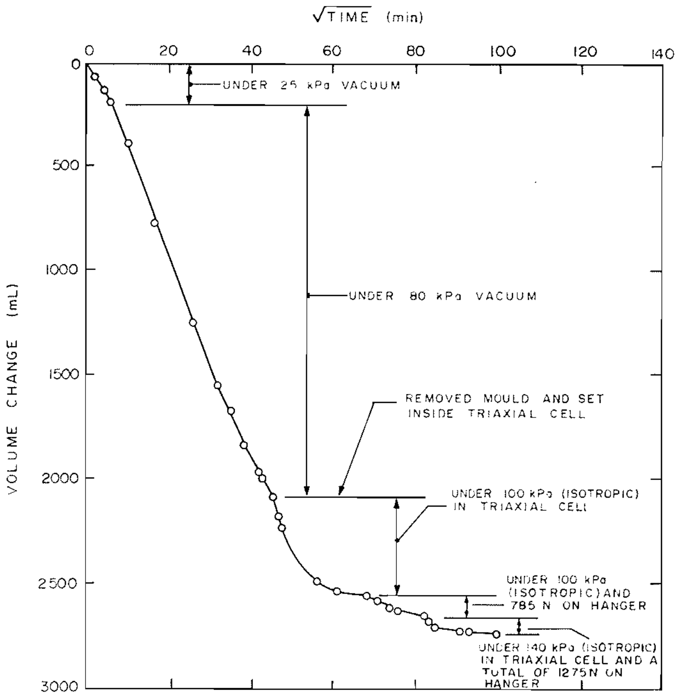

FIG. 11. Typical volume change – square root time relationship during triaxial consolidation of clay slurry.

sample. The duration of each load increment was sufficient to allow primary consolidation to occur as defined by conventional curve-fitting techniques. Ageing due to secondary compression was negligible. For each increment, volume change and pore pressure were monitored so that the end of primary consolidation, and thus the achievement of a specific effective stress state, was known. A volume change – time relationship for all consolidation increments on a typical slurry sample is shown in Fig. 11. In total, approximately 2.7 L (volumetric strain of 48%) of water was expelled, over a period of 165 h, from the slurry during consolidation to the desired effective axial and radial stresses. It is noted that 75% of the total volume change occurred during the initial consolidation increment in the split mould. Although the subsequent volume change resulted in the membrane becoming wrinkled, the surface of the consolidated sample was observed to be relatively smooth after the filter paper strips were removed.

The test program involved "manufacturing" and testing a normally consolidated (NC) sample and an overconsolidated (OC) sample. The stress history for each sample is summarized as follows:

	"Current" effective stress (kPa)			Yield (maximum) stress (kPa)	
Type	Axial (σ'_{a0})	Radial (σ'_{r0})	$\sigma'_{r0}/\sigma'_{a0}$	Axial (σ'_{ay})	Radial (σ'_{ry})
NC	223	140	0.63	223	140
OC	104	70	0.67	215	140

For the case of the NC clay specimen, the 150 mm diameter sample of clay slurry was consolidated to an effective stress state corresponding to σ'_r/σ'_a equal to 0.63. The maximum imposed axial and radial effective stresses were 223 and 140 kPa respectively. On completion of primary consolidation at these stresses, the sample was completely off-loaded (drainage valves closed), removed from the triaxial chamber, and stored in a humid room.

To prepare the OC sample with an OCR of about two, the clay slurry was consolidated under maximum axial and radial stresses of 215 and 140 kPa, respectively. The sample was then off-loaded and allowed to undergo primary swelling (with drainage valves open) to "current" axial and radial stresses of 104 and 70 kPa respectively. The sample was then completely off-loaded (no swelling) and stored in a humid room.

Oedometer test program

Four oedometer specimens (50 mm diameter, 12 mm high) were trimmed from the central portions of each of the 150 mm diameter samples. Two specimens were trimmed from a horizontal plane (HTO) and two were trimmed vertically (VTO), one each from two perpendicular vertical planes. The oedometer tests were carried out using conventional equipment with fixed oedometer rings and double drainage. The oedometer rings were greased to reduce side friction. Both conventional load increment ratio (LIR = 1) and smaller load increment ratio (LIR < 1) tests were carried out. The LIR < 1 tests were required to better define the W–stress relationships in the vicinity of both the current and yield stresses. During the oedometer tests, each individual load increment was applied for a sufficient period to permit primary consolidation to occur and to minimize secondary compression effects. Typically, the loading duration varied between 30 and 90 min. To this end, plots of dial reading versus log time and (or) square root time were maintained during testing. In some tests involving small load increment ratios, the resulting dial reading – log time curves were not of the classical "S" shape, thereby making the determination of the end of primary consolidation by conventional methods ambiguous and uncertain. In these cases, the rectangular hyperbola fitting method (RHM) proposed by Sridharan and Sreepada Rao (1981) and Sridharan and Prakash (1985) was utilized (Fig. 12*a*). The relationship between t/δ and for t for a load increment is plotted using arithmetic scales until a straight line with a slope "m" can be defined. A line is then drawn from the origin at a slope equal to 1.24 m; the intersection of this line with the actual laboratory curve defines the elapsed time required for an average degree of consolidation equal to 98% (i.e., t_{98}).

To confirm the validity of this method, the RHM-predicted times to end of primary consolidation for natural and remoulded Beaufort Sea clay samples were compared with those obtained by conventional interpretation of dial reading – log time curves with a well-defined "S" shape. The results obtained using the two methods are shown in Fig. 12*b* and good correspondence is evident. Figure 13 presents typical RHM results obtained whenever a small load increment ratio was used. The log time plot (Fig. 13*a*) indicates no deviation from a straight-line relationship and thus determininination of the end to primary consolidation is not possible. On the other hand, the characteristic plot associated with the rectangular hyperbola fitting method is still evident (Fig. 13*b*). Therefore, the rectangular hyperbola fitting method was found to be extremely useful in defining the end of primary consolidation for small load increment ratios.

Results of tests on normally consolidated clay

The known stress history and results of work per unit volume interpretation of the oedometer test data for normally consoli-

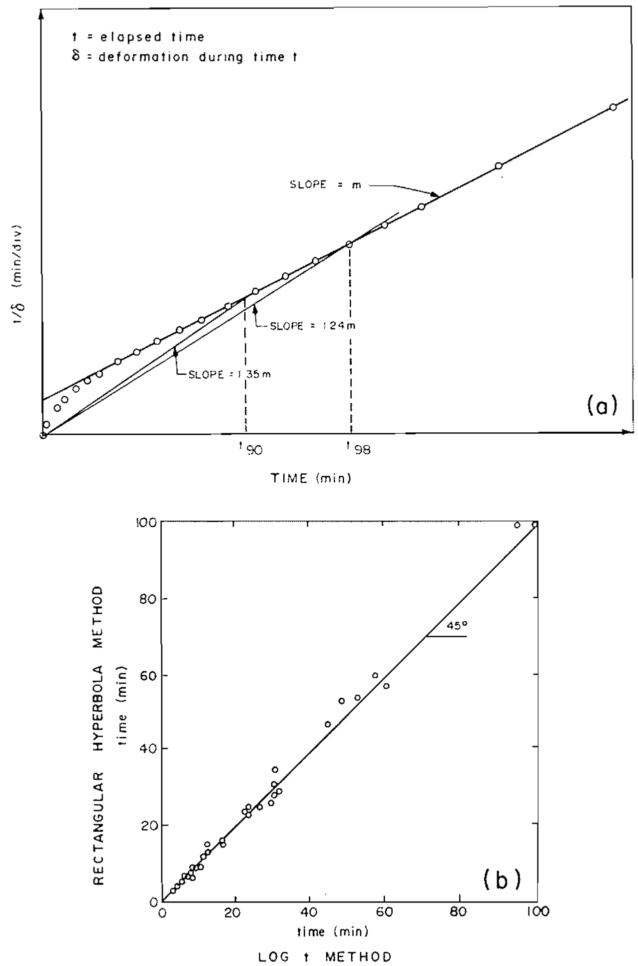

FIG. 12. (*a*) The rectangular hyperbola fitting method (RHM) for the determination of end of primary consolidation (after Sridharan and Sreepada Rao 1981). (*b*) Comparison of the time to end of primary consolidation determined by RHM with log time technique for "S"-shaped deformation – time relationships.

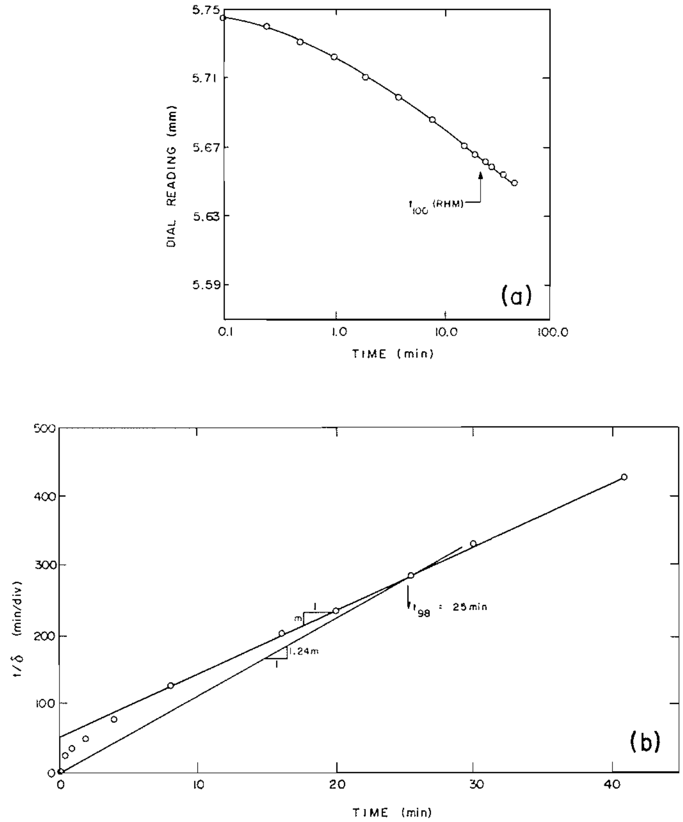

FIG. 13. Deformation–time relationships for small load increment ratios: (*a*) conventional dial reading – log time relationship; (*b*) rectangular hyperbola fitting method.

dated samples are summarized in Table 1. The initial water contents and void ratios of the individual specimens are very similar, indicating reasonable homogeneity in terms of stress history. Repeat tests were carried out: one using a conventional LIR equal to one and the other using smaller load increment ratios. The data for these tests (Fig. 14) exhibit distinctly linear postyield W–stress relationships. Also, the preyield data can be reasonably approximated by linear relationships in all tests. The intersections of the lines were approximately equal to the known maximum stresses in the axial and radial directions. There is little difference in the yield stress values determined from the LIR = 1 and LIR < 1 tests; both indicate predicted yield stresses within $\pm 5\%$ of the known yield stresses. However, there is a tendency for the test results associated with the LIR < 1 tests to provide more accurate predictions.

Results of tests on overconsolidated clay

The stress history and results of the work per unit volume interpretation of the oedometer test data on the overconsolidated samples are also summarized in Table 1. The imposed maximum axial and radial effective stresses were 215 and 140 kPa respectively. The sample was unloaded and allowed to undergo primary swelling to "current" axial and radial

TABLE 1. Summary of oedometer test results

Test	w_n (%)	e_0	Current and yield stresses (kPa) σ'_{a0}	σ'_{ay}	σ'_{r0}	σ'_{ry}
N/C specimen			223*	223*	140*	140*
HTO-1	35.3	0.960	210	210	—	—
HTO-2	36.0	0.980	220	220	—	—
VTO-1	36.3	0.987	—	—	160	160
VTO-2	36.1	0.982	—	—	150	150
O/C specimen			104*	215*	70*	140*
HTO-1	40.6	1.104	100	200	—	—
HTO-2	40.5	1.102	108	209	—	—
VTO-1	41.2	1.121	—	—	75	145
VTO-2	40.0	1.088	—	—	75	149

*Imposed stress.

effective stresses of 104 and 70 kPa respectively. Four oedometer tests were performed on specimens trimmed in the same manner as described for the normally consolidated clay. These four samples had very similar initial void ratios. A LIR < 1 was adopted at stress levels below 300 kPa, with LIR = 1 thereafter. Tests were carried out on two vertical and two horizontal specimens to evaluate repeatability.

The W–stress plots (Fig. 15) indicate the three different deformation responses characteristic of overconsolidated natural clays. Again, linear W–stress relationships that best approximate the preyield data could be drawn to include all of the data points. The W–stress relationships for the postyield data were distinctly linear. The intersections of the lines

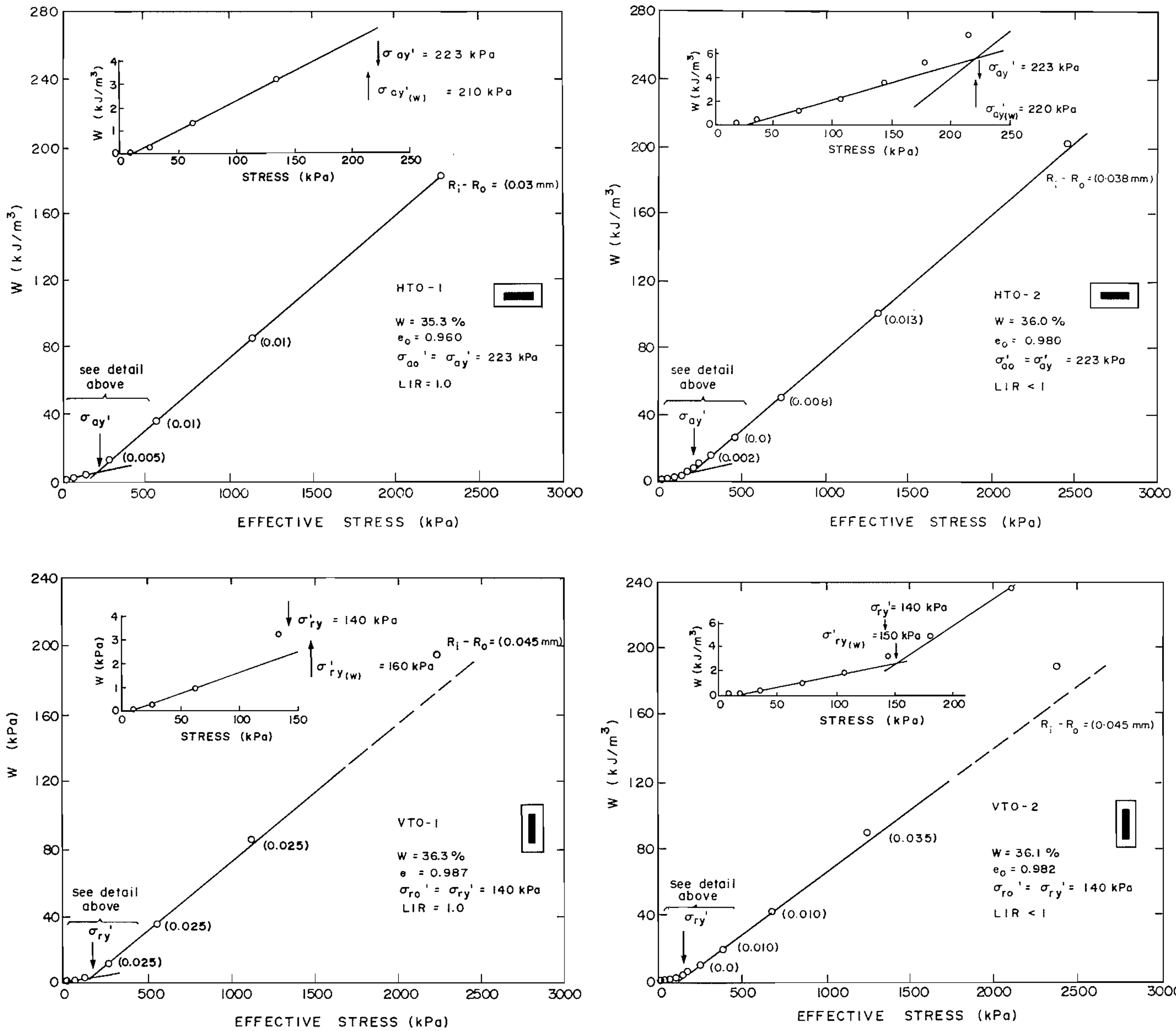

FIG. 14. Work per unit volume interpretation of oedometer tests on reconstituted normally consolidated samples.

defined at low stress levels ($\sigma' < \sigma_0'$) and the lines defined at high stress levels ($\sigma' > \sigma_y'$) accurately reflect the maximum applied (yield) stresses. The inset figures showing the linear approximation to the data points at low stress levels clearly indicate distinct changes in response at stresses corresponding quite closely to the known "current" stresses. Thus, both VTO tests reflect a distinct break at a stress of approximately 70 kPa (i.e., σ_{r0}') and both HTO tests indicate a change at about 104 kPa (i.e., σ_{a0}').

Discussion

A comparison between the known "current" and yield stresses and those determined using the work per unit volume approach for all of the control test data is shown in Fig. 16*a*. The agreement between the known stresses and those interpreted using the work per unit volume approach is good, with the latter values all lying within 10% of the known stresses. Also shown in Fig. 16*b* is a typical *e* − log stress curve for one of these tests. The apparently rounded nature of the curve would cause some difficulty in the application of the Casagrande approach to achieve the same degree of correspondence.

As shown in Figs. 14 and 15, the computed *W* values at stresses greater than approximately 1200 kPa in some cases plot slightly above the postyield line that passes through the preceding three to four data points. In other instances, the postyield line passes through all the computed *W*−stress points (Fig. 17). This difference is associated with "flow or squeezing out" of the clay between the loading cap − porous stone and the oedometer ring at high stress levels. The width of this annulus was not the same for all of the oedometer equipment and flow of the clay at high stress levels was more predominant in cells with a wider annulus. The magnitude of the yield stress of the clay is also a factor: at the same stress level, clays with a lower yield stress will have a greater tendency to flow given the same annulus dimension. Because of this phenomenon, the measured vertical strain is larger and hence the computed work per unit volume is higher than would be the case if no flow

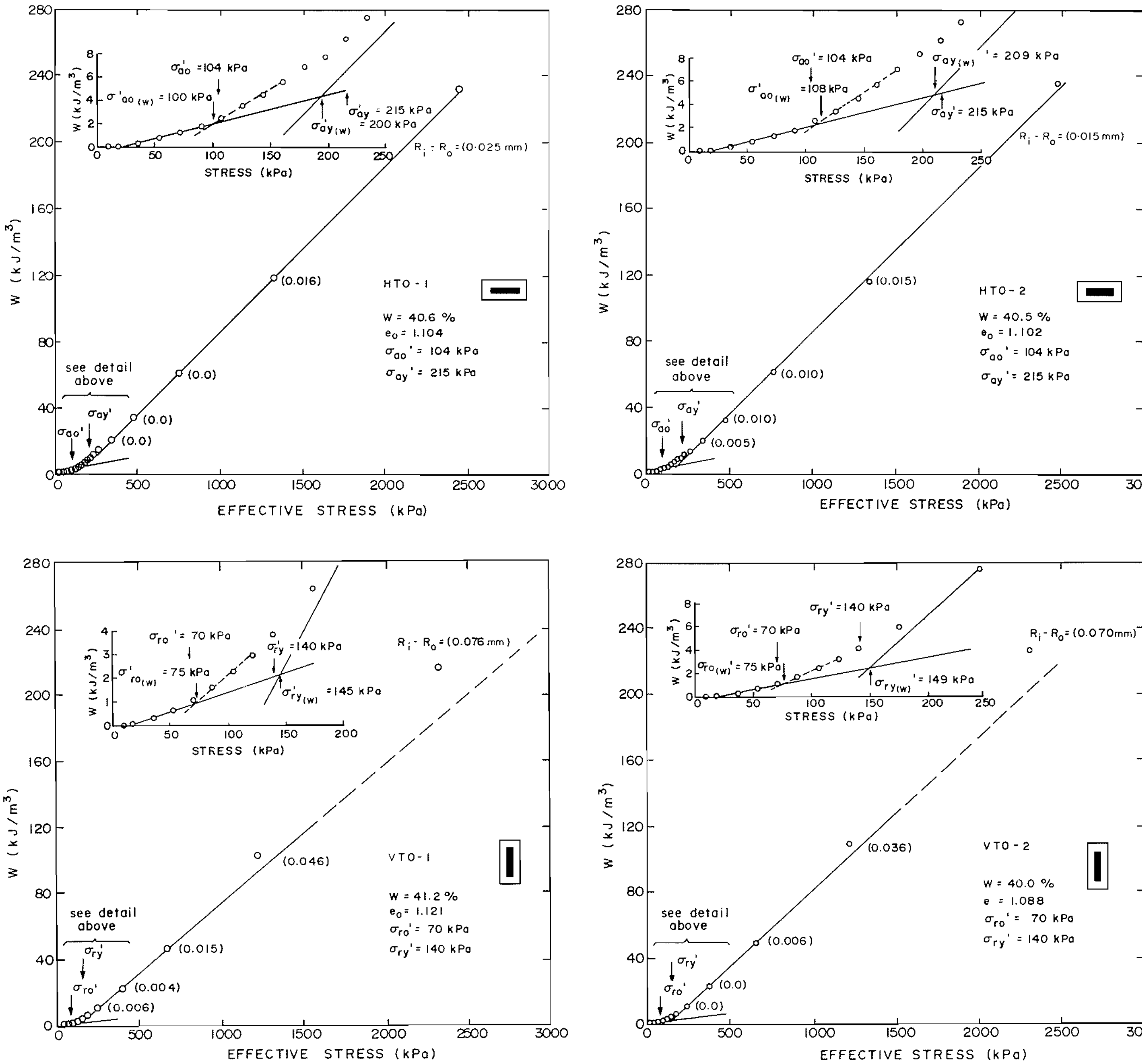

FIG. 15. Work per unit volume interpretation of oedometer tests on reconstituted overconsolidation samples.

occurred. The effect can be partially quantified by determining the magnitude of (R_i − R_0) for each point, where R_i is the actual initial dial gauge reading at the beginning of a given loading increment and R_0 is the "theoretical" initial dial gauge reading based on conventional log time or square root time constructions. Small values of (R_i − R_0) indicate negligible flow of the clay into the annular space; large values of (R_i − R_0) indicate significant flow. The magnitude of (R_i − R_0) for some loading increments is indicated beside the corresponding point in W−stress space in Figs. 14 and 15. In general, the value of (R_i − R_0) increases with increasing vertical stress. For comparatively small values of (R_i − R_0), the postyield line passes directly through all computed W−stress points; however, for larger values of (R_i − R_0) the computed W−stress points start to diverge (i.e., lie above) the postyield line. It is noted that for cases in which (R_i − R_0) generally exceeded 0.04 mm, flow of the clay into the annulus was visually observed when the test equipment was dismantled. These observations greatly enhanced the "fitting" of the postyield data by indicating which points should be considered suspect, the postyield line being defined on the basis of points having low values of (R_i − R_0). In general, the postyield line is defined by the computed W−stress points corresponding to the lower stress portion of the line (Fig. 17).

The determination of yield based on the work criterion is not greatly influenced by LIR, since yield is accurately defined by the intersection of the initial portion of the W−stress relationship at low stress levels (approximated by a straight line) and the line defined at higher stress levels. The points defined by LIR = 1 are therefore adequate for the determination of yield. However, definition of current effective stresses generally requires more loading increments in the vicinity of this stress level than is provided by LIR = 1. Variation in the load increment ratio, however, has been reported to have a significant effect on the interpretation made regarding the shape of the void ratio − log stress curve (Leonards 1962). At stress levels

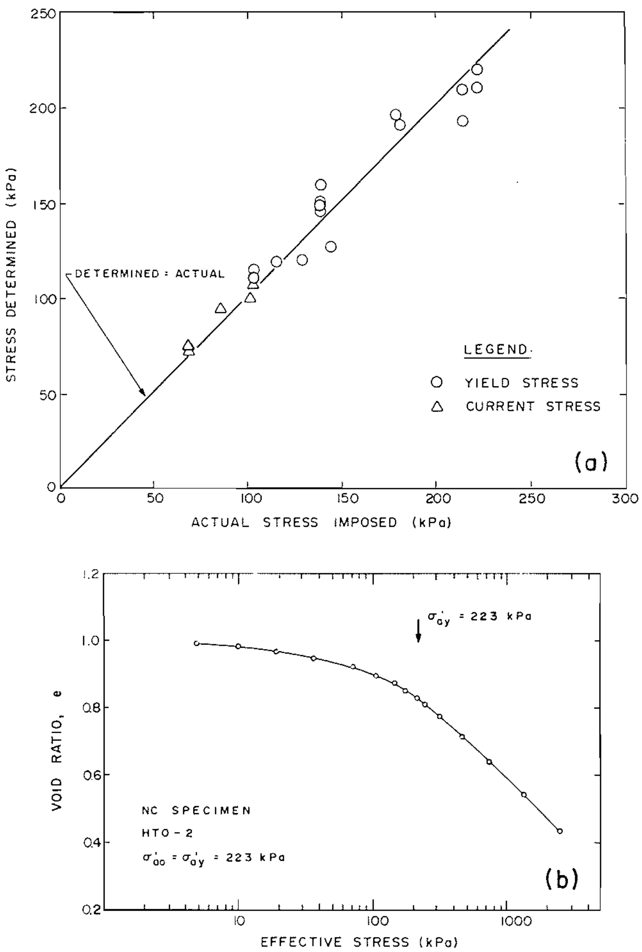

FIG. 16. (*a*) Comparison of stresses determined from work per unit volume interpretation of oedometer tests with the known stresses imposed on reconstituted samples. (*b*) Example of void ratio − log stress relationship from oedometer tests on reconstituted clay.

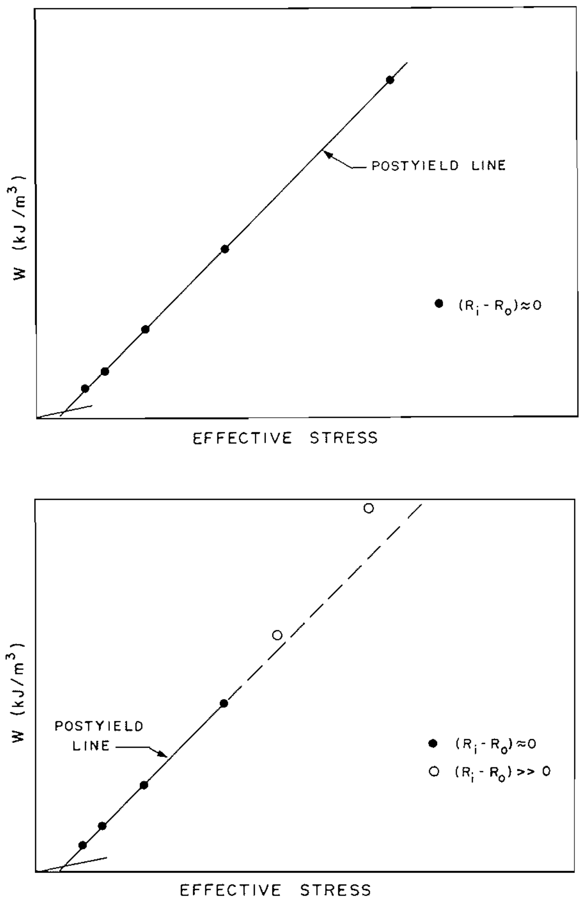

FIG. 17. Criterion for the determination of the postyield line.

below and well beyond σ'_p, the difference in void ratio obtained from an oedometer test using LIR = 1 and from a test using LIR < 1 is small. However, in the vicinity of the preconsolidation (yield) pressure, the interpretation of the shape of the curve between measured points can be different particularly if long load durations are used, which allows greater secondary compression deformations to occur. This is particularly true for "structured" clays, which undergo very distinct "structural breakdown" and exhibit an e − log stress curve with a well-defined break. For less structured clays, the interpretation of the actual curve between actually measured points is subject to less variation. For example, Fig. 18 presents the results of oedometer tests conducted on reconstituted clay specimens trimmed from adjacent locations in the same triaxial sample. One test series used a LIR of unity and the other employed smaller, varying load increment ratios. The results are presented in terms of both e − log stress and W−stress plots. There is essentially no effect of LIR on the resultant relationships, even in the vicinity of yield. This agreement suggests that additional W−stress data points can be accurately obtained by discretizing the e − log stress curves obtained using LIR = 1.

In the field, the vertical and horizontal directions are associated with stress- and strain-controlled boundary conditions respectively. In the conventional oedometer test (HTO) the imposed boundary conditions reflect *in situ* boundary conditions and, as such, the inferred stresses from the work per unit volume interpretation should accurately reflect σ'_{v0} and σ'_p. However, in the case of a VTO test, the boundary conditions during the test are not those that would exist *in situ*. The influence of this boundary condition change on the accuracy of measured horizontal stresses (σ'_{h0} and σ'_{hy}) is not known at this time; for the reasons discussed below, the effect may not be significant. In the triaxial cell, both the axial and radial boundaries are stress controlled. The oedometer tests, which were conducted on reconstituted samples consolidated in the triaxial cell, did not satisfy these boundary conditions. However, the work per unit volume interpretation of the HTO and VTO test data accurately defined the imposed current and yield stresses in both the vertical and radial directions. Further work is required to fully assess the significance of boundary conditions. For example, a clay slurry could be consolidated in a large oedometer cell with instrumented walls to known stress conditions, followed by HTO and VTO testing with subsequent work per unit volume interpretation.

Comparison with other oedometer data interpretation techniques

In the absence of a fully formulated comprehensive stress−strain−time model for clays, the interpretation of significant soil properties from the results of laboratory tests must rely on empirical plotting techniques (e.g., yield stresses in oedometer tests). Therefore, it is not a question of which technique is "correct"; rather the issue is which technique provides the most repeatable result and is least ambiguous. In the case of

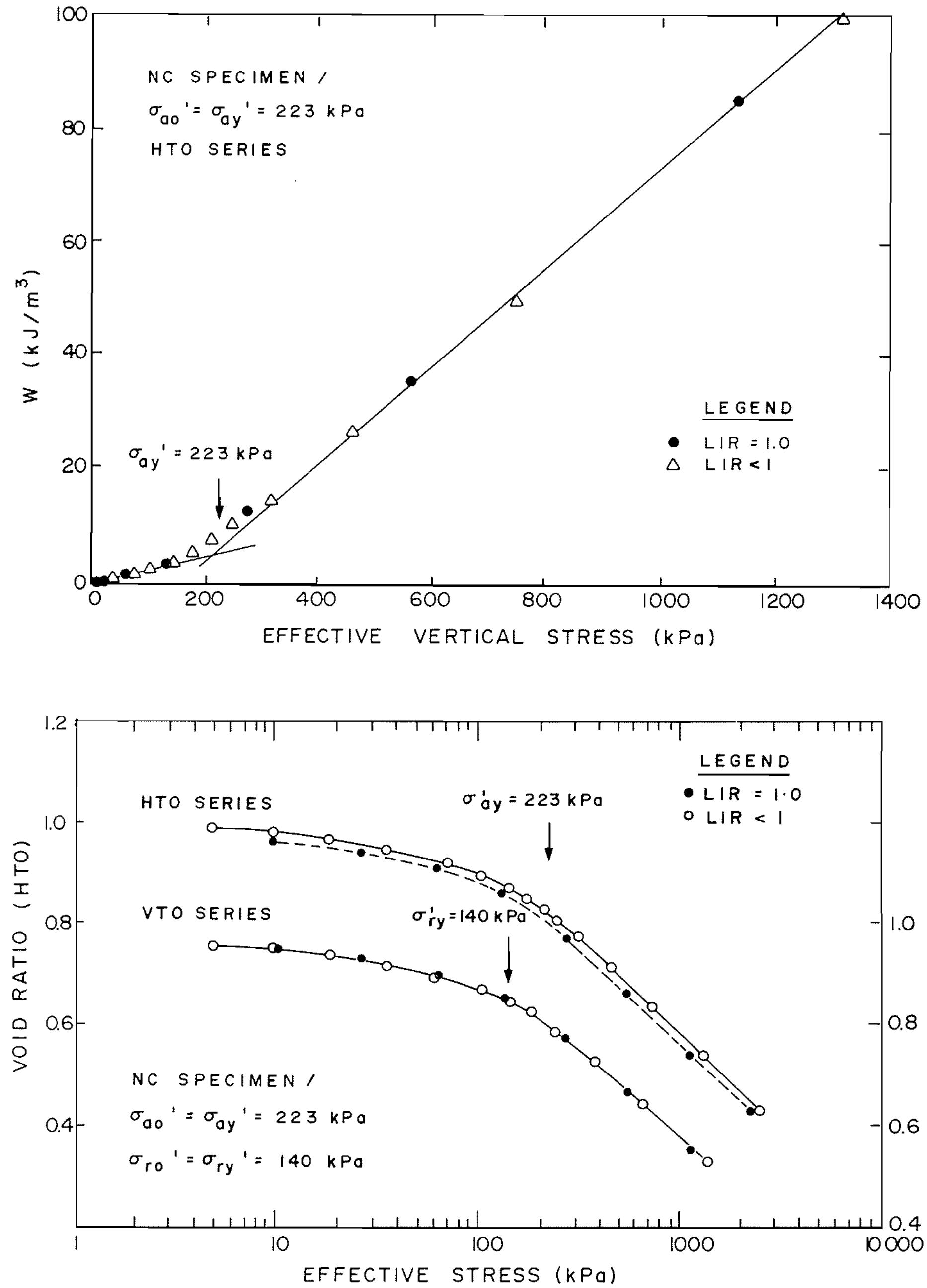

FIG. 18. Effect of load increment ratio (LIR) on oedometer test results.

oedometer test interpretation, it can be readily demonstrated that the same relationship between the stress–strain data will be preserved regardless of whether the data are plotted in terms of void ratio – log stress, log specific volume – log stress, or work per unit volume – stress. However, natural strains must be used if the same form of relationships is to be preserved in strain – log stress plots; the use of engineering strains causes distortion of the test data particularly at high strains. It is also noted that the oedometer test data should be based on primary consolidation strains only; inclusion of arbitrary amounts of secondary compression strain is not acceptable if a repeatable reference line is to be developed.

Based on the above, it is evident that the only difference between the plotting techniques will be the enhancement of data presentation. It is considered that the work per unit volume approach provides this advantage for the following reasons. Firstly, the quantities—work per unit volume and stress—are plotted on arithmetic scales. Thus, the potential for error in defining the stresses at which changes in strain response occur is greatly minimized. The use of logarithmic scales for plotting stress data normally introduces significant uncertainty. Secondly, the effect of integration, which is inherent in the work per unit volume approach, is to enhance the data presentation. As illustrated by the postyield data for the tests presented in this paper, linear relationships are clearly evident. In the preyield range, the effect of integration is to reduce the curvature of the relationship, particularly in the stress range between σ_0' and σ_y'. This facilitates approximation of the data as linear relationships, which allows a reliable measure of the *in situ* stress state to be made.

It is normally the case that the W–stress data below σ_0' form a reasonably well-defined linear approximation, whereas the postyield data exhibit a distinctly linear relationship. Therefore, definition of yield stresses using the work per unit volume approach can be used with a high degree of confidence.

In some cases the data in the stress range between σ_0' and σ_y' form a more pronounced curved relationship and as such some degree of subjectivity is involved in approximating these data as a linear relationship. Thus, in these cases the degree of confidence involved in predicting current *in situ* stresses is less than is the case for yield stress predictions. It is possible that further examination of the work per unit volume approach will result in a better selection of the optimum test conditions to minimize this uncertainty. The degree of subjectivity involved in defining σ_0' can be reduced by placing less emphasis on data below 30 kPa and by using the data included in the stress range immediately beyond σ_0' where σ_0' is indicated by the first significant divergence of the data from the initial linear approximation of the data below σ_0'. Further, data points corresponding to stresses approaching the yield stress should receive less attention while fitting this intermediate line.

Conclusions

A method of interpreting oedometer test data using work per unit volume as a criterion is presented. For normally consolidated clays, the relationship between work per unit volume and stress, when plotted using arithmetic scales, can be approximated by two straight lines, the intersection of which defines the yield stress. For overconsolidated samples, the data below σ_0' form a reasonably well-defined linear approximation to the actual curved relationship, whereas the postyield (i.e., above σ_y') data exhibit a distinctly linear relationship. The intersection of these two fitted lines represents the yield stress. The data points corresponding to stresses slightly above σ_0' are observed to diverge in another linear approximation from the initial linear approximation of the data below σ_0'. The point of this departure, defined as the intersection of these two lines, represents the current effective stress, σ_0'.

The above forms of work per unit volume relationships apply to both conventionally (horizontally) trimmed samples and to vertically trimmed oedometer samples. Therefore, the respective vertical and horizontal *in situ* effective stresses and yield stresses can be determined. The accuracy with which *in situ* effective and yield stresses are determined was demonstrated to be within 10% of known stresses based on the following testing:

σ_{v0}'—for natural clays by comparing with computed values (i.e., $\gamma' z$).

σ_{h0}'—for natural clays based on comparison with self-bored pressuremeter test data.

σ_{vy}'—for natural clays with well-defined "knees" on the e − log stress curves, work per unit volume predictions agreed well with predictions of preconsolidation based on the Casagrande construction.

—for remoulded clays in an oedometer test subjected to two off-loading cycles; the maximum loads were accurately predicted in each case.

σ_{a0}', σ_{ay}', σ_{r0}', σ_{ry}'—a remoulded clay was consolidated anisotropically from a slurry in a triaxial chamber to known effective axial and radial stresses (normally consolidated sample). A second sample was overconsolidated by allowing the sample to swell to a known "current" effective stress state. The "current" effective and yield stresses on horizontal and vertical planes through the sample were accurately determined using the work per unit volume method.

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AMERICAN SOCIETY OF CIVIL ENGINEERS. 1975. *In situ* measurement of soil properties. Proceedings, ASCE Specialty Conference of the Geotechnical Engineering Division, Raleigh, NC.

BECKER, D. E. 1981. Settlement analysis of intermittently-loaded structures founded on clay subsoils. Ph.D. thesis, Faculty of Engineering Science, University of Western Ontario, London, Ont.

BECKER, D. E., CROOKS, J. H. A., JEFFERIES, M. G., and MCKENZIE, K. 1984. Yield behaviour and consolidation. Part 2. Strength gain. Proceedings, ASCE Geotechnical Engineering Division Symposium on Sedimentation Consolidation Models, Predictions and Validation. San Francisco. *Edited by* R. N. Yong and F. C. Townsend. pp. 382−398.

BECKER, D. E., CROOKS, J. H. A., and BEEN, K. 1987. Interpretation of the field vane test in terms of *in situ* and yield stresses. American Society for Testing and Materials. International Symposium on Laboratory and Field Vane Shear Strength Testing, Tampa, FL.

BISHOP, A. W., and HENKEL, D. J. 1962. The measurement of soil properties in the triaxial test. 2nd ed. Edward Arnold (Publishers) Ltd., London, England.

BJERRUM, L., and ANDERSON, K. H. 1972. *In situ* measurement of lateral pressures in clay. Proceedings, 5th European Conference on Soil Mechanics and Foundation Engineering, Madrid, Vol. 1, pp. 11−20.

BROOKER, E. W., and IRELAND, H. O. 1965. Earth pressures at rest realted to stress history. Canadian Geotechnical Journal, **2**; 1−15.

BURMISTER, D. M. 1942. Laboratory investigations of soils at Flushing Meadow Park. Transactions of the American Society of Civil Engineers, **107**: 187.

——— 1951. The application of controlled test methods in consolidation testing. Symposium on Consolidation Testing of Soils. American Society for Testing and Materials, Special Technical Publication 126, p. 83.

CASAGRANDE, A. 1936. The determination of the preconsolidation load and its practical significance. Proceedings, First International Conference on Soil Mechanics and Foundation Engineering, Cambridge, Vol. 3, pp. 60−64.

CHAN, A. C. Y., and MORGENSTERN, N. R. 1986. Measurement of lateral stress in a lacustrine clay deposit. Proceedings, 39th Canadian Geotechnical Conference, Ottawa, pp. 285−290.

CROOKS, J. H. A., and GRAHAM, J. 1976. Geotechnical properties of the Belfast estuarine deposits. Géotechnique, **26**: 293−315.

CROOKS, J. H. A., BECKER, D. E., JEFFERIES, M. G., and MCKENZIE, K. 1984. Yield behaviour and consolidation. Part 1. Pore pressure response. Proceedings, ASCE Geotechnical Engineering Division Symposium on Sedimentation Consolidation Models Predictions and Validation, San Francisco. *Edited by* R. N. Yong and F. C. Townsend. pp. 356−381.

FOLKES, D. J., and CROOKS, J. H. A. 1985. Effective stress paths and yielding in soft clays below embankments. Canadian Geotechnical Journal, **22**: 357−374.

GRAHAM, J., NOONAN, M. L., and LEW, K. V. 1983. Yield states and stress−strain relationships in a natural plastic clay. Canadian Geotechnical Journal, **20**: 502−516.

HUGHES, J. M. O. 1973. An instrument for *in situ* measurement of soft clays. Ph.D. thesis, University of Cambridge, Cambridge, England.

JANBU, N., TOKHEIM, O., and SENNESET, K. 1981. Consolidation test with continuous loading. Proceedings, 10th International Conference on Soil Mechanics and Foundation Engineering, Stockholm, Vol. 1, pp. 645–654.

JEFFERIES, M. G., CROOKS, J. H. A., BECKER, D. E., and HILL, P. R. 1987. Independence of geostatic stress from overconsolidation in some Beaufort Sea clays. Canadian Geotechnical Journal, **24**: 342–356.

LADD, C. C., and FOOTT, R. 1974. New design procedure for stability of soft clays. ASCE Journal of the Geotechnical Engineering Division, **100**(GT7): 763–786.

LADD, C. C., FOOTT, R., ISHIHARA, K., SCHLOSSER, F., and POULOS, H. G. 1977. Stress-deformation and strength characteristics. State-of-the-Art Report, 9th International Conference on Soil Mechanics and Foundation Engineering, Tokyo, pp 421–494.

LARSSON, R. 1980. Undrained shear strength in stability calculation of embankments and foundations on soft clays. Canadian Geotechnical Jorunal, **17**: 591–602.

LEONARDS, G. A. 1962. Engineering properties of soils. *In* Foundation engineering. *Edited by* G. A. Leonards. McGraw-Hill Book Company, Inc., New York, NY, chap. 2, pp. 66–240.

MARCHETTI, S. 1975. A new *in situ* test for the measurement of horizontal soil deformation. Proceedings, ASCE Specialty Conference on *In Situ* Measurement of Soil Properties. Raleigh, NC, Vol. 2, pp. 255–259.

——— 1980. *In situ* tests by flat dilatometer. ASCE Journal of the Geotechnical Engineering Division, **106**(GT3): 299–321.

MASSARSCH, K. R., HOLTZ, R. D., HOLM, B. G., and FREDRIKSSON, A. 1975. Measurement of horizontal *in situ* stresses. Proceedings, ASCE Specialty Conference on *In Situ* Measurement of Soil Properties, Raleigh, NC, Vol. 1, pp. 266–285.

MORI, H. 1981. Soil exploration and sampling—General report. Proceedings, 10th International Conference on Soil Mechanics and Foundation Engineering, Stockholm, Vol. 4, pp. 399–412.

PERLOFF, W. H., and BARON, W. 1976. Soil mechanics: Principles and applications. Ronald Press Company, New York, NY, pp. 26–91.

ROSCOE, K. H., and BURLAND, J. B. 1968. On the generalized stress–strain behaviour of wet clay. *In* Engineering plasticity. Cambridge University Press, London, England, pp. 535–609.

SAADA, A. S., and TOWNSEND, F. C. 1980. State-of-the-art: Laboratory strength testing of soils. *In* Laboratory shear strength of soil. American Society for Testing and Materials, Special Technical Publication 740, pp. 7–77.

SCHMERTMANN, J. M. 1955. The undisturbed consolidation of clay. Transactions of the American Society of Civil Engineers, **120**: 1201.

SOOS, P. V., and SALLFORS, G. 1981. Laboratory testing—General report. Proceedings, 10th International Conference on Soil Mechanics and Foundation Engineering, Stockholm, Vol. 4, pp. 291–305.

SRIDHARAN, A., and PRAKASH, K. 1985. Improved rectangular hyperbola method for the determination of coefficient of consolidation by rectangular hyperbola. Geotechnical Testing Journal, **8**(1): 37–40.

SRIDHARAN, A., and SREEPADA RAO, A. 1981. Rectangular hyperbola fitting method for one dimensional consolidation. Geotechnical Testing Journal, **4**(4): 161–168.

TAVENAS, F. A., BLANCHETTE, G., LEROUEIL, S., ROY, M., and LAROCHELLE, P. 1975. Difficulties in the *in situ* determination of K_0 in soft sensitive clays. Proceedings, ASCE Specialty Conference on *In Situ* Measurement of Soil Properties, Raleigh, NC, Vol. 1, pp. 450–476.

TAVENAS, F., DES ROSIERS, J. P., LEROUEIL, S., LA ROCHELLE, P., and ROY, M. 1979. The use of strain energy as a yield and creep criterion for lightly overconsolidated clays. Géotechnique, **29**: 285–303.

WROTH, C. P. 1984. The interpretation of *in situ* soil tests. The 24th Rankine Lecture. Géotechnique, **34**: 449–489.

List of symbols

H_0	initial height of oedometer test specimen
H_i	specimen height at end of i loading increment
H_{i-1}	specimen height at end of $i - 1$ loading increment or the height of specimen at beginning of i loading increment
HTO	horizontally trimmed oedometer (conventional)
I'_0	$\sigma'_{v0}(1 + 2K_0)/3$
I'_y	$(\sigma'_{vy} + 2\sigma'_{hy})/3$
K_0	ratio of *in situ* horizontal stress to vertical effective stress, $\sigma'_{h0}/\sigma'_{v0}$
LIR	load increment ratio in oedometer test
NC	normally consolidated
OC	overconsolidated
OCR	overconsolidated ratio, σ'_p/σ'_{v0}
RHM	rectangular hyperbola fitting method
R_i	actual initial dial gauge reading at beginning of a loading increment
R_0	"theoretical" initial dial gauge reading at beginning of a loading increment
VTO	vertically trimmed oedometer (rotated 90° to conventional)
VCL	virgin consolidation line
W	work done per unit volume
t_{98}, t_{100}	time for 98% and 100% primary consolidation
w_n	water content
σ'_{v0}	initial vertical overburden effective stress
σ'_{h0}	initial horizontal effective stress
σ'_{vy}	yield stress in the vertical direction
σ'_{hy}	yield stress in the horizontal direction
σ'_p	preconsolidation pressure
σ'_v	vertical effective stress
σ'_0	initial effective stress
σ'_a	axial effective stress
σ'_r	radial effective stress
σ_1, σ_2, σ_3,	principal stresses
σ'_i, σ'_{i+1}	effective stress at end of i and $i + 1$ loading increment
$d\epsilon_1$, $d\epsilon_2$, $d\epsilon_3$	incremental principal strains
ϵ_i, ϵ_{i+1}	natural strains at end of i and $i + 1$ loading increment ($\epsilon_i = (H_0 - H_i)/H_{i-1}$)