Module 7: Shear strength of soils (part 1)

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Tentative schedule

Day	Date	Topic	Lab.
W	1/19/2022	Class introduction, syllabus, policies	Soil
F	1/21/2022	Invited speaker: Topic TBD	components
M	1/24/2022	Introduction: The geological cycle, soil origin	Grain size
W	1/26/2022	Introduction: Site investigation	dist.
F	1/28/2022	Index properties: Phase relationships	
M	1/31/2022	Index properties: Grain size distribution, Atterberg limits	Atterberg
W	2/2/2022	Index properties: Soil classification	limits
F	2/4/2022	Compaction	
M	2/7/2022	Quiz 1: Introduction, index properties, compaction, in-situ testing	Visual
W	2/9/2022	Water in soils: Groundawater table, pore pressure, total and effective stresses	classification
F	2/11/2022	Water in soils: Darcy's law	
M	2/14/2022	Water in soils: Permeability and hydraulic conductivity	Compaction
W	2/16/2022	Water in soils: One-dimensional seepage	
F	2/18/2022	Water in soils: 2D-3D seepage, flow nets, pore pressure, uplift force, seepage force	
M	2/21/2022	President's day: no class	In-situ
W	2/23/2022	Water in soils: piping	density
F	2/24/2022	Quiz 2: Water in soils	
M	2/28/2022	Induced stress: Approximations, Bousinesq's elastic solution	Permeability
W	3/2/2022	Induced stress: Bousinesq's elastic solution, superposition	
F	3/4/2022	Induced stress: Stress tensor, elastic deformations	
M	3/7/2022	Consolidation: Oedometer test, primary and secondary consolidation	Site
W	3/9/2022	Consolidation: Preconsolidation pressure, OCR	investigation
F	3/11/2022	Consolidation: Primary consolidation parameters	
M	3/14/2022	Spring break: no class	
W	3/16/2022	Spring break: no class	
F	3/18/2022	Spring break: no class	
M	3/21/2022	Consolidation: rate of consolidation	Bonus
W	3/23/2022	Consolidation: preloading, radial consolidation	
F	3/25/2022	Quiz 3: Induced stress and consolidaiton	
M	3/28/2022	State of stress: 2D stresses and Mohr's circle	Consolidation
W	3/30/2022	State of stress: principal stresses, stress invariants, rotations	
F	4/1/2022	State of stress:: Usage of Mohr's circle	
M	4/4/2022	State of stress: stress paths, simple shear, triaxial compression	Settlement
W	4/6/2022	Quiz 4: State of stress	estimates
F	4/8/2022	Shear strength: Mohr-Coulomb failure criteria	
M	4/11/2022	Shear strength: drained and undrained behavior	Unconfined
W	4/13/2022	Shear strength: Shear strength of clays	compression
F	4/15/2022	Shear strength: Shear strength of sands	test
M	4/18/2022	Quiz 5: Shear strength	Direct
W	4/20/2022	Lateral earth pressure: at-rest, passive, and active conditions [2]	shear
F	4/22/2022	Intro to slope stability [2]	
M	4/25/2022	Intro to bearing capacity [2]	Direct
W	4/27/2022	Maine's day: no class	shear
F	4/29/2022	Classes end: Q&A session	
M	5/2/2022	Final exam (1:30 PM- 3:30 PM) Williams Hall 110	

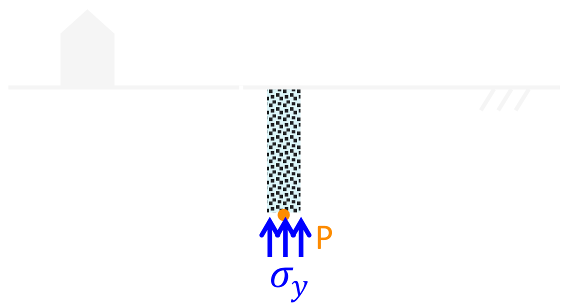
M: Monday - W: Wednesday - F: Friday

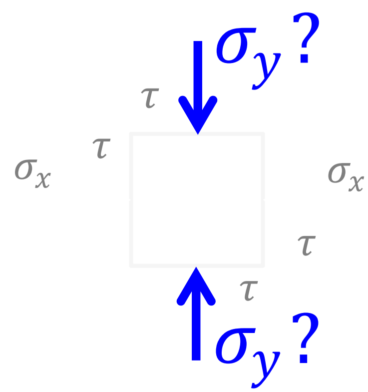
[2]This items may or may not be covered. It will be determined by how far the course has progressed.

- We learned about Terzaghi's consolidation theory.
- We learned how to calculate primary consolidation settlements vs. time.
- We learned how to calculate the coefficient of consolidation.
- We learned about the importance of the drainage boundary conditions.
- We learned about some miigation techniques used in practice.
- Today we will discuss about the state of stress in soils.

- Stress tensor.
- State of stress.
- 2D state of stress.
- Geometric interpretation (Mohr's circle)
- Stress paths: triaxial and simple shear test.

We have learned how to compute total/effective **vertical stresses** (normal to the horizontal plane)



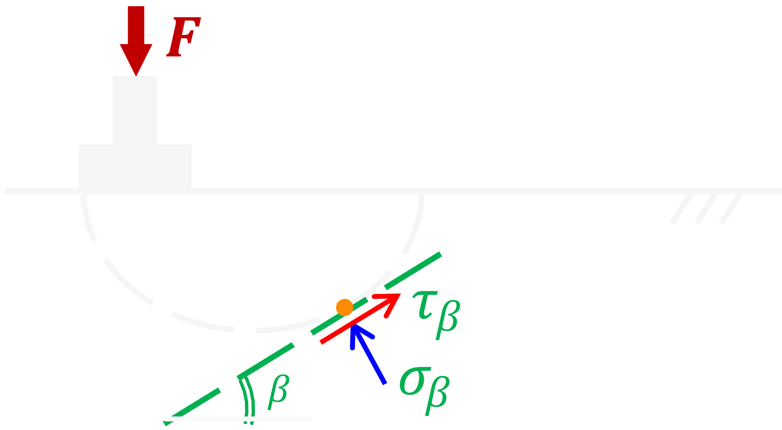


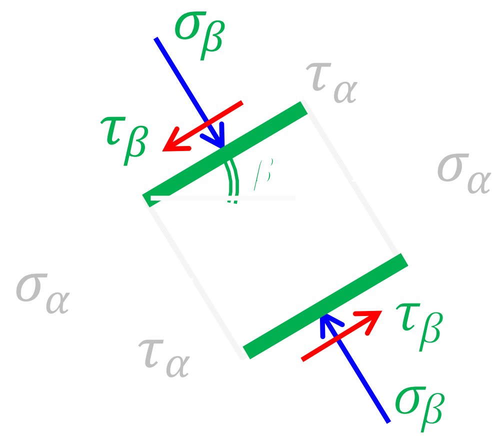
- What about the **Horizontal stress**?
- We use the concept of the **coefficient of lateral earth pressure**
- When soil is "**at rest**" (no lateral movement), we call it the **at-rest** coefficient K_0.

$$K_0 = \frac{\sigma'_h}{\sigma'_v}$$

- σ'_h = horizontal effective stress
- σ'_v = vertical effective stress

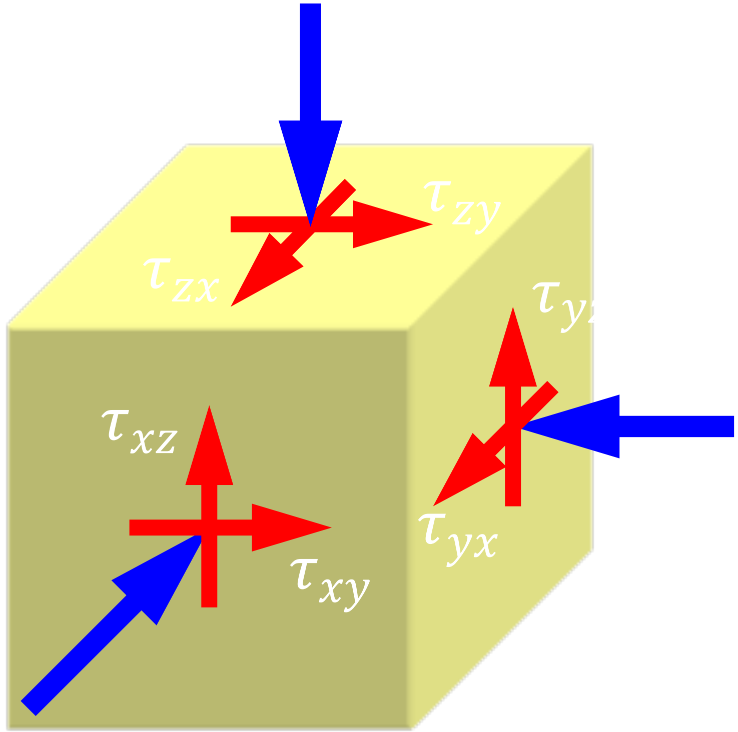
Geotechnical engineers often require to analyse stresses in different angles. E.g: to calculate the bearing capacity of a foundation.





$$\alpha = \beta + 90$$

≡



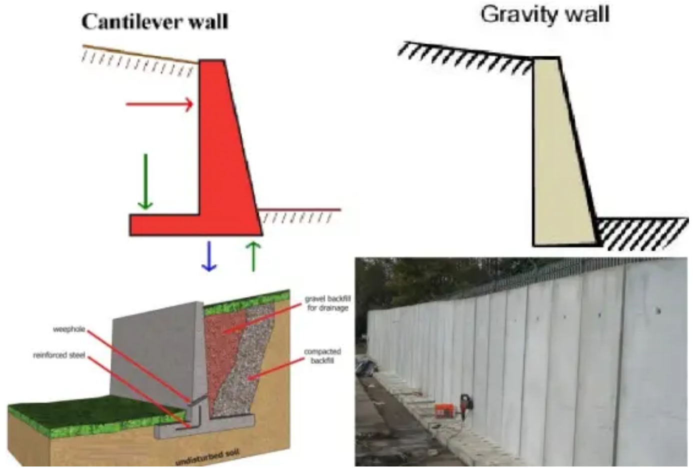
$$\begin{matrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{matrix}$$

$$\tau_{xy} = \tau_{yx}$$

Symmetric → $\tau_{xz} = \tau_{zx}$

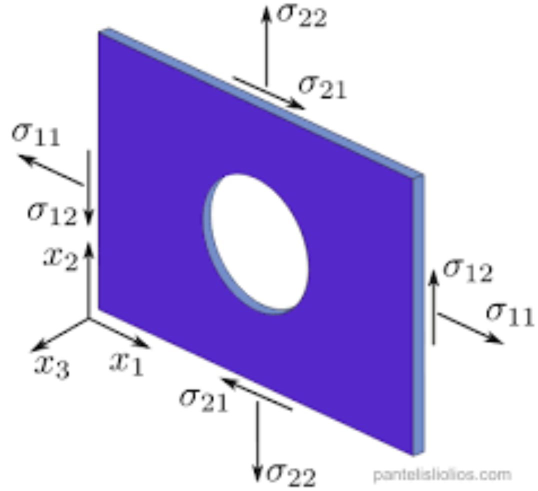
$$\tau_{yz} = \tau_{zy}$$

Plane strain



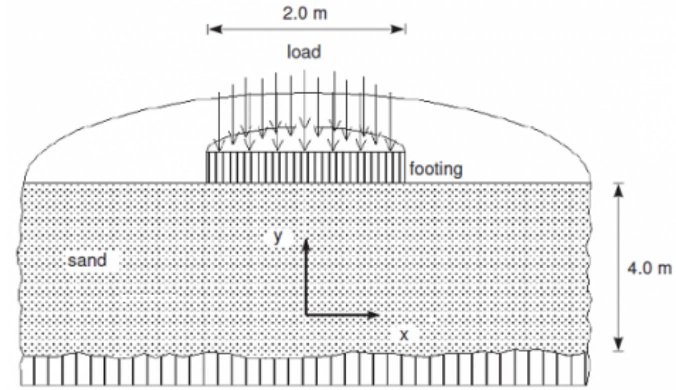
$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & 0 \\ \tau_{yx} & \sigma_{yy} & 0 \\ 0 & 0 & \sigma_{zz} \end{bmatrix}$$

Plane stress

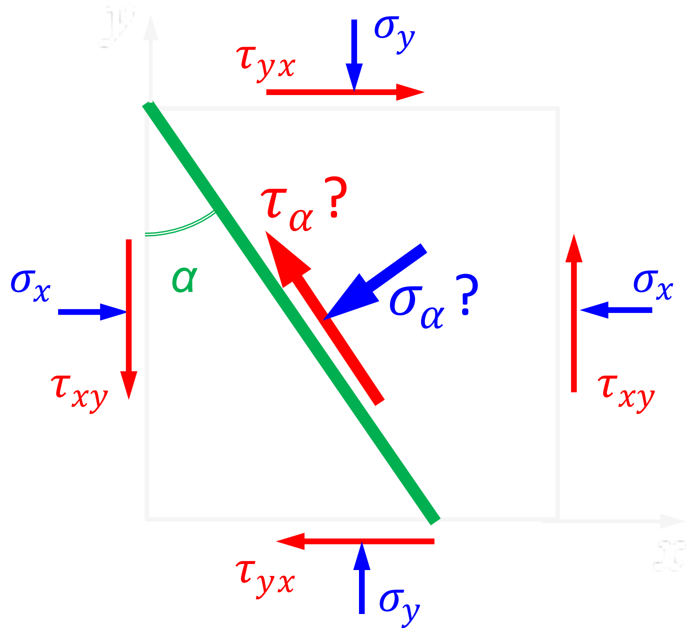


$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & 0 \\ \tau_{yx} & \sigma_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Axisymmetric



$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{xx} & \tau_{xr} & 0 \\ \tau_{rx} & \sigma_{rr} & 0 \\ 0 & 0 & \sigma_{\theta\theta} \end{bmatrix}$$



- Normal stress **acting at angle α** is σ_α
- $\sigma_\alpha = \dfrac{\sigma_y + \sigma_x}{2} + \dfrac{\sigma_y - \sigma_x}{2} \cos 2\alpha + \tau_{xy} \sin 2\alpha$
- Shear stress **acting at angle α** is τ_α
- $\tau_\alpha = \dfrac{\sigma_y - \sigma_x}{2} \sin 2\alpha - \tau_{xy} \cos 2\alpha$

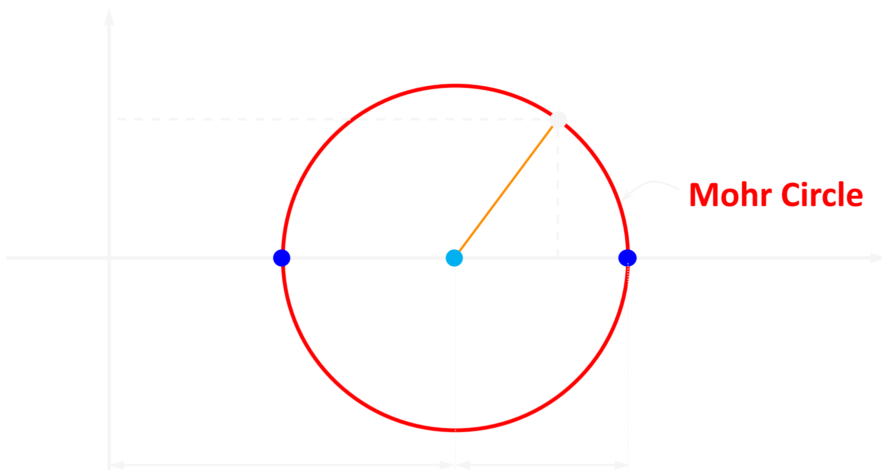
≡

There **always exists** an angle α that satisfies $\tau = 0$. Such that:

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$$

- These stresses are the so-called **principal stresses**. We named them so that $\sigma_1 \leq \sigma_2 \leq \sigma_3$.
- σ_1 is the **major pincipal stress** and σ_3 is the **minor principal stress**
- In plane strain conditions, the stresses acting on the plane of interest are σ_1 and σ_3. σ_2 acts on the direction perpendicular to the plane.

For **2D stresses**, the state of stress can be represented using Mohr's circle



- $\sigma_\alpha = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_3}{2} \cos 2\alpha$
- $\tau_\alpha = \frac{\sigma_1 - \sigma_3}{2} \sin 2\alpha$

≡

Are stresses that do not change depending on the orientation. For example:

- $\frac{\sigma_1+\sigma_3}{2}$ = the mean stress. Note that the mean stress that matters is the 3D mean stress $p = \sigma_1 + \sigma_2 + \sigma_3/3$.
- $\sigma_d = \sigma_1 - \sigma_3$ the deviator stress. **This is what causes failure**.
- The principal stresses.

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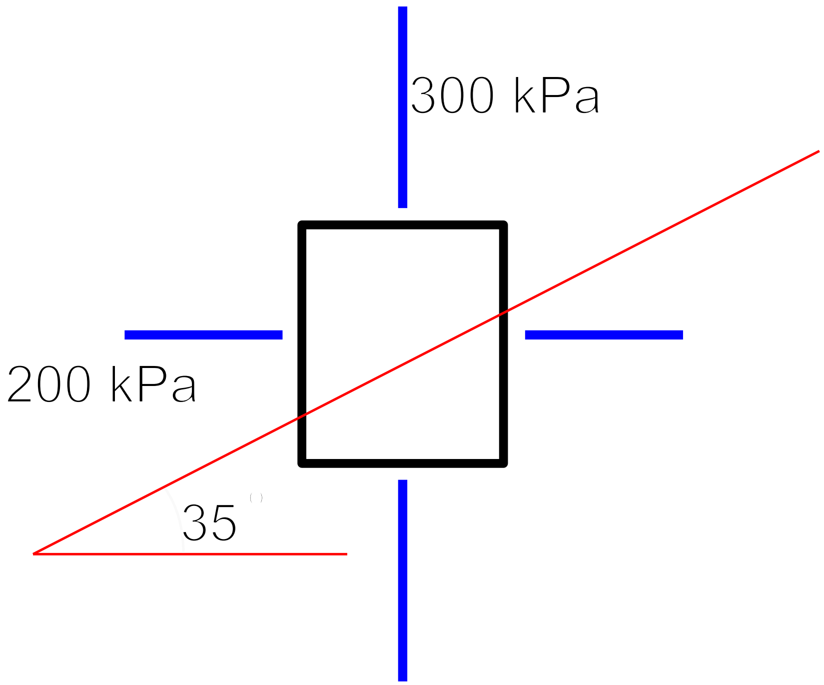
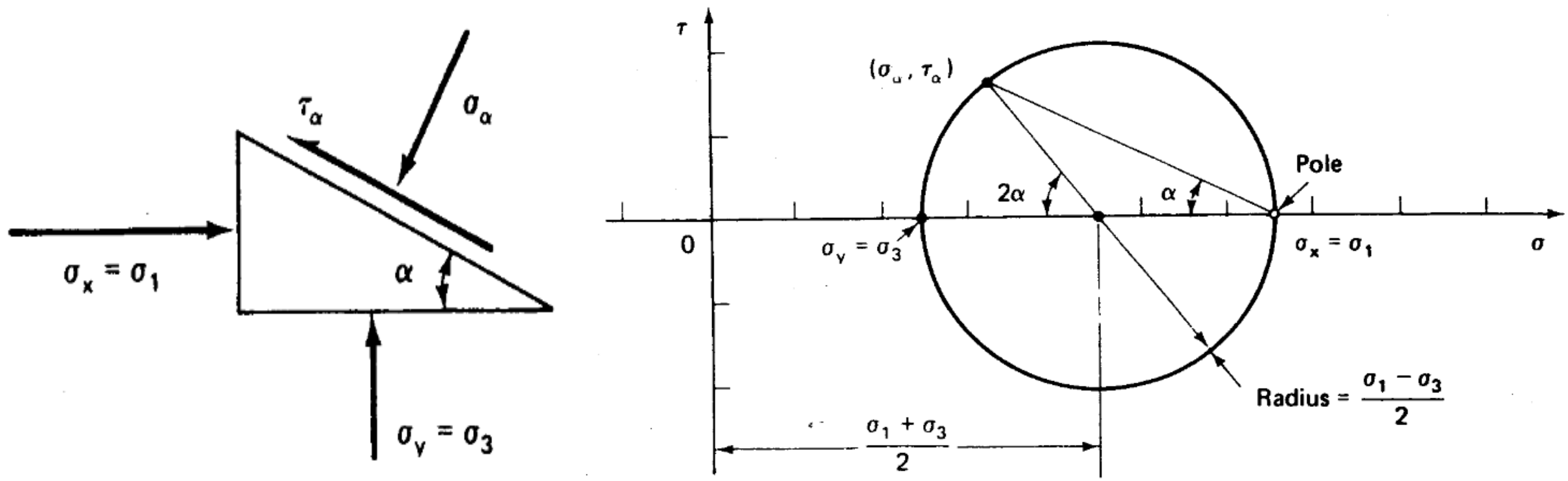
For the element in the figure find the normal and shear stress on a plane inclined at $\alpha = 35^o$ from the horizontal reference plane.



One useful graphical consequence of Mohr's circle is the existence of the **pole**. It links the **physical angular coordinate system** and **Mohr's circle angular coordinate system**



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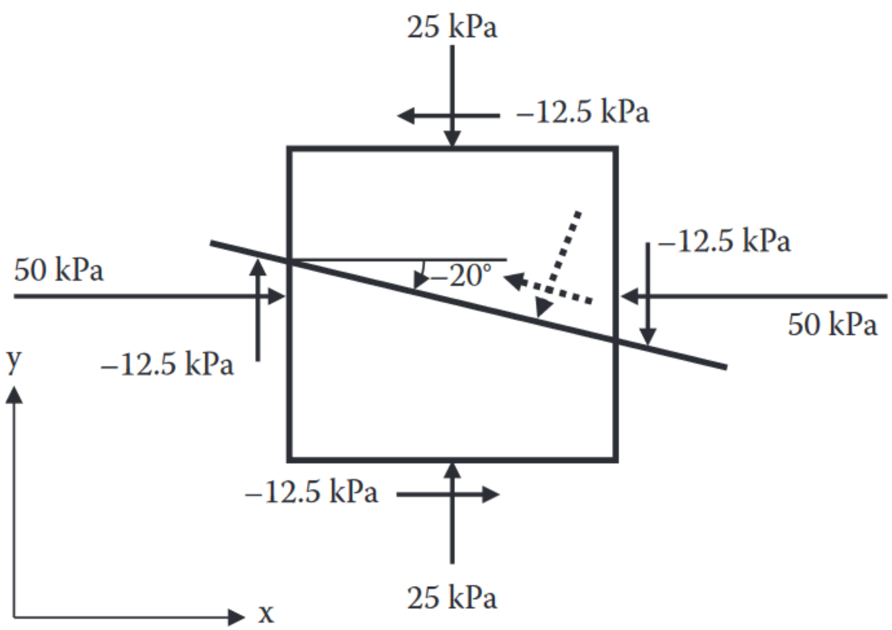
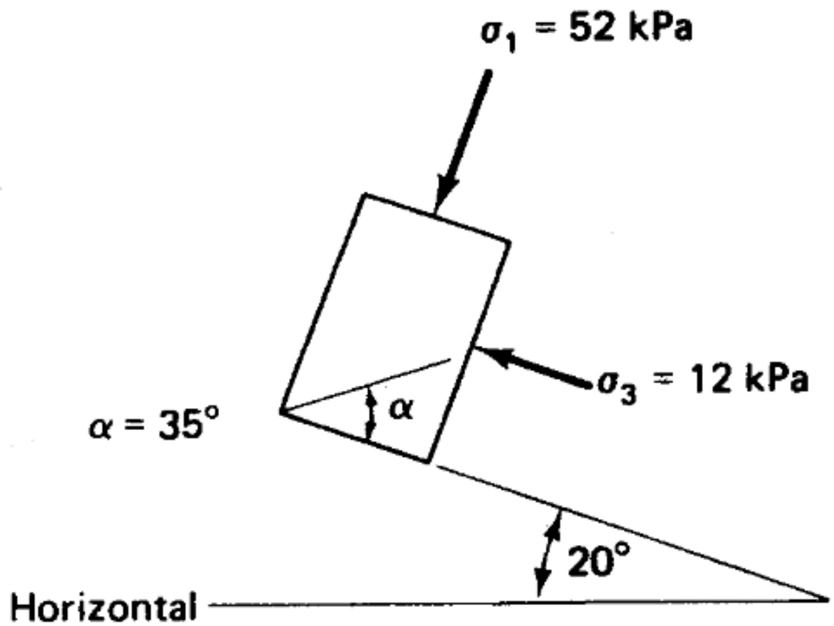
For the element in the figure below, find the principal stresses and their orientation using the equations and the Pole's method. Find the stresses acting at the indicated angle.



Same as Example 7.2 for the figure below



(a)

A simplistic, but fundamental model relating stress-strain behavior.

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = 1/E \begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ -\nu & 1 & -\nu & 0 & 0 & 0 \\ -\nu & -\nu & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2(1+\nu) & 0 & 0 \\ 0 & 0 & 0 & 0 & 2(1+\nu) & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(1+\nu) \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix}$$

Using linear elasticity we can find an **elastic solution for $K_0 = \nu/(1-\nu)$** (more after shear strength)

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For the soil profile shown in the figure below plot the profile of in-situ vertical and horizontal total and effective stresses. Plot the Mohr's circle of stresses for point A