How Volatility Dynamics Change Path-Dependent Prices

Executive Summary

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November 7, 2025

This project investigates how different volatility dynamics like constant (GBM) and time-varying (GARCH(1,1)) affect the pricing of arithmetic Asian call options. Using historical SPY data from `yfinance`, both models were calibrated and used to generate Monte Carlo price paths under the risk-neutral assumption.

An arithmetic Asian call option payoff is

$$\text{Payoff} = \max(\bar{S} - K, 0), \quad \bar{S} = \frac{1}{N}\sum_{i=1}^{N} S_{t_i}.$$

Under the GBM model, prices evolve with a constant volatility σ as

$$S_t = S_0 \exp[(r - \frac{1}{2}\sigma^2)t + \sigma\mathcal{N}(0,1)]$$

Under the GARCH(1,1) model, volatility evolves dynamically as

$$r_t = \mu + \epsilon_t, \quad \epsilon_t \sim \mathcal{N}(0, h_t)$$

$$h_t = \omega + \alpha\epsilon_{t-1}^2 + \beta h_{t-1}$$

Each model simulated 100,000 price paths with daily steps over $T = 0.5$years. The Asian option price was estimated by the discounted Monte Carlo mean

$$P = e^{-rT}\mathbb{E}\left[\max(\bar{S} - K, 0)\right]$$

Results and Market Implications

- The GBM model consistently produced higher Asian call option prices because constant volatility amplifies dispersion in the average price.
- GARCH(1,1) generated smaller prices since volatility clustering and mean revision reduce the effective variance of the path average.
- The GBM model is ideal for quoting and hedging because it is fast, stable and compatible with implied-volatility surfaces.
- GARCH(1,1) captures realistic volatility clustering, which is valuable for understanding of volatility behavior, forcasting and refining risk estimates.