

How Volatility Dynamics Change Path-Dependent Prices

Arithmetic Asian Call Options

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November 7, 2025

Arithmetic Asian Call Options

- ▶ Derivative-based financial asset.
- ▶ Payoff depends on the average price.

$$\text{Payoff} = \max(\bar{S} - K, 0), \quad \bar{S} = \frac{1}{T} \int_0^T S_t dt$$

- ▶ Path-dependent! Averaging reduces sensitivity to volatility spikes.
- ▶ Less volatility \rightarrow Less expensive.
- ▶ In class: Stock paths follow Geometric Brownian Motion (GBM)
- ▶ Are there other stock path models?
 - ▶ Generalized Autoregressive Conditional Heteroskedasticity (GARCH)

The GARCH(1,1) Model

Why use this model?

- ▶ Models time-varying volatility.
- ▶ Volatility clustering.
- ▶ Simple recursion procedure!

Model Returns (r) Structure:

$$r_t = \mu_{\Delta t} + \epsilon_t, \quad \epsilon_t \sim \mathcal{N}(0, h_t)$$

$$h_t = \omega + \alpha \epsilon_{t-1}^2 + \beta h_{t-1}$$

- ▶ $\mu_{\Delta t}$ expected return per day under the chosen measure.
- ▶ ϵ_t = random innovation (shock).
- ▶ h_t = conditional volatility.
- ▶ ω, α, β = free parameters.

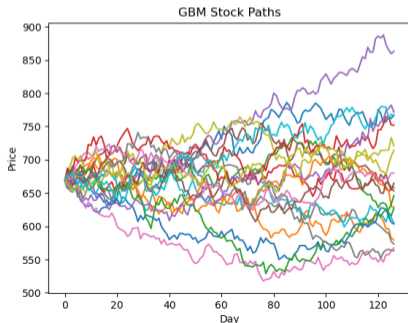
Data and parameters

- Used SPY historical data

- GARCH(1,1) params. by maximize

$$\ell = -\frac{1}{2} \sum_{t=1}^T \left[\log(2\pi) + \log(h_t) + \frac{(r_t - \mu)^2}{h_t} \right]$$

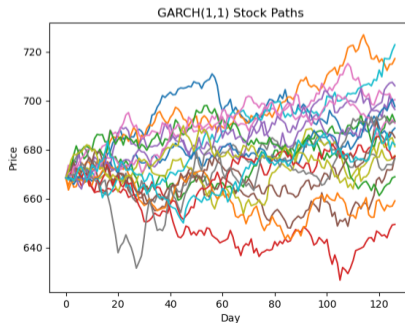
$$S_t = S_{t-1} e^{(r - \frac{1}{2}\sigma^2)dt + \sigma\sqrt{dt}\mathcal{N}(0,1)}$$



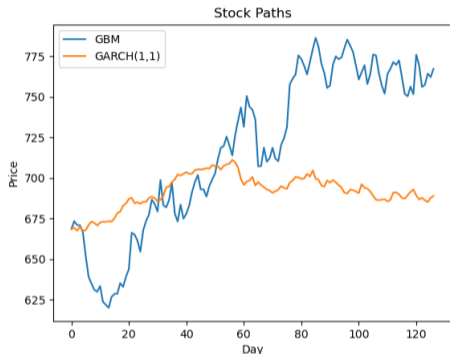
Parameters

| GBM | $\sigma = 0.179$ | | | |
|------------|--------------------|----------|---------|--------------------|
| GARCH(1,1) | ω | α | β | μ |
| | 1×10^{-6} | 0.05 | 0.85 | 5×10^{-4} |

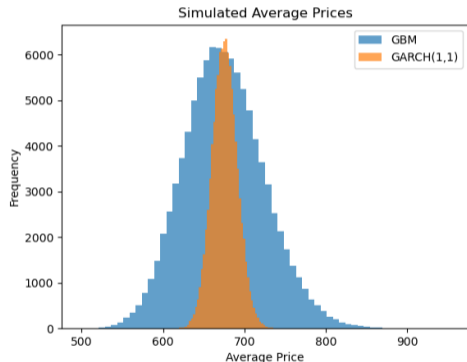
$$S_t = S_{t-1} e^{\mu\Delta t + \sqrt{h_t}\mathcal{N}(0,1)}$$



Market-Maker Perspective GBM vs GARCH(1,1)



- ▶ GBM path has uniform volatility.
- ▶ GARCH(1,1) shows clustered volatility.
- ▶ Averaging smooths out clustered volatility spikes.



Trading Implications

| Arithmetic Asian Call Option Prices | | |
|-------------------------------------|-------------|----------------|
| | Price (USD) | Error (95% CL) |
| GBM | 22.74 | 0.20 |
| GARCH(1,1) | 10.14 | 0.07 |

- ▶ GBM (Black-Scholes) → Used for quoting and hedging.
- ▶ GARCH(1,1) → Realistic and risk assessment.

