Matrix Multiplication

$$\text{If } A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \qquad B = \begin{bmatrix} p & q & r \\ s & t & u \\ v & w & x \end{bmatrix}$$

To find any entry ij in the product matrix, multiply numbers in row i in matrix A and column j in matrix B, and add the terms.

Finding elements of first row of product matrix

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \qquad B = \begin{bmatrix} p & q & r \\ s & t & u \\ v & w & x \end{bmatrix}$$

$$AB = \begin{bmatrix} a*p + b*s + c*v & a*q + b*t + c*w & a*r + b*u + c*x \\ & & \end{bmatrix}$$

1st row of A with 1st column of B

1st row of A with 2nd column of B

1st row of A with 3rd column of B

Finding elements of second row of product matrix

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \qquad B = \begin{bmatrix} p & q & r \\ s & t & u \\ v & w & x \end{bmatrix}$$

$$AB = \begin{bmatrix} a*p + b*s + c*v & a*q + b*t + c*w & a*r + b*u + c*x \\ d*p + e*s + f*v & d*q + e*t + f*w & d*r + e*u + f*x \end{bmatrix}$$

2nd row of A with 1st column of B

2nd row of A with 2nd column of B

2nd row of A with 3rd column of B

Finding elements of third row of product matrix

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \qquad B = \begin{bmatrix} p & q & r \\ s & t & u \\ v & w & x \end{bmatrix}$$

3rd row of A with 1st column of B; 3rd row of A with 2nd column of B; 3rd row of A with 3rd column of B

$$AB = \begin{bmatrix} a*p + b*s + c*v & a*q + b*t + c*w & a*r + b*u + c*x \\ d*p + e*s + f*v & d*q + e*t + f*w & d*r + e*u + f*x \\ g*p + h*s + i*v & g*q + h*t + i*w & g*r + h*u + i*x \end{bmatrix}$$

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Finding square of adjacency matrix given in the example.

$$A = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$A^2 = A \times A = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \quad X \quad \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$A^2 = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0*0 + 1*0 + 0*0 + 1*0 & 0*1 + 1*0 + 0*0 + 1*0 & 0*0 + 1*1 + 0*0 + 1*1 & 0*1 + 1*1 + 0*0 + 1*0 \\ 0*0 + 0*0 + 1*0 + 1*0 & 0*1 + 0*0 + 1*0 + 1*0 & 0*0 + 0*1 + 1*0 + 1*1 & 0*1 + 0*1 + 1*0 + 1*0 \\ 0*0 + 0*0 + 0*0 + 0*0 & 0*1 + 0*0 + 0*0 + 0*0 & 0*0 + 0*1 + 0*0 + 0*1 & 0*1 + 0*1 + 0*0 + 0*0 \\ 0*0 + 0*0 + 1*0 + 0*0 & 0*1 + 0*0 + 1*0 + 0*0 & 0*0 + 0*1 + 1*0 + 0*1 & 0*1 + 0*1 + 1*0 + 0*0 \end{bmatrix} \end{array}$$

$$A^2 = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & 0 & 2 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$