Circuit Theory and Electronics Fundamentals

Engineering Physics
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Lab 1: Circuit Analysis Methods

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Contents

1 Introduction

The objective of this laboratory assignment is to study a circuit containing eight nodes numbered from 1 to 7, plus GND. In the present circuit, we can find seven resistors, R_1 to R_7, one voltage source, V_a, one current source, I_d, one voltage controlled current source, I_b and, finally, one current controlled voltage source, V_c, connected as seen in Figure 1.

In Section 2, we present a theoretical analysis of the circuit using the mesh method, as well as the nodal method and, in Section 3, we compare these results to the ones obtained by simulation using Ngspice. Furthermore, in Section 4, we summarise the results of the simulations and their agreement with the theoretical predictions.

2 Theoretical Analysis

In this section, the circuit shown in Figure 1 is analysed theoretically, using mesh and nodal methods.

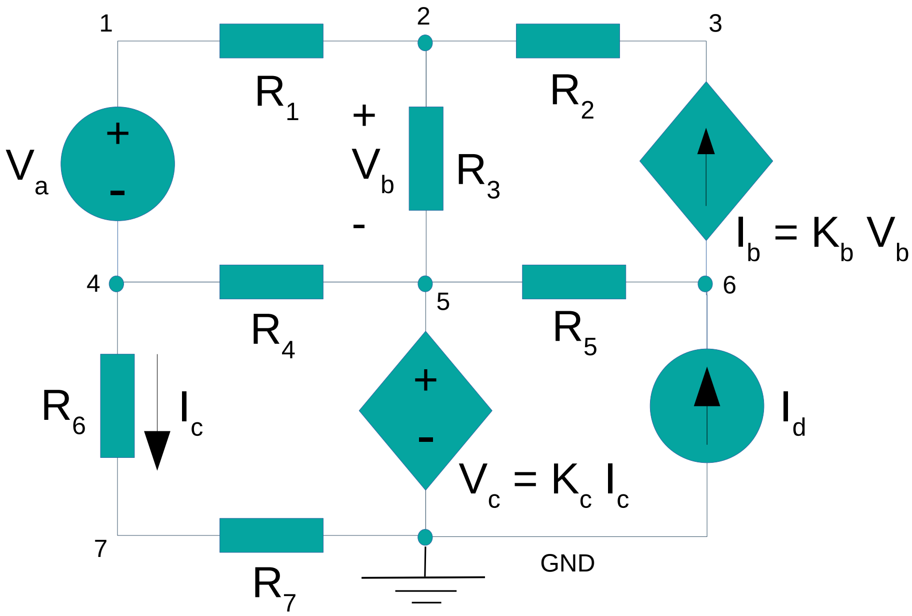

Figure 1: Electric circuit.

2.1 Mesh Analysis

Mesh analysis is a well-organized method for solving a circuit, which makes use of Kirchhoff's Voltage Law and Ohm's Law to deduce a set of equations guaranteed to be solvable if the circuit has a solution.

In order to solve the circuit in study resorting to this method, we started by arbitrarily assigning mesh currents in the essential meshes, I_1 and I_3 looping in a clockwise direction and I_2 and I_4 in an anti-clockwise direction, as shown in the Figure 2.

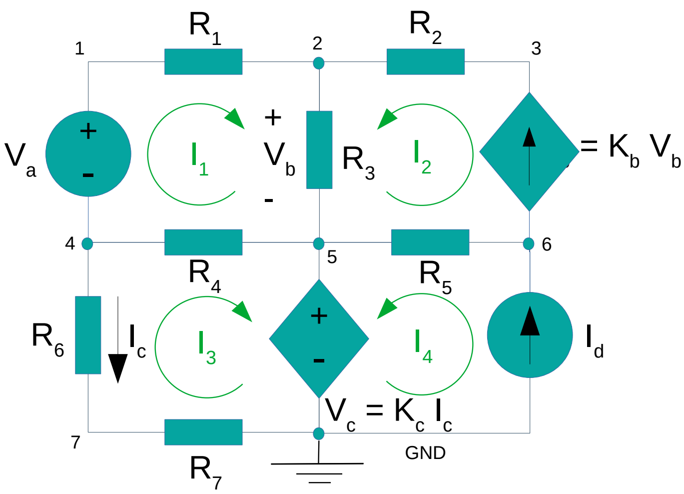

Figure 2: Mesh Analysis.

Hence, applying KVL to meshes 1, 2 and 3 and equaling I_4 to current source value I_d, we deduced the following system of linear equations, whose solution is the set of assigned currents:

$$\begin{bmatrix} K_bR_3 & K_bR_3-1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ R_1+R_3+R_4 & R_3 & -R_4 & 0 \\ -R_4 & 0 & R_4+R_6+R_7-K_c & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 0 \\ I_d \\ V_a \\ 0 \end{bmatrix}$$

The solution vector for this system was obtained using the Octave maths tool and is presented in Table 1:

Name	Value [A]
I1	2.337076849703084E-01
I2	-2.448213188275633E-01
I3	-9.915169208779064E-01
I4	1.018900832110000E+00

Table 1: Solution vector for assigned currents.

From these data, we were able to determine the rest of the circuit's characteristics, which are shown in Table 2:

Name	Value [A or V]
@gb[i]	-2.448213188275633E-01
@id[current]	1.018900832110000E+00
@r1[i]	2.337076849703084E-01
@r2[i]	2.448213188275633E-01
@r3[i]	-1.111363385725495E-02
@r4[i]	-1.225224605848215E+00
@r5[i]	-1.263722150937563E+00
@r6[i]	9.915169208779064E-01
@r7[i]	9.915169208779064E-01
n1	8.364839171063712E+00
n2	8.130067164915477E+00
n3	7.629171867647544E+00
n4	3.118567456673712E+00
n5	8.164424366367525E+00
n6	1.197105789448569E+01
n7	1.040218654656905E+00

Table 2: Circuit's characteristics using mesh method. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

2.2 Node Analysis

Similarly, node analysis is used to deduce a set of equations to determine the nodes potential in an electric circuit, this time using Kirchhoff's Current Law and Ohm's Law.

In order to solve the circuit in study resorting to this method, we applied KCL in nodes not connected to voltage sources, 2, 3, 6 and 7, and wrote additional equations for nodes related by voltage sources, 1, 4 and 5. This way, we were able to write 6 linearly independent equations, for what we needed one more to determine all the seven variables (excluding GND). Therefore, we managed to relate nodes 5 and GND using KCL to get the last equation needed, and obtained the following system of linear equations, whose solution is the set of nodes potentials:

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{K_c}{R_6} & 1 & 0 & \frac{K_c}{R_6} \\ \frac{1}{R_1} & -\frac{1}{R_1}-\frac{1}{R_2}-\frac{1}{R_3} & \frac{1}{R_2} & 0 & \frac{1}{R_3} & 0 & 0 \\ 0 & -\frac{1}{R_2}-K_b & \frac{1}{R_2} & 0 & K_b & 0 & 0 \\ 0 & -K_b & 0 & 0 & \frac{1}{R_5}+K_b & -\frac{1}{R_5} & 0 \\ 0 & 0 & 0 & \frac{1}{R_6} & 0 & 0 & -\frac{1}{R_6}-\frac{1}{R_7} \\ 0 & \frac{1}{R_3} & 0 & \frac{1}{R_4} & -\frac{1}{R_3}-\frac{1}{R_4}-\frac{1}{R_5} & \frac{1}{R_5} & \frac{1}{R_7} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \end{bmatrix} = \begin{bmatrix} V_a \\ 0 \\ 0 \\ 0 \\ -I_d \\ 0 \\ I_d \end{bmatrix}$$

Since GND potential isn't an unknown variable, taking the value 0, we excluded it from this system, in order to simplify the calculations.

Once again, the solution vector for this system was obtained using Octave. The results are shown in Table 4:

Name	Value [V]
V1	8.364839171063709E+00
V2	8.130067164915467E+00
V3	7.629171867647522E+00
V4	3.118567456673709E+00
V5	8.164424366367514E+00
V6	1.197105789448569E+01
V7	1.040218654656904E+00

Table 3: Solution vector for nodes' potentials.

Like above, we were able to calculate the rest of the circuit's characteristics, which are presented in Table 4:

Name	Value [A or V]
@gb[i]	-2.448213188275692E-01
@id[current]	1.018900832110000E+00
@r1[i]	2.337076849703159E-01
@r2[i]	2.448213188275688E-01
@r3[i]	-1.111363385725521E-02
@r4[i]	-1.225224605848213E+00
@r5[i]	-1.263722150937564E+00
@r6[i]	9.915169208779054E-01
@r7[i]	9.915169208779052E-01
n1	8.364839171063709E+00
n2	8.130067164915467E+00
n3	7.629171867647522E+00
n4	3.118567456673709E+00
n5	8.164424366367514E+00
n6	1.197105789448569E+01
n7	1.040218654656904E+00

Table 4: Circuit's characteristics using nodal method. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

3 Simulation Analysis

In addition to theoretical analysis, we computed the characteristics of the circuit using Ngspice. The simulated operating point results are then shown in Table 5.

Name	Value [A or V]
@gb[i]	-2.448213188275818e-01
@id[current]	1.018900832110000e+00
@r1[i]	2.337076849703071e-01
@r2[i]	2.448213188275788e-01
@r3[i]	-1.111363385725579e-02
@r4[i]	-1.225224605848216e+00
@r5[i]	-1.263722150937587e+00
@r6[i]	9.915169208779069e-01
@r7[i]	9.915169208779065e-01
n1	8.364839171063712e+00
n2	8.130067164915479e+00
n3	7.629171867647514e+00
n4	3.118567456673713e+00
n5	8.164424366367529e+00
n6	1.197105789448577e+01
n7	1.040218654656905e+00
n8	1.040218654656905e+00

Table 5: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

Taking into account that the voltage source V_c is controlled by the current I_c, which flows through resistor R_6, and Ngspice requires a voltage source with current I_c to implement this component, we considered an additional voltage source of $0V$ in series with the resistor R_6, working as an amperemeter measuring the aforementioned current (Table 3). That's the reason an additional node (8) appears in the table above.

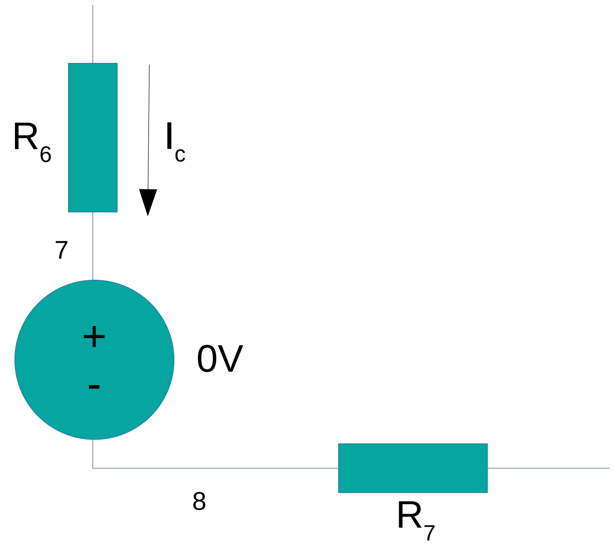

Figure 3: Additional voltage source.

Compared to the theoretical analysis results, one notices that the values only differ from the 13th decimal place onwards for both mesh and nodal method. Taking into account that the previously known characteristics of the circuit only had 11 decimal places, it's clear that these results match with great precision with the ones obtained by simulation.

4 Conclusion

In this laboratory assignment, the objective of analysing an electric circuit has been achieved. Its characteristics were determined theoretically, resorting to mesh and nodal methods and using the Octave maths tool, as well as simulated using Ngspice. The simulation results matched the theoretical ones precisely, which can be explained by the fact that this is a straightforward circuit containing only linear components.