CS/COE 0447

Data Representation: Binary, Hexadecimal, and Arbitrariness

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Binary

How to convert to and from Base-2 numeral systems

Positional Number Systems

- The numbers we use are written **positionally**: the position of a digit within the number has a meaning.

Most Significant   Least Significant

$$1{,}234 = \begin{array}{l} 1 \times 10^3 + \\ 2 \times 10^2 + \\ 3 \times 10^1 + \\ 4 \times 10^0 \end{array}$$

1000s	100s	10s	1s
10^3	10^2	10^1	10^0

- How many digit **symbols** do we have in our number system?
  - 10: 0, 1, 2, 3, 4, 5 ,6 ,7, 8, 9

Ranges of Representation

- Suppose we have a 4-digit numeric display.
- What is the smallest number it can show?
- What is the biggest number it can show?
- How many *different* numbers can it show?
  - 9999 - 0 + 1 = 10,000
- What power of 10 is 10,000?
  - 10^4
- **With n digits**:
  - We can **represent** 10^n numbers
  - The **largest number** is 10^n-1

Numeric Bases

- These 10s keep popping up… and for good reason
- We use a **base-10 (decimal)** numbering system
  - 10 different digits, and each place is a power of 10
- But we can use (almost) any number as a base!
- The most common bases when dealing with computers are **base-2 (binary)** and **base-16 (hexadecimal)**
- When dealing with multiple bases, you can write the base as a subscript to be explicit about it:

$$5_{10} = 101_2$$

Let's make a base-2 system

- Given base **B**,
  - There are **B** digit symbols
  - Each place is worth $\mathbf{B^i}$, starting with **i = 0** on the right
  - Given **n** digits,
    - You can represent $\mathbf{B^n}$ numbers
    - The largest representable number is $\mathbf{B^n - 1}$
- So how about base-2?

Binary (base-2)

- We call a **B**inary dig**IT** a **bit** – a single 1 or 0
- When we say an *n*-bit number, we mean one with *n* binary digits

MSB							LSB
1	0	0	1	0	1	1	0
2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
128s	64s	32s	16s	8s	4s	2s	1s

$$1001\ 0110 = 1 \times 128 + 0 \times 64 + 0 \times 32 + 1 \times 16 + 0 \times 8 + 1 \times 4 + 1 \times 2 + 0 \times 1 = 150_{10}$$

To convert binary to decimal: ignore 0s, add up place values wherever you see a 1.

Making Change

- You want to give someone $9.63 in change, using the fewest bills and coins possible. How do you count it out?

$5×1 $1×4 25¢×2 10¢×1 5¢×0 1¢×3

Left: $9.63 -$5= $4.63 -$4= $0.63 -50¢= $0.13 -10¢= $0.03 -0¢= $0.03 -3¢= $0.00



- Biggest to smallest
- Most significant to least significant
- WHERE COULD THIS BE GOING…

Converting Decimal to Binary

- You want to convert the number 83_{10} to binary.

128s	64s	32s	16s	8s	4s	2s	1s

Left: 83 - 0 = 83 - 64 = 19 - 0 = 19 - 16 = 3 - 0 = 3 - 0 = 3 - 2 = 1 - 1 = 0

0 1 0 1 0 0 1 1

- For each place from **MSB**:
- If place value < remainder:
  - digit = 1
  - remainder = remainder – place

64 < 83, therefore:

digit = 1

remainder = remainder–64 (83-64=19)

19 > 32, however, so:

digit = 0 (remainder stays the same)

Bits, bytes, nybbles, and words

- A ***bit*** is one binary digit, and its unit is **lowercase** b.
- A ***byte*** is an 8-bit value, and its unit is **UPPERCASE** B.
  - This is why your 30 mega*bit* (Mb/s) internet connection can only give you at most 3.75 mega*bytes* (MB) per second!
- A ***nybble*** (awww!) is 4 bits – half of a byte.
  - Corresponds nicely to a single hex digit.
- A ***word*** is the "most comfortable size" of number for a CPU.
- When we say "32-bit CPU," we mean its ***word*** size is 32 bits.
  - This means it can, for example, add two 32-bit numbers at once.
- **BUT WATCH OUT:**
  - Some things (Windows, x86) use **word** to mean **16 bits** and **double word** (or **dword**) to mean **32 bits.**

Why binary? *Whynary?*

- cause it's the easiest thing to implement. :P
- arithmetic also becomes really easy (as we'll see in several weeks)
- so, everything on a computer is represented in binary.
  - **everything.**
  - ***<u>EVERYTHING.</u>***
  - ***01000101 01010110 01000101 01010010 01011001 01010100 01001000 01001001 01001110 01000111 00101110***
    - ***(“EVERYTHING.”)***

Hexadecimal

How to convert to and from Base-16 numeral systems

Shortcomings of Binary/Decimal

- Binary numbers can get really long, really quickly.
  - **$3{,}927{,}664_{10}$ = $11\ 1011\ 1110\ 1110\ 0111\ 0000_2$**
- But nice "round" numbers in binary look arbitrary in decimal.
  - **1000000000000000_2 = $32{,}768_{10}$**
- This is because 10 is not a power of 2!
- We could use base-4, base-8, base-16, base-32, etc.
  - Base-4 is not much terser than binary
    - e.g. **$3{,}927{,}664_{10}$ = $120\ 3331\ 2323\ 0000_4$**
  - Base-32 would require 32 digit symbols. Yeesh.
    - They do, oddly, have their place… but not really in this context.
  - **Base-8** and **base-16** look promising!

Let's make a base-~~2~~ 16 system

- Given base **B**,
  - There are **B** digit symbols
  - Each place is worth $\mathbf{B^i}$, starting with **i = 0** on the right
  - Given **n** digits,
    - You can represent $\mathbf{B^n}$ numbers
    - The largest representable number is $\mathbf{B^n - 1}$
- So how about base-16?

Hexadecimal or “hex” (base-16)

- Digit symbols after 9 are A-F, meaning 10-15 respectively.
- Usually we call one hexadecimal digit a *hex digit*. No fancy name :(

$$\underset{16^7}{0}\;\underset{16^6}{0}\;\underset{16^5}{3}\;\underset{16^4}{B}\;\;\underset{16^3}{E}\;\underset{16^2}{E}\;\underset{16^1}{7}\;\underset{16^0}{0} =$$

$$\begin{aligned} &0 \times 16^7 + \\ &0 \times 16^6 + \\ &3 \times 16^5 + \\ &11 \times 16^4 + \\ &14 \times 16^3 + \\ &14 \times 16^2 + \\ &7 \times 16^1 + \\ &0 \times 16^0 = \\ &3{,}927{,}664_{10} \end{aligned}$$

To convert hex to decimal: use a dang calculator lol

Converting from Binary to Hex

- **Four bits** are equivalent to **one hex digit.**
- Converting between them is easy!
- Say we had this binary number:

1110111110111001110000_2

- Starting **from the LSB**, divide into groups of 4 bits (put 0s before the first digits if there are leftovers). Then use the table.

00 11 1011 1110 1110 0111 0000

0x3 B E E 7 0

(this is common notation for hex, derived from the C language.)

Bin	Hex	Bin	Hex
0000	0	1000	8
0001	1	1001	9
0010	2	1010	A
0011	3	1011	B
0100	4	1100	C
0101	5	1101	D
0110	6	1110	E
0111	7	1111	F

know this table.

Binary to Hex

0100 1100 1010 0010 0000 0010 0110 0001

4 C A 2 0 2 6 1

0x4CA20261

32-bits! (Not so bad…)

Base-8??

- base-8, **octal**, used to be commonplace but isn't anymore
- each octal digit (0-7) corresponds to *three* bits
  - this made it a nice fit for 9-, 12-, 18-, and 36-bit machines
- buuuut no one cares about octal anymore ☹
- SORRY OCTAL
  - it's okay, it has its revenge from time to time
  - try this out in Java sometime: (leading zeroes mean octal, yikes!)

```
System.out.println("012345 = " + 012345);
```

The Powers of Two

- Memorize **at least** the powers up to ~2^8 or 2^{10}.
  - If you can't remember one, just add the previous one to itself.
- These are the place values for binary, and they are also nice "round" numbers in binary and hex.
- What is the **largest number** that an 8-bit value can hold? What is that in hexadecimal?
  - 255: 0xFF
- How about a 16-bit value?
  - 65535: 0xFFFF
- "0xFFFF" is kinda like "9999" in decimal.
  - What happens if we go beyond that?

	Dec	Hex
2^0	1	0x1
2^1	2	0x2
2^2	4	0x4
2^3	8	0x8
2^4	16	0x10
2^5	32	0x20
2^6	64	0x40
2^7	128	0x80
2^8	256	0x100

Overflow

- In computers, **numbers are finite.**
- Let's say our 4-digit display was counting up: 9997, 9998, 9999...
- What comes "next"?
  - What does this "0000" *really mean?*
  - It **wrapped around.**
- This is **overflow:** the number you are *trying* to represent is **too big to be represented.**
- Essentially, **all arithmetic on the computer is modular arithmetic!**
  - This causes a *lot* of software bugs.

What do we dooo???

- if there's an overflow, that's bad right?
- there are basically three options:
  - **store**
  - **ignore**
  - fall on the **floor** and scream and cry about it
    - (i.e. crash the program)
      - (I* really struggled to come up with a rhyme)
- each of these has pros and cons
- we'll learn more about em later but keep them in your brainmeats

* Jarrett did this, not me

Summing it up (well, soon...)

- everything's in binary, and hex is convenient shorthand
- numbers don't really act like numbers?
- actually, how does the computer know a number is a number?
- how does it know how to add two numbers?
- how does it know how to manipulate strings (for instance, text)?
- how does it know if one pattern of bits is a string or a number or a video or a program or a file or an icon or

IT DOESN'T

Arbitrariness

How binary data is interpreted (or not. I mean who really knows?)

What is means to be “arbitrary”

- it means there's **no reason for it to be that way.**
- we just kinda ***agree*** that it's how things are.
- One of the biggest things I want you to know is:

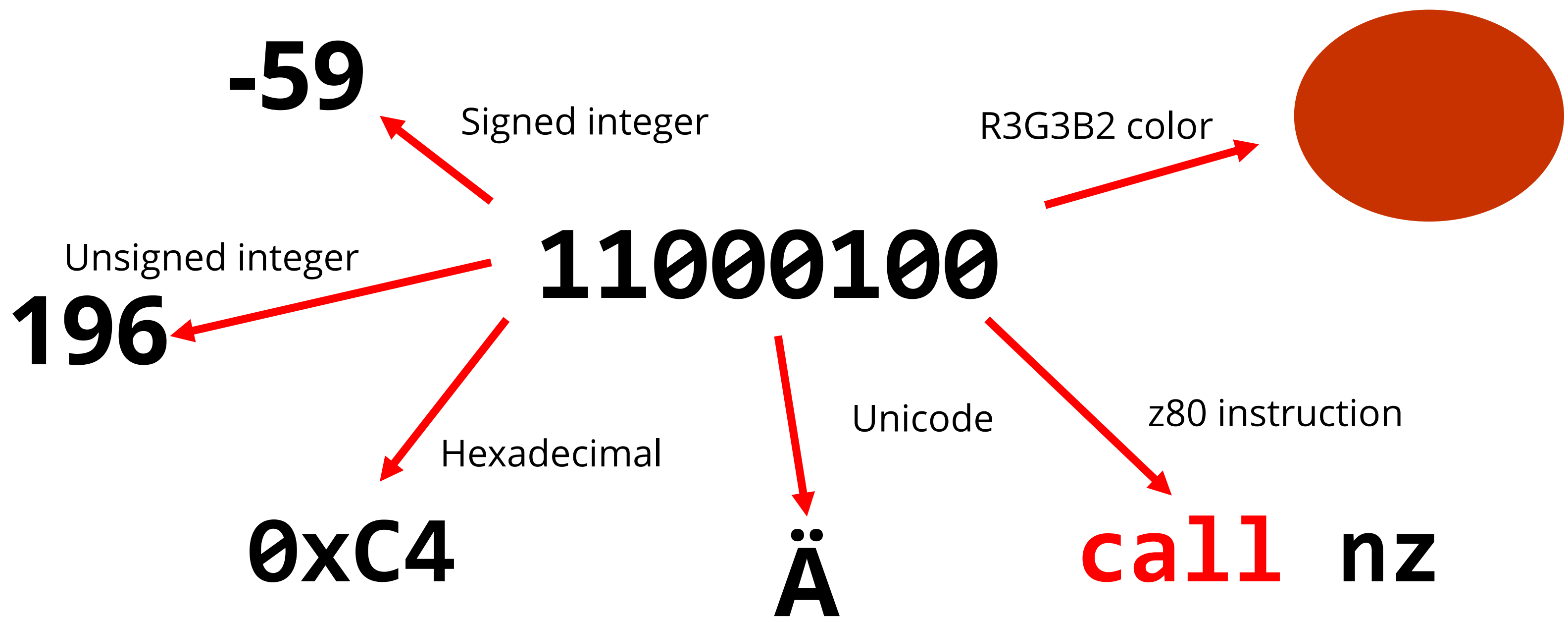
What a pattern of bits *means* is arbitrary.

- As a corollary:

The *same* pattern of bits can be *interpreted* many different ways.

One bit pattern, many meanings

- **All information** on computers is stored as patterns of bits, but…
- How these bits are **interpreted**, **transformed**, and **displayed** is up to the programmer and the user.



The computer doesn't know (or care)

- when writing assembly (and C!) programs, **the computer has no idea what you mean, cause nothing means anything to it**
- "my program assembles/compiles, why is it crashing?"
  - cause the computer is stupid
    - it's a big fast calculator
- there's no difference between nonsense code and useful code
- it's good at doing fun things with bit patterns
- but don't confuse what it does with intelligence
- **every "smart" thing a computer does, it does because a human programmed it to act like that**

Next time…

- Looking at how programs are represented
  - Speaking of arbitrary…
    - We will look at MIPS, tho, specifically
- First taste of Assembly
  - How values are stored, retrieved, manipulated.