Notes on LES Modeling and Numerical Method in LES3D-MP  
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January, 2010

## 1. Dynamic modelling

The dynamic model provides a methodology to determine an appropiate local value for to be used with a Smagorinsky type model. The underlying principle is to extract information via a double filtering operation in physical space (Germano). Two grid filters are now introduced: the grid filter and the test filter . The grid filter’s width is usually taken to be that of the grid spacing, , or for better resolution. The operation of grid filtering is denoted by an overbar, e.g.,

|  |  |  |
| --- | --- | --- |
|  |  | (1) |

The test filter, of larger width, is defined as (for instance ).

|  |  |  |
| --- | --- | --- |
|  |  | (2) |

Since is unknown in a LES calculation,it is more relevant to consider the test filter applied to , to yield the doubly filtered quantity . Thus, for both filters, the double-filtering operation can be written as

|  |  |  |
| --- | --- | --- |
|  |  | (3) |

Accordingly, the subtest-scale stresses and scalar flux appear which are paremetrized as

|  |  |  |
| --- | --- | --- |
|  |  | (4) |
|  |  | (5) |

In the dynamic SGS model, these are modeled in the same manner as the residual terms,

|  |  |  |
| --- | --- | --- |
|  |  | (6) |
|  |  | (7) |

Test filtering the residual stress tensor (eqn no) and subtracting it from subtest-scale stress tensor defines Germano’s identity. It yields the known Leonard subtest scale stress also known as the resolved (turbulent) stress. Similarly, subtracting the filtered residual heat flux vector from the subtest-scale scalar flux yields,

|  |  |  |
| --- | --- | --- |
|  |  | (8) |
|  |  | (9) |

Its main advantage is that it is known in terms of , whereas , , and aren’t. They are the contribution to the residual stress (and scalar flux) from the largest resolved motions (with respect to the test filter). By substituting the eddy viscosity and diffusivity models for and , one finds that the coefficient and are determined by relations,

|  |  |  |
| --- | --- | --- |
|  |  | (10) |
|  |  | (11) |

Similary the double filtered deformation tensor is defined and its characteristic counter part also as . Further, taking to be uniform, a scale similar tensor was previously defined and can be confirmed by re-expressing the Leonards stress and obtaining the following expression which containes : , yielding the final relationship,

|  |  |  |
| --- | --- | --- |
|  |  | (12) |

We may observe that both and are known in terms of , which can be used to determine for the present modelling technique. Also, a single coefficient cannot be chosen to match 5 independent equations. Therefore, a mean square error is minimized. Following Lilly the square of the residual is required to be minimal and an equation for the local value of is:

|  |  |  |
| --- | --- | --- |
|  |  | (13) |

With this procedure the eddy viscosity tends to zero near the wall without the use of a Van Driest type of damping function. In addition, can take on negative values and apparently capture the effects of backscattering. Similarly, the eddy diffusivity model coefficient is now computed based on similar formulation,

|  |  |  |
| --- | --- | --- |
|  |  | (14) |

where the coefficient is averaged using Lagrangian averaging and is limited to positive values.

# 1. Extension to variable density

This section includes the principles and methodology to account for the effects of variable density in LES3D-MP. It is largely based on the work carried out by Dr. Charles Pierce PhD Thesis at Stanford University, 2001. Recall that in this work, filtering is implicitely defined by the computational grid used for the large-scale equations and that it is not invoked explicitely. Quantities per unit volume are treated using the Reynold’s decompositon,

|  |  |  |
| --- | --- | --- |
|  |  | (15) |

while quantities per unit mass are best described by a Favre (density weighted) decomposition,

|  |  |  |
| --- | --- | --- |
|  |  | (16) |

where

|  |  |  |
| --- | --- | --- |
|  |  | (17) |

## 1. Governing Equations and Filtering

With Favre decomposition, filtered variables represent “mixed mean” averages over subgrid volumes. This ensures that the filtering process does not alter the form of the conservation laws. Applying this procedures to the working equations, the Favre LES equations are now written as:

Continuity:

|  |  |  |
| --- | --- | --- |
|  |  | (18) |

Momentum:

|  |  |  |
| --- | --- | --- |
|  |  | (19) |

Using the decomposition , where the first term is the LES grid resolved convective flux and the second term the corresponding SGS contribution. The filtered viscous stress tensor is modeled as:

|  |  |  |
| --- | --- | --- |
|  |  | (20) |

where . It is also costumary (in the literature) to define the stress tensor as , where now the deviatoric deformation tensor is defined as . Incorporating the decomposition yields the following momentum equations,

|  |  |  |
| --- | --- | --- |
|  |  | (21) |

where are the same terms that must be modeled.

Scalar Transport:

|  |  |  |
| --- | --- | --- |
|  |  | (22) |

Again, we can use the standard LES decomposition: , where the first term is the LES grid resolved convective flux of scalar transport and the second term the corresponding SGS contribution. We approximate , where , which finally yields the following filtered LES, Favre equation:

|  |  |  |
| --- | --- | --- |
|  |  | (23) |

where,

State relation:

|  |  |  |
| --- | --- | --- |
|  |  | (24) |

## 2. Dynamic Modeling

The extension to variable density of the SGS dynamic modeling is presented by inspection of the turbulent (and scalar flux) stress constituents

|  |  |  |
| --- | --- | --- |
|  |  | (25) |

The anisotropic part is modeled based on a eddy-viscosity concept:

|  |  |  |
| --- | --- | --- |
|  |  | (26) |

Invoking the classical Smagorinsky model, where is the model coefficient that is dynamically computed from the LES solution

|  |  |  |
| --- | --- | --- |
|  |  | (27) |

The anisotropic part is as follows,

|  |  |  |
| --- | --- | --- |
|  |  | (28) |

The isotropic constituent of the SGS stress tensor is typically modeled based on the Yoshiwaza relation (Yoshiwaza, 1986). From which , we have

|  |  |  |
| --- | --- | --- |
|  |  | (29) |

and the entire modeled term becomes

|  |  |  |
| --- | --- | --- |
|  |  | (30) |

where is a model coefficient. Erlebacher et al, neglected the isotropic term on the grouds that it is negligible compared to the thermodynamic pressure. Pierce, (PhD Thesis) also left this term out since he decoupled pressure from the thermodynamic variables. However, Moin et al modeled this term and presented excellent results (Moin, POF 1991) in comparison DNS data. Furthermore, researchers in the past have use a combination of both and in the Smagorinsky model (ie., Martin and Piomelli 2000, Moin 1992, Elechbacher 1992, T.Ma 2007). However, recent papers published by Pierce and Moin suggest to use the deviatoric definition for both. Currently, a low-mach number formulation is utilized which neglects acoustic interactions and compressibility effects. This present approach therefore leaves out the isotropic term and the modeling becomes:

|  |  |  |
| --- | --- | --- |
|  |  | (31) |

In order to close the momentum equations, must be modeled: (Using )

|  |  |  |
| --- | --- | --- |
|  |  | (32) |
|  |  | (33) |

Utilization of the spectral data is performed with the introduciton of the test filter in the resolved field. This has a larger width than the resolved grid filter, generating a region with larger scales which can be used dynamically. This width is denoted as . Therefore, the test filtered stresses are defined as:

|  |  |  |
| --- | --- | --- |
|  |  | (34) |

Using Germano’s identity, the Leonard stresses can be expressed in terms of and

|  |  |  |
| --- | --- | --- |
|  |  | (35) |

This identity is useful because it provides a results (rhs) which can be obtained from the filtered variables. Knowing and reexpressing it interm of its modeled terms will yield a useful expression for the dynamic coefficient . The anisotropic part of the Leonard stress tensor therefore becomes:

|  |  |  |
| --- | --- | --- |
|  |  | (36) |

where

|  |  |  |
| --- | --- | --- |
|  |  |  |

This equation corresponds to 5 independent relations for making it overspecified. Therefore, a least squares approach is followed to calculate the model coefficients in analogy to the incompressible case.

|  |  |  |
| --- | --- | --- |
|  |  | (37) |

### 1. SGS scalar flux

A similar solution is obtained through the use of the eddy diffusivity concept to model the SGS heat flux, .

|  |  |  |
| --- | --- | --- |
|  |  | (38) |

where . Reynolds and Favre averaging yields the following model terms:

|  |  |  |
| --- | --- | --- |
|  |  | (39) |

The subtest scalar flux now takes the following form:

|  |  |  |
| --- | --- | --- |
|  |  | (40) |

Through the use of Germano’s identity we obtain the scalar Leonards tensor and the Modeling tensor as: **[Trial mode]**Similarly, to the incompressible version we can solve this overdefined system through the least-squares method for **[Trial mode]** **[Trial mode]**