Notes on LES Modeling and Numerical Method in LES3D-MP  
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## 1. SGS scalar flux

A similar solution is obtained through the use of the eddy diffusivity concept to model the SGS heat flux, .

|  |  |  |
| --- | --- | --- |
|  |  | (1) |

where . Reynolds and Favre averaging yields the following model terms:

|  |  |  |
| --- | --- | --- |
|  |  | (2) |

The subtest scalar flux now takes the following form:

|  |  |  |
| --- | --- | --- |
|  |  | (3) |

Through the use of Germano’s identity we obtain the scalar Leonards tensor and the Modeling tensor as:

|  |  |  |
| --- | --- | --- |
|  |  | (4) |
|  |  | (5) |

Similarly, to the incompressible version we can solve this overdefined system through the least-squares method for

|  |  |  |
| --- | --- | --- |
|  |  | (6) |

# 1. Numerical Method

## 1. Runge Kutta

The three stage, Runge Kutta algorithm is now presented as applied to a fractional step method. An implicit second-order Crank-Nicholson method is used for wall normal diffusion while the other terms are advanced with the Runge Kutta method. The three steps (k=1,2,3) fractional step, time advancement is therefore:

|  |  |  |
| --- | --- | --- |
|  |  | (7) |

|  |  |  |
| --- | --- | --- |
|  |  | (8) |

|  |  |  |
| --- | --- | --- |
|  |  | (9) |

|  |  |  |
| --- | --- | --- |
|  |  | (10) |

where the explicit terms, , are

|  |  |  |
| --- | --- | --- |
|  |  | (11) |

At the first substep, when k=1, , and at the end of the third substep, the solution is . The coefficients of the time advancement scheme are

This time advancement scheme (withouth the additional LES cross-terms) is stable under the following conditions:

|  |  |  |
| --- | --- | --- |
|  |  | (12) |
|  |  | (13) |

where for and for . When the LES cross terms are included, the limit on the timestep is lower.

Spatial derivatives are approximated by a second order central differences on a staggered grid. The choice of the staggered grid has the advantage of being conservative. On the other hand, the use of a staggered grid induces additional complications when defining boundaries.

## 2. Discrete Equation

The governing equations are discretized using a finite differences approach. Velocity components are staggered with respect to pressure in both space and time. The density is co-located at the pressure points. Thus, the calculation of the mass flux (momentum per unit volume) requires spatial interpolation and is given the symbol . The density is staggered in time, so that effectively the density is calculated at time . Conversion between and is accomplished using the following

|  |  |  |
| --- | --- | --- |
|  |  | (14) |

The fully discrete equations are now presented in the following form,

Continuity:

|  |  |  |
| --- | --- | --- |
|  |  | (15) |

Momentum:

|  |  |  |
| --- | --- | --- |
|  |  | (16) |

Scalar Transport:

|  |  |  |
| --- | --- | --- |
|  |  | (17) |

For clarity the above compact notation can be expanded in more conventional form. For instance, the continuity equation in two dimensions for a staggered cell in time-space would expanded as,

|  |  |  |
| --- | --- | --- |
|  |  | (18) |

Generalizing the above equations for non-uniform grids (stretched), the interpolation and differencing operators are defined as,

|  |  |  |
| --- | --- | --- |
|  |  | (19) |

where are the interpolation weights.These defenitions maintain formal second order accuracy of the interpolation operator, but they do not satisfy the discrete product rules exactly so that the secondary conservation is only approximately satisfied. For stability, we will use equally weighted interpolation (i.e. c=1/2).

# 2. Summary