Follow the Perturbed Leader: Model Selection for COVID-19 Market

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Contents

1 Introduction

Since the financial crisis of 2007-2008, the stock market had been experiencing the longest bull run in history. For context, the S&P saw a rise of 330% in the last 10 years after taking 5 years to fully recover from the crash of 2008. Economists and experts believed that the bullish market was here to stay [1], but the outbreak of COVID-19 triggered a global financial crisis, forcing industries to shut down operations due to stay at home orders in order to contain the virus. Although markets seem to be recovering, this event caused prices to fall to a 10-year record low [2], with some industries still struggling to recover. Therefore, it is of interest to understand how to model the events that occurred in the stock market following the COVID crisis.

Modeling the financial market is an important task, as models can provide accurate predictions on price movements of specific stocks (SPY, AAPL, MSFT, etc.) and inform traders when executing trades. Specifically, statistical models are used extensively in finance, as their mathematical foundation allow them to be robust, accurate, and lend themselves to automation that has helped shape the world of high-frequency trading today. As Salisu et al.[2] suggest, modeling only tells one side of the story: evidence continues to show that news are a key factor in predicting movements in the stock market.

Times of high uncertainty brought by the COVID-19 pandemic are examples in history when financial models are prone to error. Especially after having such a long track of growth, an exogenous variable, such as a pandemic, could not be factored in by typical statistical models. In general, major events that occur in history are characterized by volatility and uncertainty in the market, making it difficult to select a model that accurately captures movements in the stock market. Therefore, there is merit in being able to dynamically select a model and still being able to make profitable trades even when model predictions contain high uncertainty.

In this work, the Follow the Perturbed Leader (FTPL) Algorithm is used to evaluate different strategies on stock market data from the November 2019 - November 2020 period. The stocks evaluated are mainly large cap stocks, including tech stocks, which profited during the pandemic, and commercial stocks, which generally took a hit during the pandemic. The aim is to understand how profitable this algorithm is, as well as comparing strategies which use long-term models such as an AutoRegressive Integrated Moving Average (ARIMA) model as opposed to a simple Mean Reversion strategy during this period.

2 Related Work

The task of predicting the movement of the stock market can be characterized by a method in machine learning known as Online Learning [3] where data is fed to a model sequentially and the results are used to update the predictor. That is, at time t, the model has observed $(X_1, Y_1), \ldots, (X_{t-1}, Y_{t-1})$ and given X_t the model is to predict $Y_t \in \{0, 1\}$.

For the task of stock market prediction, it would be desirable to model the approach in an adversarial fashion. There should be no requirement that the observations made are independent and identically distributed, but rather the sequence of prices is any bounded sequence. The relative performance is then measured to some benchmark rather than directly comparing to the adversarial sequence.

2.1 Prediction with Expert Advice

The problem of prediction with expert advice will be set up in the same fashion as Rigollet [3]. We let $\mathcal{A}$ denote the set of actions we can take and $\mathcal{Z}$ the adversary's moves. The action taken $a_t \in \mathcal{A}$ is revealed alongside the adversary's move $z_t \in \mathcal{Z}$. We denote the loss associated with each timestep as $\ell(a_t, z_t)$.

The set of decisions we can make at each timestep $\mathcal{A}$ can be thought of the different models we can choose from. For example, we can have our 'experts' being AutoRegressive models $AR(1)$, $AR(2)$, and $AR(3)$. In general, we say that we have k experts to choose from at each timestep.

Rather than trying to minimize $\sum_{t=1}^{n} \ell\left(a_t, z_t\right)$, we define our benchmark to be the expert with minimum loss to the adversary. Define an expert $e \in \mathcal{A}^n$ to be a vector of length T, the time horizon of our predictions. With k experts, our benchmark can be defined as follows:

$$\text{benchmark } = \min_{1 \leq j \leq K} \sum_{t=1}^{T} \ell\left(b_t^{(j)}, z_t\right)$$

In prediction with expert advice in an adversarial setting, it is common to define regret, a metric that compares an algorithm's performance with the performance of the best expert in the set of experts.

Define R_T, the regret at time horizon T:

$$R_T = \sum_{t=1}^{T} \ell\left(a_t, z_t\right) - \min_{1 \leq j \leq K} \sum_{t=1}^{T} \ell\left(b_t^{(j)}, z_t\right)$$

With this construction, we can refocus on trying to minimize regret in an adversarial scenario.

There are a number of algorithms that suitable for this problem, and Rigollet mentions several of them: Exponential Weighting (EW), Weighted Majority (WM), Randomized WM [3].

2.2 Follow the Perturbed Leader

2.2.1 Problem Formulation

Follow the Perturbed Leader algorithm builds upon Follow the Leader, which is a much simpler algorithm that simply picks the expert that minimizes the current loss up to timestep $t-1$ when making a prediction at time t. Rigollet provides a counterexample as to how this strategy provides linear regret in a two expert setting with adversarial outputs [3].

To address this, FTPL regularizes FTL by adding a perturbation in the form of a random variable P with support $\left[0, \frac{1}{\eta}\right]^k$ to the loss of each expert.

The appendix contains a brief overview of the algorithm in psuedocode as described by Rabadi [4].

2.2.2 Proof

In comparison to FTL, FTPL is able to achieve sub-linear $o(T)$ regret. If $\eta = \frac{1}{\sqrt{kT}}$, then it is shown by Rigollet that the expected regret of FTPL is as follows:

$$E[R_T] \leq 2\sqrt{2kT}$$

The proof involves bounding the Be the Leader (BTL) algorithm, which bounds the best expert in hindisght each step then bounding the difference between FTPL and BTL expected regrets by the perturbation introduced. Rigollet provides a more in depth proof that is considered outside the scope of the paper and thus further reading for the reader [3].

2.2.3 Use in Finance

The FTPL algorithm finds its place in finance as it is time series prediction algorithm for online learning with sublinear regret. The proof does not make any distributional assumptions, so it is useful to use this algorithm in a time of uncertainty in a time series, such as the COVID-19 pandemic.

3 Experiment Design

This section will describe the models used as well as the stocks that were analyzed.

3.1 Models Used

3.1.1 Mean Reversion

Mean reversion strategies work under the assumption that there is an underlying stable trend of an asset. The strategy provides a signal based on the mean of the current window. If the incoming price is higher than the mean, the signal is that it will decrease, and vice-versa. A mean reversion strategy takes in a parameter n, which is the size of the window to perform the mean reversion on. For the experiment, there will be a pool of 7 experts with n ranging from 10 day to 34 day mean reversion in increments of 4 days.

3.1.2 Momentum

Momentum strategies work under the assumption that there exists a general trend pattern of the assets price. This can be characterized as a 'buy low, sell high' strategy. Like mean reversion, a momentum strategy also uses a parameter n which is the size of the window to perform the momentum on. The signal provided by this strategy is given by the general trendline by plotting the last n prices observed. If the trend is positive, then there is a signal to buy, and vice versa. This strategy can be prone to buying right before the dip, or selling right before the spike, but nevertheless it is frequently used as an indicator for its simplicity. For the experiment, there will be a pool of 5 experts with n ranging from 14 days to 30 days in increments of 4 days.

3.1.3 ARIMA Model

ARIMA is a statistical model for analyzing and forecasting time series data. Unlike the previous models, these models need to be trained on a set of data in order to make good predictions. The model is described by three parameters: p, q, d which control the lag order, the degree of differencing, and the size of the moving average window. The models work under the assumption that the time series is stationary. If it is not, then the data must be transformed, or a degree of differencing must be introduced to the ARIMA process to properly fit the data. For the experiment, ARIMA$(x, 1, 0)$ models were tested, where x ranges from 1 to 9. The choice of 1 for the degree of differencing will be explained in the results section.

3.2 Stocks Used

3.2.1 Technology

The COVID-19 pandemic has been characterized as a time where technology sector experienced growth amid the uncertainty, given that people were for the most part at home due to nation and state wide lockdown mandates for at least one third of 2020. Companies like Amazon, Apple, and Zoom were able to expand in sales and therefore valuation. For the experiments, Amazon (AMZN), Apple (AAPL), Google (GOOG) will be analyzed.

3.2.2 Commercial

On the opposite end, the commercial sector suffered a great deal from the pandemic. The period of March to June can be characterized by a number of cancellations of flights, cruises, and summer vacations. Companies that were most affected include American Airlines, Carnival Cruise Lines, and Airbnb. Many of these companies chose to layoff numerous employees amid a pandemic to remain operable given the situation. For the experiments, US Global Jets ETF (JETS), Disney (DIS), Carnival Corp (CCL) will be analyzed.

3.3 Implementation Details

As Rabadi suggests, FTPL is used to select a model hyperparameter rather than choose a model class [4]. Thus, when running simulations of FTPL, only models of the same class were ran together and were allowed to be selected by the algorithm.

The data used by the algorithm is daily stock price data provided by Yahoo Finance ranging from 11/19/2019 to 11/19/2020. A column is added to the data for daily returns which is taken by taking the difference of close and open prices. The underlying assumption in these models is that each model predicts a direction of the stock price, and the position is on a daily period. For example, if the direction is long, then the asset is assumed to be purchased at start of day and sold at the end of the day, and the model's return is the underlying's daily return.

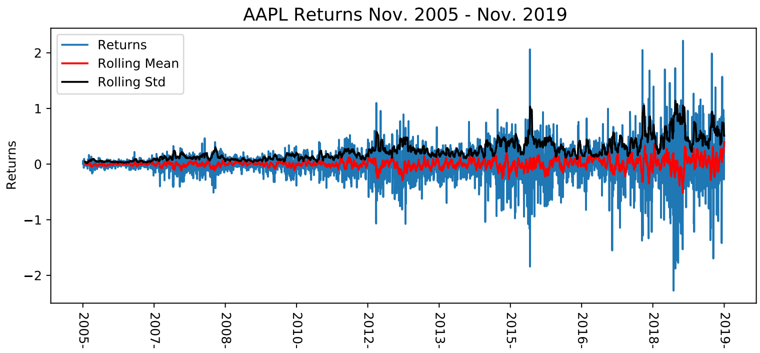
As mentioned in section 3.1.3, ARIMA models must be trained on previous data. The data used to train these models was the underlying's data from November 2005 to November 2019, also gathered from Yahoo Finance.

4 Results

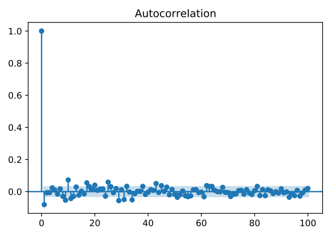
4.1 ARIMA Model Analysis

As mentioned in section 3.1.3, a good ARIMA model class needed to be found for the time series data. Figure 1 shows an analysis of AAPL stock returns by showing the rolling mean and standard deviation, an Autocorrelation function (ACF) plot, and results from a Dickey-Fuller Test. For AAPL data, although the p-value was low, the critical values were not close to the ADF statistic, so it was concluded that the time series was not stationary. This was true for the other 5 stocks that were analyzed in the study, and their analysis can be seen in section 7.2.

In order to make the time series stationary, a logarithmic and cube-root transform were attempted, but the results were unsuccessful for logarithmic due to negative values and cube-root due to failure of the Dickey-Fuller Test. Instead, it was identified that a value of 1 for the degree of differencing was able to fit the data correctly as opposed to having no degree of differencing. There was no observable difference in the time series plots for the 6 stocks from the dates 2005-2018 (2015-2019 for JETS). The ACF plots all suggested that the first lag was extremely significant, with the remaining lags rarely passing the significance threshold. This might explain why a time difference of 1 makes the time series stationary.



(a) 1a



(b) 1b

ADF Statistic: -9.144971717661393
p-value: 2.78377974663320O5e-15
Critical Values
1%: -3.4322245674254277
5%: -2.8623682773183745
10%: -2.5672109065140374

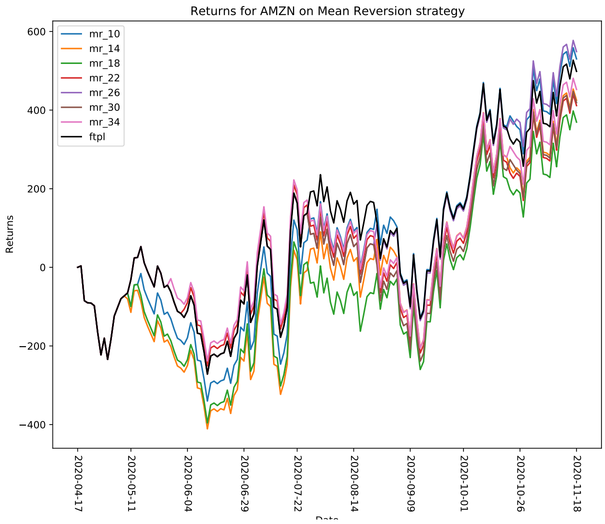
(c) 1c

Figure 1: AAPL Returns and ACF

4.2 Technology

The tech stocks that were simulated using the FTPL algorithm were highly profitable stocks throughout the months of November 2019 to November 2020, with their stock prices growing from 60-80% overall. Using the three strategies mentioned in section 3.1, we see the returns that were achieved for Amazon's stock in Figure 2. It is seen that the mean reversion and ARIMA model strategies were profitable, but a momentum strategy was not. A possible explanation for this is due to the point made in 3.1.2, which is that momentum models might keep buying stock right before a crash, which occurred often throughout the pandemic, as prices were volatile in the short term, even for tech stocks that saw massive growth.

It is interesting to see that a mean reversion strategy was almost as profitable, or at times even more profitable (as seen in Figure 2) in comparison to an ARIMA model. For the Amazon ARIMA simulation, the AR(1) process seems to be the most profitable, but the other 'experts' with higher lag order cause the FTPL algorithm to limit its profits. This shows that an AR(1) process best fit the data, and this can be seen by the data in section 7.2 with ACF plots.



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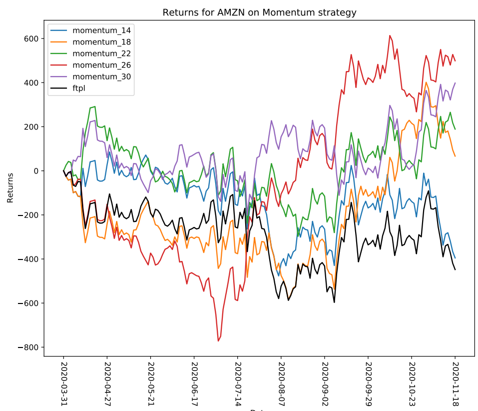
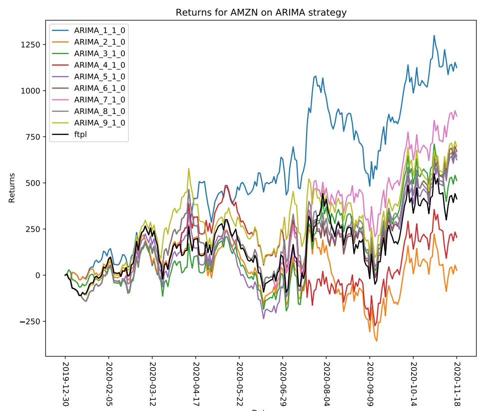


(b) 2b



(c) 2c

Figure 2: AMZN FTPL Returns

In general, the best strategy for technology stocks was an ARIMA model, but a mean reversion strategy was almost as profitable with the advantage of being a much simpler model that does not require to be trained. It is possible that, given a much more uncertain period, a mean reversion strategy would out perform an ARIMA model due to it not predicting new prices accurately.

4.3 Commercial

On the other hand, commercial stocks that were simulated using the FTPL algorithm mostly went down, with Carnival Cruise Corp., for example, suffering a 70% loss in their stock value from November 2019 to November 2020. JETS suffered a loss of 30%, and Disney has not changed much, but this is because they have had a U-shaped recovery as recent openings of their parks and their other businesses (streaming, movies and TV) have kept them operating and regained trust in investors.

The results from the three different strategies on Carnival Cruise Corp. are outlined in Figure 3. The results are similar to those seen for technology stocks, as the mean reversion and ARIMA strategies tend to be the most profitable and momentum strategies tend to be unprofitable. Although the results are a similar on a per strategy basis in comparison to technology stocks, it is noted that for the commercial stocks, the AR(1) process was among the least profitable, and models with higher lag were more profitable. This can be seen in the ACF plots in section 7.2.

5 Conclusion

The Follow The Leader algorithm was evaluated on 3 technology stocks (AAPL, AMZN, GOOG) and 3 commercial stocks (CCL, JETS, DIS) using 3 different strategies (Mean Reversion, Momentum, ARIMA) on timeseries data from the COVID-19 pandemic. It was found that, although there was uncertainty in the periods evaluated, an ARIMA model tended to be the most profitable strategy. Nevertheless, mean reversion was always profitable as long as ARIMA was profitable, and was able to make within 10-50% of the profits of the ARIMA model.

The experiments performed can be extended to other periods of uncertainty, such as the 2007 economic crisis. This analysis can also be extended beyond stock data, as commodities (Gold, Currencies) and other indices (S&P500, Dow Jones Industrial Average, Nasdaq Composite) can be studied under this model to get a better look at the overall market as opposed to individual stocks.

6 Reflection

I am open to any kind of comments from the reader. I enjoyed working through the implementation and analysis, and I hope you enjoyed reading through the discoveries made in the simulations I performed and the math discussion on the theory behind FTPL. I am hoping that this is a suitable paper for the final submission, but if it is not then I would appreciate any advice on reaching that state. :)

References

[1] James Chen, *Market Milestones as the Bull Market Turns 10*, Investopedia, 10/16/2019

[2] Salisu, Afees A., and Xuan Vinh Vo., *Predicting stock returns in the presence of COVID-19 pandemic: The role of health news.* International Review of Financial Analysis vol. 71 (2020): 101546. doi:10.1016/j.irfa.2020.101546

[3] Rigollet, P. (2015). *Mathematics of Machine Learning*, Lecture 15-16. MIT.

[4] Michael Rabadi (2020), *Model Selection For Finance*, BAM.

7 Appendix

7.1 FTPL Psuedocode

Explanation of variables:

$k-$ number of experts

$T-$ time horizon

$c_{t,i}-$ the cumulative loss for expert i at time t

$p_{t,i}-$ the perturbation for expert i at time t (drawn from η)

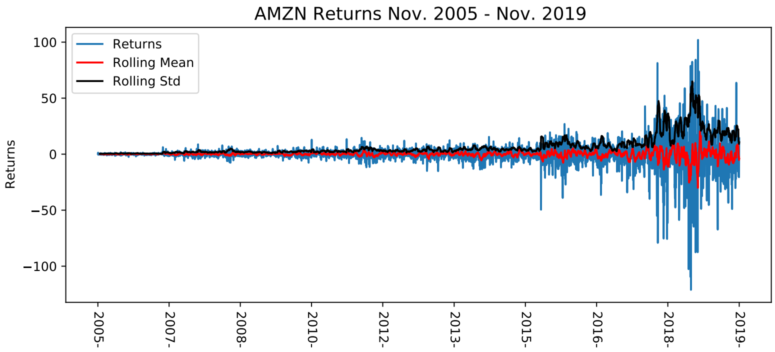
i_t*- expert chosen at timestep t

$\hat{y}_t-$ prediction caused by expert i_t^*

```
for i ← 1 : k do
|   c_{1,i} ← 0
end
for t ← 1 : T do
  for i ← 1 : k do
  |   p_{t,i} ~ P
  end
| i_t^* ← arg min_{i=1}^k c_{t,i} − p_{t,i}
| ŷ_t ← ŷ_i^*
  for i ← 1 : k do
  |   c_{t,i} ← c_{t,i} + L(ŷ_{t,i}, y_t)
  end
end
```

where P is uniform distribution with support $\left[0, \frac{1}{kT}\right]^k$.

7.2 Stock ACF and Dickey-Fuller Test



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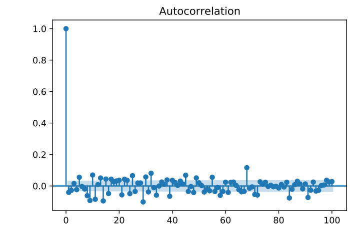
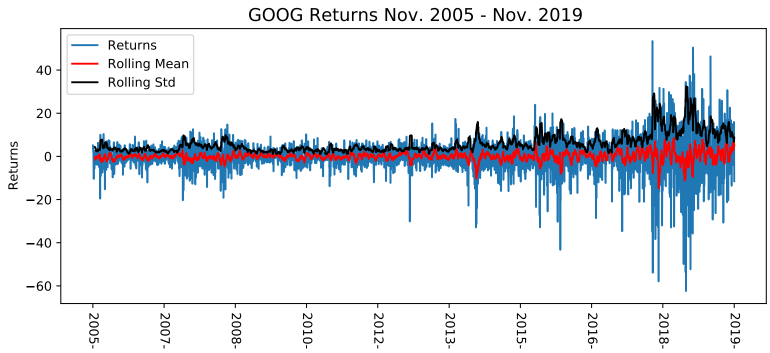


(b) 3b

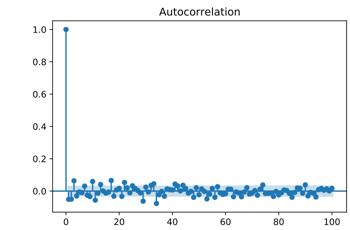
ADF Statistic: -10.690535521073713
p-value: 3.735971082277776e-19
Critical Values
1%: -3.432224030212334
5%: -2.8623680400255576
10%: -2.5672107801861626

(c) 3c

Figure 3: AMZN Returns and ACF



(a) 4a

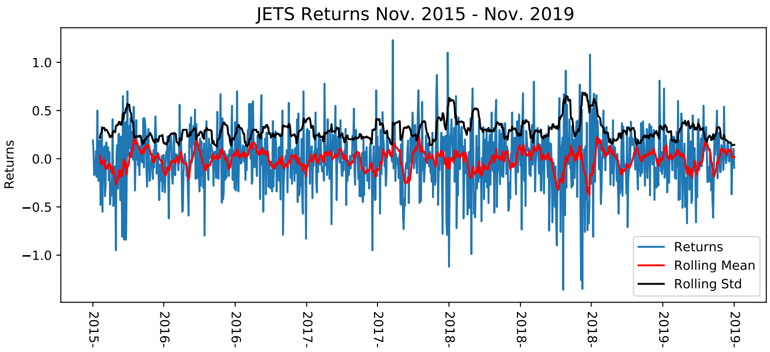


(b) 4b

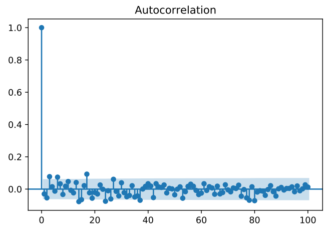
ADF Statistic: -10.3292307759665
p-value: 2.8714995592752695e-18
Critical Values
1%: -3.432223493307061
5%: -2.8623678028686657
10%: -2.567210653930658

(c) 4c

Figure 4: GOOG Returns and ACF



(a) 5a

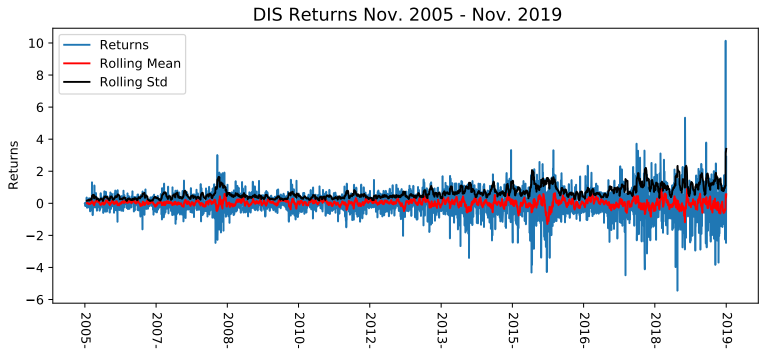


(b) 5b

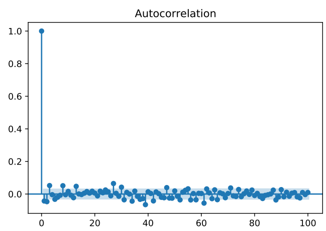
ADF Statistic: -7.207624112440697
p-value: 2.2783185400296593e-10
Critical Values
1%: -3.436979275944879
5%: -2.8644668170148058
10%: -2.5683283824496153

(c) 5c

Figure 5: JETS Returns and ACF



(a) 6a

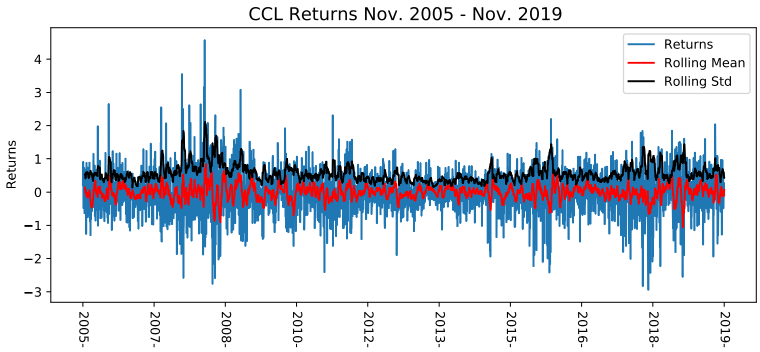


(b) 6b

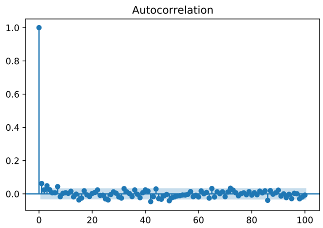
ADF Statistic: -15.626312625938283
p-value: 1.7070917961798887e-28
Critical Values
1%: -3.4322149444714065
5%: -2.8623640267493293
10%: -2.567208643634359

(c) 6c

Figure 6: DIS Returns and ACF



(a) 7a



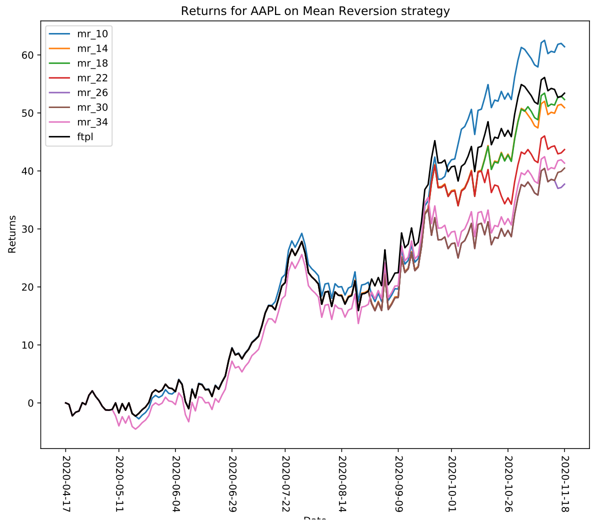
(b) 7b

ADF Statistic: -31.58298255088435
p-value: 0.0
Critical Values
1%: -3.4322096409547136
5%: -2.862361684119379
10%: -2.5672073964871087

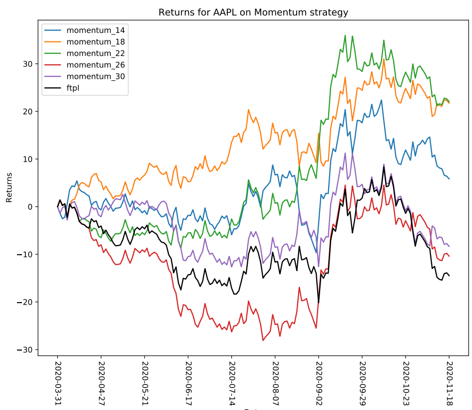
(c) 7c

Figure 7: CCL Returns and ACF

7.3 FTPL Returns



(a) 8a



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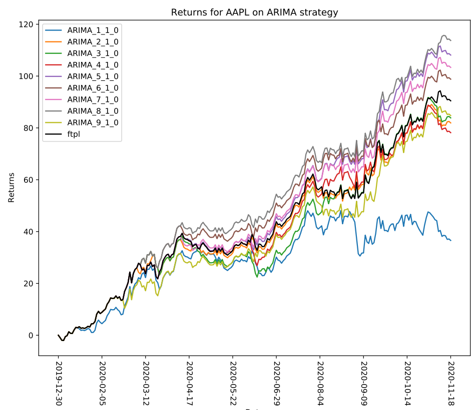

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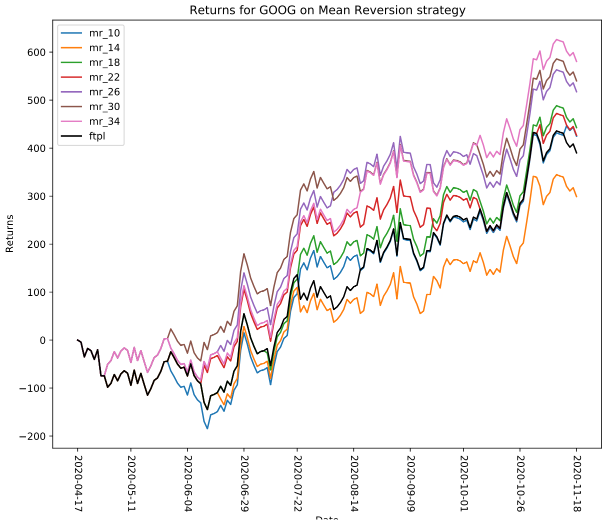
Figure 8: AAPL FTPL Returns



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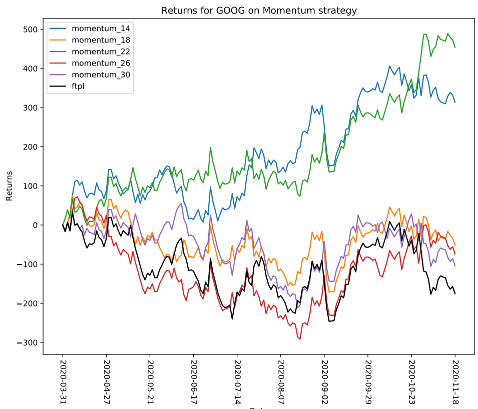

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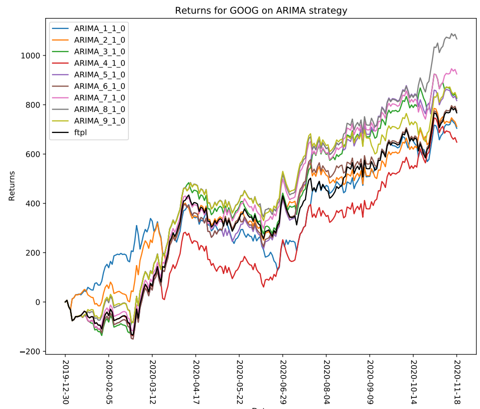
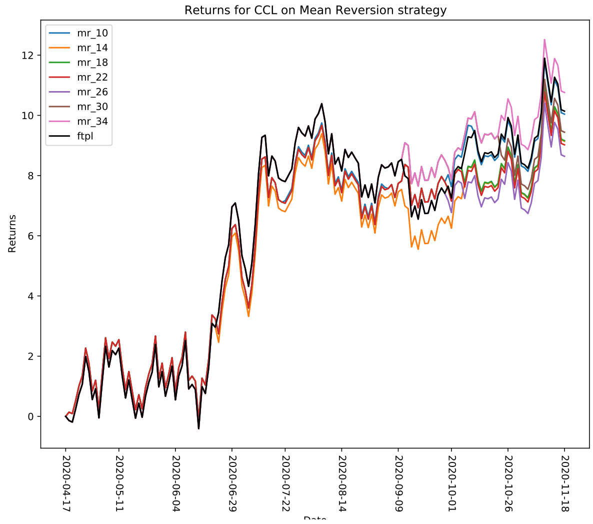


(c) 9c

Figure 9: GOOG FTPL Returns



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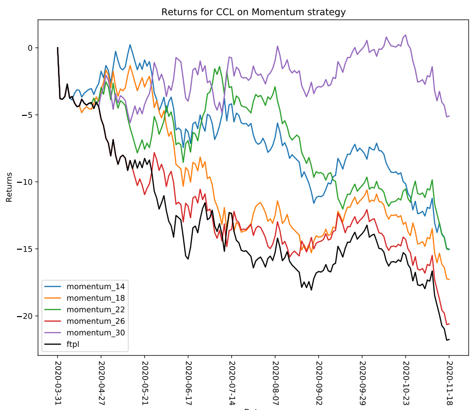

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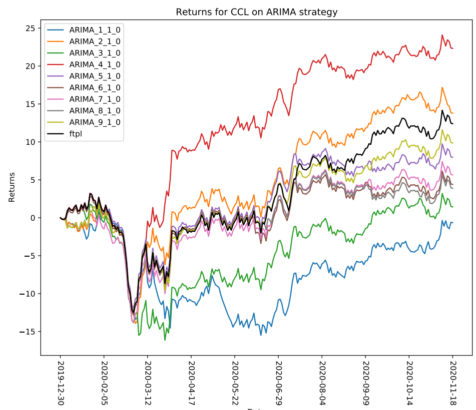
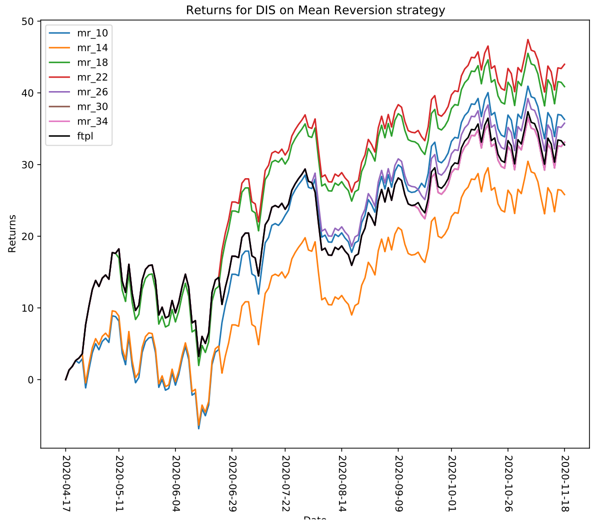


(c) 10c

Figure 10: CCL FTPL Returns



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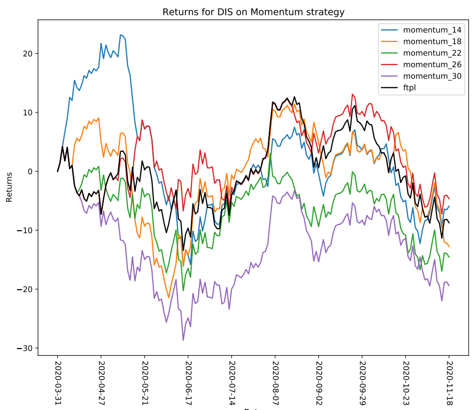

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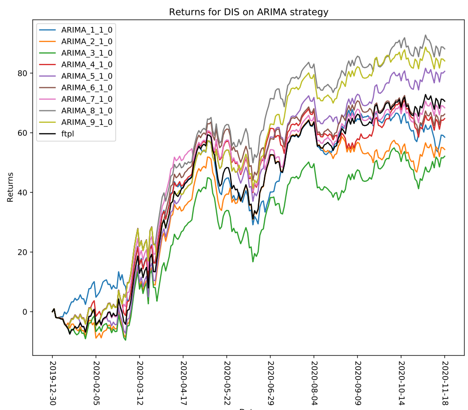
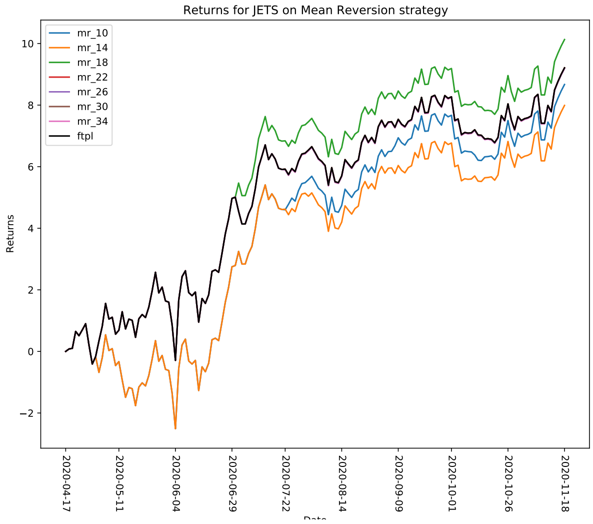


(c) 11c

Figure 11: DIS FTPL Returns



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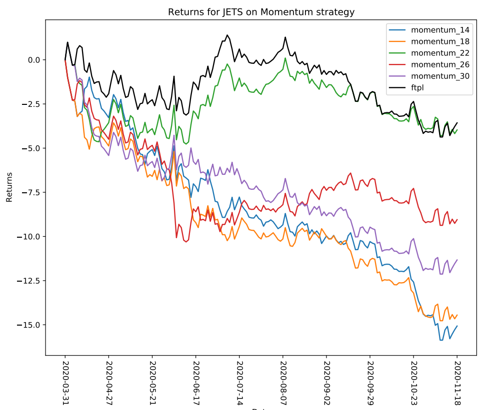
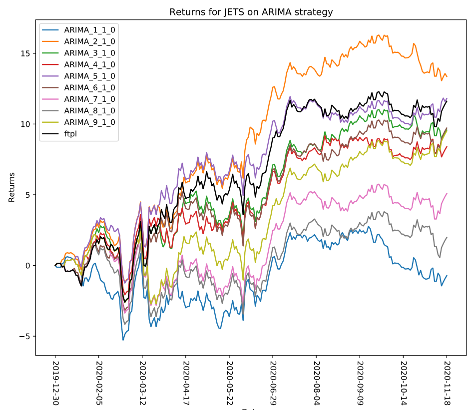


(b) 12b



(c) 12c

Figure 12: JETS FTPL Returns