

Polinomio Interpolación Lineal

$$f(x) = \frac{1}{x}$$

x	2	2.5	4
$f(x)$	0.5	0.4	0.25

$$P(x) = \frac{f(x_1) - f(x_0)}{x_1 - x_0} (x - x_0) + f_0$$

$$P(x) = \left(\frac{0.4 - 0.5}{2.5 - 2} \right) (x - 2) + 0.5$$

$$P(x) = -0.2(x - 2) + 0.5$$

$$P(x) = -0.2x + 0.4 + 0.5$$

$$P(x) = -0.2x + 0.9$$

$$\text{R/ } P(x) = -0.2x + 0.9$$

Polinomio interpolación cuadrático.

$$f(x) = \frac{1}{x}$$

x	2	2.5	4
$f(x)$	0.5	0.4	0.25

$$b_0 = 0.5$$

$$b_1 = \frac{0.4 - 0.5}{2.5 - 2}, \quad b_1 = -0.2$$

$$b_2 = \frac{\left(\frac{0.25 - 0.4}{4 - 2.5} \right) - \left(\frac{0.4 - 0.5}{2.5 - 2} \right)}{4 - 2}$$

$$b_2 = 0.05$$

$$P(x) = 0.5 + (-0.2)(x-2) + (0.05)(x-2)(x-2.5)$$

$$P(x) = 0.5 - 0.2x + 0.4 + (0.05)(x^2 - 4.5x + 5)$$

$$P(x) = 0.5 - 0.2x + 0.4 + 0.05x^2 - 0.225x + 0.25$$

$$P(x) = 0.05x^2 - 0.425x + 1.15$$

$$R/ \quad P(x) = 0.05x^2 - 0.425x + 1.15$$

Interpolación de Lagrange.

x	4	0	-6	1	-4
$f(x)$	808	4	1438	10	160

Polinomio de grado 1

$$\begin{aligned}P_1 &= (808) \left(\frac{x-0}{4-0} \right) + (4) \left(\frac{x-4}{0-4} \right) \\&= (808) \left(\frac{x}{4} \right) + (4) \left(\frac{x-4}{0-4} \right) \\&= (808) \left(\frac{1}{4}x \right) + 4 \left(\frac{1}{-4}x - \frac{4}{-4} \right) \\&= 202x + 4 \left(\frac{x}{-4} + 1 \right) \\&= 202x + 4 - x \\&= 202x + 4 - x \\&= 201x + 4\end{aligned}$$

$$R/ \quad P_1 = 201x + 4.$$

Polinomio de grado 2

x	4	0	-6	1	-4
$f(x)$	808	4	1438	10	160

$$P_2 = (808) \left(\frac{(x-0)(x+6)}{(4-0)(4+6)} \right) + (4) \left(\frac{(x-4)(x+6)}{(0-4)(0+6)} \right)$$

$$+ (1438) \left(\frac{(x-4)(x+0)}{(-6-4)(-6-0)} \right)$$

$$P_2 = (808) \left(\frac{(x)(x+6)}{(4)(10)} \right) + (4) \left(\frac{(x-4)(x+6)}{(-4)(6)} \right) + (1438) \left(\frac{(x-4)(x)}{(-10)(-6)} \right)$$

$$P_2 = (808) \left(\frac{x^2 + 6x}{40} \right) + (4) \left(\frac{(x^2 + 6x - 4x - 24)}{-24} \right) + (1438) \left(\frac{x^2 - 4x}{60} \right)$$

$$P_2 = (808) \left(\frac{x^2}{40} + \frac{6x}{40} \right) + (4) \left(\frac{x^2 + 2x - 24}{-24} \right) + (1438) \left(\frac{x^2}{60} - \frac{4x}{60} \right)$$

$$P_2 = 20.2x^2 + 121.2x - 0.166666666667x^2 - x + 4$$

$$+ 23.96666667x^2 - 95.86666667x + \cancel{44x^2} + \cancel{24.33333333x} + 4$$

$$R/ P_2 = 44x^2 + 24.33333333x + 4.$$

Polinomio grado 3.

x	4	0	-6	1	-4
$f(x)$	808	4	1438	10	160

$$P_3 = (808) \left(\frac{(x-0)(x+6)(x-1)}{(4-0)(4+6)(4-1)} \right) + (4) \left(\frac{(x-4)(x+6)(x-1)}{(0-4)(0+6)(0-1)} \right) \\ + (1438) \left(\frac{(x-4)(x-0)(x-1)}{(-6-4)(-6-0)(-6-1)} \right) + (10) \left(\frac{(x-4)(x-0)(x+6)}{(1-4)(1-0)(1+6)} \right)$$

$$P_3 = (808) \left(\frac{(x)(x+6)(x-1)}{(4)(10)(3)} \right) + (4) \left(\frac{(x-4)(x+6)(x-1)}{(-4)(6)(-1)} \right) \\ + (1438) \left(\frac{(x-4)(x-0)(x-1)}{(-10)(-6)(-1)} \right) + (10) \left(\frac{(x-4)(x)(x+6)}{(-3)(1)(7)} \right)$$

$$P_3 = (808) \left(\frac{(x)(x+6)(x-1)}{120} \right) + (4) \left(\frac{(x-4)(x+6)(x-1)}{24} \right) \\ + (1438) \left(\frac{(x-4)(x)(x-1)}{-420} \right) + (10) \left(\frac{(x-4)(x)(x+6)}{-21} \right)$$

$$P_3 = (808) \left(\frac{(x)(x^2 - x + 6x - 6)}{120} \right) + (4) \left(\frac{(x-4)(x^2 + 6x - x - 6)}{24} \right) \\ + (1438) \left(\frac{(x)(x^2 - x - 4x + 4)}{-420} \right) + (10) \left(\frac{(x)(x^2 + 6x - 4x - 24)}{-21} \right)$$

$$P_3 = (808) \left(\frac{(x)(x^2 + 5x - 6)}{120} \right) + (4) \left(\frac{(x-4)(x^2 + 5x - 6)}{24} \right) \\ + (1438) \left(\frac{(x)(x^2 - 5x + 4)}{-420} \right) + (10) \left(\frac{(x)(x^2 + 2x - 24)}{-21} \right)$$

$$P_3 = (808) \left(\frac{x^3 + 5x^2 - 6x}{120} \right) + (4) \left(\frac{x^3 + 5x^2 - 6x - 4x^2 - 20x + 24}{24} \right) \\ + (1438) \left(\frac{x^3 - 5x^2 + 4x}{-420} \right) + (10) \left(\frac{x^3 + 2x^2 - 24x}{-21} \right)$$

$$P_3 = (808) \left(\frac{x^3}{120} + \frac{5x^2}{120} - \frac{6x}{120} \right) + (4) \left(\frac{x^3}{24} + \frac{x^2}{24} - \frac{26x}{24} + \frac{24}{24} \right) \\ + (1438) \left(\frac{x^3}{-420} - \frac{5x^2}{-420} + \frac{4x}{-420} \right) + (10) \left(\frac{x^3}{-21} + \frac{2x^2}{-21} - \frac{24x}{-21} \right)$$

$$P_3 = 6.733333333x^3 + 33.66666667x^2 - 40.4x + \\ 0.166666667x^3 + 0.166666667x^2 - 4.333333333x + 4 \\ - 3.423809524x^3 + 17.11904762x^2 - 13.6952381x \\ - 0.4761904762x^3 - 0.9523809224x^2 + 11.42857143x$$

~~(R/P₃)~~

$$R/P_3 = 3x^3 + 50.00000033x^2 - 47x + 4$$

Polinomio de grado 4.

x	4	0	-6	1	-4
$f(x)$	808	4	1438	10	160

$$\begin{aligned}
 P_4 = & (808) \left(\frac{(x-0)(x+6)(x-1)(x+4)}{(4-0)(4+6)(4-1)(4+4)} \right) + \\
 & (4) \left(\frac{(x-4)(x+6)(x-1)(x+4)}{(0-4)(0+6)(0-1)(0+4)} \right) + \\
 & (1438) \left(\frac{(x-4)(x-0)(x-1)(x+4)}{(-6-4)(-6-0)(-6-1)(-6+4)} \right) + \\
 & (10) \left(\frac{(x-4)(x-0)(x+6)(x+4)}{(-1-4)(-1-0)(-1+6)(-1-4)} \right) + \\
 & (160) \left(\frac{(x-4)(x-0)(x+6)(x-1)}{(-4-4)(-4-0)(-4+6)(-4-1)} \right)
 \end{aligned}$$

$$\begin{aligned}
 P_4 = & (808) \left(\frac{(x)(x+6)(x-1)(x+4)}{(4)(10)(3)(8)} \right) + \\
 & (4) \left(\frac{(x-4)(x+6)(x-1)(x+4)}{(-4)(6)(-1)(4)} \right) + \\
 & (1438) \left(\frac{(x-4)(x)(x-1)(x+4)}{(-10)(-6)(-7)(-2)} \right) + \\
 & (10) \left(\frac{(x-4)(x)(x+6)(x+4)}{(-3)(1)(7)(5)} \right) + \\
 & (160) \left(\frac{(x-4)(x)(x+6)(x-1)}{(-8)(-4)(2)(-5)} \right)
 \end{aligned}$$

$$\begin{aligned}
 P_4 = & (808) \left(\frac{(x)(x+6)(x-1)(x+4)}{960} \right) + \\
 & (4) \left(\frac{(x-4)(x+6)(x-1)(x+4)}{96} \right) + \\
 & (1438) \left(\frac{(x-4)(x)(x-1)(x+4)}{840} \right) + \\
 & (10) \left(\frac{(x-4)(x)(x+6)(x+4)}{-105} \right) + \\
 & (160) \left(\frac{(x-4)(x)(x+6)(x-1)}{-320} \right)
 \end{aligned}$$

$$\begin{aligned}
 P_4 = & (808) \left(\frac{(x)(x+6)(x^2+4x-x-4)}{960} \right) + \\
 & (4) \left(\frac{(x-4)(x+6)(x^2+4x-x-4)}{96} \right) + \\
 & (1438) \left(\frac{(x-4)(x)(x^2+4x-x-4)}{840} \right) + \\
 & (10) \left(\frac{(x-4)(x)(x^2+4x+6x+24)}{-105} \right) + \\
 & (160) \left(\frac{(x-4)(x)(x^2-x+6x-6)}{-320} \right)
 \end{aligned}$$

$$\begin{aligned}
 P_4 = & (808) \left(\frac{(x)(x+6)(x^2+3x-4)}{960} \right) + \\
 & (4) \left(\frac{(x-4)(x+6)(x^2+3x-4)}{96} \right) + \\
 & (1438) \left(\frac{(x-4)(x)(x^2+3x-4)}{840} \right) + \\
 & (10) \left(\frac{(x-4)(x)(x^2+10x+24)}{-105} \right) + \\
 & (160) \left(\frac{(x-4)(x)(x^2+5x-6)}{-320} \right)
 \end{aligned}$$

$$\begin{aligned}
 P_4 = & (808) \left(\frac{(x)(x^3+3x^2-4x+6x^2+18x-24)}{960} \right) + \\
 & (4) \left(\frac{(x-4)(x^3+3x^2-4x+6x^2+18x-24)}{96} \right) + \\
 & (1438) \left(\frac{(x)(x^3+3x^2-4x-4x^2-12x+16)}{840} \right) + \\
 & (10) \left(\frac{(x)(x^3+10x^2+24x-4x^2-40x-96)}{-105} \right) + \\
 & (160) \left(\frac{(x)(x^3+5x^2-6x-4x^2-20x+24)}{-320} \right)
 \end{aligned}$$

$$\begin{aligned}
 P_4 = & (808) \left(\frac{(x)(x^3+9x^2+14x-24)}{960} \right) + (4) \left(\frac{(x-4)(x^3+9x^2+14x-24)}{96} \right) + \\
 & (1438) \left(\frac{(x)(x^3-x^2-16x+16)}{840} \right) + (10) \left(\frac{(x)(x^3+6x^2-16x-96)}{-105} \right) + \\
 & (160) \left(\frac{(x)(x^3+x^2-26x+24)}{-320} \right)
 \end{aligned}$$

$$\begin{aligned}
 P_4 = & (808) \left(\frac{x^4 + 9x^3 + 14x^2 - 24x}{960} \right) + \\
 & (4) \left(\frac{x^4 + 9x^3 + 14x^2 - 24x - 4x^3 - 36x^2 - 56x + 96}{96} \right) + \\
 & (1438) \left(\frac{(x^4 - x^3 - 16x^2 + 16x)}{840} \right) + (10) \left(\frac{x^4 + 6x^3 - 16x^2 - 96x}{-105} \right) + \\
 & (160) \left(\frac{x^4 + x^3 - 26x^2 + 24x}{-320} \right)
 \end{aligned}$$

$$\begin{aligned}
 P_4 = & (808) \left(\frac{x^4}{960} + \frac{9x^3}{960} + \frac{14x^2}{960} - \frac{24x}{960} \right) + (4) \left(\frac{x^4}{96} + \frac{5x^3}{96} - \frac{22x^2}{96} - \frac{80x}{96} + \frac{96}{96} \right) \\
 & + (1438) \left(\frac{x^4}{840} - \frac{x^3}{840} - \frac{16x^2}{840} + \frac{16x}{840} \right) + \\
 & (10) \left(\frac{x^4}{-105} + \frac{6x^3}{-105} - \frac{16x^2}{-105} - \frac{96x}{-105} \right) + \\
 & (160) \left(\frac{x^4}{-320} - \frac{x^3}{-320} - \frac{26x^2}{-320} + \frac{24x}{-320} \right)
 \end{aligned}$$

$$\begin{aligned}
 P_4 = & 0.84166667x^4 + 7.575x^3 + 11.78333333x^2 - 20.20x + \\
 & 0.041666667x^4 + 0.208333333x^3 - 0.916666667x^2 - 3.333333333x \\
 & + 4 + 1.71904762x^4 - 1.71904762x^2 - 27.39047619x^2 \\
 & + 27.39047619x - 0.5x^4 + 0.5x^3 + 13x^2 - 12x
 \end{aligned}$$

$$P_4 = 2.000014714x^4 + 6x^3 - 2.000000003x^2 + x + 4. \quad R/$$