Systems of Equations

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Overview

We will use MATLAB to demonstrate some properties of solutions of linear systems.

Systems of equations with two unknowns

Systems of equations involving two unknowns can be analyzed graphically. For example, consider the system,

$$\begin{aligned} 2x + 3y &= 1 \\ x - y &= 2. \end{aligned}$$

These are two linear equations. To see this we can solve for y to get,

$$\begin{aligned} y &= \frac{1}{3} - \frac{2}{3}x \\ y &= x - 2. \end{aligned}$$

To plot the graphs of these equations together, type the following:

```
>> ezplot('1/3-2*x/3',[-10,10])

>> hold on

>> ezplot('x-2',[-10,10])

>> title('My plot')

>> grid

>> hold off
```

- From the figure, does the linear system have a solution? Many solutions? No solution?
- If there is a unique solution, give an estimate of it's value by reading off the coordinates of the point where the lines cross.

The matrix equation: Ax = b

The system of equations above can be rewritten using Matrix-vector notation as

$$\begin{pmatrix} 2 & 3 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}.$$

We usually refer to this notation to $Ax = b$. To solve for the solution of the system

```
>> A = [2, 3; 1, -1]; b = [1; 2];

>> x = A\b
```

Row Echlon Form

Another way that you can solve for the system of equations in MATLAB is to use the reduced row echelon form. Our system of equations has the *augmented matrix*,

$$\left(\begin{array}{cc|c} 2 & 3 & 1 \\ 1 & -1 & 2 \end{array}\right)$$

In MATLAB, you can type

```
>> B = [2, 3, 1; 1, -1, 2];
```

```
>> rref(B)
```

MATLAB yields the *reduced row echelon form* of the matrix. From this, we have a solution $x = 1.4, y = -0.6$.
Over the next few labs, we will write our own code to solve systems of equations!

In-Class Exercises

Start a diary file. For the following system of equations, use the command **ezplot** to determine if the linear system has a solution. Estimate the solution, if only one solution exists, with the comment '%' or edit the diary file. Otherwise, indicate how many solutions exist. Solve the system of equations, if only one solution exists, use the backslash (\) operator and the reduced row echelon function to show the solution to the system of equations. (*rref*).

1. $$\begin{aligned} -x + 2y &= 3 \\ 2x - 4y &= -6 \end{aligned}$$

2. $$\begin{aligned} x - 7y &= -11 \\ 5x + 2y &= -18 \end{aligned}$$

Submission Guidelines

- Turn in a diary file as a .txt file.
- Make sure that your submission is .txt and do not compress the file(s).
- Include all .m files, do not compress these, submit them individually.
- You do not need to submit a graph.

Magic Matrices

A magic matrix is a square matrix with integer entries in which all rows, columns and the two diagonals have the same sum. For example

$$\begin{pmatrix} 8 & 1 & 6 \\ 3 & 5 & 7 \\ 4 & 9 & 2 \end{pmatrix}$$

is a magic matrix because each row, each column and the diagonals adds up to 15. Suppose you are given a partial magic matrix. Can you fill in the rest? For example

$$\begin{pmatrix} 8 & 1 & 3 \\ ? & ? & ? \\ ? & ? & ? \end{pmatrix} \rightarrow \begin{pmatrix} 8 & 1 & 3 \\ x_1 & x_2 & x_3 \\ x_4 & x_5 & x_6 \end{pmatrix}$$

We can label all the unknown entries with variables. Since the top row adds up to 12, each row, column and diagonal adds up to 12. Therefore, we have the following system of equations:

$$\begin{aligned} x_1 + x_2 + x_3 &= 12 \\ x_4 + x_5 + x_6 &= 12 \\ 8 + x_1 + x_4 &= 12 \\ 1 + x_2 + x_5 &= 12 \\ 3 + x_3 + x_6 &= 12 \\ 8 + x_2 + x_6 &= 12 \\ 3 + x_2 + x_4 &= 12 \end{aligned}$$

Note that this is a system of seven equations with six unknowns. Therefore, it is over-determined and might not have a solution. We have the augmented matrix:

$$\left(\begin{array}{cccccc|c} 1 & 1 & 1 & 0 & 0 & 0 & 12 \\ 0 & 0 & 0 & 1 & 1 & 1 & 12 \\ 1 & 0 & 0 & 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 & 1 & 0 & 11 \\ 0 & 0 & 1 & 0 & 0 & 1 & 9 \\ 0 & 1 & 0 & 0 & 0 & 1 & 4 \\ 0 & 1 & 0 & 1 & 0 & 0 & 9 \end{array}\right).$$

The reduced row echelon is

$$\left(\begin{array}{cccccc|c} 1 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 1 & 0 & 0 & 0 & 9 \\ 0 & 0 & 0 & 1 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 & 1 & 0 & 7 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right).$$

Using **rref**, we have that the only possibility is

$$\begin{pmatrix} 8 & 1 & 3 \\ -1 & 4 & 9 \\ 5 & 7 & 0 \end{pmatrix}.$$