Assignment 8

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The goal of this assignment is write an algorithm that can check if a matrix is diagonalizable over the real numbers, and if so, give an orthonormal basis of eigenvectors for the image of a square matrix.

Complete the following steps:

Step 1. Calculate the characteristic polynomial of A by using >>p = poly(A)
(the polynomial is a vector, with the first number being the coefficient of the largest power and the last number is the constant term).

Step 2. Find the roots of p by calling >>v = roots(p), then sort the roots by using >>sort(v) (For all my examples the eigenvalues will be integers, you may want to use Matlab's round function to make the values nice).

Step 3. For each different eigenvalue λ, find the corresponding eigenspace E_λ. (You should use matlab's unique function on v, this is so that you only do this for each <u>different number</u> in V.
To do this, calculate the null space of $A - \lambda * I$ by using Matlab's null(B) function.

Step 4. For each eigenvalue λ check if the dimension of E_λ is the algebraic multiplicity of λ (how many times λ appears in v). If these two values are not equal for an eigenvalue, throw an error saying "The matrix is not diagonalizable.".

Step 5. For each eigenvalue λ use your function grams.m from lab 8 to find an orthonormal basis of the eigenspace E_λ, these will be eigenvectors for λ.

Step 6. Let P be a matrix the same size as A, let the columns of P be all the orthonormal eigenvectors from all passes of step 5. Make sure to group the columns of P so that left most columns are in the eigenspace of the smallest eigenvalue, and so on (the order within each block of eigenvectors with the same eigenvalue does not matter).

Step 7. Let D be a matrix the same size as A, let D have all zeros on the off diagonal and the diagonal should be v. >>D = diag(v) works.

Step 8. P is in fact orthonormal since eigenvectors of different eigenvalues are perpendicular. So check that P is orthonormal by checking that $P^T P = I$ the identity matrix. Hence $P^{-1} = P^T$. Lastly check that $A = PDP^T$.

Thus, we have $D = P^{-1}AP$, a diagonalization of A. Notice then that $AP = PD$, so for a column p_i of P, $Ap_i = D(i,i)p_i$ where $D(i,i)$ is the eigenvalue for p_i; this is the definition of an eigenvalue-eigenvector pair.

Example: Let $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$, then the eigenvalues are $0, 5$ and $E_5 = span\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right)$ and $E_0 = span\left(\begin{bmatrix} -2 \\ 1 \end{bmatrix}\right)$ (Matlab might give you not nice numbers if you call $null(A - 5I)$, but it will be a scalar multiple of $span\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right)$). So one choice for P is $P = \begin{bmatrix} -2/\sqrt{5} & 1/\sqrt{5} \\ 1/\sqrt{5} & 2/\sqrt{5} \end{bmatrix}$.

Check your function on the 3 matrices given in the .txt file on blackboard in your diary.

Submit a .m file for your function and a .txt file for your diary.