Lab Project: Markov Chains

General Information: Markov Chains are designed to model systems that change from state to state. These models can be used to predict future conditions based on probabilistic information.

Instructions:

1. Let P be the following transition matrix of a Markov chain,

$$P = \begin{bmatrix} 0.10 & 0.05 & 0 & 0.25 & 0.33 \\ 0.20 & 0.35 & 0 & 0.25 & 0.32 \\ 0.30 & 0.10 & 0.35 & 0.25 & 0 \\ 0.15 & 0.40 & 0.55 & 0.25 & 0 \\ 0.25 & 0.10 & 0.10 & 0 & 0.35 \end{bmatrix}.$$

   (a) What is the probability that an individual at site 2 (the initial state vector is $(0, 1, 0, 0, 0)^T$) will move to site 5 in three steps?

   (b) Suppose 100 individuals are uniformly distributed at the five sites initially. How will the individuals be distributed after four steps?

   (c) Find the steady state vector of P.

2. *Modeling Influenza* At Malady College, a college campus of 5000 students, the spread of influenza is rampant. In this problem, we will call each student either susceptible or infected with influenza. During a given year, the percentage of the U.S. population that will get the flu each year, on average, is between 5% and 20%. At Malady, if a student is not infected with influenza the chance they will catch the flu on a given day is 16%. If a student has the flu, the chance they will recover and return to susceptible on a given day is 40%.

   (a) Create a matrix, A (called the transition matrix) whose columns represent the current state of a student, either susceptible or infected, and whose rows represent the state of a student, either susceptible or infected, the next day. Here, $A_{i,j}$ is the probability of a student going from current state j to the state i the next day.

   (b) If 100 students have the flu initially, how many students have the flu on the second day? How many students have the flu on the tenth day?

   (c) How many students have the flu initially if there are 1400 students with the flu on the third day?