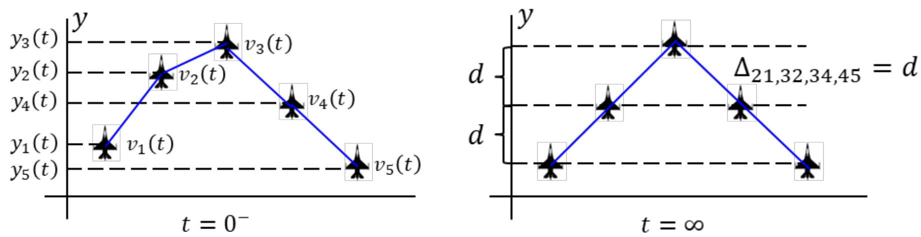
ELCT 222
Signals and Systems
Computer Assignment 6

Notes:

- Unclear or illegible work will not receive full credit.
- Label all sketches and plots completely and clearly.
- Where appropriate, "box in" your final answer.

Consider a flight formation scenario where the jets can *only* talk with its adjacent neighbors, as indicated by the blue lines below.



In this scenario, the ith jet adjusts its velocity, i.e., $v_i(t)$, on the direction of y-axis as

$$\frac{dv_i(t)}{dt} = -\frac{1}{|N_i|}\sum_{j\in N_i} \alpha\big(y_i(t) - y_j(t) - \Delta_{ij}\big) + \beta\left(v_i(t) - v_j(t)\right), \qquad (1)$$

where α and β are the stiffness and damping coefficients, respectively, $y_i(t)$ is the position of the ith jet on the y-axis, and it can be expressed as

$$y_i(t) = y_i(0) + \int_0^t v_i(t)\,dt, \qquad (2)$$

N_i is the set of neighbor indices of the ith jet, $|N_i|$ is the cardinality of the set N_i (i.e., the number of neighbors of the ith jet), and Δ_{ij} is the desired distance between the ith jet and the jth jet for $t \to \infty$. For example, for this scenario, $\Delta_{12} \coloneqq \lim_{t\to\infty} y_1(t) - y_2(t) = -d$ and $\Delta_{21} \coloneqq \lim_{t\to\infty} y_2(t) - y_1(t) = d$. (Please pay attention to the signs in your expressions.)

For the initial positions $\big(y_1(0), y_2(0), y_3(0), y_4(0), y_5(0)\big) = (0,20,40,60,80)$, initial velocities $\big(v_1(0), v_2(0), v_3(0), v_4(0), v_5(0)\big) = (500,500,500,500,500)$, $\alpha = 1, \beta = 2$, and $d = 10$,

1. (25 pts) Determine $\lim_{t\to\infty} v_i(t)$ with MATLAB for all i
2. (25 pts) Determine $V_i(s)$ with MATLAB for all i
3. (25 pts) With WolframAlpha, calculate the inverse Laplace transform of $V_3(s)$ and plot $v_3(t)$ in MATLAB
4. (25 pts) By using the approximation $\frac{dv_i(t)}{dt} \approx \frac{v_i(t+\Delta t) - v_i(t)}{\Delta t}$ in (1),
    - Develop a MATLAB code that obtains $v_i(t)$ numerically for $t \in [0,20]$ seconds for i (Hint: Choose $\Delta t = 0.001$ and use it in (1) and (2))
    - Plot $v_i(t)$ for all the jets (Hint: $v_3(t)$ should match with the result in part 3)
    - Plot $y_2(t) - y_1(t)$, $y_3(t) - y_2(t)$, $y_3(t) - y_4(t)$, and $y_4(t) - y_5(t)$ (Hint: They should approach $d = 10$ as $t \to \infty$)