

Problem a:

Computer Assignment 9.

a. Derive the Fourier transform of the source $i_s(t)$ for $A = 5 \text{ mA}$, & $T = 3 \text{ s}$

even / symmetric function

$$I_S(\omega) = 2 \int_0^{T/2} i_s(t) \cos(\omega t) dt$$

$m = \frac{0-A}{\frac{T}{2}-0} = -\frac{2A}{T}$ slope

$$i_s(t) = A + mt = A - \frac{2A}{T}t = A \left(1 - \frac{2t}{T}\right)$$

$$i_s(t) = \begin{cases} A \left(1 - \frac{2t}{T}\right), & 0 \leq t \leq \frac{T}{2} \\ 0, & \text{otherwise} \end{cases}$$

$$I_S(\omega) = 2A \int_0^{T/2} \left(1 - \frac{2t}{T}\right) \cos(\omega t) dt$$

from Wolfram Alpha:

$$\int_0^{T/2} \left(1 - \frac{2t}{T}\right) \cos(\omega t) dt = \frac{4 \sin^2\left(\frac{T\omega}{4}\right)}{T\omega^2}$$

$$2A \left[\frac{4 \sin^2\left(\frac{T\omega}{4}\right)}{T\omega^2} \right] = \frac{8A \sin^2\left(\frac{T\omega}{4}\right)}{T\omega^2}$$

$$\frac{0.04 \sin^2\left(\frac{3\omega}{4}\right)}{3\omega^2} = I_S(\omega)$$

For problem a, we are tasked with deriving the Fourier transform of the source $i_s(t)$ using values $A = 5 \text{ mA}$ and $T = 3 \text{ seconds}$. We are able to simplify the integration by realizing it is an even/symmetric function, allowing us to change the integration bounds and pull 2 out of the integral. I then determine the slope and split $i_s(t)$ into parts depending on where it is in the time domain, from $-T/2$ to $T/2$, it has some type of slope, otherwise, its value is 0. I use Wolfram Alpha to solve the complex integral, then manually simplify it into an easy-to-manage form.

Problem b:

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b. Determine the transfer function $H(\omega) = \frac{V_{out}(\omega)}{I_s(\omega)}$ for $R_1 = 500 \Omega$, $R_2 = 2000 \Omega$, $C = \frac{1}{3} \text{ mF}$

$Z_2 = R_2 + Z_C = R_2 + \frac{1}{j\omega C}$

Admittance at the node:

$$Y_{eq} = Y_1 + Y_2 = \frac{1}{R_1} + \frac{1}{Z_2} \rightarrow V_1 = \frac{I_s}{Y_{eq}} = \frac{I_s}{\frac{1}{R_1} + \frac{1}{Z_2}}$$

$V_{out} = V_1 \frac{Z_C}{Z_2} \rightarrow H(\omega) = \frac{V_{out}}{I_s} = \frac{Z_C}{R_1 + Z_2}$

$\frac{V_{out}}{I_s} = \frac{V_1}{I_s} \times \frac{Z_C}{Z_2} = \frac{1}{\frac{1}{R_1} + \frac{1}{Z_2}} \times \frac{Z_C}{Z_2} = \frac{R_1 Z_C}{R_1 + Z_2} = \frac{R_1 Z_C}{R_1 + R_2 + \frac{1}{j\omega C}}$

$= \frac{R_1 \cdot \frac{1}{j\omega C}}{R_1 + R_2 + \frac{1}{j\omega C}} \rightarrow \frac{R_1}{1 + j\omega C(R_1 + R_2)}$

$R_1 = 500 \Omega$, $R_2 = 2000 \Omega$, $C = .003 \text{ F}$

$= \frac{500}{1 + j\omega \cdot .0033(2500)} \rightarrow H(\omega) = \frac{500}{1 + j\omega(8.33)}$

Problem b asks us to determine the transfer function $H(\omega)$ using values $R_1 = 500$, $R_2 = 2000$, and $C = \frac{1}{3} \text{ mF}$. I first compute the impedance of the parallel branch and name it Z_2 . Using the admittance, I am able to calculate the input voltage, which I can later use to calculate the output voltage. I then compute the transfer function by dividing v_{out} by I_s , which gives the transfer function expression. Using the given values, I can approximate the transfer function to be used in later steps of the assignment.

Problem c:

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c. Obtain $V_{out}(t)$ by using Fourier analysis

$I_s(\omega) = \frac{.04 \sin^2(\frac{3\omega}{4})}{3\omega^2}$, $H(\omega) = \frac{500}{1 + j\omega(\frac{8}{3})}$

$V_{out}(\omega) = H(\omega) I_s(\omega)$

$= \left[\frac{500}{1 + j\omega(\frac{8}{3})} \right] \left[\frac{.04 \sin^2(\frac{3\omega}{4})}{3\omega^2} \right]$

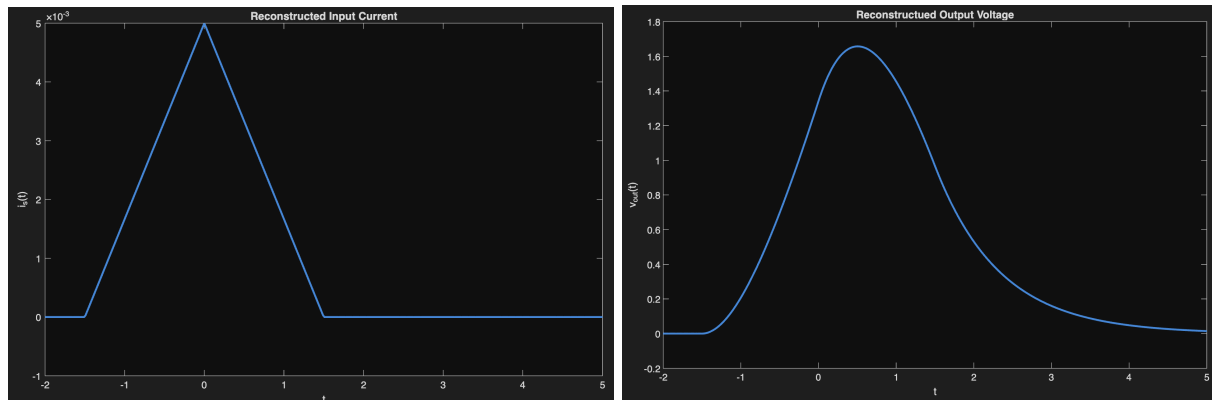
$V_{out}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} V_{out}(\omega) e^{j\omega t} d\omega$

$V_{out}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[\frac{500}{1 + j\omega(\frac{8}{3})} \right] \left[\frac{.04 \sin^2(\frac{3\omega}{4})}{3\omega^2} \right] e^{j\omega t} d\omega$

→ left as analytical expression

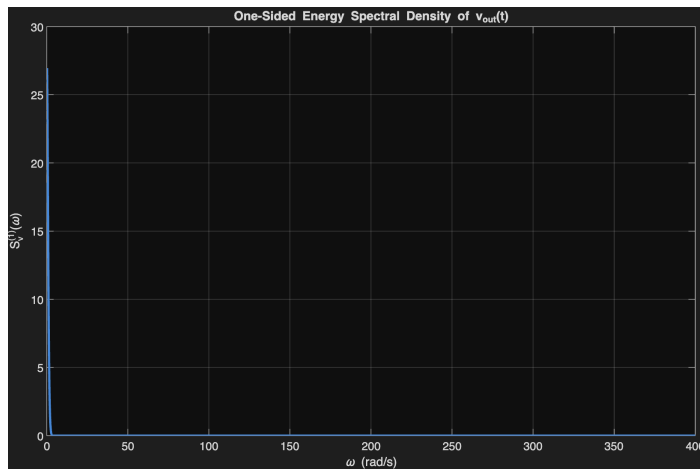
For problem c, we must find $v_{out}(t)$, which we can do by considering $V_{out}(\omega) = H(\omega)I_s(\omega)$. Multiplying these two terms gives us $V_{out}(\omega)$, which we can use in the inverse Fourier transform. We leave this as is without solving the integration, as the assignment specifically states I can leave it as an “analytical expression”.

Problem d:



For problem d, we must reconstruct the input and output signals. I first create a time vector from -2 to 5 to capture the desired change. I then make zero matrices for i_s and v_{out} and use a for loop to calculate each value for a given omega value. I created a variable wMax, as when I tried to do this step from -inf to inf, it caused issues with MATLAB. To solve this, I realized that setting wMax set at 400 did not limit the output. I set i_s and v_{out} accordingly using their functions, which I found in earlier steps of the assignment. I then plot both of these filled-in vectors on their own figure, hence the plots above.

Problem e:



For problem e, we are asked to plot the one-sided energy spectral density of v_{out} . I was able to solve this problem by first creating a time vector from 0 to our previously set wMax value of 400. I then calculated the two-sided energy spectral density, using $V_{out}(w)$, which we found in part c. I took the magnitude of this and squared it to get the desired value. To solve for the one-sided version, I multiplied each value by 2, except for the DC component, which should stay as it is. Once I had a vector of all of the one-sided energy spectral density components, I plotted this vector over my time vector, generating the attached plot.

Appendix:

% Luis Kligman --- Computer Assignment 9

clear all;

close all;

clc;

%% Parameters

A = 5e-3; % 5 mA

T = 3; % Period (seconds)

R1 = 500;

R2 = 2000;

C = (1/3) * 1e-3; % 1/3 mF

tau = C*(R1 + R2); % time constant = 5/6 seconds

% Define I_s(omega)

I_s = @(w) ...

(w==0).*(A*T/2) + ...

(w~=0).*((8*A ./ (T*w.^2)) .* sin(T*w/4).^2);

% Transfer function H(omega)

H = @(w) R1 ./ (1 + 1j*w*tau);

% Output spectrum V_out(omega)

V_out = @(w) H(w).*I_s(w);

%% Part D: plot functions with a numeric integration from -inf to inf

% Time vecotr

t = linspace(-2,5,400);

i_s = zeros(size(t));

v_out = zeros(size(t));

wMax = 400;

for k = 1:length(t)

ti = t(k);

i_s(k) = real(integral(@(w) I_s(w) .* exp(1j*w*ti), -wMax, wMax) / (2 * pi));

v_out(k) = real(integral(@(w) V_out(w) .* exp(1j*w*ti), -wMax, wMax) / (2 * pi));

end

% Plot i_s(t)

figure;

plot(t, i_s, 'LineWidth', 2);

xlabel('t');

ylabel('i_s(t)');

title('Reconstructed Input Current');

% Plot v_out(t)

figure;

plot(t, v_out, 'LineWidth', 2);

xlabel('t');

ylabel('v_{out}(t)');

title('Reconstructued Output Voltage');

%% Part E: Plot one-sided energy spectral density of v_out(t)

w = linspace(0, wMax, 2000); % rad/s

% Two-sided energy spectral density

Sv_two = abs(V_out(w)).^2;

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% One-sided energy spectral density
% double everything except DC
Sv_one = 2 * Sv_two;
Sv_one(1) = Sv_two(1); % Do not double the DC component
% Plot one-sided energy spectral density
figure;
plot(w, Sv_one, 'LineWidth', 2);
xlabel('\omega (rad/s)');
ylabel('S_v^{(1)}(\omega)');
title('One-Sided Energy Spectral Density of v_{out}(t)');
grid on;

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