ELCT 222
Signals and Systems
Computer Assignment 4

Important:

- This assignment must be typed (e.g., a Word document) – no handwritten work accepted.
- Providing only MATLAB code or only MATLAB output without any discussion will receive 0 points.
- Unclear or illegible work will not receive full credit.
- For answers, provide your discussions or your approach to solving the problems and describe in clear English how your routine works in a few sentences.
- Include MATLAB code at the END of the assignment as an appendix.
- Label all axes in MATLAB.

Fibonacci numbers $F_0, F_1, F_2, \ldots$ satisfy the following recurrence relation:

$$F_{n+1} = F_n + F_{n-1}\ (n \geq 1, F_0 = 0, F_1 = 1)$$

The sequence begins with 0,1,1,2,3,5 .... Now, consider the polynomial defined by

$$f(t) \coloneqq \frac{F_0}{0!} + \frac{F_1}{1!}t + \frac{F_2}{2!}t^2 + \frac{F_3}{3!}t^3 \ldots = \sum_{k=0}^{\infty} \frac{F_k}{k!}t^k$$

This is also known as exponential generating function (EGF) of Fibonacci sequence.

1. (10 pts) Show the nth Fibonacci number can be obtained as

$$F_n = \frac{d^n}{dt^n} f(t)|_{t=0}$$

2. (10 pts) By using the recurrence relation, prove that the following relationships holds:

$$\frac{d^2 f(t)}{dt^2} = \frac{df(t)}{dt} + f(t)$$

3. (20 pts) Calculate the Laplace transform of $f(t)$ from the differential equation above.
4. (20 pts) Solve $f(t)$ by using partial fractions expansion of $F(s)$ and identify the zeros and poles of $F(s)$.
5. Plot $f(t)$ in MATLAB between for $t \in [-2,2]$:
    a. (2 pts) By using EGF expression above by using terms $F_0, \ldots, F_5$
    b. (8 pts) By using the exact expression obtained by using the inverse Laplace transform.
6. (20 pts) Obtain a closed-form expression for F_n by using the closed-form expression obtained from the inverse Laplace transform and $F_n = \frac{d^n}{dt^n} f(t)|_{t=0}$ (note that $u(t) = 1, t \geq 0, u(t) = 0, t < 0$).
7. (10 pts) Express F_{19} by using the closed-form expression and calculate its value based on this expression in MATLAB.