Stochastic Lattice Lotka-Volterra Predator-Prey Models

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Population dynamics is the study of interacting particle systems typically involving a number of different species, and their time evolution.

LV competition model that describes the population densities of predators *a* and prey *b*:

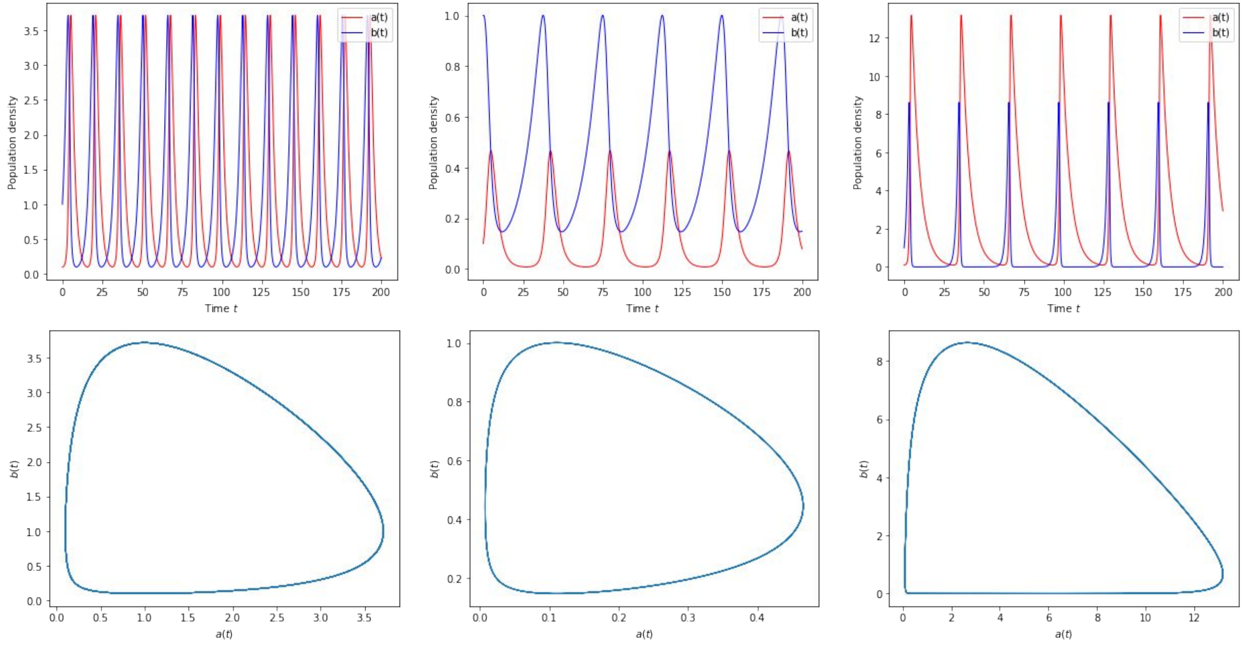
$$\frac{da(t)}{dt} = a(t)[\lambda b(t) - \mu], \frac{db(t)}{dt} = b(t)[\sigma - \lambda a(t)]$$

The parameters λ, μ, and σ describe the phenomenological predation, predator death, and prey reproduction rates, respectively.

CLASSIC LOTKA-VOLTERRA

$$\frac{da(t)}{dt} = a(t)[\lambda b(t) - \mu], \frac{db(t)}{dt} = b(t)[\sigma - \lambda a(t)]$$

λ = 0.5 μ=0.5 σ=0.5   *λ = 0.9 μ=0.4 σ=0.1*   *λ = 0.3 μ=0.2 σ=0.8*



The microscopic and more general LV reaction rules from which equations derive are given by:

$$B \xrightarrow{\sigma} B + B, \quad A \xrightarrow{\mu} \emptyset, \quad A + B \xrightarrow{\lambda} A + A$$

The prey *B* reproduce with rate $\sigma > 0$; the predators *A* spontaneously die with rate $\mu > 0$; and upon encountering each other in their immediate vicinity, both species may interact with predation rate $\lambda > 0$.

Adding spatial freedom: If implemented on a lattice spatial correlation is achieved. Invasion between species, spatio-temporal patterns, diffusivity...

Diffusivity can be implemented by neighbouring site interactions or with a nearest site hopping rate *D*.

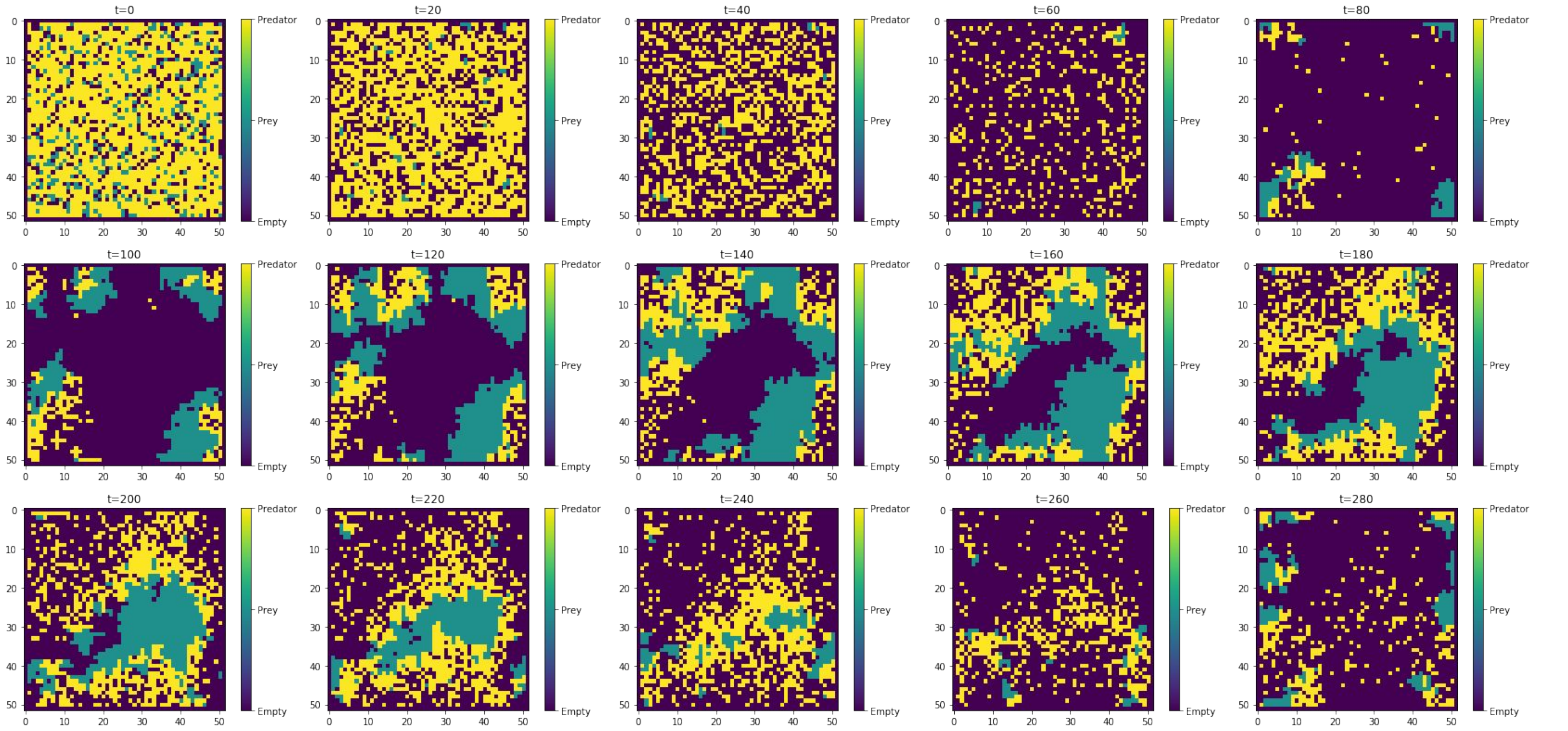
Adding stochasticity: Changes in the behaviour of the system.

Spatio-temporal Lotka-Volterra in a 2D-Lattice with particle restriction and periodic boundary conditions:

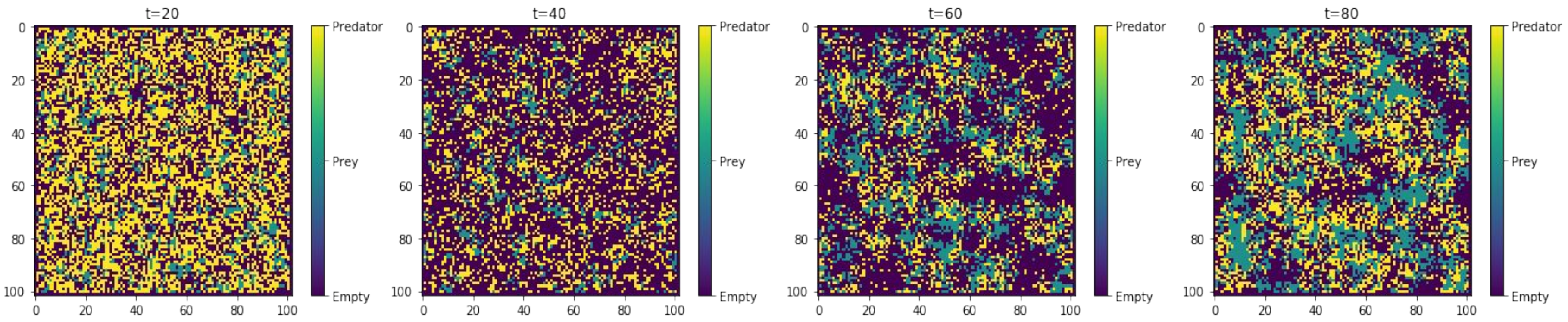
Select a lattice occupant at random and generate a random number *r* in the range [0,1] to perform one of the following possible reactions:

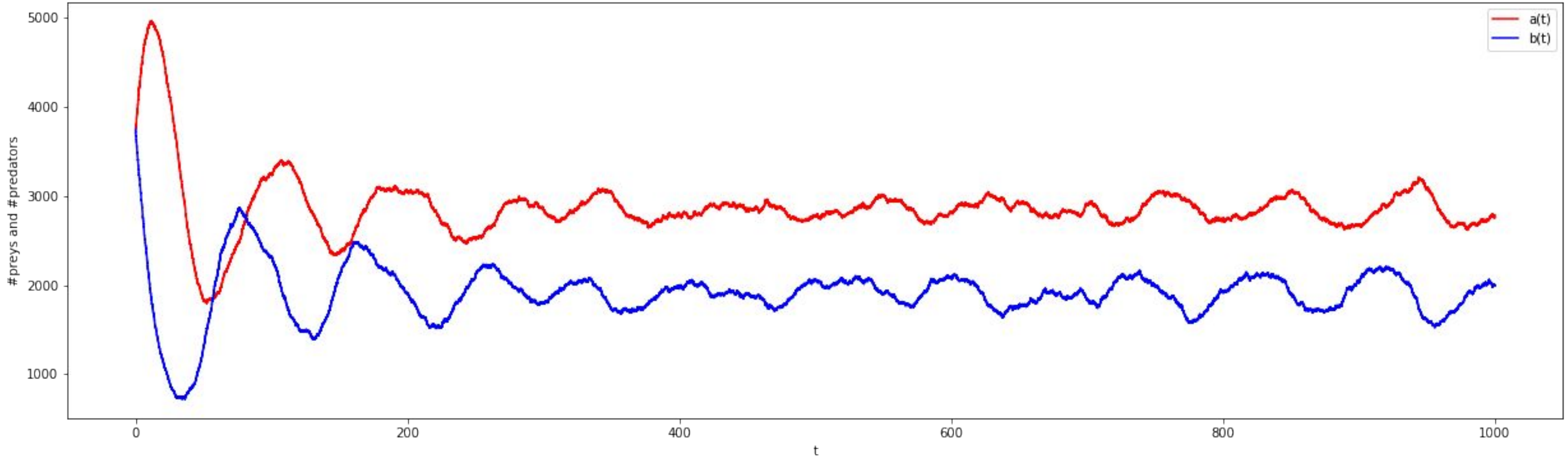
1. If $r < .25$, choose randomly one adjacent site and move the occupant there with probability *D*, if it's empty.
2. If $.25 < r < .5$ and the occupant is an *A*, the occupant die with probability μ.
3. If $.5 < r < .75$ and the occupant is an *A*, choose randomly one adjacent site and if there is a *B*, with probability λ replace it with an *A* particle.
4. If $.75 < r < 1$ and the occupant is a *B*, choose randomly one adjacent site and generate an offspring with probability σ if it's empty.

Lattice 50x50; $a(0)=0.8$ & $b(0)=0.2$; $\lambda=1$, $\mu=0.05$, $\sigma=0.6$, $D=0$.

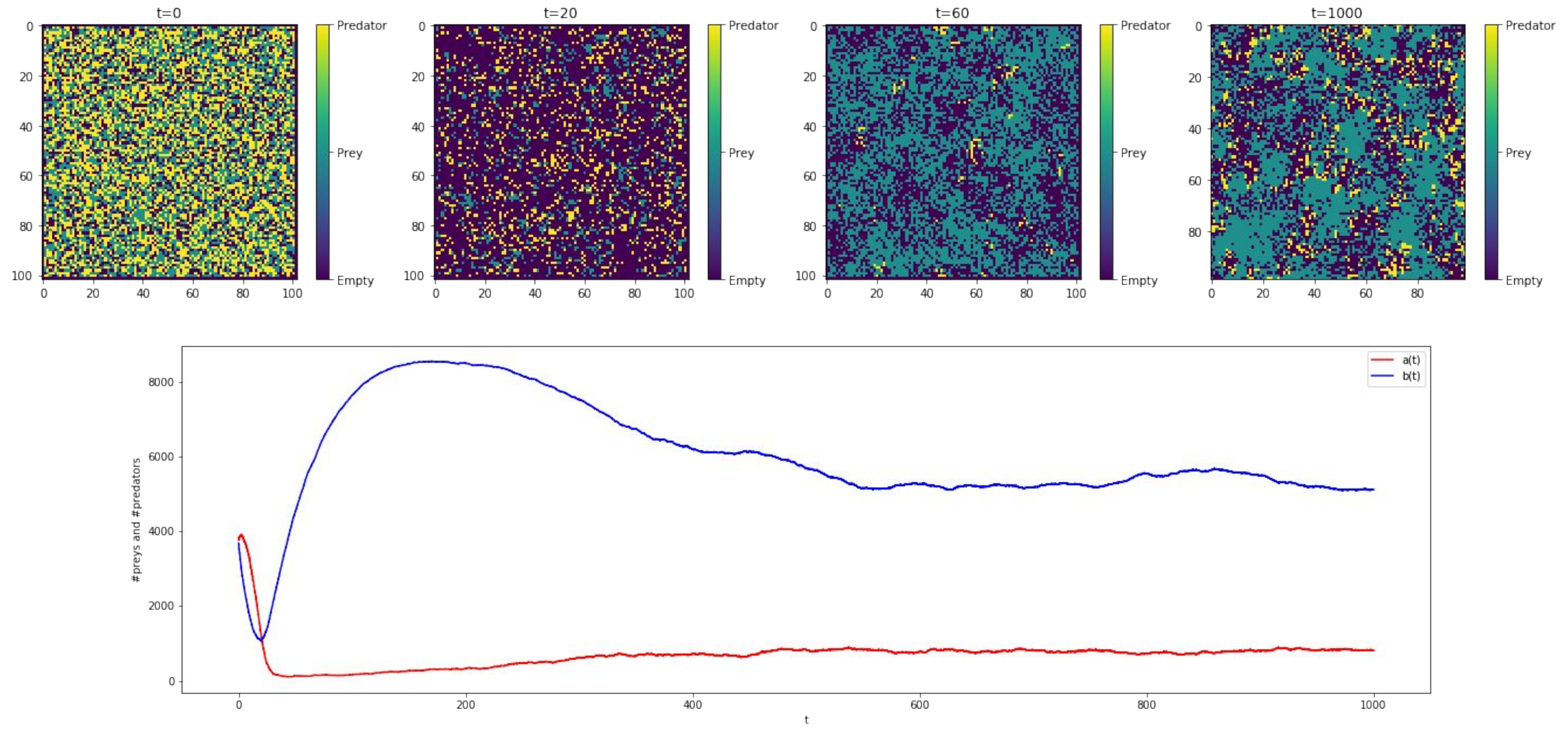


Lattice 100x100; a(0)=0.5 & b(0)=0.5; λ=1, μ=0.1, σ=0.4, D=1.

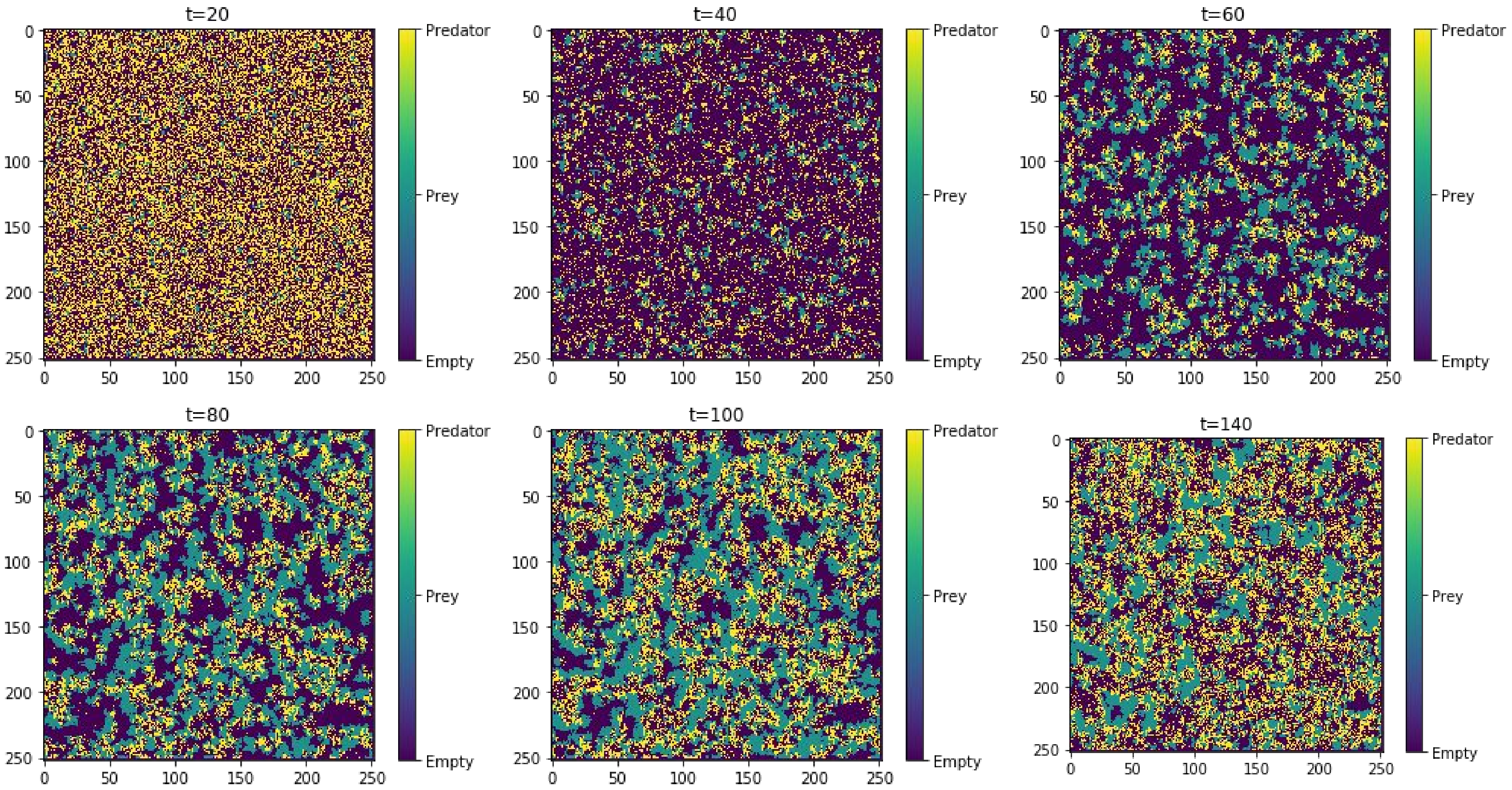




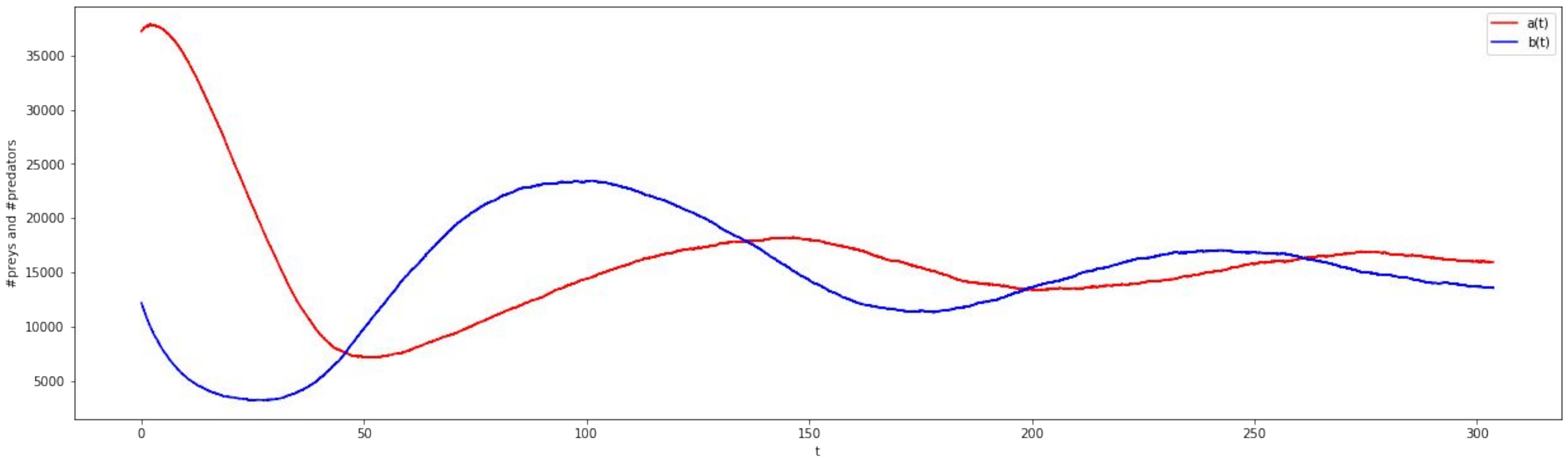
Lattice 100x100; a(0)=0.5 & b(0)=0.5; λ=1, μ=0.2, σ=0.1, D=1.



Lattice 250x250; a(0)=0.7 & b(0)=0.3; λ=1, μ=0.1, σ=0.5, D=0.



Lattice 250x250; $a(0)=0.7$ & $b(0)=0.3$; $\lambda=1$, $\mu=0.1$, $\sigma=0.5$, $D=0$.



THANK YOU

for your attention