PHYS 1310

Chapter 7

Potential Energy and Energy Conservation

Chapter 7 in a Nutshell

- Potential Gravitational Energy: $\Delta U_{grav} = mg\Delta h$
- Potential Elastic Energy: $\Delta U_{el} = \frac{1}{2}\mathrm{k}x^2$

- Conservation of Energy:

$$\Delta E = \Delta K + \Delta U = \frac{1}{2}\mathrm{m}(\Delta v)^2 - mg\Delta h = 0$$

- Determining Force from Potential Energy:

$$F(x) = -\frac{d}{dx}U(x) = -\left(\frac{\delta U}{\delta x}\hat{\imath} + \frac{\delta U}{\delta y}\hat{\jmath} + \frac{\delta U}{\delta z}\hat{z}\right)$$

CH 7 Objectives

- How to use gravitational potential energy in problems that involve vertical motion.

- How to use elastic potential energy in problems that involve a moving object attached to a stretched or compressed spring.

- The distinction between conservative and non-conservative forces. (Conservative forces always have a corresponding potential-energy function.)

- How to use energy diagrams to understand how an object moves under the influence of a conservative force.

Introduction

- How do energy concepts apply to the descending sandhill crane?
- We will see that we can think of energy as being stored and **transformed** from one form to another.

Gravitational Potential Energy (1 of 3)

- When a particle is in the gravitational field of the earth, there is a gravitational potential energy associated with the particle:

Gravitational potential energy associated with a particle

$$U_{\text{grav}} = mgy$$

Vertical coordinate of particle (y increases if particle moves upward)

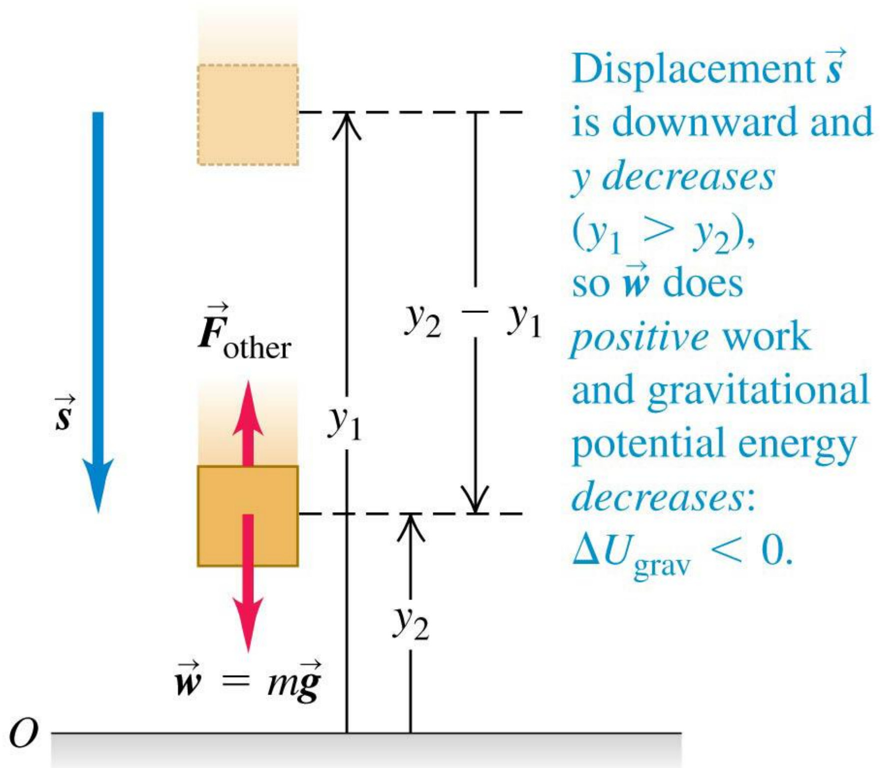
Mass of particle

Acceleration due to gravity

- As the basketball descends, gravitational potential energy is converted to kinetic energy and the basketball's speed increases.

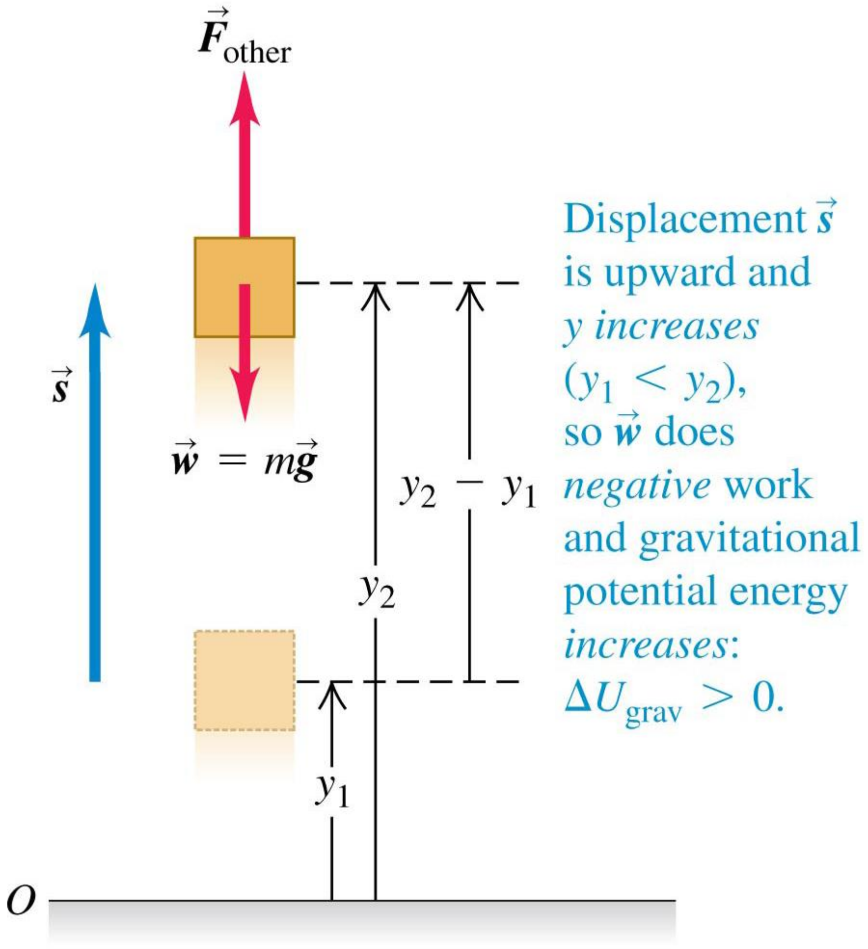
Gravitational Potential Energy (2 of 3)

- The change in gravitational potential energy is related to the work done by gravity.



Gravitational Potential Energy (3 of 3)

- When the object moves up, *y* increases, the work done by the gravitational force is negative, and the gravitational potential energy increases.



The Conservation of Mechanical Energy (1 of 2)

- The total **mechanical energy** of a system is the sum of its kinetic energy and potential energy.
- A quantity that always has the same value is called a **conserved** quantity.
- When only the force of gravity does work on a system, the total mechanical energy of that system is conserved.
- This is an example of the **conservation of mechanical energy**.

If only the gravitational force does work, total mechanical energy is conserved:

Initial kinetic energy $K_1 = \frac{1}{2}mv_1^2$

Initial gravitational potential energy $U_{grav,1} = mgy_1$

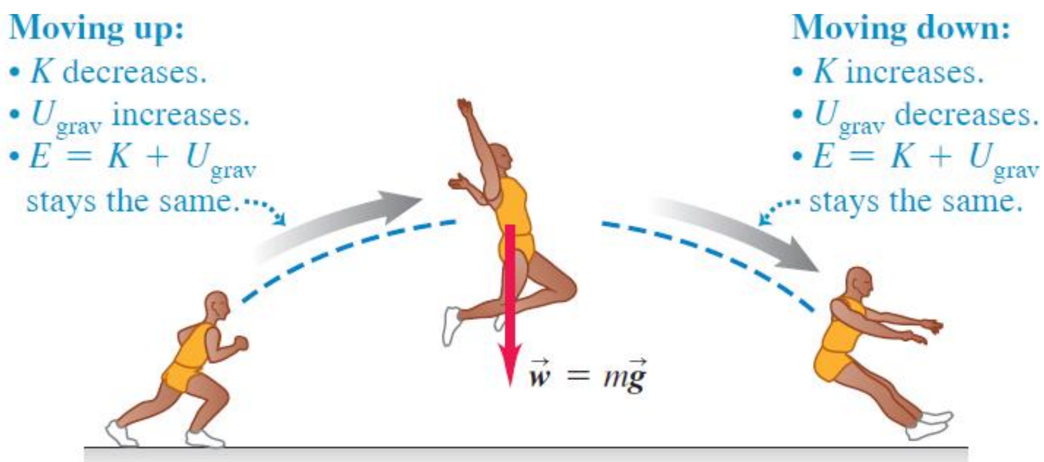
$$K_1 + U_{grav,1} = K_2 + U_{grav,2}$$

Final kinetic energy $K_2 = \frac{1}{2}mv_2^2$

Final gravitational potential energy $U_{grav,2} = mgy_2$

The Conservation of Mechanical Energy (2 of 2)

- When only the force of gravity does work on a system, the total mechanical energy of that system is conserved.



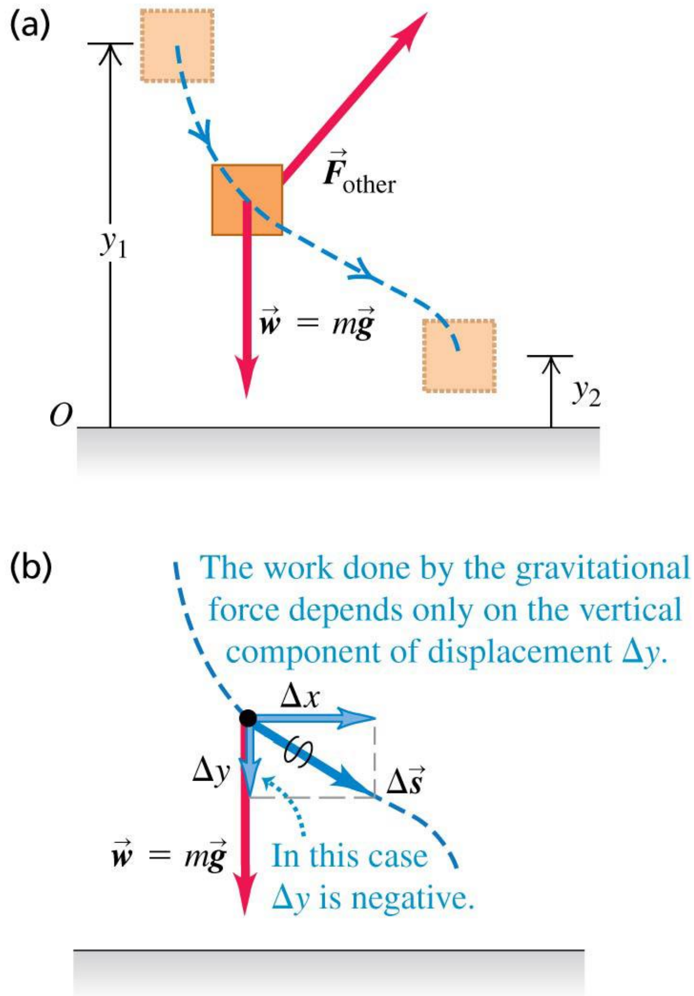
When Forces Other Than Gravity Do Work

- $\vec{F}_{\text{other}}$ and $\vec{s}$ are opposite, so $W_{\text{other}} < 0$.
- Hence $E = K + U_{\text{grav}}$ must decrease.
- The parachutist's speed remains constant, so K is constant.
- The parachutist descends, so U_{grav} decreases.

Work and Energy Along a Curved Path

- We can use the same expression for gravitational potential energy whether the object's path is curved or straight.

- $W_{grav} = mgy_1 - mgy_2$



Elastic Potential Energy (1 of 2)

- A object is **elastic** if it returns to its original shape after being deformed.
- **Elastic potential energy** is the energy stored in an elastic object, such as a spring:

Elastic potential energy stored in a spring

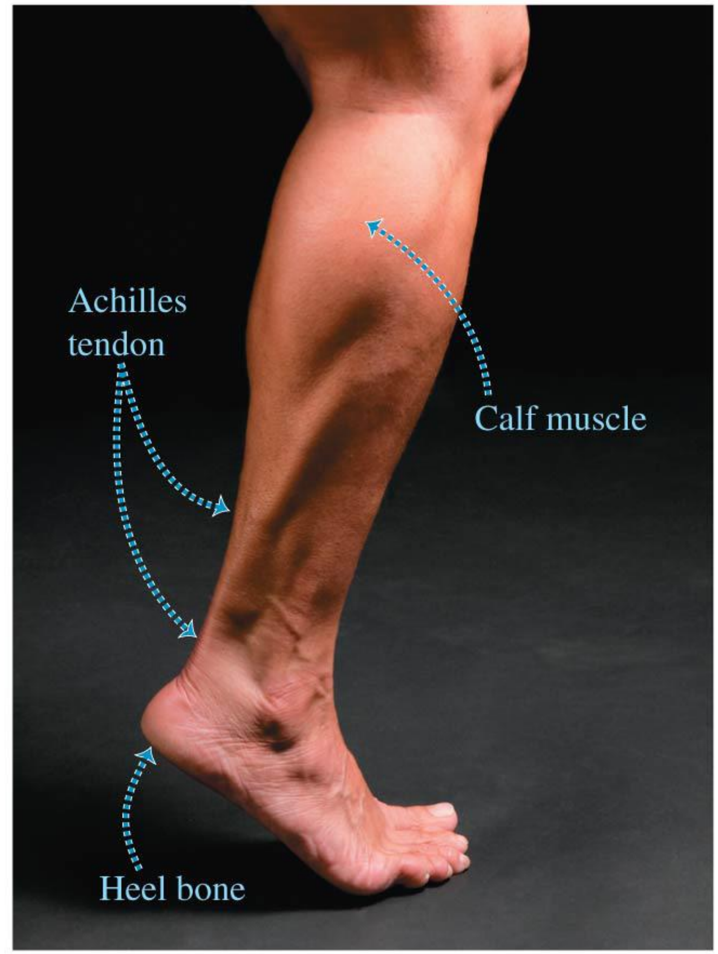
$$U_{\mathrm{el}} = \tfrac{1}{2}kx^2$$

Force constant of spring

Elongation of spring ($x > 0$ if stretched, $x < 0$ if compressed)

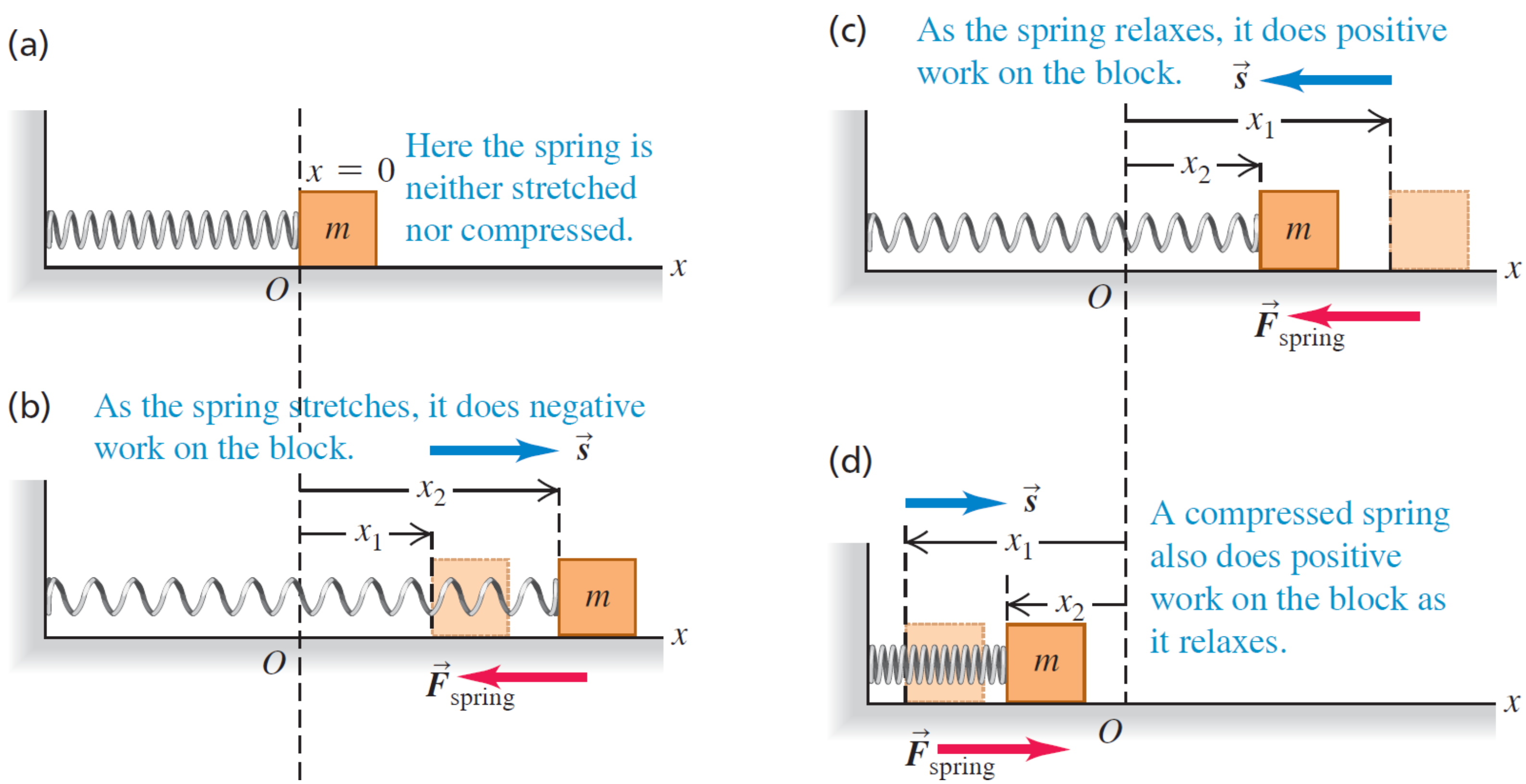
Elastic Potential Energy (2 of 2)

- The Achilles tendon acts like a natural spring.
- When it stretches and then relaxes, this tendon stores and then releases elastic potential energy.
- This spring action reduces the amount of work your leg muscles must do as you run.



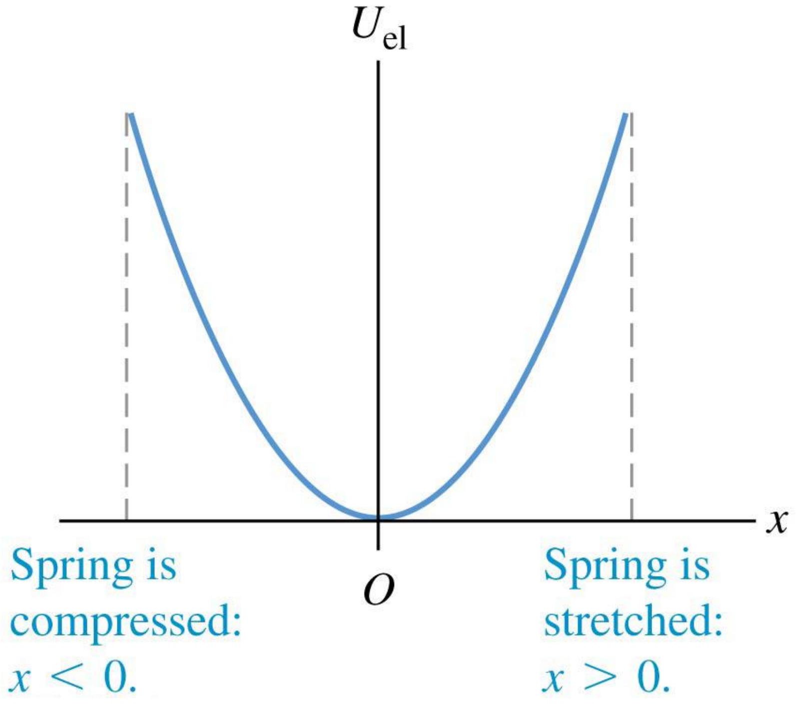
Work Done by a Spring

- Figure 7.13 below shows how a spring does work on a block as it is stretched and compressed.



Elastic Potential Energy

- The graph of elastic potential energy for an ideal spring is a parabola.
- *x* is the extension or compression of the spring.
- Elastic potential energy is never negative.



Situations with Both Gravitational and Elastic Forces

- When a situation involves both gravitational and elastic forces, the total potential energy is the **sum** of the gravitational potential energy and the elastic potential energy:

$$U = U_{\text{grav}} + U_{\text{el}}$$

Conservative vs Non-Conservative Forces

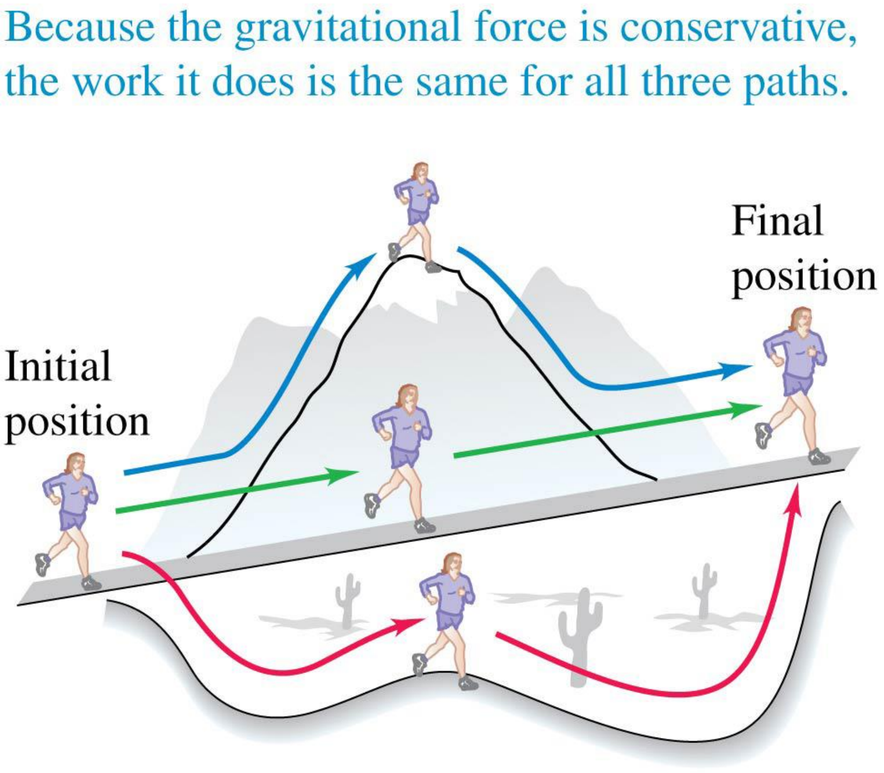
- A **conservative force** allows conversion between kinetic and potential energy. Gravity and the spring force are conservative.

- The work done between two points by any conservative force
  a) can be expressed in terms of a **potential energy function**.
  b) is reversible.
  c) is independent of the path between the two points.
  d) is zero if the starting and ending points are the same.

- A force (such as friction) that is not conservative is called a **nonconservative force**, or a **dissipative force**.

Conservative Forces

- The work done by a conservative force such as gravity depends on only the endpoints of a path, not the specific path taken between those points.



Non-Conservative Forces

- As an automobile tire flexes as it rolls, nonconservative internal friction forces act within the rubber.
- Mechanical energy is lost and converted to internal energy of the tire.
- This causes the temperature and pressure of a tire to increase as it rolls.
- That’s why tire pressure is best checked before the car is driven, when the tire is cold.

Conservation of Energy

- Nonconservative forces do not store potential energy, but they do change the **internal energy** of a system.
- **The law of conservation of energy** means that energy is never created or destroyed; it only changes form.
- This law can be expressed as: $\Delta K + \Delta U + \Delta U_{\text{int}} = 0.$

$$\Delta E = \Delta K + \Delta U \sim \frac{1}{2}\mathrm{m}(\Delta v)^2 - mg\Delta h = 0$$

Force and Potential Energy in One Dimension (1 of 3)

- In one dimension, a conservative force can be obtained from its potential energy function using:

Force from potential energy: In **one-dimensional motion,** the value of a conservative force at point x ...

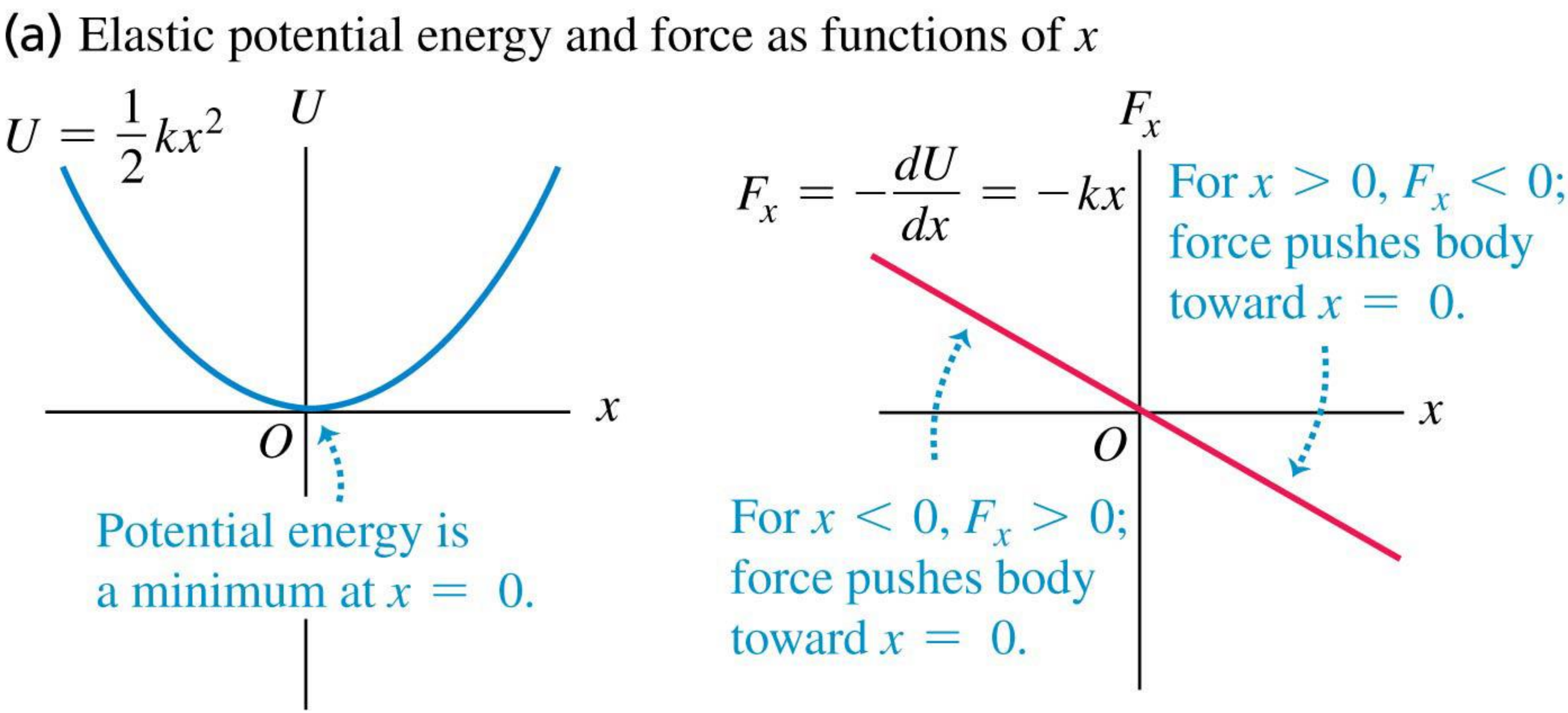
$$F_x(x) = -\frac{dU(x)}{dx}$$

... is the negative of the derivative at x of the associated potential-energy function.

- In regions where $U(x)$ changes most rapidly with x, this corresponds to a large force magnitude.
- Also, when $F_x(x)$ is in the positive x-direction, $U(x)$ decreases with increasing x.
- A conservative force always acts to push the system toward **lower** potential energy.

Force and Potential Energy in One Dimension (2 of 3)

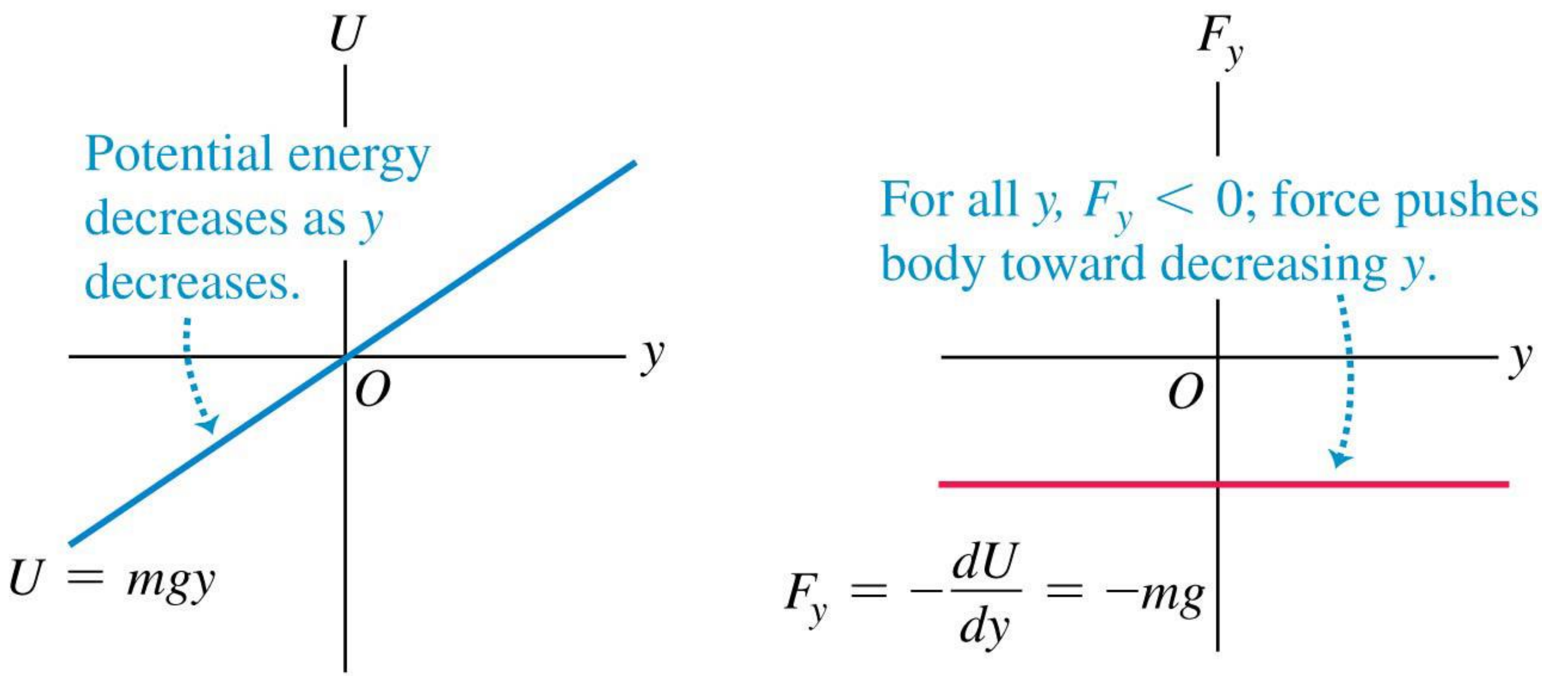
- Elastic potential energy and force as functions of *x* for an ideal spring.



Force and Potential Energy in One Dimension (3 of 3)

- Gravitational potential energy and the gravitational force as functions of *y*.

(b) Gravitational potential energy and force as functions of y



Force and Potential Energy in Three Dimensions (1 of 2)

- In three dimensions, the components of a conservative force can be obtained from its potential energy function using partial derivatives:

Force from potential energy: In **three-dimensional motion,** the value at a given point of each component of a conservative force ...

$$F_x = -\frac{\partial U}{\partial x} \qquad F_y = -\frac{\partial U}{\partial y} \qquad F_z = -\frac{\partial U}{\partial z}$$

... is the negative of the partial derivative at that point of the associated potential-energy function.

Force and Potential Energy in Three Dimensions (2 of 2)

- When we take the partial derivative of *U* with respect to each coordinate, multiply by the corresponding unit vector, and then take the vector sum, this is called the **gradient** of *U*:

Force from potential energy: The vector value of a conservative force at a given point ...

$$\vec{F} = -\left(\frac{\partial U}{\partial x}\hat{\imath} + \frac{\partial U}{\partial y}\hat{\jmath} + \frac{\partial U}{\partial z}\hat{k}\right) = -\vec{\nabla} U$$

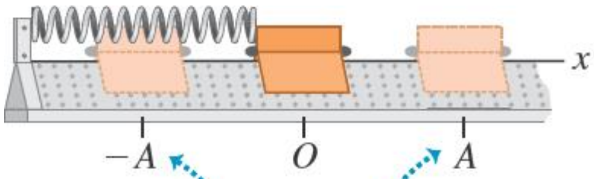
... is the negative of the gradient at that point of the associated potential-energy function.

Force and Potential Energy

- The greater the elevation of a hiker in Canada’s Banff National Park, the greater the gravitational potential energy U_{grav}.
- Where the mountains have steep slopes, U_{grav} has a large gradient and there’s a strong force pushing you along the mountain’s surface toward a region of lower elevation (and hence lower U_{grav}).
- There’s zero force along the surface of the lake, which is all at the same elevation.

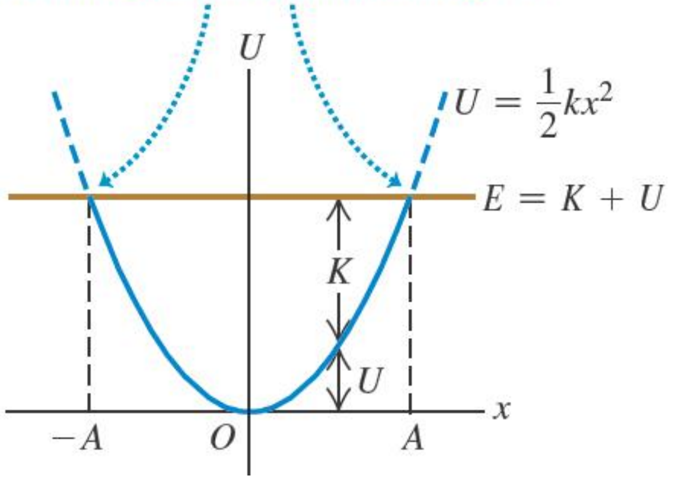
Energy Diagrams

- An **energy diagram** is a graph that shows both the potential-energy function $U(x)$ and the total mechanical energy E.
- The figure illustrates the energy diagram for a glider attached to a spring on an air track.



The limits of the glider's motion are at $x = A$ and $x = -A$.

On the graph, the limits of motion are the points where the U curve intersects the horizontal line representing total mechanical energy E.



Force and a Graph of Its Potential-Energy Function

- For any graph of potential energy versus x, the corresponding force is: $$F_x = -\frac{dU}{dx}.$$
- Whenever the slope of U is zero, the force there is zero, and this is a point of equilibrium.
- When U is at a minimum, the force near the minimum draws the object **closer** to the minimum, so it is a **restoring force**. This is called **stable equilibrium**.
- When U is at a maximum, the force near the maximum draws the object **away** from the maximum. This is called **unstable equilibrium**.

Unstable Equilibrium

- Each of these acrobats is in unstable equilibrium.
- The gravitational potential energy is lower no matter which way an acrobat tips, so if she begins to fall she will keep on falling.
- Staying balanced requires the acrobats' constant attention.