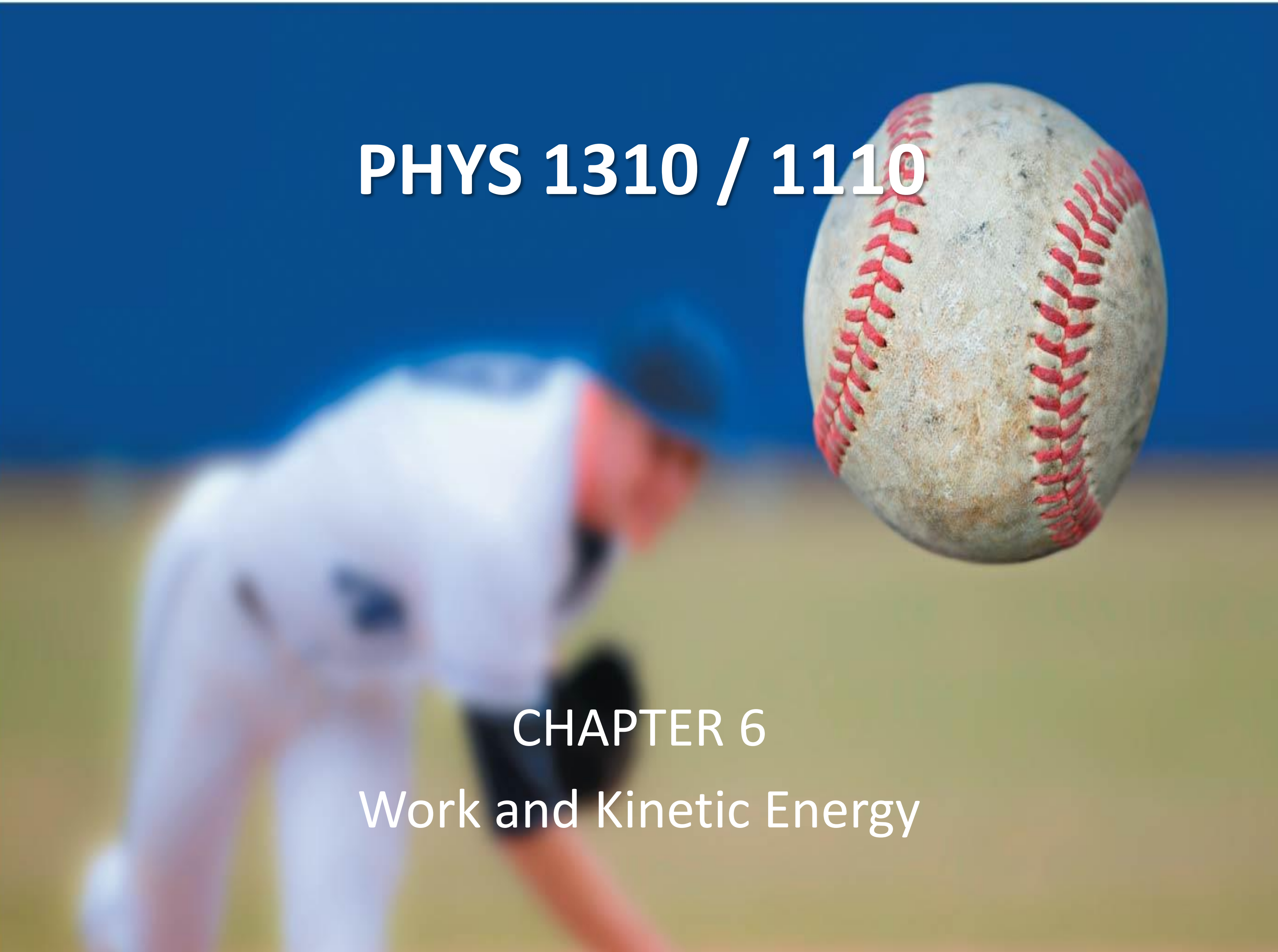


PHYS 1310 / 1110

CHAPTER 6

Work and Kinetic Energy

CH6 in a Nutshell

- Hook's Law: $F = k \cdot \Delta x$ $\quad [N = \frac{kg}{s^2} \cdot m]$

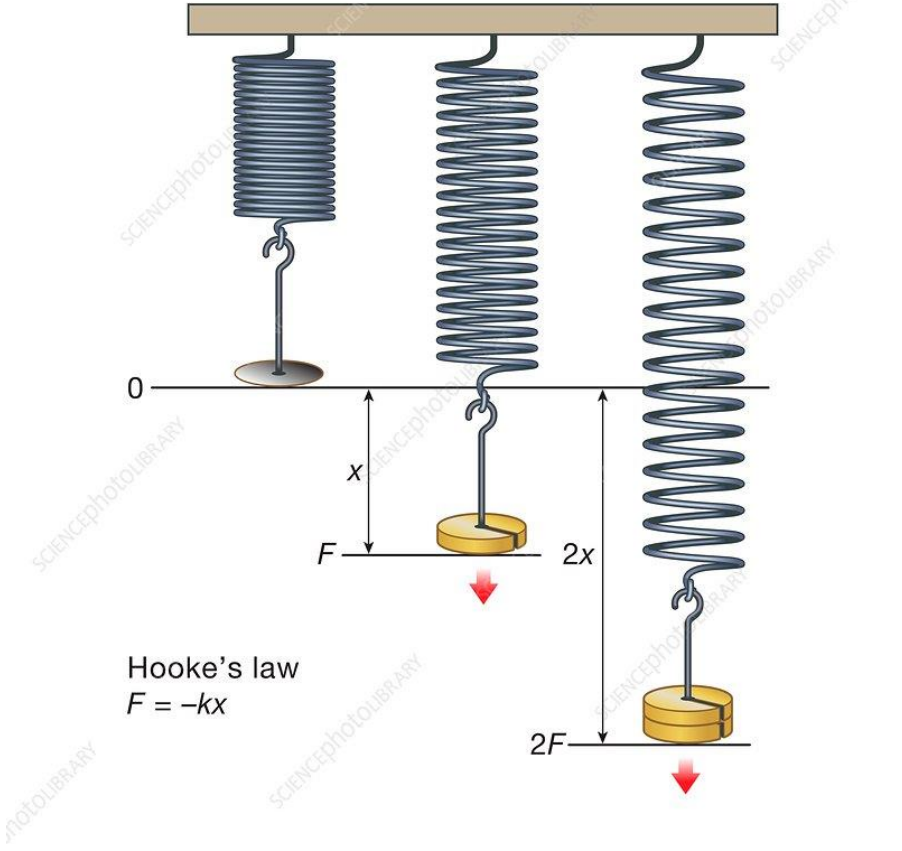
- Work: $W = Fx = \int F\, dx = \frac{1}{2} kx^2$ $\quad [J = \mathrm{N} \cdot m]$

- Kinetic Energy: $K = \frac{1}{2} mv^2$ $\quad [J = \mathrm{N} \cdot m]$

- Power: $P = \frac{W}{\Delta t} = F \frac{\Delta x}{\Delta t} = F \cdot v$ $\quad \left[W = \frac{J}{s}\right]$

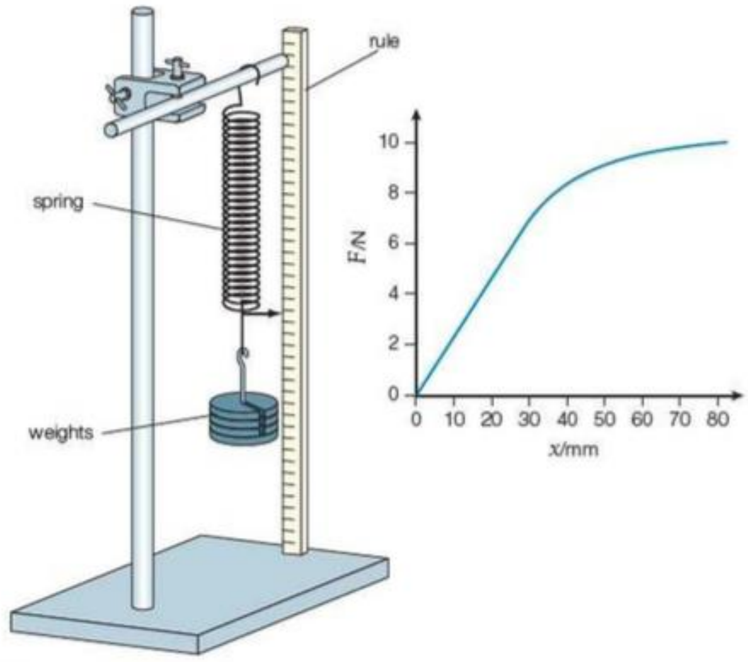
Hooke's Law

- … states that the strain of the material is proportional to the applied stress within the elastic limit of that material.



$$F = k \cdot \Delta x$$

where k is the stiffness of the spring [N/m].



Overview

- 6.1 Work
- 6.2 Kinetic Energy
- 6.3 Work and Energy
- 6.4 Power

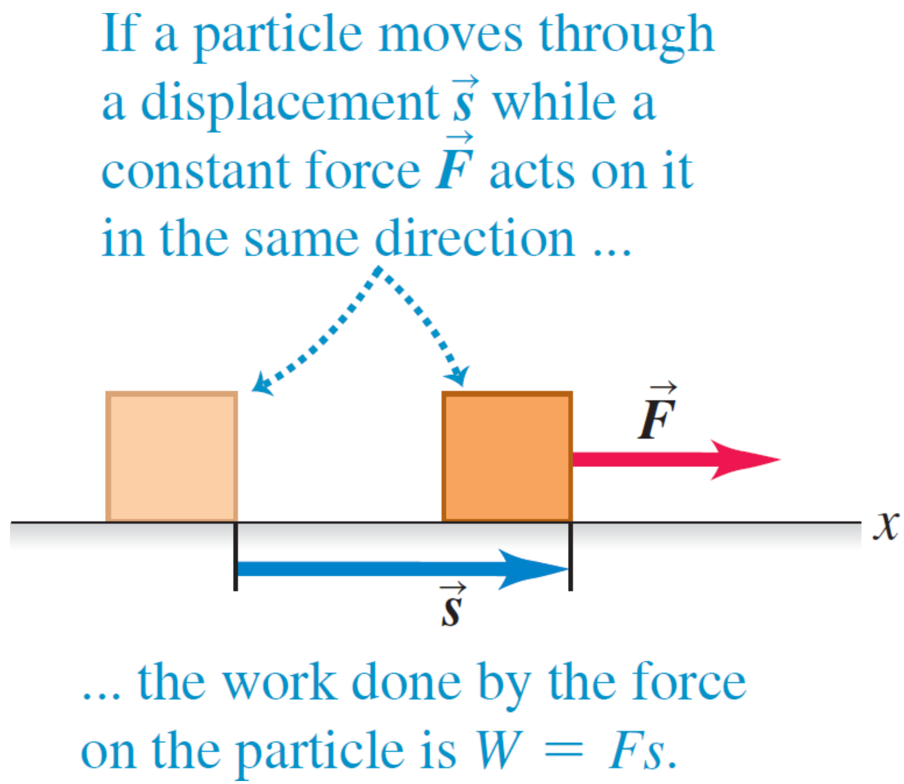
Introduction

- A baseball pitcher does work with his throwing arm to give the ball a property called **kinetic energy**.
- In this chapter, the introduction of the new concepts of **work**, **energy**, and the **conservation of energy** will allow us to deal with problems in which Newton's laws alone aren't enough.

Work

- A force on an object does **work** if the object undergoes a displacement.

These people are doing work as they push on the car because they exert a force on the car as it moves.



Units of Work

$$1\,J = \frac{kg{\cdot}m^2}{s^2}$$

- The SI unit of work is the **joule** (named in honor of the 19th-century English physicist James Prescott Joule).

- Since $W = Fx$, the unit of work is the unit of force multiplied by the unit of distance.

- <u>In SI units</u>:

1 joule = (1 newton) (1 meter) or 1 J = 1 N · m

- If you lift an object with a weight of 1 N a distance of 1 m at a constant speed, you do 1 J of work on it.

Work Done by a Constant Force (1 of 2)

- The work done by a constant force acting at an angle to the displacement is:

Work done on a particle by **constant force $\vec{F}$** during **straight-line displacement $\vec{s}$**

$$W = Fs\cos\phi$$

Magnitude of $\vec{F}$

Angle between $\vec{F}$ and $\vec{s}$

Magnitude of $\vec{s}$

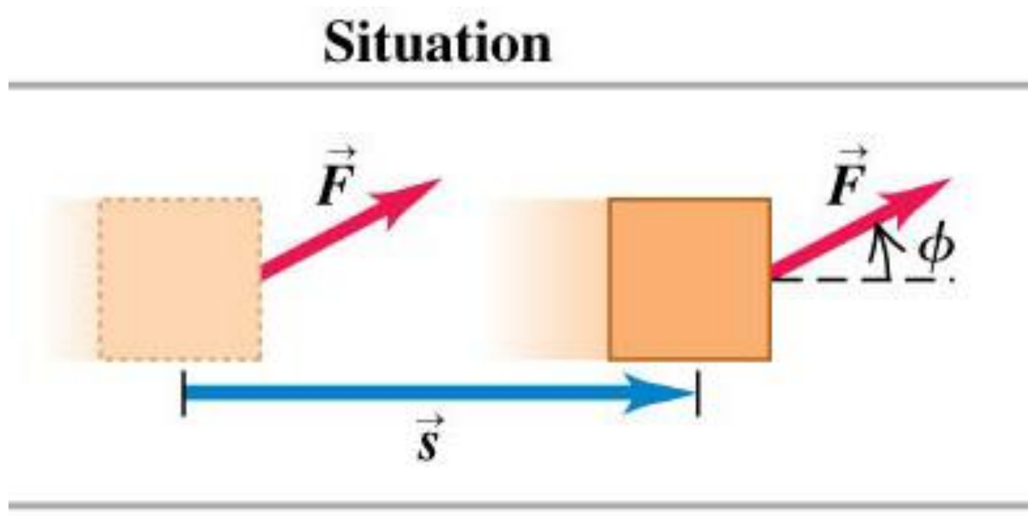
- This can be written more compactly as:

Work done on a particle by **constant force $\vec{F}$** during **straight-line displacement $\vec{s}$**

$$W = \underbrace{\vec{F} \cdot \vec{s}}_{\text{Scalar product (dot product) of vectors } \vec{F} \text{ and } \vec{s}}$$

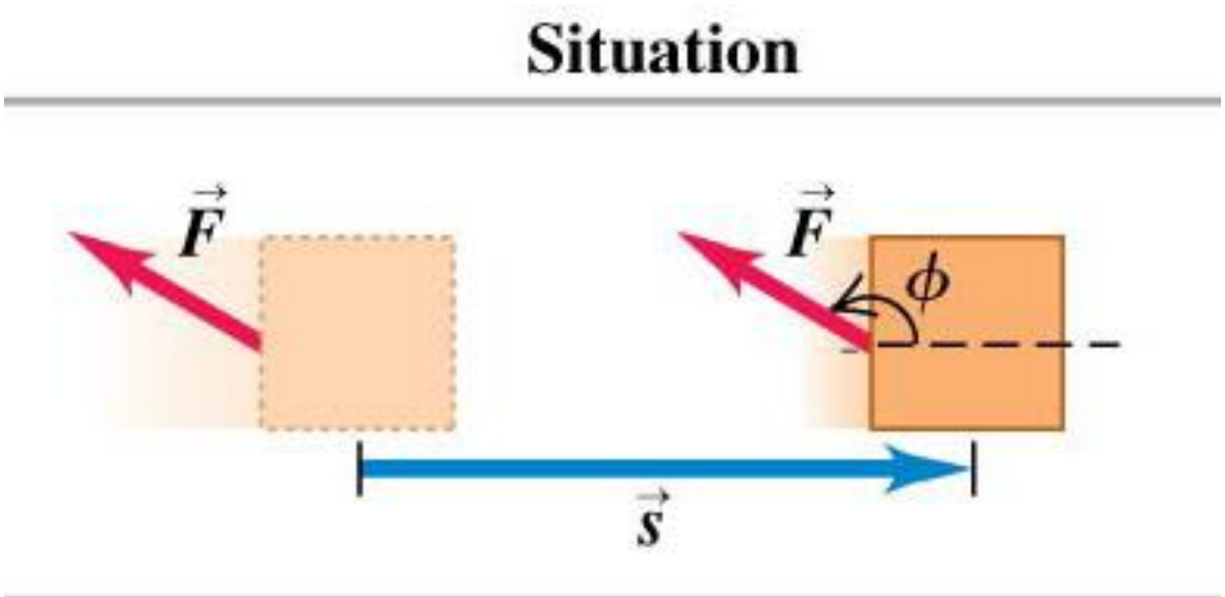
Positive Work

- When the force has a component in the direction of the displacement, work is **positive**.



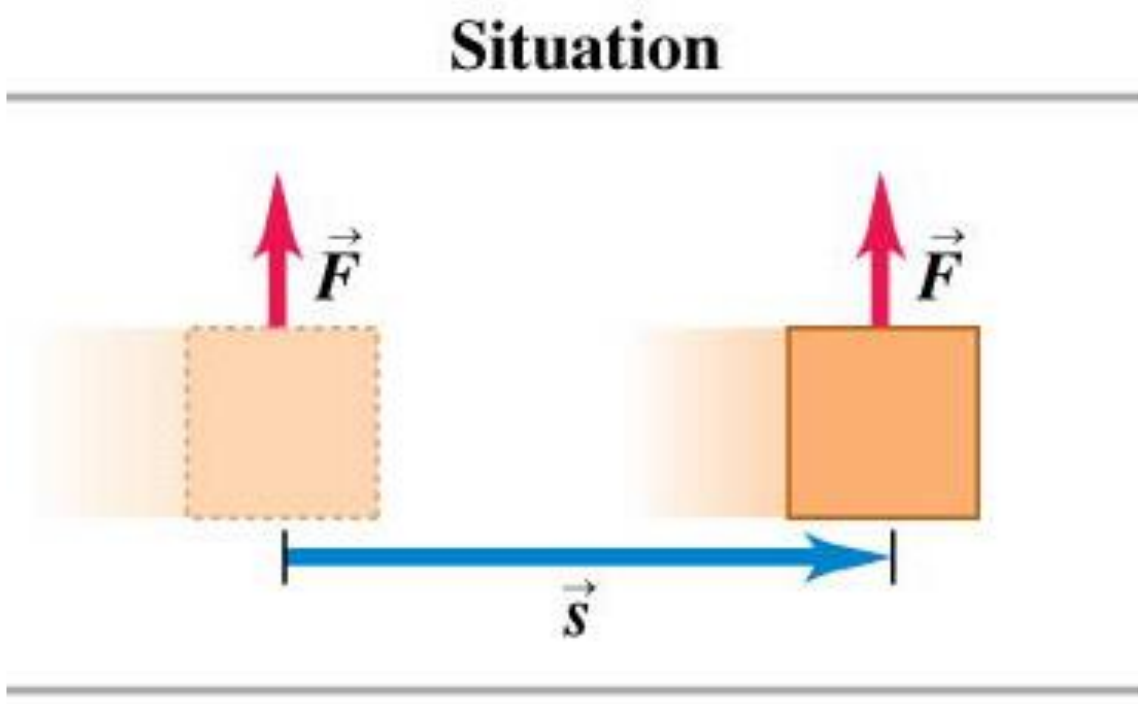
Negative Work

- When the force has a component opposite to the direction of the displacement, work is **negative**.



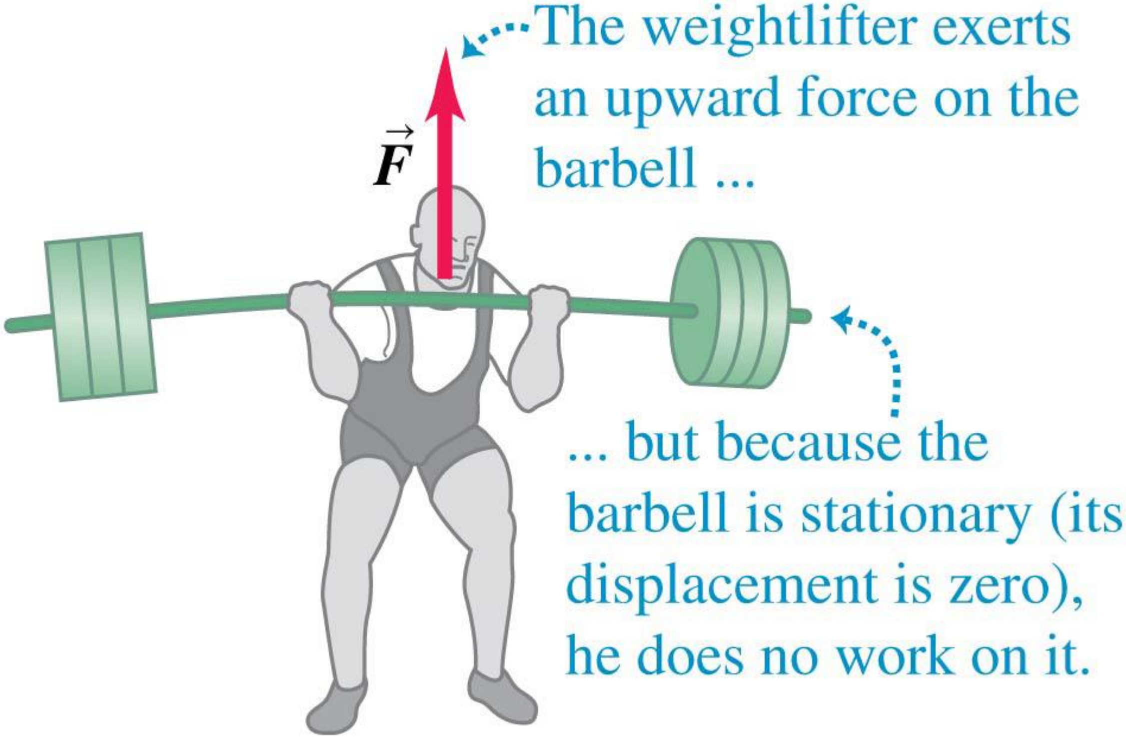
Zero Work (1 of 2)

- When the force is perpendicular to the direction of the displacement, the force does **no** work on the object.



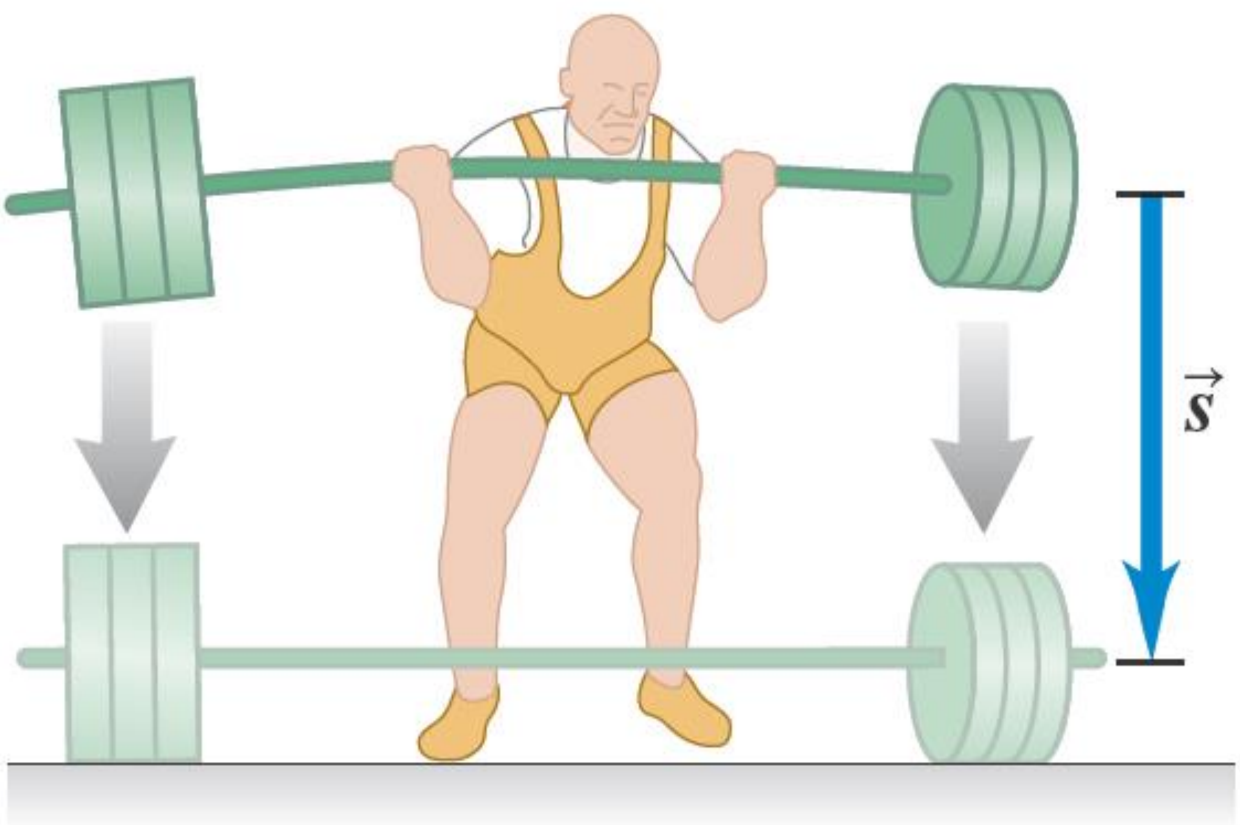
Zero Work (2 of 2)

- A weightlifter does no work on a barbell as long as he holds it stationary.

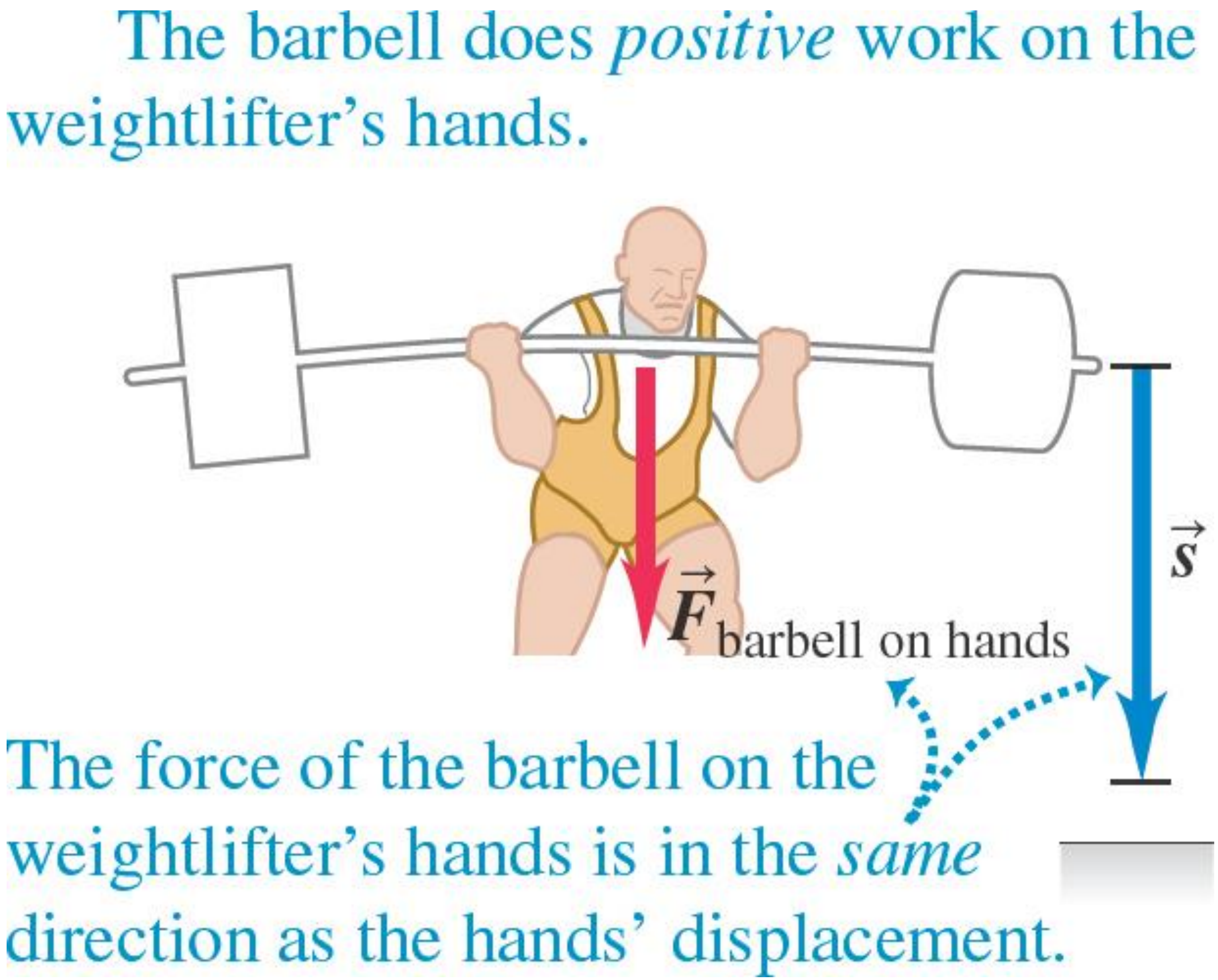


Lowering the Barbell to the Floor: Slide 1

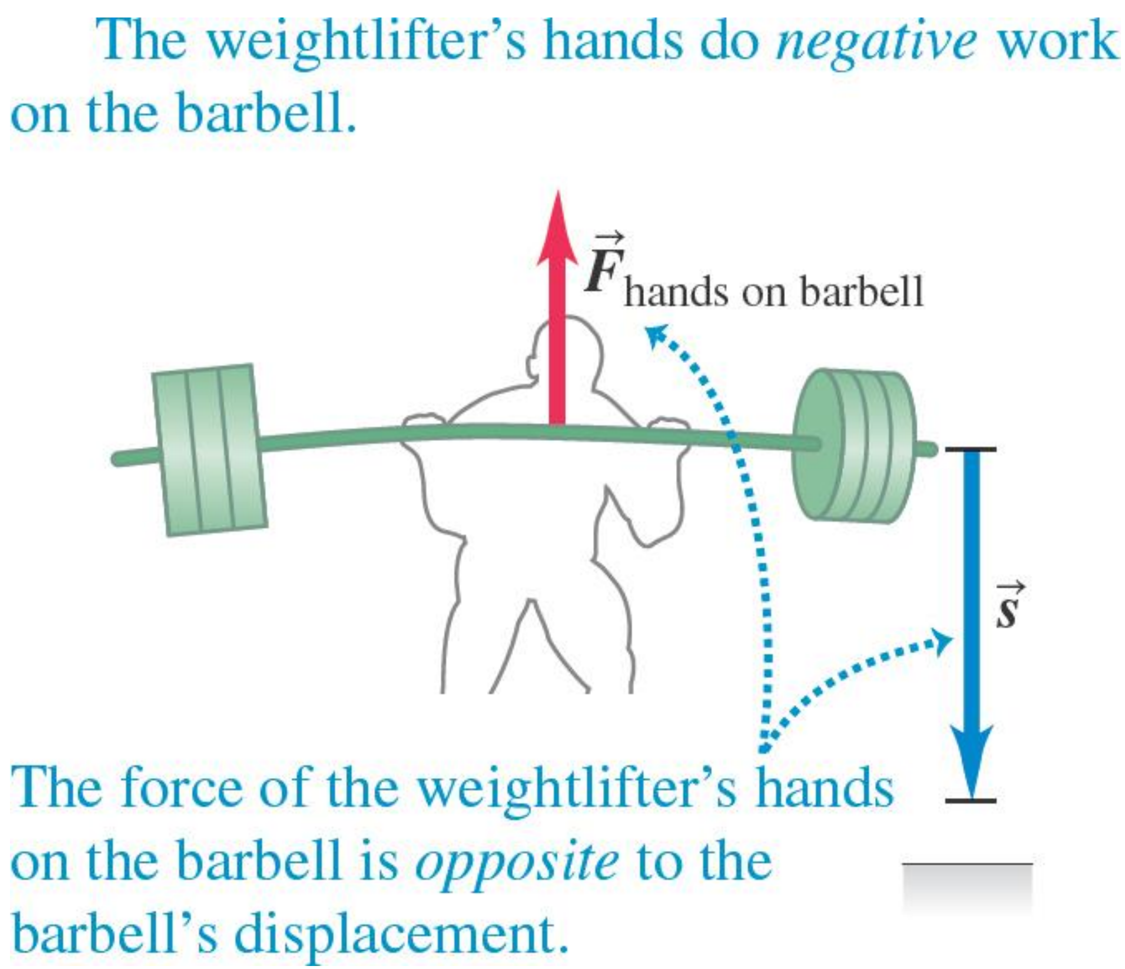
A weightlifter lowers a barbell to the floor.



Lowering the Barbell to the Floor: Slide 2

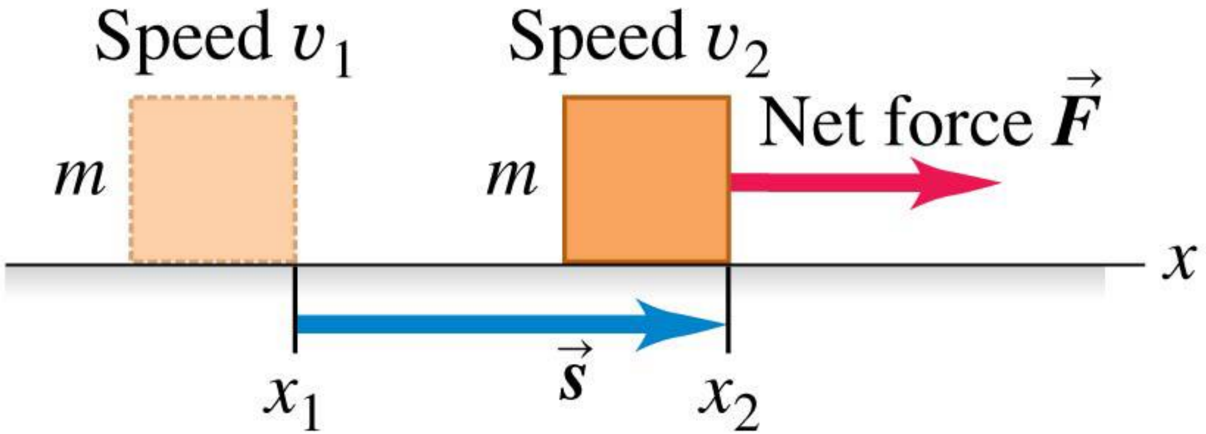


Lowering the Barbell to the Floor: Slide 3



Total Work

- The work done by the net force on a particle as it moves is called the **total work** W_{tot}.
- The particle speeds up if $W_{tot} > 0$, slows down if $W_{tot} < 0$, and maintains the same speed if $W_{tot} = 0$.



Kinetic Energy (1 of 4)

- The energy of motion of a particle is called **kinetic energy**

Kinetic energy of a particle

$$K = \frac{1}{2}mv^2$$

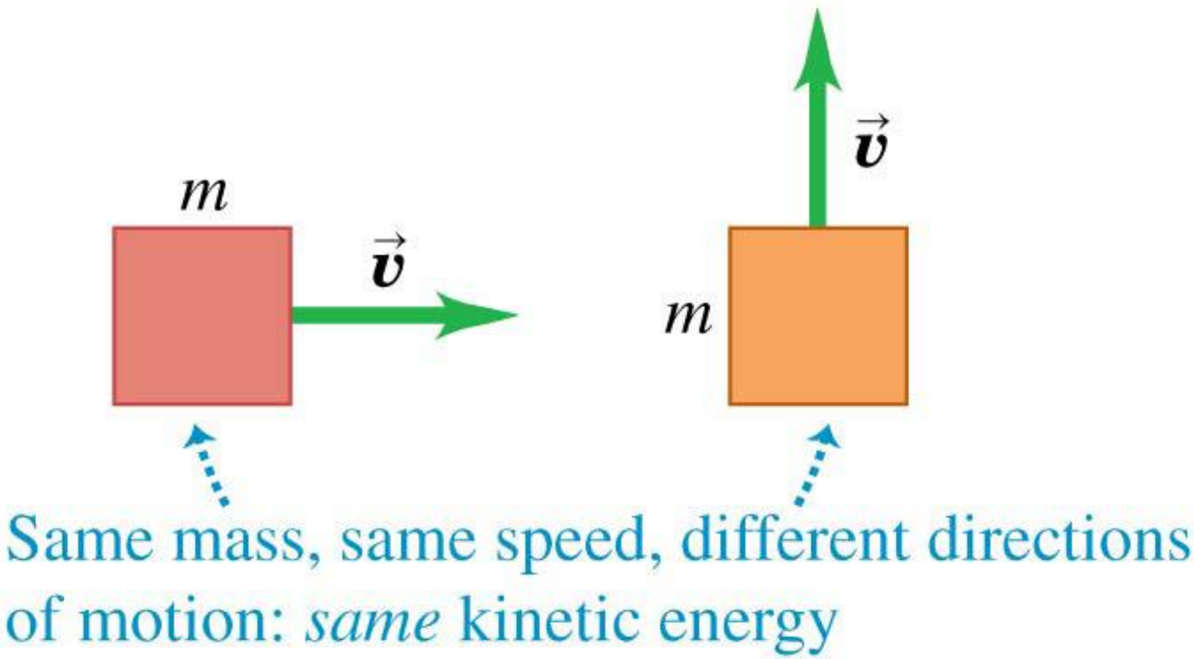
Mass of particle

Speed of particle

- Like work, the kinetic energy of a particle is a scalar quantity; it depends on only the particle's mass and speed, not its direction of motion.
- Kinetic energy can never be negative, and it is zero only when the particle is at rest.
- The SI unit of kinetic energy is the joule.

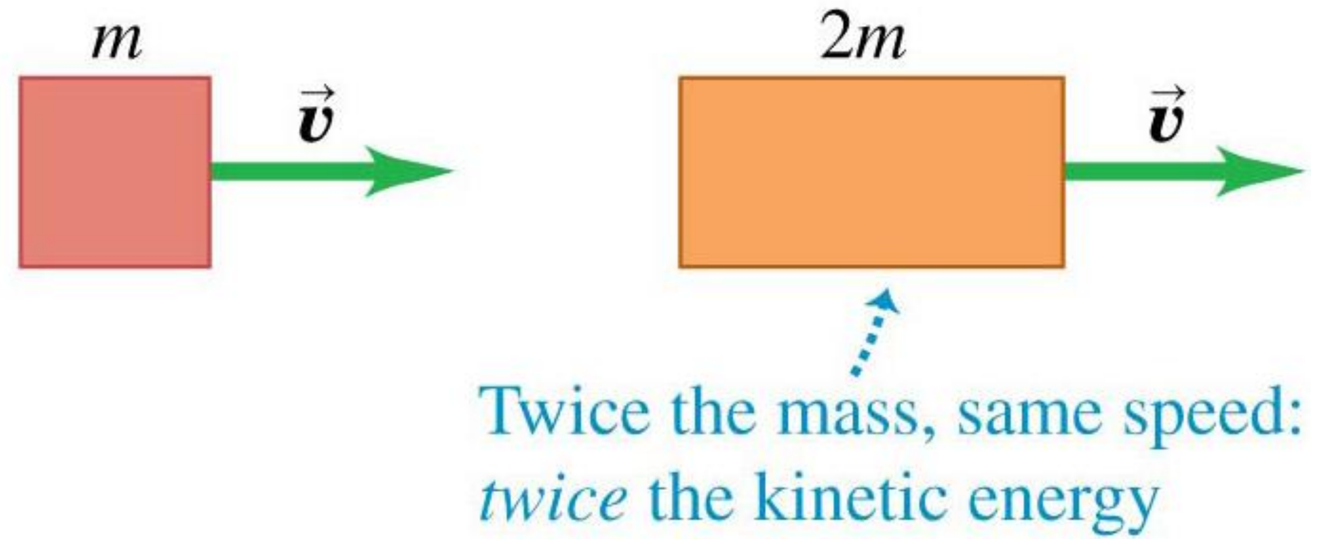
Kinetic Energy (2 of 4)

- Kinetic energy does not depend on the direction of motion.



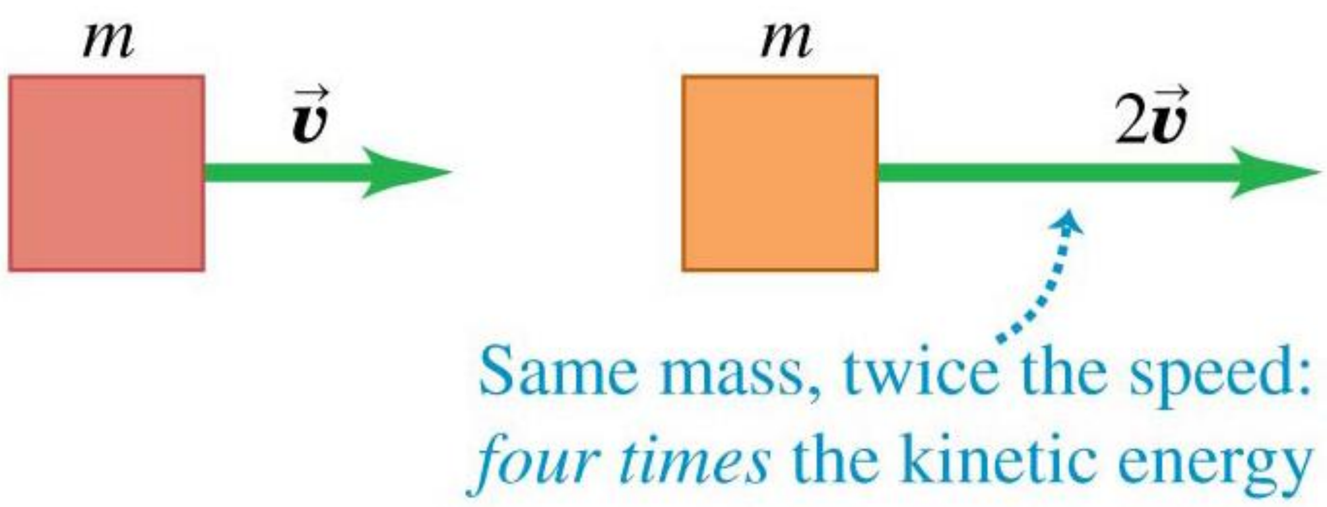
Kinetic Energy (3 of 4)

- Kinetic energy increases linearly with the mass of the object.



Kinetic Energy (4 of 4)

- Kinetic energy increases with the **square** of the speed of the object.



The Work-Energy Theorem

- The **work-energy theorem**: The work done by the net force on a particle equals the change in the particle's kinetic energy.

Work–energy theorem: Work done by the net force on a particle equals the change in the particle's kinetic energy.

Total work done on particle = work done by net force

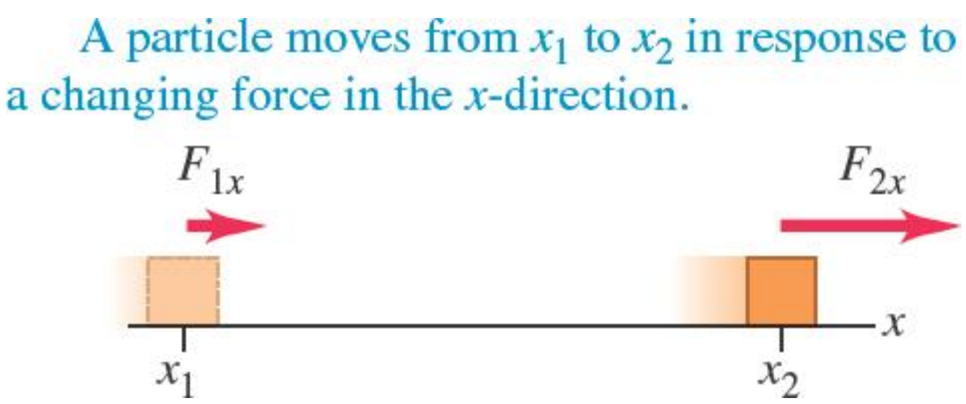
$$W_{\text{tot}} = K_2 - K_1 = \Delta K$$

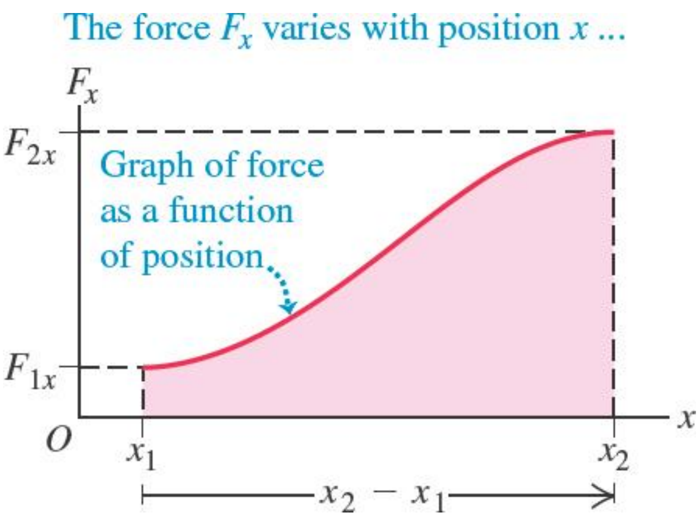
Final kinetic energy (K_2); Initial kinetic energy (K_1); Change in kinetic energy (ΔK)

Ahead: We will be using the work-energy theorem to formulate the conservation of energy.

Work and Energy with Varying Forces (1 of 3)

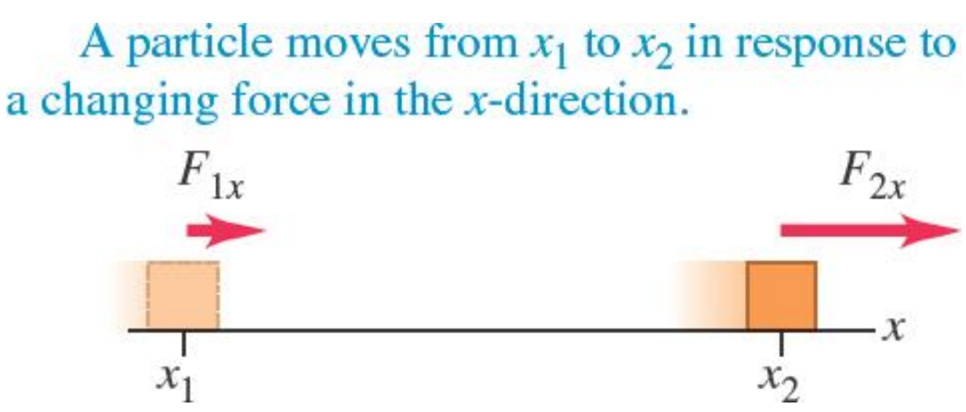
- Many forces are not constant.
- Suppose a particle moves along the x-axis from x_1 to x_2.

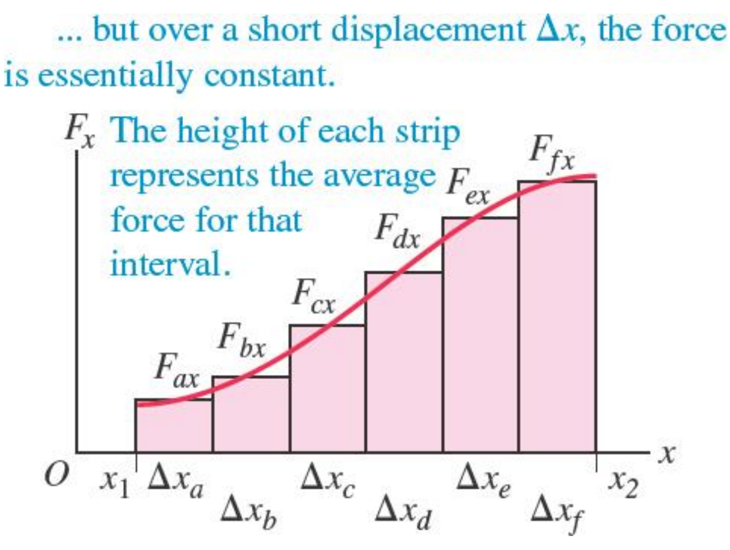




Work and Energy with Varying Forces (2 of 3)

- We calculate the approximate work done by the force over many segments of the path.
- We do this for each segment and then add the results for all the segments.





Work and Energy with Varying Forces (3 of 3)

- The work done by the force in the total displacement from x_1 to x_2 is the integral of F_x from x_1 to x_2:

Work done on a particle by a **varying x-component of force F_x** during **straight-line displacement** along x-axis

$$W = \int_{x_1}^{x_2} F_x \, dx$$

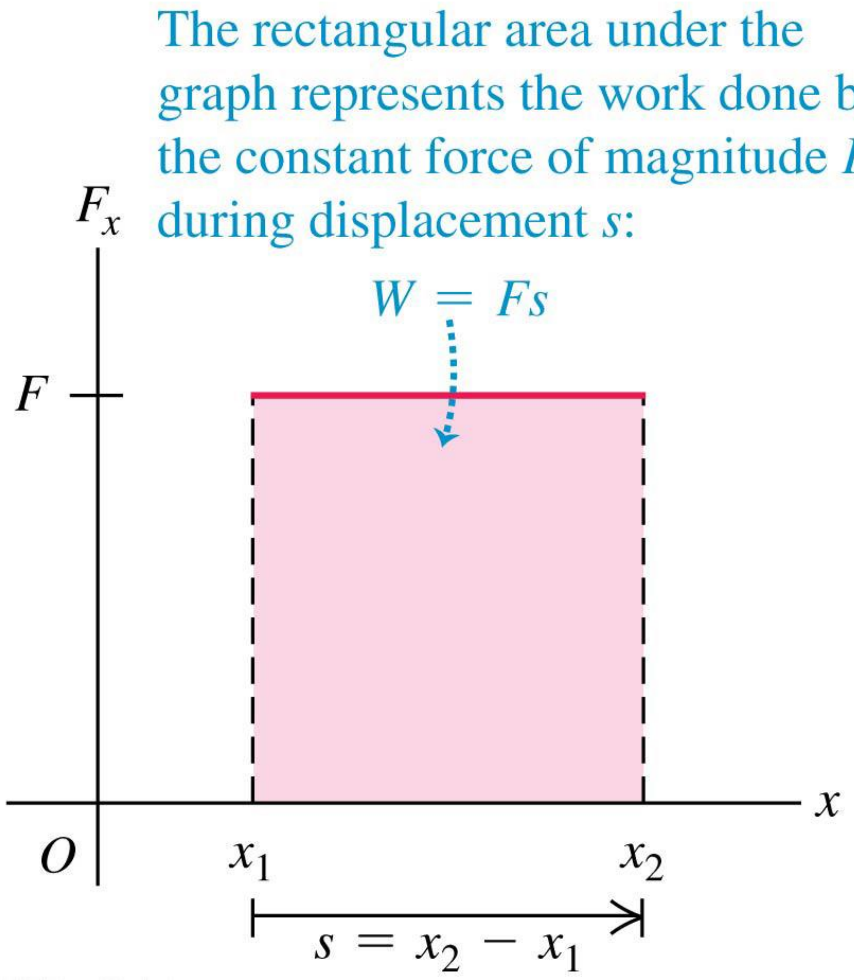
Upper limit = final position

Integral of x-component of force

Lower limit = initial position

- On a graph of force as a function of position, the total work done by the force is represented by the **area** under the curve between the initial and final positions.

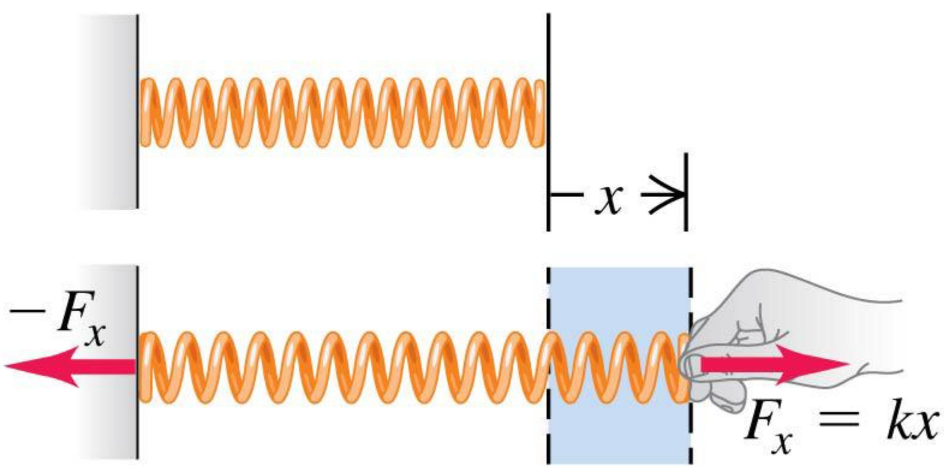
Work Done by a Constant Force (2 of 2)

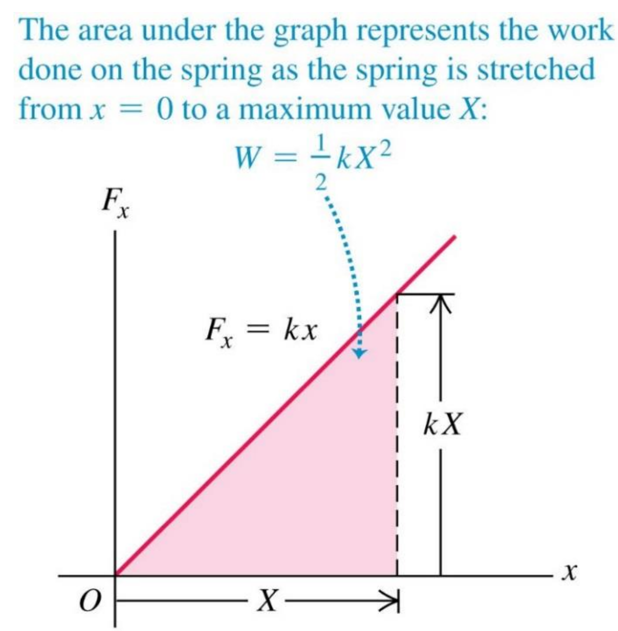


Hooke's Law: Stretching a Spring

- The force required to stretch a spring a distance *x* is proportional to *x*: $\boldsymbol{F_x = kx}$.
- The area under the graph represents the work done on the spring to stretch it a distance:

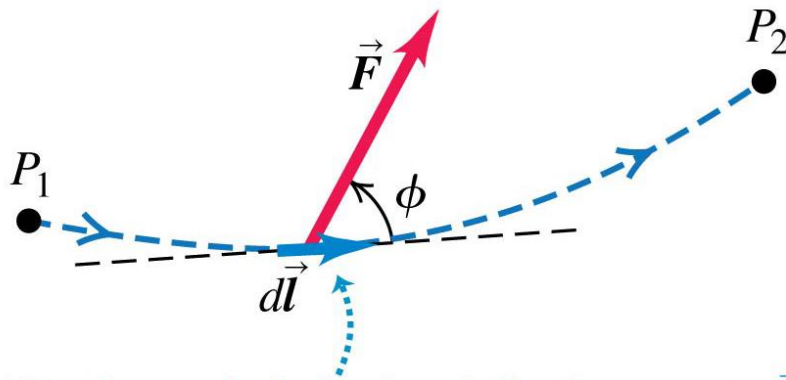
$$W = \frac{1}{2}kx^2$$





Work–Energy Theorem for Motion Along a Curve

- A particle moves along a curved path from point P_1 to P_2, acted on by a force that varies in magnitude and direction.
- The work can be found using a **line integral**:



During an infinitesimal displacement $d\vec{l}$, the force $\vec{F}$ does work dW on the particle:

$$dW = \vec{F} \cdot d\vec{l} = F\cos\phi\, dl$$

Upper limit = final position

Scalar product (dot product) of $\vec{F}$ and displacement $d\vec{l}$

Work done on a particle by a **varying force** $\vec{F}$ along a **curved path**

$$W = \int_{P_1}^{P_2} \vec{F} \cdot d\vec{l} = \int_{P_1}^{P_2} F\cos\phi\, dl = \int_{P_1}^{P_2} F_{\parallel}\, dl$$

Lower limit = initial position

Angle between $\vec{F}$ and $d\vec{l}$

Component of $\vec{F}$ parallel to $d\vec{l}$

Power (1 of 2)

- **Power** is the **rate** at which work is done.
- **Average power** is:

Average power during time interval Δt

$$P_{av} = \frac{\Delta W}{\Delta t}$$

ΔW: Work done during time interval

Δt: Duration of time interval

- **Instantaneous power** is:

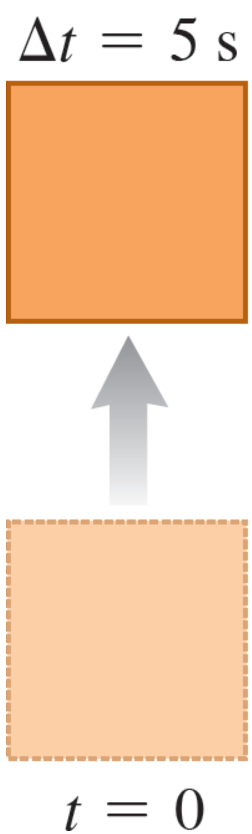
Instantaneous power

$$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \underbrace{\frac{dW}{dt}}_{\text{Time rate of doing work}}$$

Average power over infinitesimally short time interval

- The SI unit of power is the **watt** (1 W = 1 J/s), but another familiar unit is the **horsepower** (1 hp = 746 W).

Power: Lifting a Box Slowly



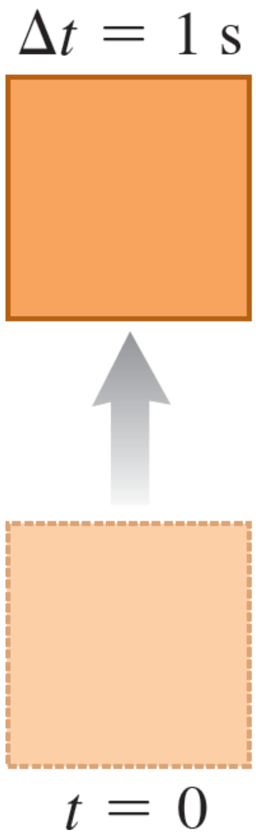
Work you do on the box to lift it in $\Delta t = 5$ s:

$$\Delta W = 100 \text{ J}$$

Your average power output:

$$P_{\text{av}} = \frac{\Delta W}{\Delta t} = \frac{100 \text{ J}}{5 \text{ s}} = 20 \text{ W}$$

Power: Lifting a Box Quickly



Work you do on the same box to lift it the same distance in $\Delta t = 1$ s:

$$\Delta W = 100 \text{ J}$$

Your average power output:

$$P_{av} = \frac{\Delta W}{\Delta t} = \frac{100 \text{ J}}{1 \text{ s}} = 100 \text{ W}$$

Power (2 of 2)

- In mechanics we can also express power in terms of force and velocity:

Instantaneous power for a force doing work on a particle

$$P = \vec{F} \cdot \vec{v}$$

Force that acts on particle

Velocity of particle

- Here is a one-horsepower (746-W) propulsion system.

Watts to horsepower conversion:

$$1\,W = \frac{1}{746}\,hp$$