Chapter 9 Rotation of Rigid Bodies

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November 24, 2023

0.1 Brief Overview

$$\text{Angular Velocity} \qquad w_{av-z} = \frac{\triangle\theta}{\triangle t} v = rw \implies w = \frac{v}{r} \tag{1}$$

$$\text{Angular Acceleration} \qquad a_z = \frac{dw_z}{dt} \quad a_{tan} = ra \implies a = \frac{a_{tan}}{r} \tag{2}$$

$$\text{Rotational Kinetic Energy} \qquad K = \frac{1}{2} I w^2 \tag{3}$$

0.2 Angular Coordinates

A car's speedometer rotates about a fixed axis,

0.2.1 Units of Angles

One revolution is equal to $360\deg = 2\pi$ radians
An angle in radians is equal to $\theta = \frac{s}{r}$

The importance of Radians vs. Degrees

Always use radians when relating linear and angular quantities, degrees cannot be used to express the Angles

0.3 Angular Velocity

The average angular velocity is defined by eq. 1 above, the subscript z means that the rotation is about the z axis. We choose the angle θ to increase in the counter-clockwise direction.

Instantaneous Angular Velocity

The instantaneous angular velocity is the limit of average angular velocity as $\triangle\theta$ approaches zero.
'angular velocity' is interchangeable with instantaneous angular velocity, not the average.

0.3.1 Angular Velocity is a Vector

Angular velocityis defined as a vector whose direction is defined by the right hand rule(Like turning a screwdriver)

0.4 Angular Accceleration

The instantaneous angular acceleration is defined by eq. 2 above, th subscript meaning the rotation is in the z direction
Angular Acceleration is a vector quantity.

0.5 Rotational Equations of Motion with Constant Angular Acceleration

Straight-Line Motion with Constant Linear Acceleration	Fixed-Axis Rotation with Constant Angular Acceleration
$a_x =$ Constant	$a_z =$ Constant
$v_x = v_{0x} + a_x t$	$w_z = w_{0z} + a_z t$
$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$	$\theta = \theta_0 + \theta_{0z}t + \frac{1}{2}a_x t^2$
$V_x^2 = V_{0x}^2 + 2a_x(x - x_0)$	$w_z^2 - w_{0z}^2 + 2a(\theta - \theta_0)$
$x - x_0 = \frac{1}{2}(v_{0x} + v_x)t$	$\theta - \theta_0 = \frac{1}{2}(w_{0z} + w_z)t$

Table 1: The relationship between straight line motion and rotiational motion

0.5.1 Relating Linear and Angular Kinematics

A point at a distance of r from the axis of rotation has a linear speed of $v = rw$
its tangential accelerationis

$$a_{tan} = ra$$

its centripetal(radial) acceleration is

$$a_{rad} = \frac{v^2}{r} = rw^2$$

0.6 Moment of Inertia

Inertia is a 'passive' property of an object which opposes external forces

Item	Moment
Slender Rod(axis through center)	$I = \frac{1}{12}ML^2$
Slender Rod(axis through one end)	$I = \frac{1}{3}ML^2$
Rectangular Plate(axis through center)	$I = \frac{1}{12}M(a^2 + b^2)$
Rectangular Plate(axis along edge)	$I = \frac{1}{3}Ma^2$

Table 2: