CH 10 Exercises:

#1 (10.7) Given: $l = 0.25\,m$, $F = 17\,N$, $\theta = 37°$

(a) $\tau = F \cdot l = r \cdot F \sin\theta$

$$= (0.25\,m)(17\,N)\sin(37°) = \boxed{2.56\ N\cdot m}$$

(b) Torque is max when $\theta = 90° \Rightarrow \sin\theta = 1$

$$\therefore\ \tau_{max} = (0.25\,m)(17\,N)\sin(90°) = \boxed{4.25\ N\cdot m}$$

#2 (10.9) Given: $I = 1.6\ kg\cdot m^2$, $\omega_0 = 0$, $\omega_{8s} = 400\ rev/min$

$$\omega_z\,(8s) = 400\,\frac{rev}{min} \times \left(\frac{1\,min}{60\,s}\right) \times \left(\frac{2\pi\,rad}{rev}\right) = \underline{41.9\ rad/s}$$

$\tau = I\alpha$ where $\alpha = \frac{\omega_z - \omega_0}{t} = \frac{41.9\ rad/s}{8\,s} = 5.24\ rad/s^2$

$$\therefore\ \tau = I\cdot\alpha = (1.6\ kg\cdot m^2)(5.24\ rad/s^2) = \boxed{8.38\ N\cdot m}$$

#3 (10.15) Given: $F = 80\,N$, $r = 0.12\,m$, $\omega_0 = 0$, $\omega_{2s} = 12\ rev/s$, $t = 2s$

Use: $\tau = r \times F = I\alpha$

where $\omega_z = \omega_0 + \alpha t \Rightarrow \alpha = \frac{\omega_z - \omega_0}{t} = \frac{(12\ rev/s)(2\pi\ rad/rev)}{2s} = \underline{37.7\ \frac{rad}{s^2}}$

Solve for moment of inertia (I):

$$I = \frac{rF}{\alpha} = \frac{(0.12m)(80N)}{37.7\ rad/s^2} = \boxed{0.255\ kg\cdot m^2}$$

#4 (10.17) Given: $m = 2.2\ kg$, $\varnothing = 1.2\ m$, $\omega = 2.6\ rad/s$

(a) $v_{cm} = r \cdot \omega = (\tfrac{1}{2}\varnothing)\omega = (0.6 m)(2.6\ rad/s) = 1.56\ m/s$

(b) $K_{tot} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2$ where $I_{hoop} = MR^2 = (2.2\ kg)(0.6\ m)^2 = 0.792\ kg\cdot m^2$

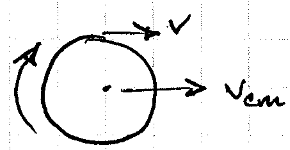
$= \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2$

$= \tfrac{1}{2}(2.2\ kg)(1.56\ m/s)^2 + \tfrac{1}{2}(0.792\ kg\cdot m^2)(2.6\ rad/s)^2$

$= 2.68\ J + 2.68\ J$

$= \boxed{5.35\ J}$

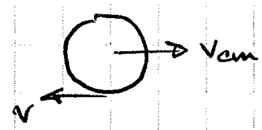
(c) (i) at highest point on the hoop:



$v = 2v_{cm} = 2(1.56\ m/s)$

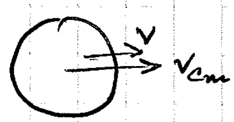
$= \boxed{3.12\ m/s}$

(ii) at lowest point on the hoop:



$\boxed{v = 0}$

(iii) a point midway to the right



$v = \sqrt{v_{cm}^2 + v_{tan}^2} = \sqrt{v_{cm}^2 + (R\omega)^2}$

$= \sqrt{2v_{cm}^2}$

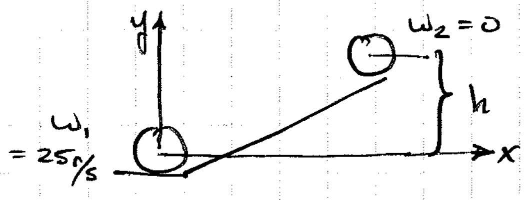
$= \sqrt{2}\, v_{cm}$

$= \sqrt{2}\,(1.5\ m/s)$

$= \boxed{2.21\ m/s}$

#5 (10.23) <u>Given:</u> $mg = 392\,N$, $\omega = 25\,rad/s$, $r_\omega = 0.6\,m$

$I_\omega = (0.8)MR^2$, $W_f = 2600\,J$



Kinetic Energy of the wheel...

$$K_\omega = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I\omega^2$$

Apply conservation of Energy ... $\Delta E = E_{final} - E_{initial} = 0$

$$\Rightarrow \quad K_{final} + U_{final} = K_{initial} + U_{initial} + W_f$$

where $K_{final} = 0$

$U_{final} = mgh$

$K_{initial} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$U_{initial} = 0$

$W_f = -2600\,J$

<u>Aside</u>: $mg = F$

$\Rightarrow m = \frac{(392\,N)}{(9.8\,m/s^2)}$

$= \underline{40\,kg}$

$v = R \cdot \omega$

$= (0.6\,m)(25\,rad/s)$

$= \underline{15\,m/s}$

So...

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 - 2600\,J$$

$$\cancel{m}h = \frac{\frac{1}{2}(40\,kg)(15\,m/s)^2 + \frac{1}{2}(0.8)(40\,kg)(0.6\,m)^2(25\,rad/s)^2 - 2600\,J}{(392\,N)}$$

$$\therefore \boxed{h = 14\,m}$$

<u>Noted</u>: $K_i = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$= \frac{1}{2}m(R\omega)^2 + \frac{1}{2}(0.8)mR^2 \cdot \omega^2$

$= \frac{1}{2}mR^2\omega^2 + \frac{1}{2}(0.8)mR^2\omega^2$

\#6. (10.27) Given: $\varnothing = 22.6\text{ cm} \Rightarrow r = 0.113\text{ m}$, $m = 0.426\text{ kg}$, $h = 5\text{ m}$, $I = \frac{2}{3}mr^2$ (thin walled hollow sphere)

Noted: rolling without slipping $\Rightarrow v_{cm} = R \cdot \omega$

Apply conservation of energy to the soccer ball ... $\Delta E = E_f - E_i = 0$

$$\Rightarrow \quad K_f + U_f = K_i + U_i + \cancel{W_f}^{0}$$

where ...
$K_f = 0$ (ball stops)
$U_f = mgh$
$K_i = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I\omega^2$
$U_i = 0$
$W_f = 0$ (rolling without slipping $\Rightarrow$ no friction)

Thus,

$$mgh = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I\omega^2 \quad \text{where} \quad \omega = v_{cm}/R$$

$$= \frac{1}{2}mv_{cm}^2 + \frac{1}{\cancel{2}}\left(\frac{\cancel{2}}{3}m\cancel{r^2}\right)\left(\frac{v_{cm}}{\cancel{r}}\right)^2$$

$$= \frac{1}{2}mv_{cm}^2 + \frac{1}{3}mv_{cm}^2$$

$$= \frac{5}{6}mv_{cm}^2$$

$$\Rightarrow \quad v_{cm} = \sqrt{\frac{6}{5}gh} = \sqrt{\frac{6}{5}(9.8\text{ m/s}^2)(5\text{ m})} = \boxed{7.67\text{ m/s}}$$

rate of rotation:

$$\omega = \frac{v_{cm}}{r} = \frac{7.67\text{ m/s}}{0.113\text{ m}} = \boxed{67.9\text{ rad/s}}$$

(b) rotational Kinetic Energy $\Rightarrow K_{rot} = \frac{1}{2}I\omega^2$

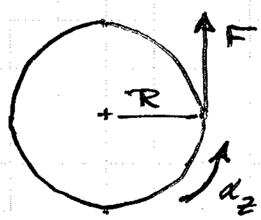
$$K_{rot} = \frac{1}{2}\left(\frac{2}{3}mr^2\right)\omega^2$$

$$= \frac{1}{3}(0.426\text{ kg})(0.113\text{ m})^2(67.9\text{ rad/s})$$

$$= \boxed{8.36\text{ J}}$$

#7. (10.29) Given: $R = 2.4\,m$, $I = 2100\ kg\cdot m^2$

(a) $F = 18\,N$, $t = 15\,s$ ... $\alpha_z = ??$ $\rightarrow$ $\omega_z\ (t = 15\,s)$



$$\sum_i \tau = I\alpha_z \quad \text{where} \quad \tau = RF$$

$$\Rightarrow \alpha_z = \frac{RF}{I} = \frac{(2.4\,m)(18\,N)}{(2100\ kg\cdot m^2)} = 0.021\ rad/s^2$$

from Eqn of motion: $\omega_z = \cancel{\omega_0}^{\,0} + \alpha_z t$

$$\Rightarrow \omega_z = \alpha_z \cdot t = (0.021\ rad/s^2)(15\,s) = \boxed{0.309\ rad/s}$$

(b) average power supplied by the child...

$$P = \tau \cdot \omega_z = (RF)\,\omega_z = (2.4\,m)(18\,N)(0.309\ rad/s) = \boxed{13.3\ Watts}$$

verify via dimensional analysis ... $\left[\frac{N\cdot m}{s}\right] = \left[\frac{kg\cdot m^2}{s^3}\right] \Rightarrow \left[\frac{J}{s}\right] \equiv$ WATTS

#8 (10.39) Given: $m = 2.8\ kg$, $r = 0.1\ m$, Solid cylinder: $I = \frac{1}{2} m r^2$

(a) $\omega_z = \frac{1200\ rev}{min} \times \left(\frac{1\ min}{60\,s}\right) \times \left(\frac{2\pi\ rad}{rev}\right) = 125.7\ rad/s$, $t = 2.5\,s$

$$\Rightarrow \alpha_z = \omega_z / t = (125.7\ rad/s)/(2.5\,s) = 50.28\ rad/s$$

$\tau = I\alpha_z$ where $I = \frac{1}{2} m r^2 = \frac{1}{2}(2.8\ kg)(0.1\,m)^2 = 0.014\ kg\cdot m^2$

$$= \frac{1}{2}(0.014\ kg\cdot m^2)(50.28\ rad/s) = \boxed{0.7\ N\cdot m}$$

(b) $\Delta\theta = \theta - \theta_0 = \frac{1}{2}(\cancel{\omega_0}^{\,0 \text{ at rest}} + \omega_z)\cdot t = \frac{1}{2}(125.7\ rad/s)(2.5\,s) = \boxed{157\ rad}$

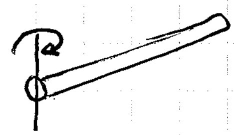
(c) $\omega = \tau \cdot \Delta\theta = (0.7\ N\cdot m)(157\ rad) = \boxed{110\ J}$

(d) Given: $\omega_z = \frac{120\ rev}{min} \times \left(\frac{1\ min}{60\,s}\right) \times \left(\frac{2\pi\ rad}{rev}\right) = 125.7\ rad/s$

$$K = \frac{1}{2} I \omega_z^2 = \frac{1}{2}(0.014\ kg\cdot m^2)(125.7\ rad/s)^2 = \boxed{110\ J}$$

Same :)

#9. (10.37) Given: $l = 15\,cm = 0.15\,m$, $m = 6\,g = 0.006\,kg$

recall:  $I = \frac{1}{3} m l^2 \Rightarrow$ moment of inertia for a slender rod when axis of rotation is at one end

(a) What is the angular momentum?

$$L = I\omega_z \quad \text{where} \quad \omega_z = \frac{1\,rev}{min} \times \left(\frac{1\,min}{60\,s}\right) \times \left(\frac{2\pi\,rad}{rev}\right) = \underline{0.1047\,rad/s}$$

thus,

$$I = \frac{1}{3}(0.006\,kg)(0.15\,m)^2 = \underline{4.5 \cdot 10^{-5}\,kg \cdot m^2}$$

$$L = I\omega_z = (4.5 \cdot 10^{-5}\,kg \cdot m^2)(0.1047\,rad/s) = \boxed{4.71 \cdot 10^{-6}\,kg \cdot m^2/s}$$

#10 (10.45) Given: $r = 2\,m$, $m = 120\,kg$, $\omega_z = 3\,rad/s$
$M = 70\,kg$ ... for a disk; $I = \frac{1}{2} m r^2$

Set up:

$$I_{initial} = \frac{1}{2} m r^2 = \frac{1}{2}(120\,kg)(2\,m)^2 = \underline{240\,kg \cdot m^2}$$

$$I_{final} = I_i + I_p = (240\,kg \cdot m^2) + MR^2 = (240\,kg\,m^2) + (70\,kg)(2\,m)^2$$

$$= \underline{520\,kg \cdot m^2}$$

via conservation of angular momentum:

$$I_i \omega_i = I_f \omega_f \Rightarrow \omega_f = \omega_i\left(\frac{I_i}{I_f}\right) = \left(3\,\frac{rad}{s}\right)\left(\frac{240\,kg \cdot m^2}{520\,kg \cdot m^2}\right) = \boxed{1.38\,rad/s}$$

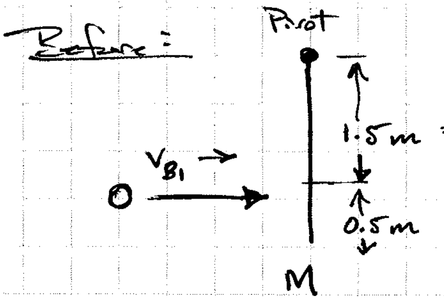
(b) $K_i = \frac{1}{2} I_i \omega_i^2 = \frac{1}{2}(240\,kg \cdot m^2)(3\,rad/s)^2 = \boxed{1080\,J}$

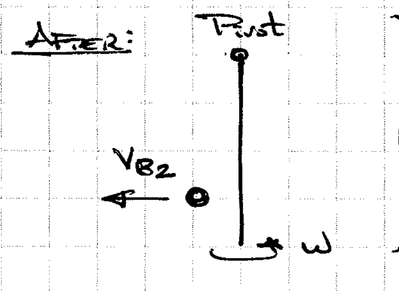
$K_f = \frac{1}{2} I_f \omega_f^2 = \frac{1}{2}(520\,kg \cdot m^2)(1.38\,rad/s)^2 = \boxed{495\,J}$

Note: the angular speed decreases because the moment of inertia increases when the parachutist is added to the system.

thus, Kinetic Energy ↓ because of the negative work done on the turntable and the parachutist by the friction force between these two objects.

#11 (10.49) Given: $L = 2\,m$, $mg = 90\,N$, $m_B = 3\,kg$, $v_{B_1} = 10\,m/s$
$v_{B_2} = -6\,m/s$





where the mass (M) of the bar

$$\Rightarrow M = \frac{(90\,N)}{(9.8\,m/s^2)} = \underline{9.18\,kg}$$

Assume moment of inertia of bar is slender rod w/ axis of rotation about end..

$$\Rightarrow I_B = \frac{1}{3}ML^2 \ldots$$

Now, via conservation of angular momentum...

$$L_f = L_i \Rightarrow I_B\,\omega - m_B v_{B2} \cdot l = m_B v_{B_1} \cdot l$$

$$\Rightarrow \frac{1}{3}ML^2\omega - m_B v_{B2}\left(\frac{3}{4}L\right) = m_B v_{B_1}\left(\frac{3}{4}L\right)$$

$$\Rightarrow \quad \omega = \frac{3\,m_B\left(\frac{3}{4}\cancel{L}\right)(v_{B2} + v_{B_1})}{ML^{\cancel{2}}}$$

$$= \frac{9}{4} \cdot \frac{(3\,kg)(10\,m/s + 6\,m/s)}{(9.18\,kg)(2\,m)}$$

$$= \frac{432\,kg \cdot m/s}{73.4\,kg \cdot m} = \boxed{5.88\,rad/s}$$

(b) During the collision, linear momentum is not conserved; there is an external force exerted by the pivot.

Nonetheless, the force on the pivot has zero torque (that is, the torque about the pivot $\tau = I \cdot \alpha = 0$), so the angular momentum of the system about the pivot is conserved.

#12 (10.52) Given: $\Omega_E = 0.5\ rad/s$, $g_m = 0.165\ g$

Noted: the precession angular speed is related to the acceleration due to gravity by the relation...

$$\Omega = \frac{mgr}{I\omega}$$ where m, r, I and ω are the same on Earth as on the moon...

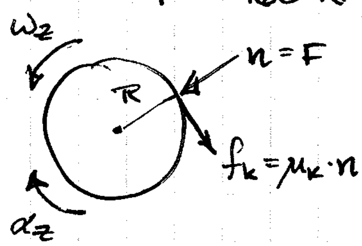
So, $$\frac{\Omega}{g} = \frac{mr}{I\omega} = \text{constant.}$$

thus, we are only concerned about the relation: $$\frac{\Omega_E}{g_E} = \frac{\Omega_m}{g_m}$$

$$\Rightarrow \Omega_m = \Omega_E \left(\frac{g_m}{g_E}\right) = (0.5\ rad/s)\left(\frac{0.165\cancel{g}}{\cancel{g}}\right) = \boxed{0.0825\ rad/s}$$

#13. Given: $\varnothing = 0.52\ m$, $M = 50\ kg$, $\omega_z = 850 \frac{rev}{min} \times \left(\frac{2\pi\ rad/rev}{60\ s/min}\right) = \underline{89\ rad/s}$

$F = 160\ N$, $t = 7.5\ s$, $\mu_k = ??$ $\qquad t = 7.5\ s$



Apply $\sum \tau = I\alpha_z$ $\qquad R = 0.26\ m$

where
$$\begin{cases} \tau = -F\cdot R = -\mu_k \cdot n \cdot (\varnothing/2) \\ \alpha_z = \dfrac{-\omega_z}{t} = \dfrac{-89\ rad/s}{7.5\ s} = \underline{-11.9\ rad/s^2} \\ I = \frac{1}{2}MR^2 \end{cases}$$

(slowing down)

Assume moment of inertia of a disk...

Thus,

$$-\mu_k n \cdot R = \left(\tfrac{1}{2}MR^2\right)\alpha_z$$

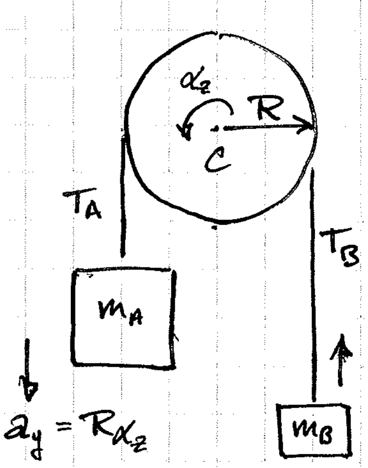
$$\Rightarrow \mu_k = \frac{-MR^{\cancel{2}}\cdot\alpha_z}{2n\cdot\cancel{R}}$$

check ✓

$$= -\frac{(50\ kg)(0.26\ m)(-11.9\ rad/s^2)}{2\,(160\ N)} \Rightarrow \left[\frac{kg\cdot m/s^2}{N}\right]$$

$$= \boxed{0.483}$$

#14. (10.59) Given: $R_w = 0.12\ m$, $I_w = 0.22\ kg\cdot m^2$, $m_A = 4\ kg$, $m_B = 2\ kg$



For blocks A and B: $\sum F_y = m a_y$

For wheel "C" $\sum \tau_z = I\alpha_z$

Block A:
$$m_A g - T_A = m_A a_y \qquad : \text{Eqn } ①$$

Block B:
$$T_B - m_B g = m_B a_y \qquad : \text{Eqn } ②$$

For the wheel "C" $\sum \tau = T_A\cdot R - T_B\cdot R = I\alpha_z = I\cdot\left(\frac{a_y}{R}\right)$

$$\Rightarrow T_A - T_B = \frac{I\cdot a_y}{R^2} \qquad : \text{Eqn } ③$$

- From Eqn ①: $\Rightarrow T_A = m_A g - m_A a_y$
- From Eqn ②: $\Rightarrow T_B = m_B a_y + m_B g$

Substitute Eqn ① and ② into ③ ...

$$\Rightarrow (m_A g - m_A a_y) - (m_B a_y + m_B g) = \frac{I}{R^2}\cdot a_y$$

$$\Rightarrow m_A g - m_B g = \left(\frac{I}{R^2}\right)a_y + m_A a_y + m_B a_y$$

Thus,
$$a_y = \frac{g\,(m_A - m_B)}{(I/R^2 + m_A + m_B)}$$

$$= \frac{(9.8\ m/s^2)(4\ kg - 2\ kg)}{\left[\frac{(0.22\ kg\cdot m^2)}{(0.12\ m)^2} + 4\ kg + 2\ kg\right]} = \boxed{0.921\ m/s^2}$$

$$\therefore\ \alpha_z = \frac{a_y}{R} = \frac{(0.921\ m/s^2)}{(0.12\ m)} = \boxed{7.68\ rad/s^2}$$