Projectile Motion

Prelab

1. When you shoot a projectile that lands at its starting height, and if air drag is not a factor, what launch angle should give it the longest range? Why?
2. When you double the launch velocity of a projectile that lands at its starting height, and if air drag is not a factor, will it land at twice the range? Why or why not?
3. When you shoot a projectile from a height above its landing height, and air drag is not a factor, should the same launch angle still give the longest range? Why or why not?
4. If air drag is a factor, how will it affect a projectile's trajectory qualitatively? Its velocity? Its acceleration?
5. Derive the equations in the purpose of experiment from the four primary equations for constant acceleration.

Experiment

Purpose of Experiment

We study projectile motion to get a better understanding of two dimensional motion. When air drag can be neglected, projectile motion is well described by the formulas

$$x = v_0 \cos \alpha_0 * t$$

$$v_x = v_0 \cos \alpha_0$$

$$y = v_0 \sin \alpha_0 * t - 1/2 g t^2$$

$$v_y = v_0 \sin \alpha_0 - gt$$

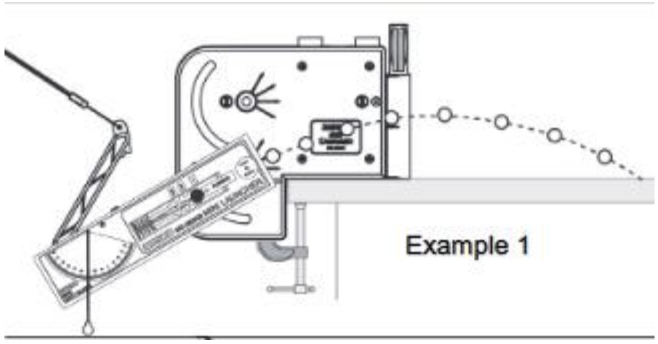
where the *x* direction refers to the distance traveled by the projectile, *y* to the height of the projectile, and α_0 to the angle from the horizontal at which the projectile was launched.

Our objective is to evaluate a number of aspects of projectile motion by using a launcher, for which we can control v_0 and α_0.

Activity 1 ~ Angle of Longest Range

General Procedure

1. The launcher is set up and a steel ball is fed into the opening. The trigger mechanism is set to one of the three available settings (three compressions of an internal spring) to provide a specific reproducible v_0.
2. The pendulum on the launcher is used to read the launching angle α_0.
3. **Be careful that the steel ball cannot hit you or others when it is being launched.**
4. Do a test shot to identify your landing zone, set up measuring tape/meter stick to measure you landing point.



Procedure

1. Place launcher at the end of the table, set trigger to first (lowest) level and let the steel ball land on the table. Be sure to have a member of your lab group in position to mark the landing and catch the ball. Make sure the outlet of the launcher is at the same height as the table.
2. Do a preliminary test to see where about the steel ball will land at your first angle setting (α_0 = 20˚). This helps you to decide where you will need to measure to. You may have to move as you vary the angle.
3. Calculate the launch speed based on the following formula: $v_0 = \sqrt{xg/\sin(2\alpha_0)}$ (derive this equation in your prelab)
4. Determine the ranges to which the ball is shot when you vary α_0 in 5˚ steps from 20˚ to 75˚.
5. Measure the distance between the launcher and the landing point. How many trials at each angle should be run?

20˚	25˚	30˚	35˚	40˚	45˚	50˚	55˚	60˚	65˚	70˚	75˚

Activity 2 ~ Angle of Largest Range when Landing Below Start Level

Procedure

1. Launch the steel ball from the table in such a direction that the ball will land on the floor.
2. Set trigger to the lowest level.
3. Vary launching angle from 20˚ to 75˚.
4. Repeat at least 5 times for each angle and measure the distance between the launcher and the landing point.

Distance	20˚	25˚	30˚	35˚	40˚	45˚	50˚	55˚	60˚	65˚	70˚	75˚
Attempt 1												
Attempt 2												
Attempt 3												

Attempt 4												
Attempt 5												
Mean												
σ												

5. Find the arithmetic average for the distances, based on the attempts for each angle.
6. Calculate the Standard Deviations.
7. Transfer your table into excel. Plot the dependence of landing distance vs. launching angle and indicate the error bars.
8. Draw a best fit curve to determine the angle of maximum range.

Analysis

1. How is the horizontal component v_x of a projectile's velocity qualitatively different from the vertical component v_y?
2. If one doubles the value of v_0 at a given launch angle θ, what happens to the projectile's initial horizontal and vertical speeds v_{0x} and v_{0y}, landing distance x_f, and maximum height y_{max}?
3. (CALC) Why does a negative second derivative indicate a maximum? Sketch a function with a local maximum ad its first and second derivatives. Explain what they tell you, and indicate how they apply to position, velocity, and acceleration.
4. Was air drag a significant factor in your experiment? Explain how your data support your answer.
5. Go back to your predictions in the prelab and, if it turns out that any of them were wrong, explain where your initial thinking went wrong and give the correct answer.
6. Discuss error in your experiment.

Experiment Extension – Simulation

Purpose

The purpose of this extension is to find the launch angle on level ground, which will maximize the height at which a projectile strikes a vertical wall, for launches at a fixed horizontal distance from the wall.

You may think of the problem as one encountered by ancient armies, who besieged city walls.

When the ball is launched at an angle from a fixed distance, x, it hits the vertical wall at a height, y, given by:

$$y = y_0 + (v_0 \sin\alpha) * t - \frac{1}{2}gt^2$$

where y_0 is the initial height of the projectile, v_0 the initial speed as it leaves the muzzle, α the inclination of the launcher above horizontal, g the acceleration due to gravity, and t the time of flight.

Procedure

1. Place the launcher at a fixed distance in such a way that the ball comfortably reaches the wall when it is past its y_{max} when shot at a moderate angle.
2. Vary the angle from 20˚ to 70˚ and measure the height at which the ball impacts the wall. For each angle, repeat twice before changing the angle to the new angle. Then change your launcher distance to 20% closer to the wall and repeat the measurements for the same v_0.
3. Transfer your table of results into excel. For each of the two launch distances from the wall, plot the impact heights y vs. launch angle α.

Launch Angle	**Height for Distance 1: m**					**Height for Distance 2: m**				
	Trial 1	**Trial 2**	**Trial 3**	**Trial 4**	**Trial 5**	**Trial 1**	**Trial 2**	**Trial 3**	**Trial 4**	**Trial 5**
20˚										
25˚										
30˚										
35˚										
40˚										
45˚										
50˚										
55˚										
60˚										
65˚										
70˚										

Analysis

1. The distance from the wall is the horizontal distance x. Rearrange the equation for x on page 1 to obtain a symbolic expression for the time of flight t. Substitute this expression into the y-equation and simplify. You should now have a formula for y, the height of impact, as a function of α, g, x, and v_0.
2. Using the average launch speed v_0 you calculated in Activity 1, overlay a smooth curve for calculated y onto each of your y-α plots.
3. If you are sure your formula is correct but the data and the theoretical curve do not agree, explain what might be wrong with your model. (Is a prominent force neglected? Might the values of your parameters v_0, g, or x be incorrect?)