Pendulum Motion

Prelab

1. It turns out that g is not a constant on the surface of the earth but depends on latitude and elevation. Calculate g for Gillette. Find the latitude and metric elevation for Gillette on the internet: (use elevation in meters for h and latitude for α)

G = 9.7803185 * (1 + 0.005278895 * $\sin^2\alpha$ – 0.000023462 * $\sin^4\alpha$) – 0.3086 * 10^{-6} * h

2. Do you predict that the pendulum period gets longer or shorter for a longer string? For a larger displacement? For a larger mass?

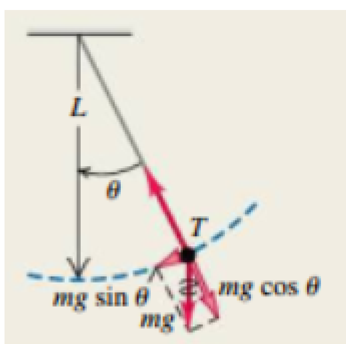
Experiment ~ Pendulum Motion

Purpose of Experiment

We use a moderately damped 'simple pendulum' to demonstrate the nature of simple periodic motion. The determination of random and systematic errors is explored.

General Procedure

A string is suspended from a support pole. The length of the string can be varied by clamping it in several positions to vary its effective free swinging length (wrap the remainder of the string around the support structure so that it will not obstruct the pendulum motion).



The clamp provides a well-defined point from which the system can swing. A set of cylindrical masses will be suspended from the end of the string at a given length of string. The period of the pendulum will be investigated (a) as a function of string length L and (b) as a function of mass m and (c) as a function of displacement angle θ. You will use a protractor to be able to displace the pendulum by a certain angle from equilibrium.

Theoretically, a simple pendulum's period T should follow the relation

$$T = \frac{1}{f} = \frac{2\pi}{\omega} \cong 2\pi\sqrt{\frac{L}{g}}$$

Where g is the gravitational acceleration and L is the length about which the mass swings. This formula applies only in the 'small angle approximation' where sin θ ~ θ. (θ < 30˚)

Activity 1 ~ Experimental Considerations and Dealing with Different Types of Error

Minimizing Random Error

Random error can be diminished through repeated measurement. Error theory is utilized to quantify the remaining random error. For our experiment in particular we have two ways to minimize random error

1. We have to decide how often we let the mass swing to get a reasonably good statistic for a single experiment. At the same time, we have to be careful that we do not add systematic errors to the data, which occur at the very end and the very beginning of the swinging process. Determine how often the pendulum swings back and forth reliably when $\theta = 20^\circ$.

2. We can repeat such an experiment several times in order to improve the statistic, while trying to reproduce the starting conditions accurately for each repetition. Repeat number 1 several times, are you consistent?

Identifying Systematic error and eliminating it, where possible, or accounting for it by estimating its effect on the data

Systematic error is error which is present in principle. It offsets the data away from the actual value and cannot be eliminated by repeated measurement.

> *An example of random error is the stopping of a stopwatch. Sometimes you will stop it too early. Sometimes you will stop it too late. Repeated measurements will eliminate the random error on a statistical basis (see error theory). On the other hand, starting a stopwatch after one has set a system into motion will always start late, if you wait until the object is moving. The operators reaction time will cause systematically too long times for the period. As a result, instead of finding the period, one measures a time that is 'period plus average reaction time'. If several operators switch roles during such a measurement the offset may differ between the data sets because the operators may have different reaction times.*

1. *Which systematic disturbances do you expect at the very beginning and near the very end of the pendulum motion?*

2. *Make a list of possible sources of systematic error in this experiment. (36 have been identified)*

3. *Divide the sources of error identified in #2 into groups such as timing errors, orientation errors, length errors, etc.*

4. *Estimate the percent offset each source of systematic error may add to the experiment. Indicate for your top 5 sources of systematic error, whether each error would tend to shift the data to too large values of period, too small values, or whether it could do either, depending on unknown parameters.*

Activity 2 ~ Influence of displacement amount

Procedure

1. Use a string of length L of 20 cm and the 100 g mass. Displace the mass to various degrees from equilibrium. Let the system swing freely and measure the cumulative period 12 times over 10 full swings.
2. Determine the period T using the equation. Calculate the arithmetic mean and standard deviation for the pendulum period you measured.
3. Measure the length of the string to the center of mass. Explain why you should do this.

Displacement	**Θ = 10˚**	**Θ = 15˚**	**Θ = 20˚**	**Θ = 25˚**
1st T				
2nd T				
3rd T				
T average				
σ				

Activity 3 ~ Dependence of period T on mass m

Procedure

1. Determine the period of the pendulum for each mass three times.
2. Calculate the arithmetic mean and standard deviation for the pendulum period you measured for each mass
3. Use a displacement angle of 20˚ and a string length of 20 cm.

Mass	**200 g**	**100 g (Transfer)**	**50 g**	**20 g**
1st T				
2nd T				
3rd T				
T average				
σ				

Activity 4 ~ Dependence of period T on string length L

Procedure

1. Determine the period of the pendulum for four string lengths (15 cm, 20 cm, 25 cm, 30 cm) for the 100 g mass. Repeat 3 times for each string length. Use a constant displacement angle of 20˚.

2. Calculate the arithmetic mean and standard deviation for the pendulum period you measured for each string length.
3. Using excel, plot period vs string length using proper error bars for data points.

String Length	**15 cm**	**20 cm (Transfer)**	**25 cm**	**30 cm**
1st T				
2nd T				
3rd T				
T average				
σ				

Analysis

1. Determine an overall experimental value for g using the simple pendulum equation. Each average period you determined is an independent observation. In excel make a table of T average, L and the resulting g from all observations. If the values of g are similar to each other, find their average. If you choose to not include any of the values in the averages, identify them and explain why you excluded them.
2. To get the uncertainty in your calculated values of g, calculate the g's that would result from each observation using $T+\sigma_T$ and L, $T-\sigma_T$ and L, $L+\sigma_L$ and T, and $L-\sigma_L$ and T. Enter each of these values in your excel table. How much does g vary when you use these different values of T and L? Does the theoretical value of g fall within these ranges?
3. Does this final 'experimental result within error' agree with the theoretical value for g? Consider the influence of latitude and elevation on $g_{gillette}$ if necessary. If your 'experimental result within error' does not agree with the theoretical value, give a discussion of the most important errors, which would likely have contributed most to the discrepancy. Make sure that the error sources you use in the question would actually shift the experimental value in the observed direction off the actual theoretical value. Use a theoretical value $g_{gillette}$ from the pre-lab.
4. In excel, plot T vs m, T vs $L^{0.5}$ and T vs θ. Use error bars and best fit lines to determine whether you can confirm the expected dependencies of T on the three properties.
5. Revisit your pre-lab predictions and evaluate them in light of what you learned in the experiment.