CH1: Exercises

#1. (1.1) Given 1 in = 2.54 cm,

(a) how many km in 1 mile?

$$1 \text{ mile} \times \left(\frac{5280 \text{ ft}}{\text{mile}}\right) \times \left(\frac{12 \text{ in}}{\text{ft}}\right) \times \left(\frac{2.54 \text{ cm}}{\text{in}}\right) \times \left(\frac{1 \text{ m}}{10^2 \text{ cm}}\right) \times \left(\frac{1 \text{ km}}{10^3 \text{ m}}\right)$$

$$= \frac{1.609 \cdot 10^5}{10^5} \text{ km} = \boxed{1.61 \text{ km}}$$

(b) how many ft in 1 km?

$$1 \text{ km} \times \left(\frac{10^3 \text{ m}}{\text{km}}\right) \times \left(\frac{10^2 \text{ cm}}{\text{m}}\right) \times \left(\frac{1 \text{ in}}{2.54 \text{ cm}}\right) \times \left(\frac{1 \text{ ft}}{12 \text{ in}}\right)$$

$$= \left(\frac{10^5}{30.48}\right) \text{ ft} = \boxed{3{,}280 \text{ ft}}$$

#2. (1.3) How many "ns" does it take light to travel 1 ft?

distance = velocity · time $(d = v \cdot t)$ where $v = c = 3 \cdot 10^8 \text{ m/s}$

Convert c to ft/s...

$$c = 3 \cdot 10^8 \text{ m/s} \times \left(\frac{10^2 \text{ cm}}{\text{m}}\right) \times \left(\frac{1 \text{ in}}{2.54 \text{ cm}}\right) \times \left(\frac{1 \text{ ft}}{12 \text{ in}}\right) = 9.84 \cdot 10^8 \text{ ft/s}$$

thus, $t = \frac{d}{c} = \frac{1 \text{ ft}}{9.84 \cdot 10^8 \text{ ft/s}} = 1.02 \cdot 10^{9} \text{ s} = \boxed{1.02 \text{ ns}}$

#3. (1.7) 1 Gs = ? yrs.

$$10^9 \text{ s} \times \left(\frac{1 \text{ min}}{60 \text{ s}}\right) \times \left(\frac{1 \text{ hr.}}{60 \text{ min}}\right) \times \left(\frac{1 \text{ day}}{24 \text{ hrs}}\right) \times \left(\frac{1 \text{ yr}}{365.24 \text{ days}}\right)$$

$$= \boxed{31.7 \text{ yrs}}$$

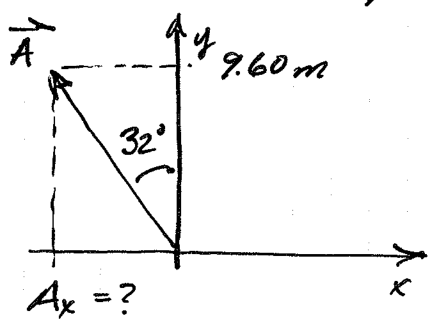
#4. (1.15) Assuming $\pi \cdot 10^7$ s $\simeq$ 1 yr, determine percent error from actual value.

$$\#\text{ of s/yr} = \left(\frac{365.24 \text{ days}}{\text{yr}}\right) \times \left(\frac{24 \text{ hrs}}{\text{day}}\right) \times \left(\frac{3600 \text{ s}}{\text{hr}}\right) = 3.156 \cdot 10^7 \text{ s/yr}$$

$$\%\text{ Error} = \frac{|\text{Theoretical} - \text{Experimental}|}{\text{Theoretical}} \times (100\%) = \frac{|3.156 - 3.142| \cdot \cancel{10^7}}{3.156 \cdot \cancel{10^7}} \times (100\%)$$

$$= \boxed{0.44\%} \qquad \therefore\ 1 \text{ yr} = \pi \cdot 10^7 \text{ s} \pm 0.44\%$$

#5. (1.29) Given: $A_y = 9.60$ m



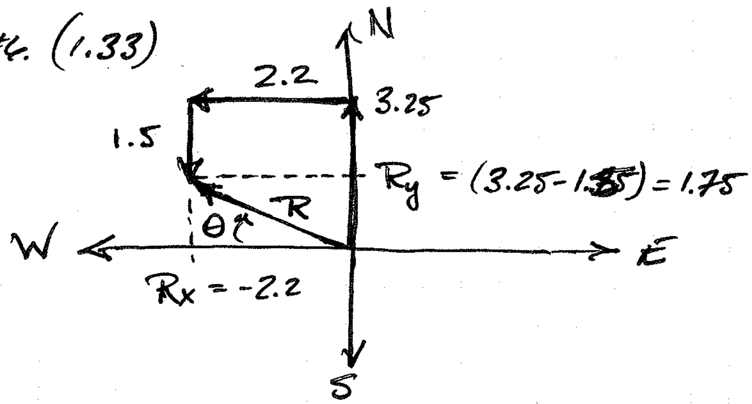
(a) What is A_x?

$$A_x = -9.6 \tan(32°) = \boxed{6.00 \text{ m}}$$

(b) What is the magnitude of $\vec{A}$?

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2} = \sqrt{128.11} = \boxed{11.3 \text{ m}}$$

#6. (1.33)



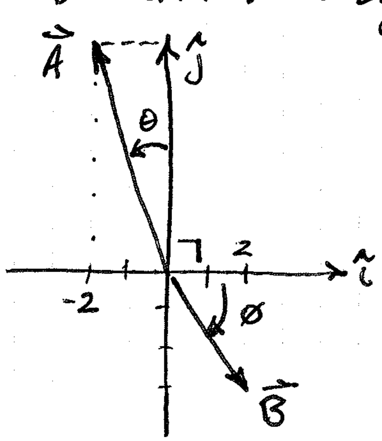
– What is the magnitude and direction of the resultant displacement?

$$|\vec{R}| = \sqrt{(1.75)^2 + (-2.2)^2}$$

$$= \boxed{2.81 \text{ km}}$$

$$\theta = \tan^{-1}\left(\frac{1.75}{2.2}\right) = \boxed{38.5° \text{ N of East}}$$

#7. (1.45) What is the angle between vector $\vec{A} = -2\hat{i} + 6\hat{j}$ and $\vec{B} = 2\hat{i} - 3\hat{j}$



$\underline{\vec{A}}$: $\tan\theta = \left(\frac{2}{6}\right) \Rightarrow \theta = \tan^{-1}\left(\frac{2}{6}\right) = 18.4°$

$\underline{\vec{B}}$: $\tan\phi = \left(\frac{3}{2}\right) \Rightarrow \phi = \tan^{-1}\left(\frac{3}{2}\right) = 56.3°$

$\therefore \angle$ between $\vec{A}$ and $\vec{B}$

$$= \theta + \phi + 90° = \boxed{164.7°}$$

#8. (1.51) A planet is discovered having a mass 5.5 greater than the Earth. It is believed to have a density similar to Neptune ($\rho = 1.76\ g/cm^3$). What is its radius in (a) kilometers, and (b) as a multiple of Earths radius?

Recall: Density $\rho = \frac{Mass}{Volume}$ where Volume $= \frac{4}{3}\pi r^3$

So, $\rho = \frac{M_x}{\frac{4}{3}\pi r^3} \Rightarrow r^3 = \frac{3}{4}\frac{M_x}{\pi \cdot \rho}$ Solve for $r = \sqrt[3]{\frac{3}{4}\frac{M_x}{\pi\rho}}$

Given: Mass of Earth: $M_E = 5.97 \cdot 10^{24}\ kg$

Radius of Earth: $R_E = 6.37 \cdot 10^6\ m$

Mass of Planet X: $M_X = 5.5 \cdot M_E = 3.28 \cdot 10^{25}\ kg$

Now, convert $\rho\ (g/cm^3) \rightarrow (kg/m^3)$

$$\rho = 1.76\ \frac{g}{cm^3} \times \left(\frac{1\ kg}{10^3\ g}\right) \times \left(\frac{10^2\ cm}{1\ m}\right)^3 = 1.76 \cdot 10^3\ kg/m^3$$

$$r_x^3 = \frac{3}{4} \cdot \frac{M_x}{\pi\rho} = \frac{3}{4\pi}\frac{(3.28 \cdot 10^{25}\ kg)}{(1.76 \cdot 10^3\ kg/m^3)} = \frac{9.84 \cdot 10^{25}}{2.21 \cdot 10^4}\ m^3$$

$$\therefore\ r_x = \sqrt[3]{4.45 \cdot 10^{21}\ m^3} = 1.64 \cdot 10^7\ m \times \left(\frac{1\ km}{10^3\ m}\right) = \boxed{1.64 \cdot 10^4\ km}$$

(b) Factor w.r.t. Earth's radius ...

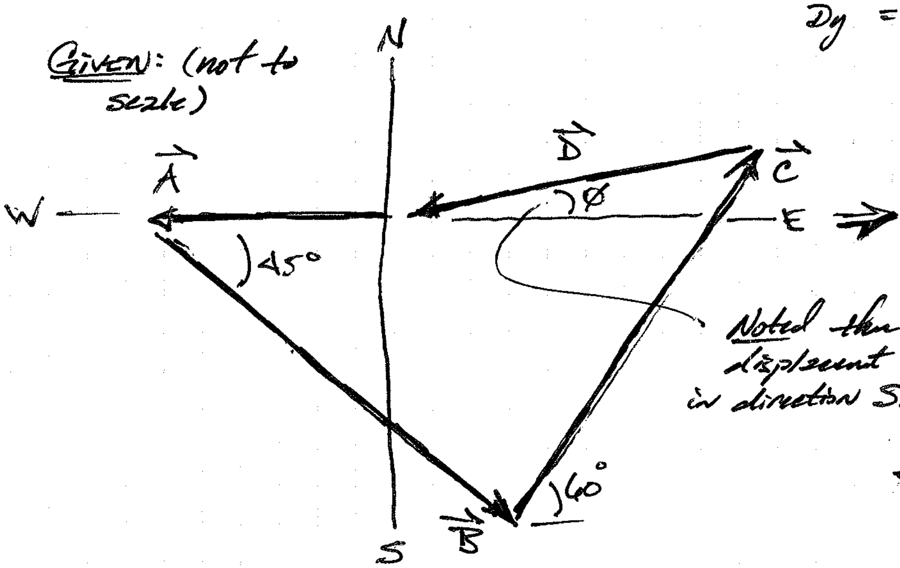
$$F_{r_x} = \frac{1.64 \cdot 10^4\ km}{6.37 \cdot 10^3\ km} R_E = \boxed{2.57\ R_E}$$

#9 (1.61) A cave diver follows a passage 180 m straight west, then 210 m in a direction E. of South, and then 280 m at 30° E. of North. After the fourth displacement, she finds herself back to where she started. Use the method of components to determine the magnitude and direction of the fourth displacement.

LET: $\vec{A}$, $\vec{B}$ and $\vec{C}$ be 3 given vectors, solve for $\vec{D}$ (the 4th displacement)
Since she ends up back where she started:

$$\vec{R} = 0 = \vec{A} + \vec{B} + \vec{C} + \vec{D} \quad \Rightarrow \quad \begin{aligned} D_x &= -(A_x + B_x + C_x) \\ D_y &= -(A_y + B_y + C_y) \end{aligned}$$

GIVEN: (not to scale)



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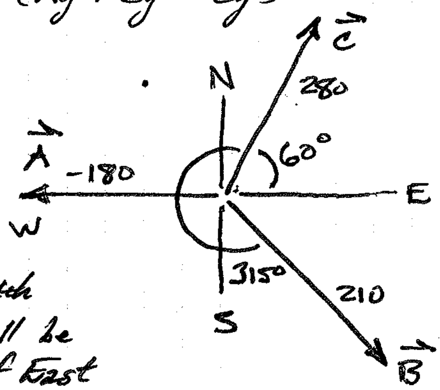


• Use tail-to-tail method to find components...

Find Components...

$$A_x = -180\ , \quad A_y = 0$$

$$B_x = B\cos(315°) = (210)\cos(315°) = +148.5\ m$$
$$B_y = B\sin(315°) = (210)\cos(315°) = -148.5\ m$$

$$C_x = C\cos(60°) = (280)\cos(60°) = +140.0\ m$$
$$C_y = C\sin(60°) = (280)\sin(60°) = +242.5\ m$$

thus, $D_x = -(A_x + B_x + C_x) = -(-180 + 148.5 + 140) = -108.5\ m$

$D_y = -(A_y + B_y + C_y) = -(0 - 148.5 + 242.5) = -94.0\ m$

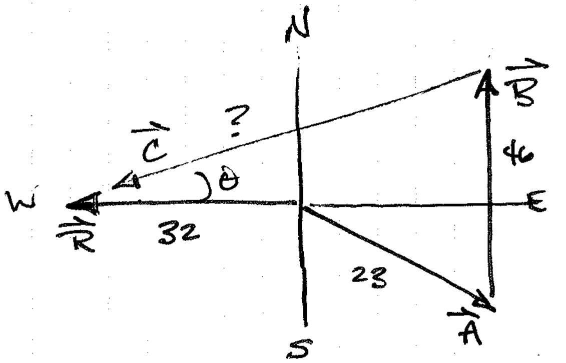
MAGNITUDE: $D = \sqrt{D_x^2 + D_y^2} = \sqrt{(-108.5)^2 + (-94)^2} = \boxed{144\ m}$

DIRECTION: $\tan Ø = \frac{D_y}{D_x} \Rightarrow Ø = \tan^{-1}\left(\frac{-94}{-108.5}\right) = \boxed{40.9° \text{ South of East}}$

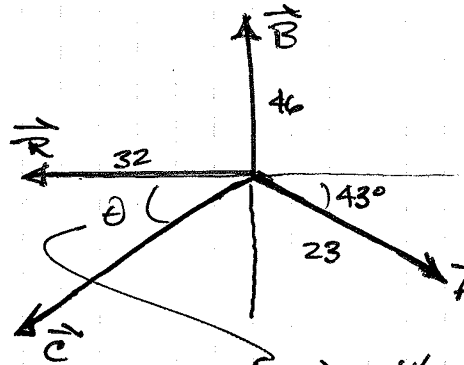
#10. (1.65) You leave College Station airport and fly 23 km in a direction 43° South of East. You then fly 46 km due North. How far and what direction must you fly to reach a private landing strip that is 32 km due West of College Station?

Given: $\vec{A}$, $\vec{B}$ and $\vec{R}$ are known vectors, solve for $\vec{C}$

where $\vec{C} = \vec{R} - (\vec{A} + \vec{B})$ $\Rightarrow \begin{cases} C_x = R_x - (A_x + B_x) \\ C_y = R_y - (A_y + B_y) \end{cases}$



$\Rightarrow$



will yield a direction South of West

Solve for vector Components:

$$A_x = A\cos(34°) = 23\cos(34°) = 19.1 \text{ km}$$
$$A_y = A\sin(34°) = 23\sin(34°) = -12.9 \text{ km}$$
$$B_x = 0, \quad B_y = 46$$
$$R_x = -32, \quad R_y = 0$$

$$C_x = R_x - (A_x + B_x) = -32 - (19.1 + 0) = \underline{-51.1 \text{ km}}$$
$$C_y = R_y - (A_y + B_y) = 0 - (-12.9 + 46) = \underline{-33.1 \text{ km}}$$

MAGNITUDE: $C = \sqrt{C_x^2 + C_y^2} = \sqrt{(-57.2)^2 + (-33.1)^2} = \boxed{60.9 \text{ km}}$

DIRECTION: $\tan\theta = \frac{C_y}{C_x} \Rightarrow \theta = \tan^{-1}\left(\frac{33.1}{51.1}\right) = \boxed{32.9° \text{ South of West}}$

#11. (1.69) You are lost at night in a large open field. Your GPS tells you are 122 m from your truck, in a direction 58° East of South. You walk 72 m due west along a ditch. How much farther, and in what direction must you walk to reach your truck?

LET: $\vec{R}$ be the vector from you to your truck ⇒ 122 m West of North

$$\vec{R} = R_x + R_y = 122\sin(58^\circ) + 122\cos(58^\circ)$$

$$\vec{A} = A_x = 72$$

Solve for: $\vec{B} = \vec{R} - \vec{A}$

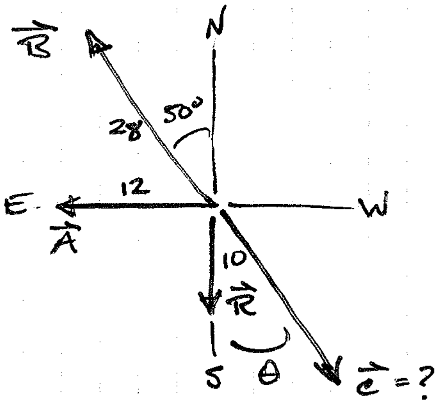
$$B_x = R_x - A_x = 122\sin(58^\circ) - 72 = \underline{31.5\text{ m}}$$

$$B_y = R_y - A_y = 122\cos(58^\circ) - 0 = \underline{64.6\text{ m}}$$

Magnitude: $B = \sqrt{B_x^2 + B_y^2} = \boxed{71.9\text{ m}}$

Direction: $\tan\theta = \dfrac{B_y}{B_x} \Rightarrow \theta = \tan^{-1}\left(\dfrac{64.6}{31.5}\right) = \boxed{64.0^\circ \text{ N of West}}$

#12. (1.75) A dog in an open field runs 12 m East and then 28 m in a direction 50° W. of North. In what direction and how must he then run to end up 10 m South of original starting point?



Given: 2 known vectors $\vec{A}, \vec{B}$ and resultant $\vec{R}$ vector.

Solve for: $\vec{C} = \vec{R} - (\vec{A} + \vec{B})$

$$A_x = -12\text{ m}, \quad A_y = 0$$

$$B_x = -B\sin(50^\circ) = -28\sin(50^\circ) = -21.5\text{ m}$$

$$B_y = B\cos(50^\circ) = 28\cos(50^\circ) = 18.0\text{ m}$$

$$R_x = 0, \quad R_y = -10\text{ m}$$

$$C_x = R_x - (A_x + B_x) = 0 - (-12 - 21.5) = \underline{9.5\text{ m}}$$

$$C_y = R_y - (A_y + B_y) = -10 - (0 + 18) = \underline{-28\text{ m}}$$

Magnitude: $C = \sqrt{C_x^2 + C_y^2} = \boxed{29.6\text{ m}}$

Direction: $\tan\theta = \dfrac{C_x}{C_y} \Rightarrow \theta = \tan^{-1}\left(\dfrac{9.5}{28}\right) = \boxed{18.7^\circ \text{ E. of South}}$