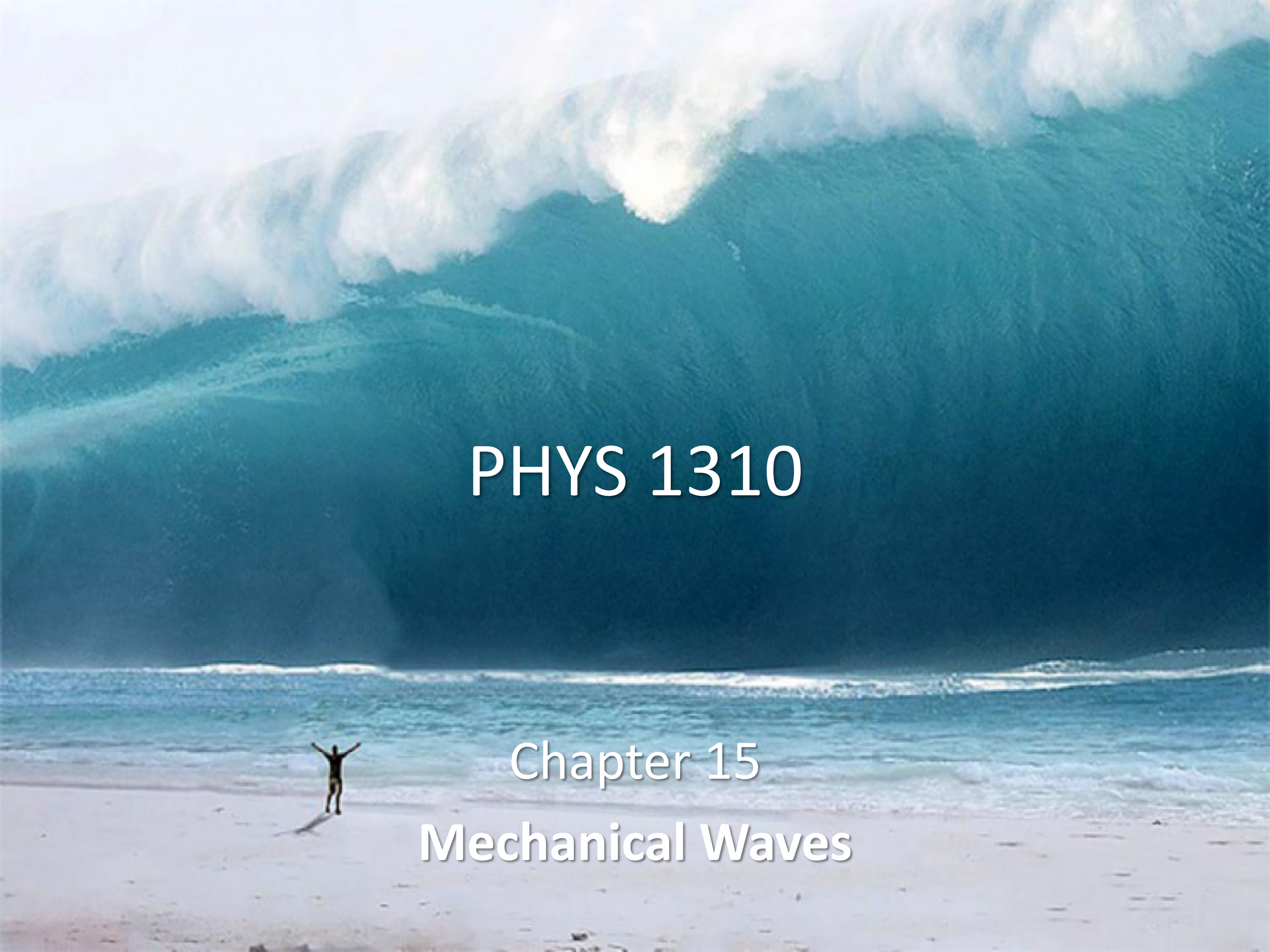


PHYS 1310

Chapter 15
Mechanical Waves

CH15 in brief

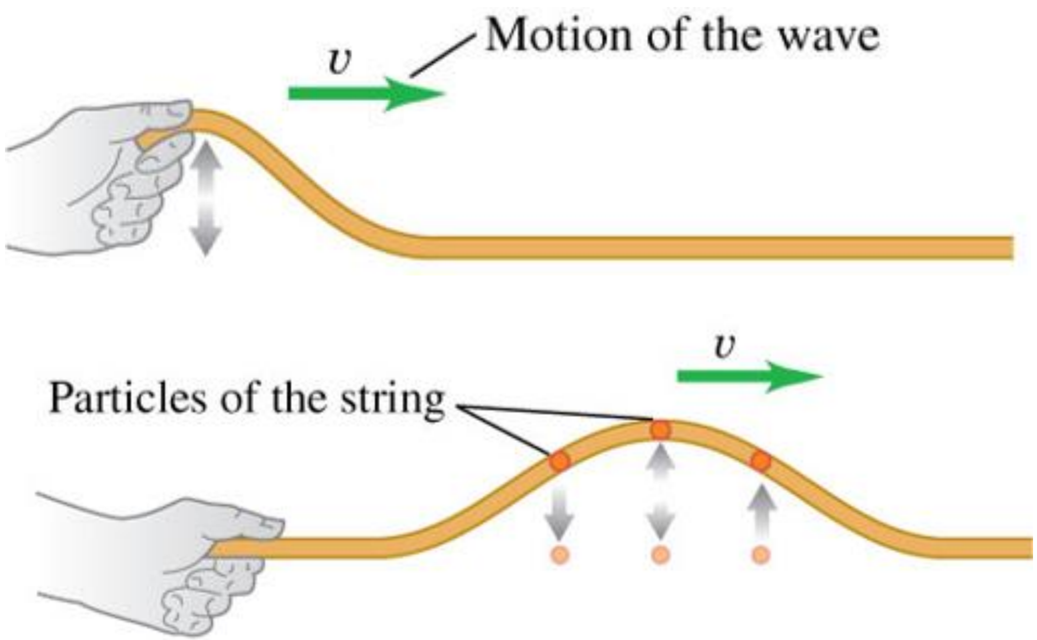
- Wave velocity: $v = \lambda f$
- Wave number: $k = 2\pi/\lambda$
- Angular frequency: $\omega = 2\pi f = vk$
- Wave function: $y(x,t) = A\cos(kx - \omega t)$
- Transverse wave speed: $v = \sqrt{F/}$
- Wave power: $P_{av} = \frac{1}{2}\sqrt{\mu F}\,\omega^2 A^2$
- Intensity: $I = \frac{Power}{area} = \frac{\frac{Energy}{time}(Watts)}{(m^2)}$
- Intensity inverse square law: $\frac{I_1}{I_2} = \frac{{r_2}^2}{{r_1}^2}$

Introduction

- Earthquake waves carry enormous power as they travel through the earth.
- Other types of mechanical waves, such as sound waves or the vibration of the strings of a piano, carry far less energy.
- Overlapping waves interfere, which helps us understand musical instruments.

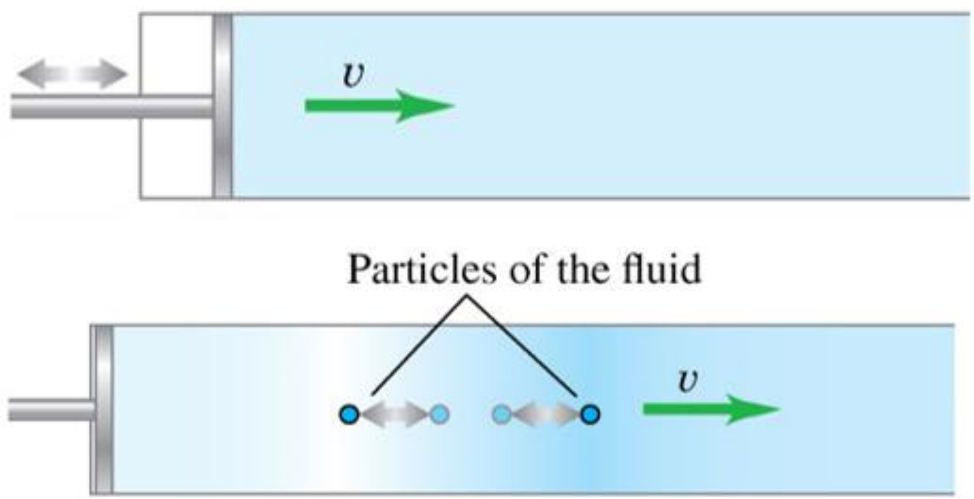
Types of Mechanical Waves (1 of 3)

- A wave on a string is a type of mechanical wave.
- The hand moves the string up and then returns, producing a transverse wave that moves to the right.



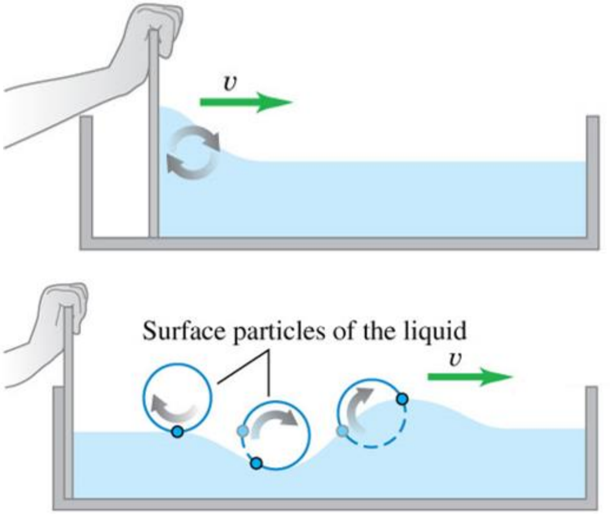
Types of Mechanical Waves (2 of 3)

- A pressure wave in a fluid is a type of mechanical wave.
- The piston moves to the right, compressing the gas or liquid, and then returns, producing a longitudinal wave that moves to the right.



Types of Mechanical Waves (3 of 3)

- A surface wave on a liquid is a type of mechanical wave.
- The board moves to the right and then returns, producing a combination of longitudinal and transverse waves.



Mechanical Waves

- Doing the wave" at a sports stadium is an example of a mechanical wave.
- The **disturbance** propagates through the crowd, but there is no transport of matter.
- None of the spectators moves from one seat to another.

Periodic Waves (1 of 2)

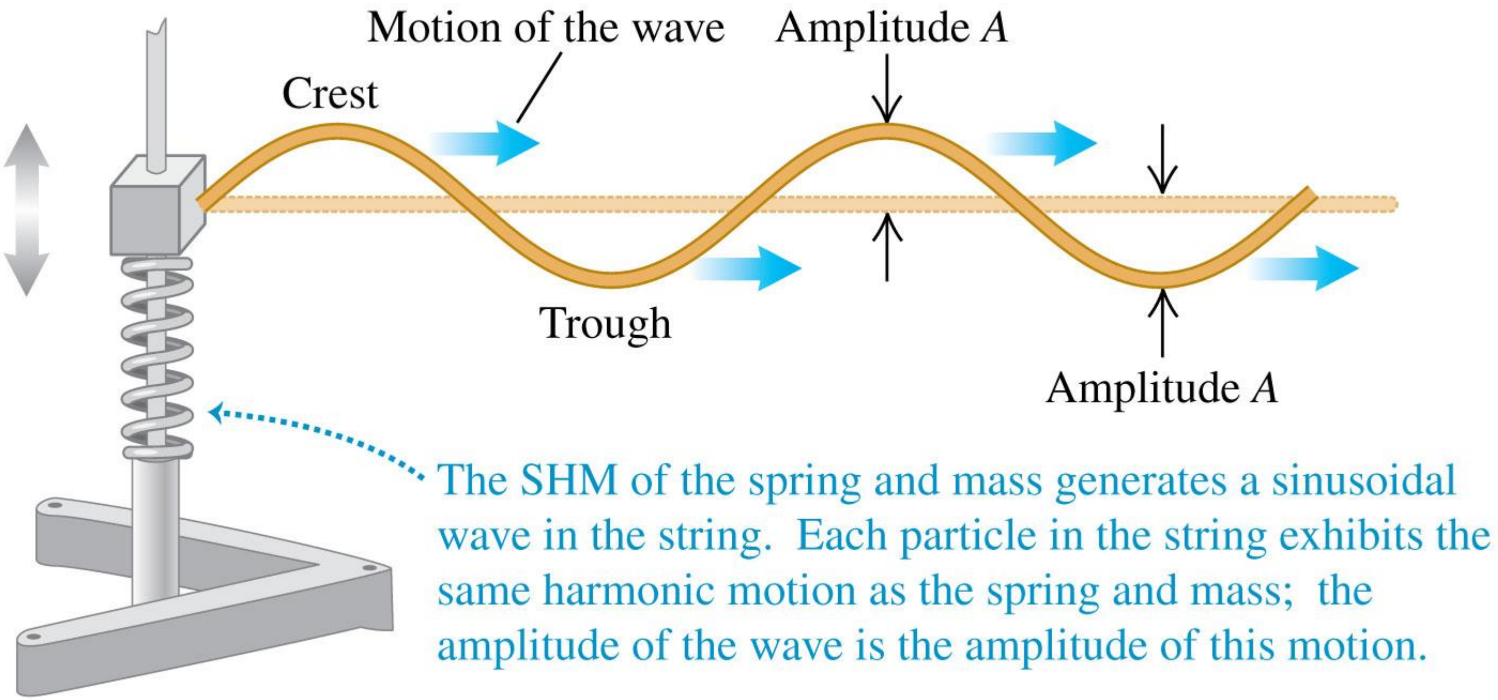
- For a **periodic wave**, each particle of the medium undergoes periodic motion.
- The **wavelength** λ of a periodic wave is the length of one complete wave pattern.
- The **speed** of any periodic wave of frequency f is:

For a **periodic wave:**

$$\underset{\text{Wave speed}}{v} = \underset{\text{Wavelength}}{\lambda}\,\underset{\text{Frequency}}{f}$$

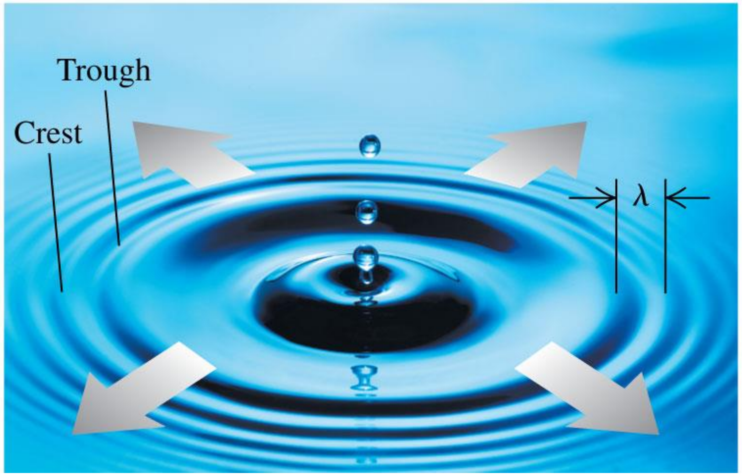
Periodic Transverse Waves

- A mass attached to a spring undergoes simple harmonic motion, producing a sinusoidal wave that travels to the right on the string.



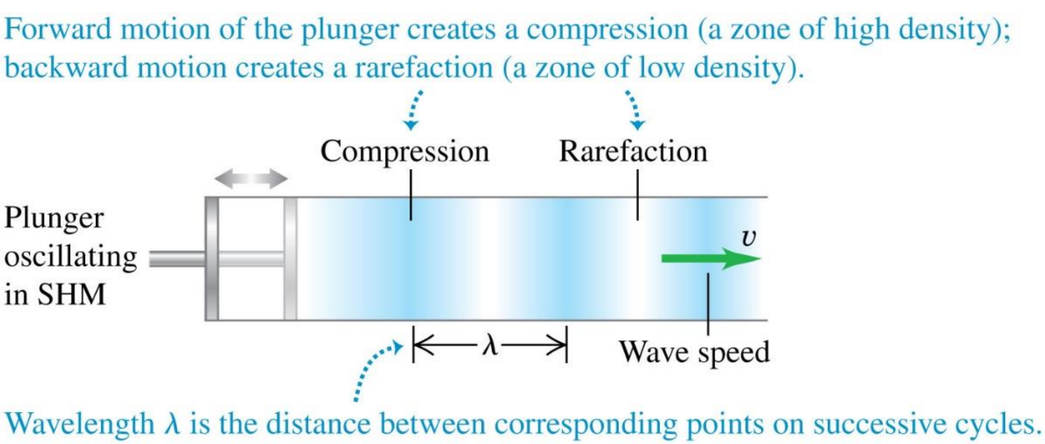
Periodic Waves (2 of 2)

- A series of drops falling into water produces a periodic wave that spreads radially outward.
- The wave crests and troughs are concentric circles.
- The wavelength λ is the radial distance between adjacent crests or adjacent troughs.



Periodic Longitudinal Waves

- Consider a long tube filled with a fluid, with a piston at the left end.
- If we push the piston in, we compress the fluid near the piston, and this region then pushes against the neighboring region of fluid, and so on, and a wave pulse moves along the tube.



Mathematical Description of a Wave

- The wave function for a sinusoidal wave moving in the +*x*-direction is given by Equation (15.7):

Wave function for a **sinusoidal wave propagating in +*x*-direction**

$$y(x, t) = \overset{\text{Amplitude}}{A}\cos(\underset{\text{Wave number } = 2\pi/\lambda}{k}\overset{\text{Position}}{x} - \underset{\text{Angular frequency } = 2\pi f}{\omega}\overset{\text{Time}}{t})$$

- In this function, *y* is the displacement of a particle at time *t* and position *x*.
- The quantity *A* is the amplitude of the wave.
- The quantity *k* is called the **wave number**, and is defined as:

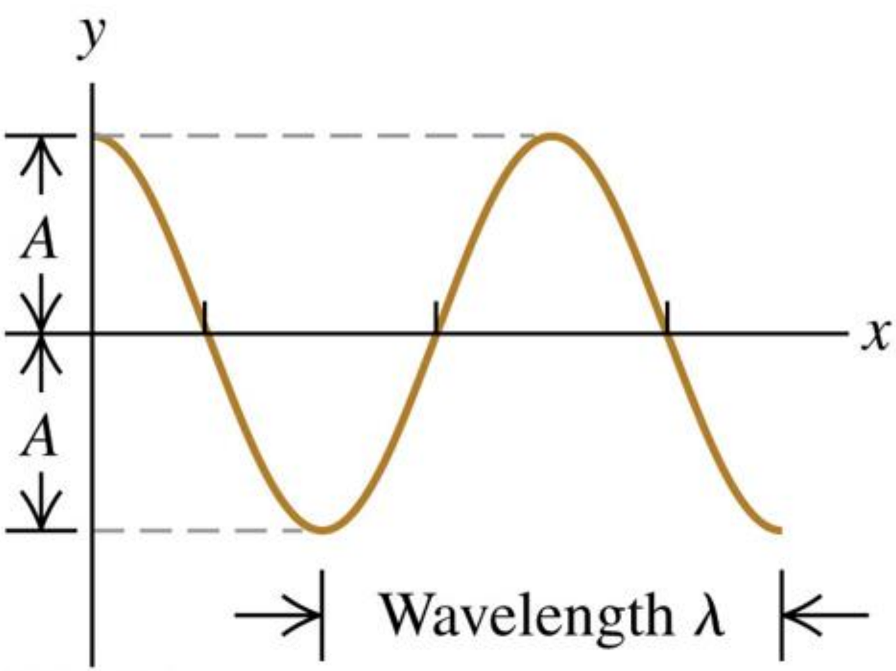
$$k = \frac{2\pi}{\lambda}.$$

- The quantity ω is called the **angular frequency**, and is defined as:

$$\omega = 2\pi f = \frac{2\pi}{T},$$

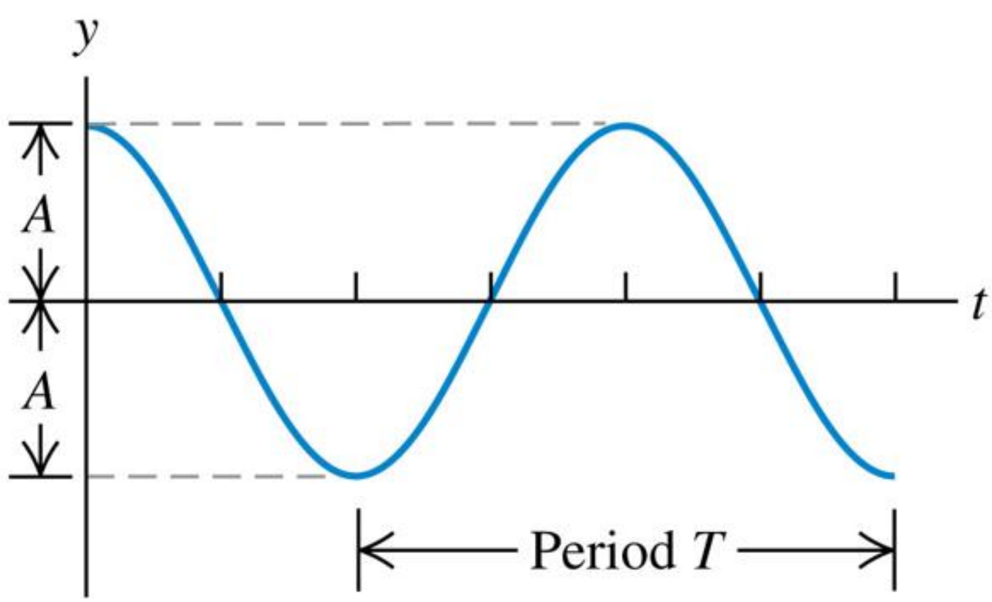
Graphing the Wave Function (1 of 2)

If we use Eq. (15.7) to plot y as a function of x for time $t = 0$, the curve shows the *shape* of the string at $t = 0$.

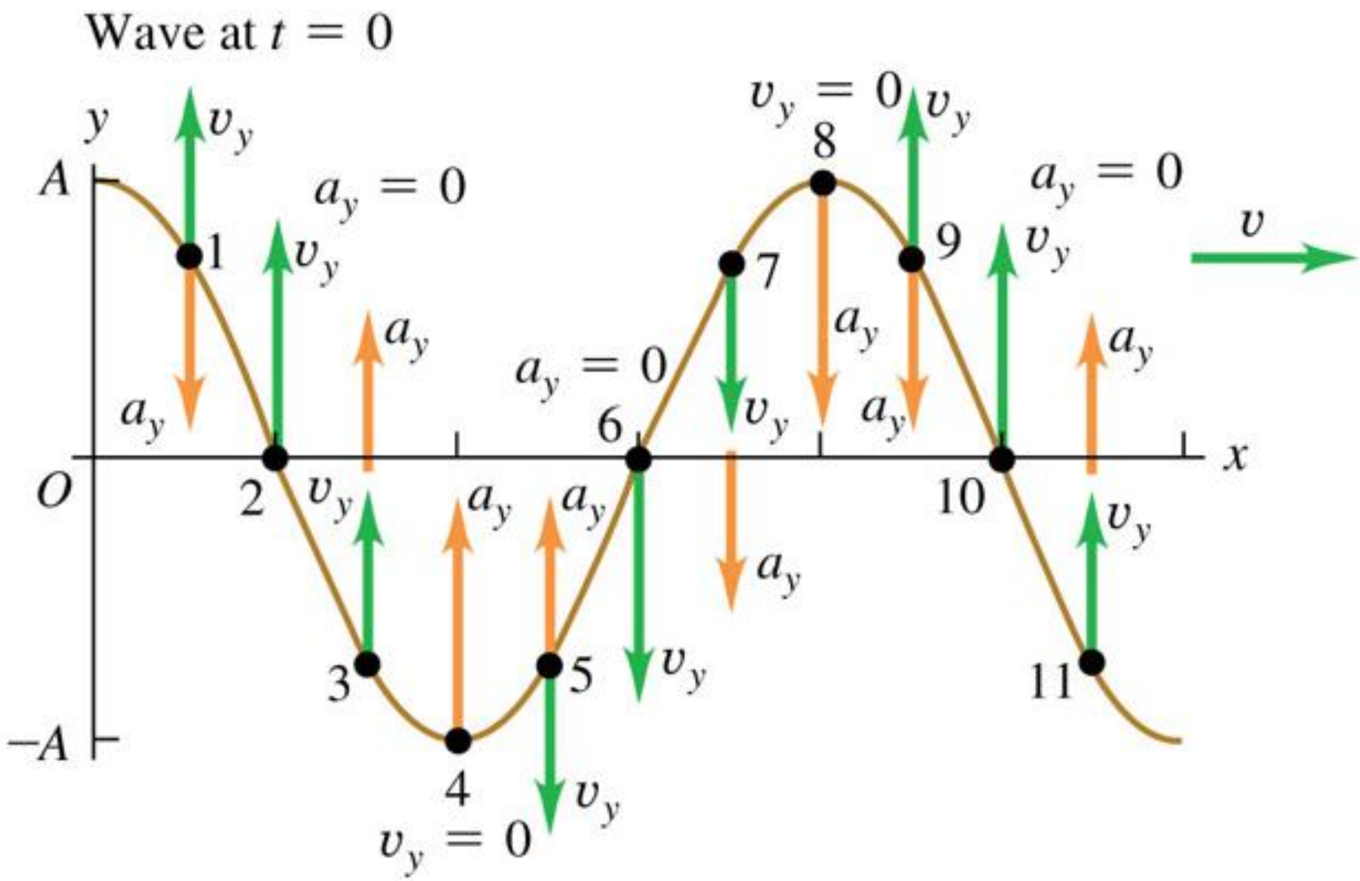


Graphing the Wave Function (2 of 2)

If we use Eq. (15.7) to plot y as a function of t for position $x = 0$, the curve shows the *displacement* y of the particle at $x = 0$ as a function of time.

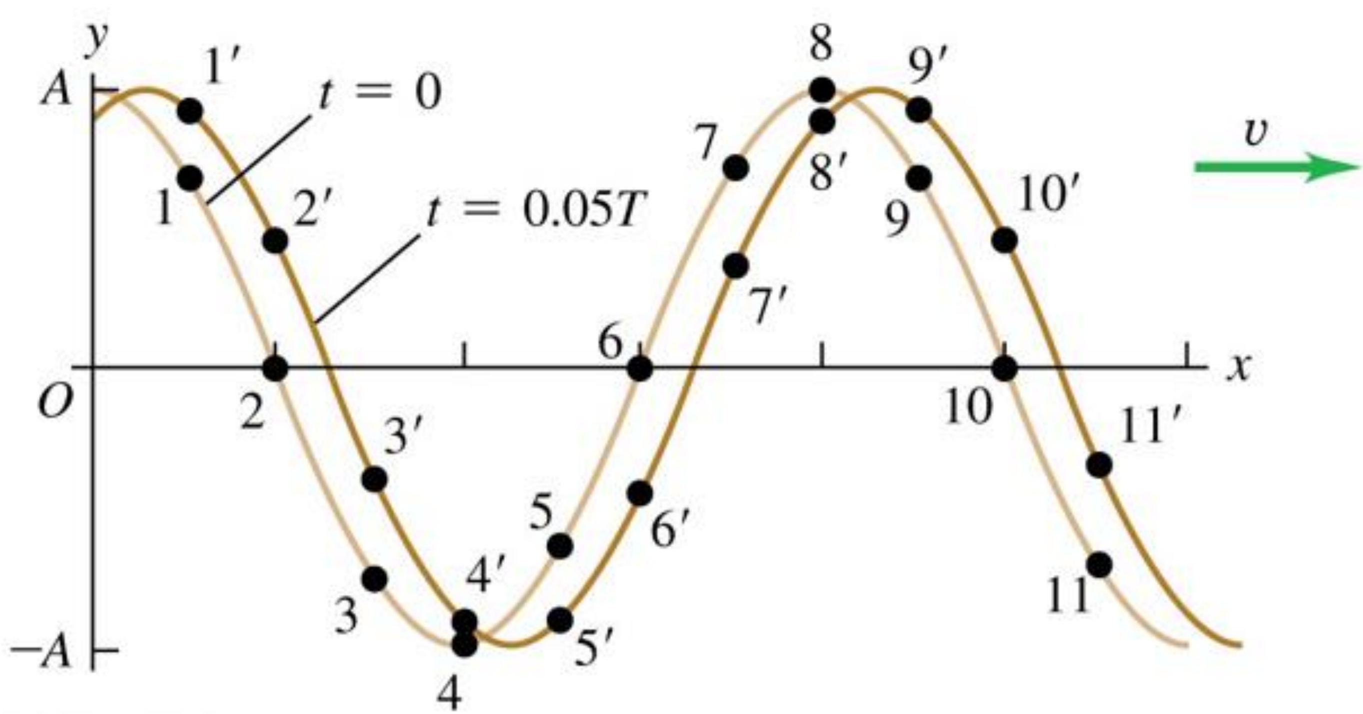


Particle Velocity and Acceleration in a Sinusoidal Wave (1 of 2)



Particle Velocity and Acceleration in a Sinusoidal Wave (2 of 2)

The same wave at $t = 0$ and $t = 0.05T$



The Speed of a Wave on a String (1 of 2)

- One of the key properties of any wave is the **wave speed**.
- Consider a string in which the tension is F and the linear mass density (mass per unit length) is μ.
- We expect the speed of transverse waves on the string v should increase when the tension F increases, but it should decrease when the mass per unit length μ increases.
- It is shown in your text that the wave speed is:

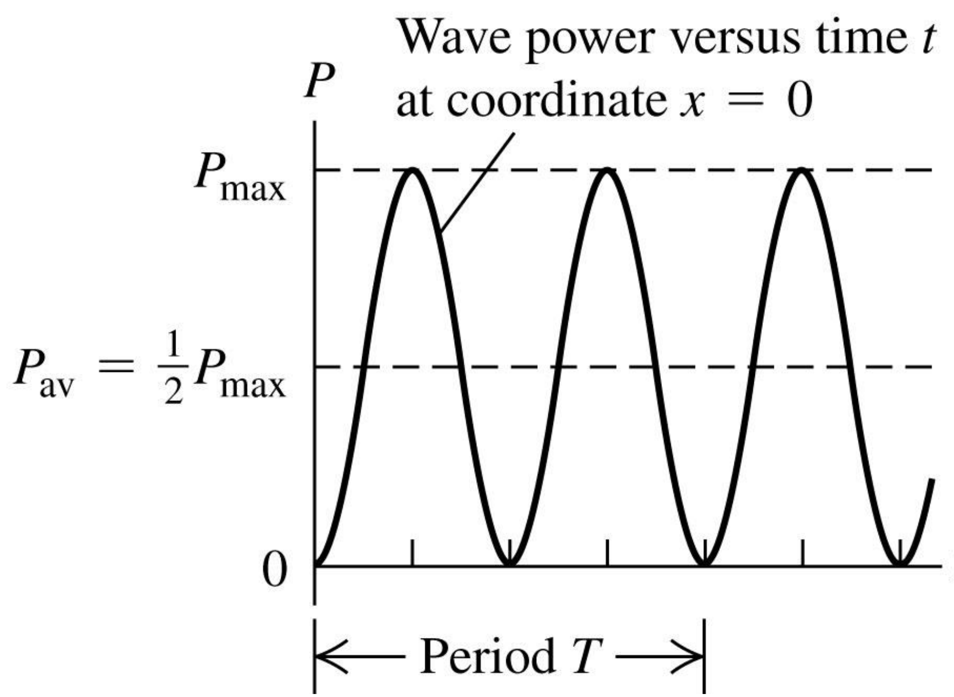
$$\underbrace{v}_{\text{Speed of a transverse wave on a string}} = \sqrt{\frac{\overbrace{F}^{\text{Tension in string}}}{\underbrace{\mu}_{\text{Mass per unit length}}}}$$

The Speed of a Wave on a String (2 of 2)

- These transmission cables have a relatively large amount of mass per unit length, and a low tension.
- If the cables are disturbed—say, by a bird landing on them—transverse waves will travel along them at a slow speed.

Power in a Wave (1 of 2)

- Shown is the instantaneous power in a sinusoidal wave.
- The power is never negative, which means that energy never flows opposite to the direction of wave propagation.



Power in a Wave (2 of 2)

- A wave transfers power along a string because it transfers energy.
- The average power is proportional to the **square** of the amplitude and to the **square** of the frequency.
- This result is true for all waves.
- For a transverse wave on a string, the average power is:

Wave angular frequency

Average power, sinusoidal wave on a string

$$P_{av} = \frac{1}{2}\sqrt{\mu F}\,\omega^2 A^2$$

Wave amplitude

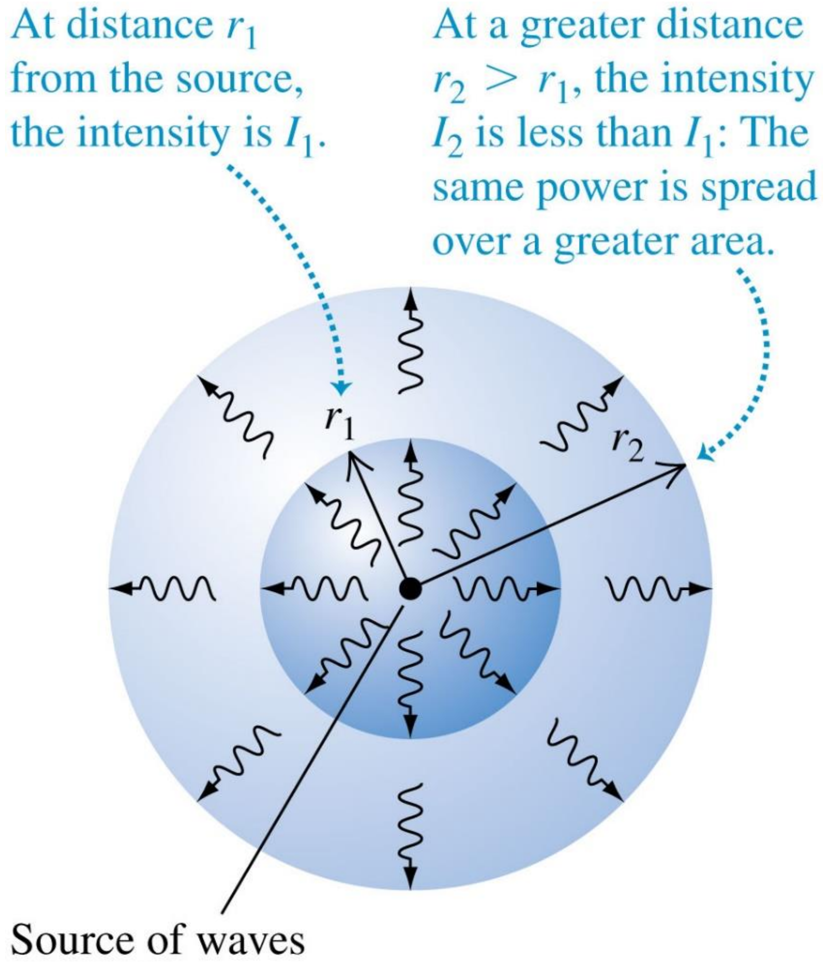
Mass per unit length

Tension in string

Wave Intensity

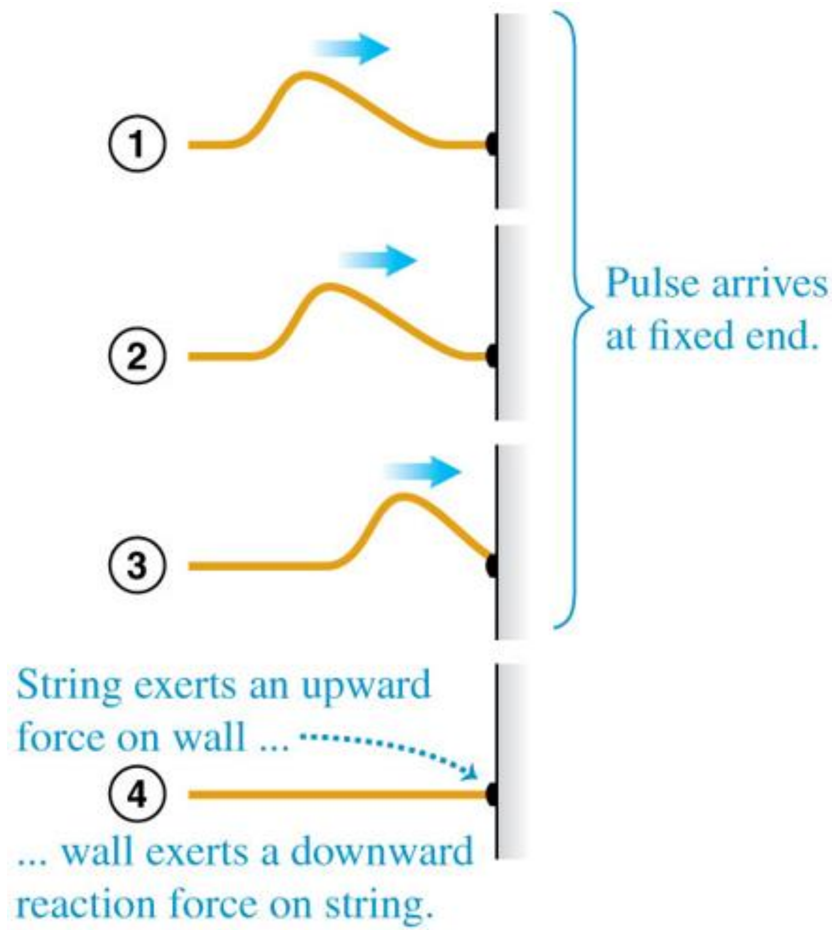
- The **intensity** of a wave is the average power it carries per unit area.
- If the waves spread out uniformly in all directions and no energy is absorbed, the intensity *I* at any distance *r* from a wave source is inversely proportional to r^2.

$$\frac{I_1}{I_2} = \frac{{r_2}^2}{{r_1}^2}$$



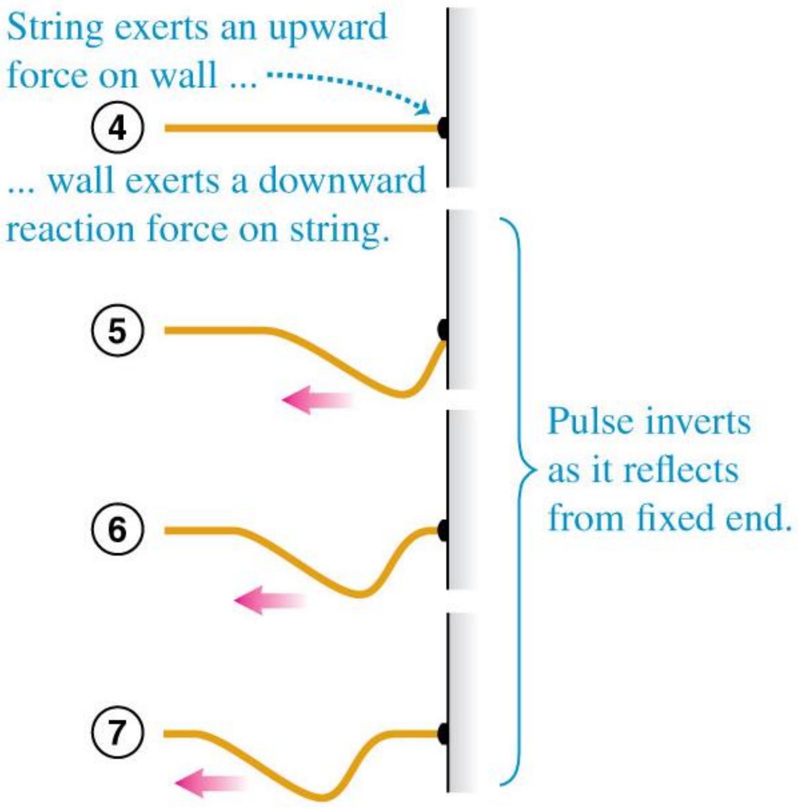
Reflection of a Wave Pulse at a Fixed End of a String (1 of 4)

- What happens when a wave pulse or a sinusoidal wave arrives at the end of the string?
- If the end is fastened to a rigid support, it is a **fixed** end that cannot move.
- The arriving wave exerts a force on the support (drawing 4).



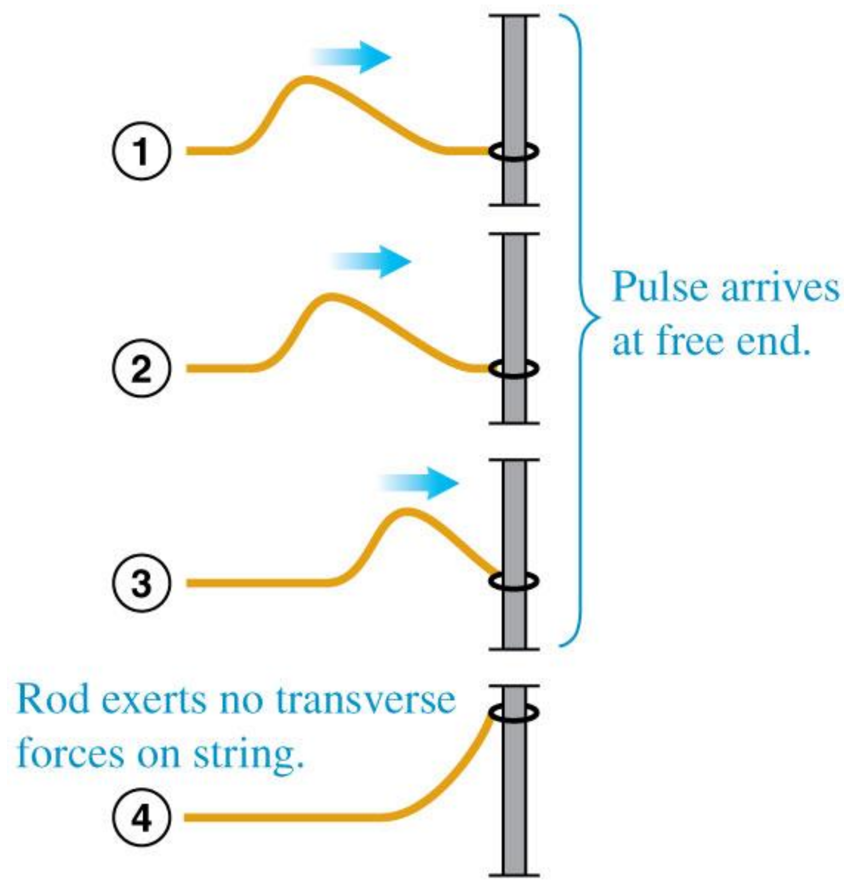
Reflection of a Wave Pulse at a Fixed End of a String (2 of 4)

- The reaction to the force of drawing 4, exerted by the support on the string, "kicks back" on the string and sets up a reflected pulse or wave traveling in the reverse direction.



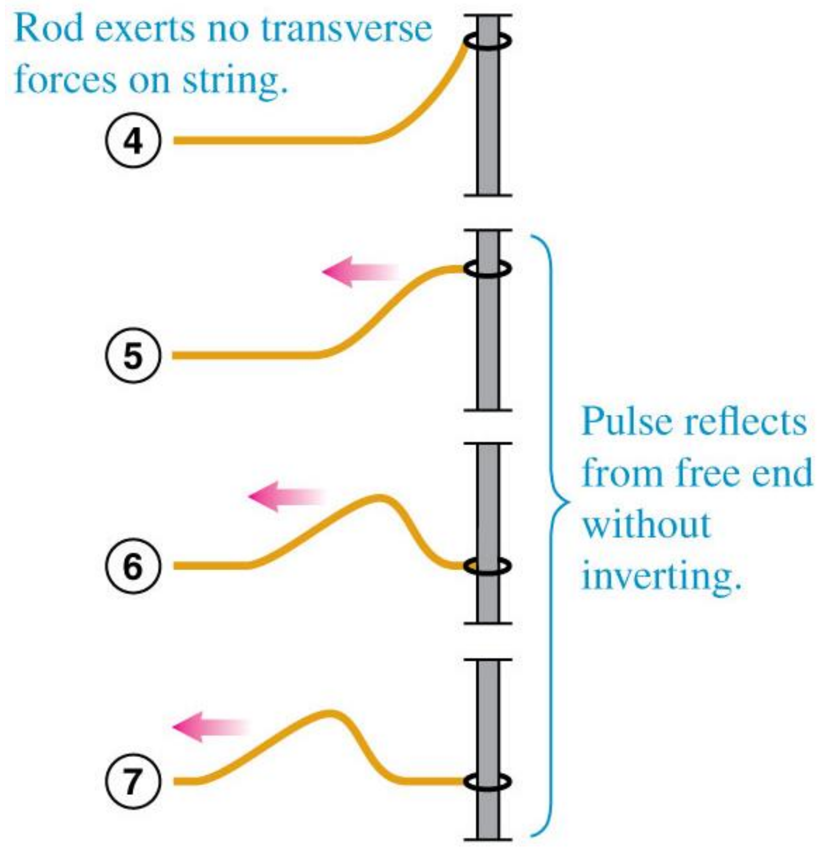
Reflection of a Wave Pulse at a Fixed End of a String (3 of 4)

- A **free end** is one that is perfectly free to move in the direction perpendicular to the length of the string.
- When a wave arrives at this free end, the ring slides along the rod, reaching a maximum displacement, coming momentarily to rest (drawing 4).



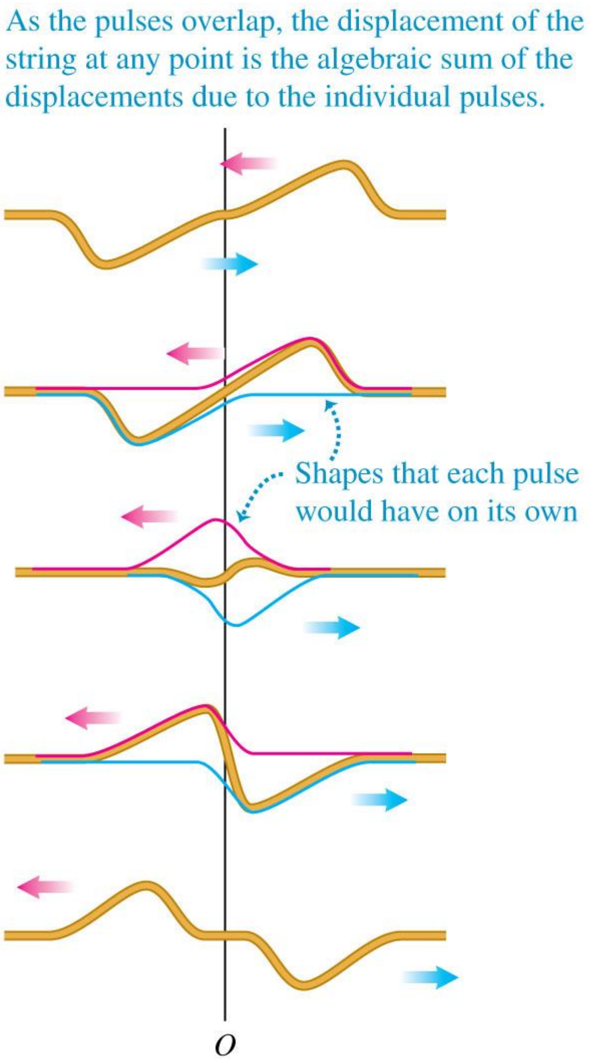
Reflection of a Wave Pulse at a Fixed End of a String (4 of 4)

- In drawing 4, the string is now stretched, giving increased tension, so the free end of the string is pulled back down, and again a reflected pulse is produced.



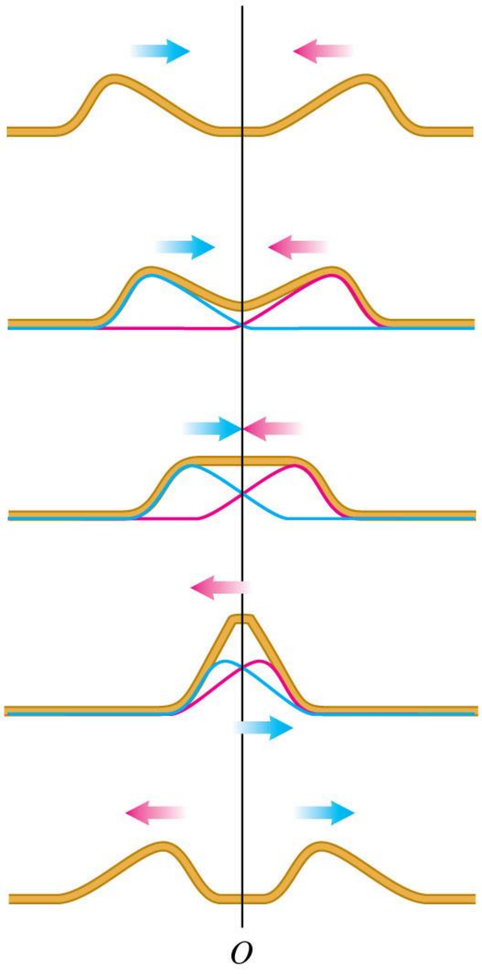
Superposition (1 of 2)

- **Interference** is the result of overlapping waves.
- **Principle of superposition**: When two or more waves overlap, the total displacement is the sum of the displacements of the individual waves.
- Shown is the overlap of two wave pulses—one right side up, one inverted—traveling in opposite directions.
- Time increases from top to bottom.



Superposition (2 of 2)

- Overlap of two wave pulses—both right side up—traveling in opposite directions.
- Time increases from top to bottom.

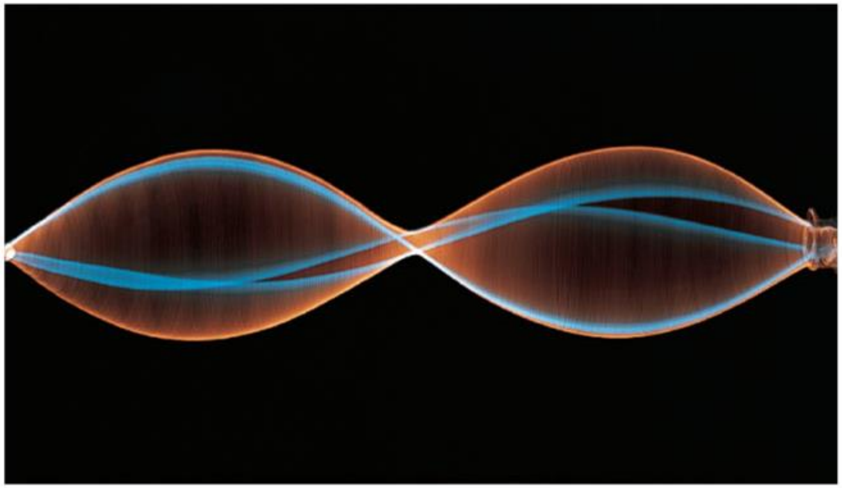


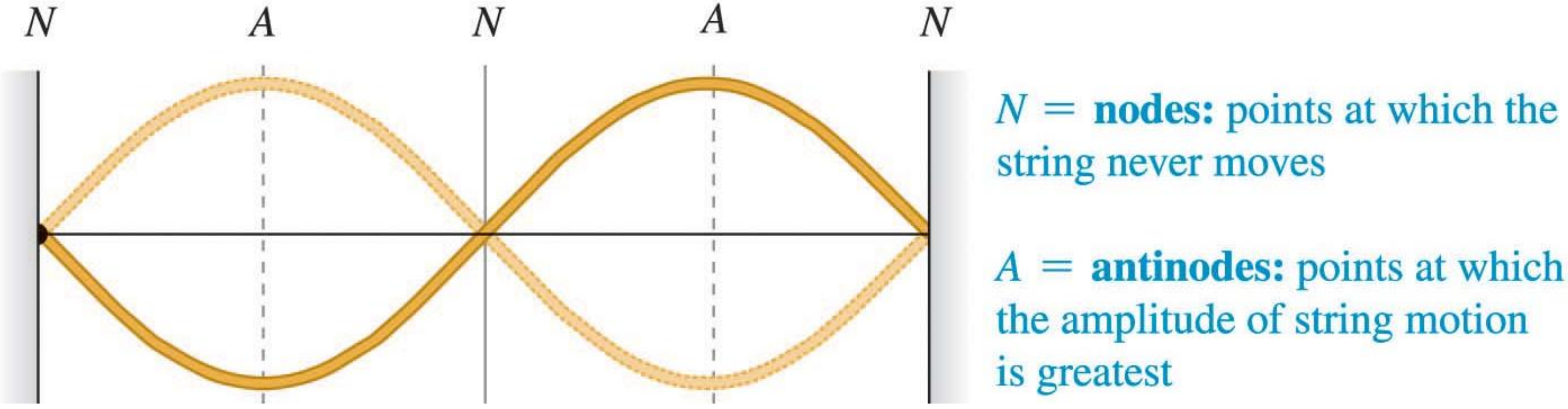
Standing Waves on a String (1 of 3)

- Waves traveling in opposite directions on a taut string interfere with each other.
- The result is a **standing wave** pattern that does not move on the string.
- **Destructive interference** occurs where the wave displacements cancel, and **constructive interference** occurs where the displacements add.
- At the **nodes** no motion occurs, and at the **antinodes** the amplitude of the motion is greatest.

Standing Waves on a String (2 of 3)

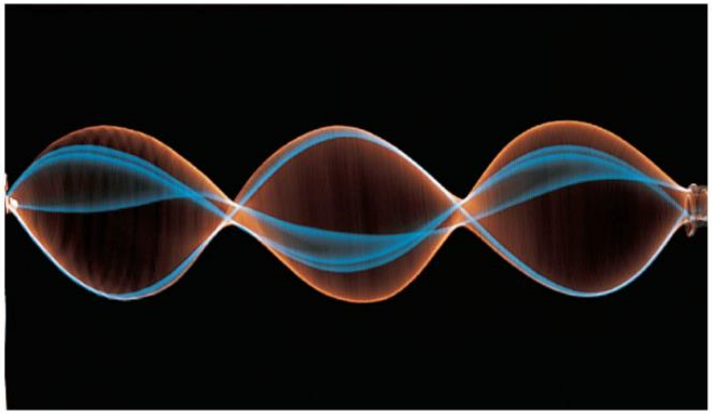
- This is a time exposure of a standing wave on a string.
- This pattern is called the **second harmonic**.

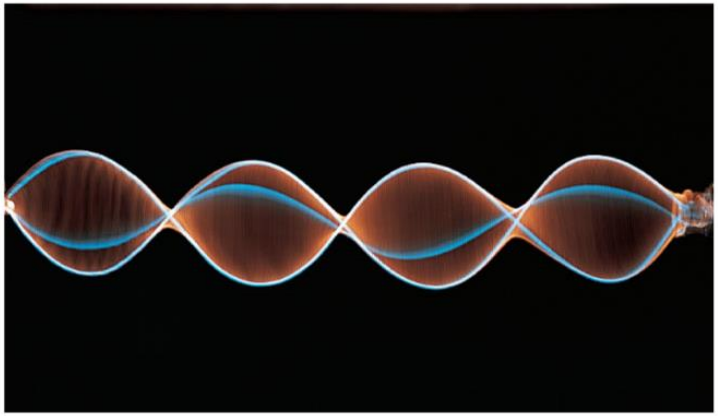




Standing Waves on a String (3 of 3)

- As the frequency of the oscillation of the right-hand end increases, the pattern of the standing wave changes.
- More nodes and antinodes are present in a higher frequency standing wave.





The Mathematics of Standing Waves

- We can derive a wave function for the standing wave by adding the wave functions for two waves with equal amplitude, period, and wavelength traveling in opposite directions.
- The wave function for a standing wave on a string in which x = 0 is a fixed end is:

Wave function Standing-wave amplitude

Standing wave on a string, fixed end at $x = 0$: $y(x, t) = (A_{SW} \sin kx) \sin \omega t$ Time

Wave number Position Angular frequency

- The standing-wave amplitude A_{SW} is twice the amplitude A of either of the original traveling waves: $A_{SW} = 2A$.

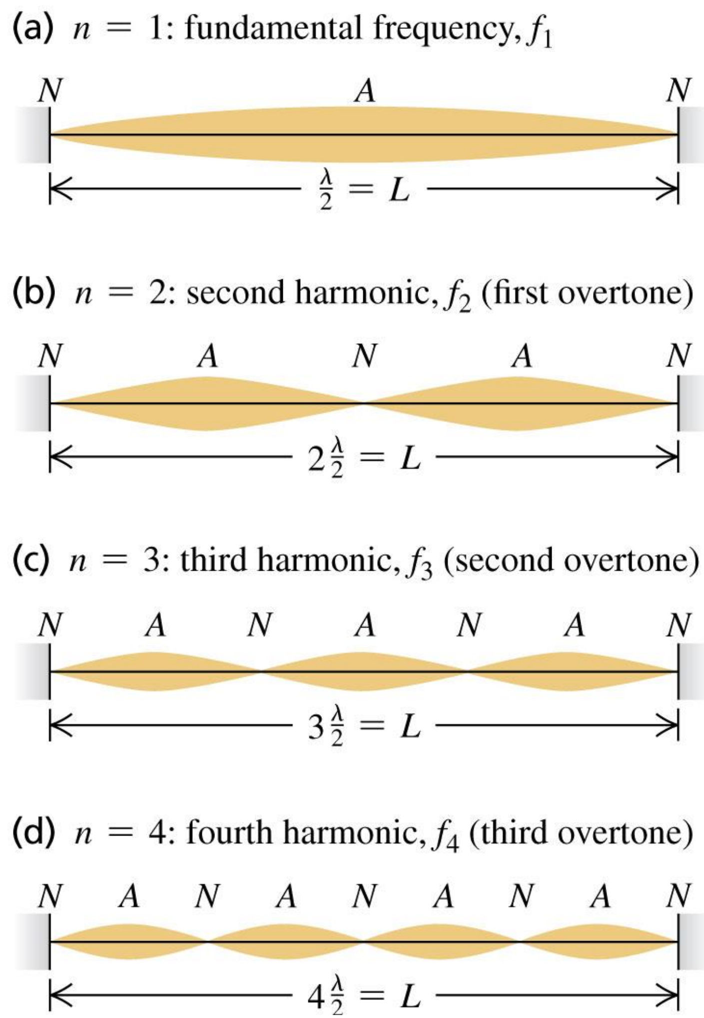
Normal Modes

- For a taut string fixed at both ends, the possible wavelengths

  are $\lambda_n = \frac{2L}{n}$ and the possible frequencies are $f_n = n\frac{v}{2L} = nf_1,$

  where n = 1, 2, 3, …

- f_1 is the **fundamental frequency**, f_2 is the second harmonic (first overtone), f_3 is the third harmonic (second overtone), etc.
- The figure illustrates the first four harmonics.



Standing Waves and String Instruments

- When a string on a musical instrument is plucked, bowed or struck, a standing wave with the fundamental frequency is produced:

Fundamental frequency, string fixed at both ends

$$f_1 = \frac{1}{2L}\sqrt{\frac{F}{\mu}}$$

Tension in string

Mass per unit length

Length of string

- This is also the frequency of the sound wave created in the surrounding air by the vibrating string.
- Increasing the tension *F* increases the frequency (and the pitch).