Chapter 10: Dynamics of Rotational Motion

Luis Mata

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0.1 Overview

Torque	$\tau = Fl = rF\sin\theta$	(1)
	$\tau = rXF$	(2)
	$\tau = I\alpha$	(3)
Kinetic Energy	$K = \frac{1}{2}Mv^2 + \frac{1}{2}I_{cm}w^2$	(4)
Work done by Torque	$W = \tau(\theta_2 - \theta_1) = \tau\triangle\theta$	(5)
Power(via Torque)	$P = \tau w_z$	(6)
Angular Momentum	$L = rXp = rXmv$	(7)
	$L = Iw$	(8)
Rotational Dynamics	$\sum \tau = \frac{dL}{dt}$	(9)

0.2 Torque

line of action: the line along which a force vector lies
lever arm: the perpendicular distance from O to the line of action of the force
Torque can be expressed as a vector using the vector product

0.2.1 Torque and Angular Acceleration: Rigid Bodies

giving an object an angular acceleration will also apply a Torque by its definition

$$\tau = I\alpha$$

Only external Torques affect a Rigid Body's rotation, this is because any two particles in any object exerting a force on each other are also going to exert an equal and opposite force on each other.

0.3 Rigid Body Rotation About a Moving Axis

$$K = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I_{cm}w^2$$

this essentially states that the energy is equal to the translation of the center of mass plus the rotation about the center of the mass.

0.3.1 Rolling without Slipping

as stated above, the total energy, or motion of an object is the translation plus the rotation, applying this to a rolling mass and adding the constraint of it not slipping give the equation:

$$v_{cm} = Rw$$

Example

Airflow around the wing of a maple seed slows the falling seed to about 1m/s and causes the seed to rotate about the center of mass, so $v_{cm} = Rw$ is true. An example of when this is not true is when a car's tires burn out on a rouad.

0.3.2 Dynamics

Acceleration of the center of mass: $\sum F_{ext} = Ma_{cm}$
The rotation motion about the center of mass: $\sum \tau = I_{cm}\alpha_z$

0.3.3 Rolling Friction

We can ignore roling friction if both the rolling object and the surface it rolls over are rigid

0.4 Rotational Work

$$W = \tau_z d_t heta$$

0.4.1 Work and Power

the total work done on an object by the torque is equal to the change in kinetic energy, and the power is: $P = \tau_z w_z$

0.5 Angular Momentum

To find the total angular Momentum of a rigid body rotating with angular speed w, first consider a 2d slice of the object Each particle in the slice has angular momentum: $L = mr^2w$

Therefore, the angular momentum of a rigid body is: $L = Iw$

0.5.1 Conservation of Angular Momentum

When the net external torque is 0, the total angular momentum is constant.

Gyroscopes and Precession

For a gyroscope, the axis of rotation changes direction, the motion of this axis is called precession. If a flywheel is initially not spinning, its initial angular momentum is zero. In each successive time interval the torque produces a change in the angular momentum in the same direction as the torque, and the flywheel axis falls.
Now starting with a rotating flywheel, it has a large initial angular momentum. Because the initial angular momentum is not zero, each change in angular momentum is perpendicular to the angular momentum.