PHYS 1310

Chapter 8
Momentum

CH8 in Brief

- Momentum (impulse): $p = mv = \mathrm{F} \cdot \Delta t$
- Force (via momentum): $F = \dfrac{dp}{dt}$
- Conservation of Momentum: $\Delta p = mv_f - mv_i = 0$
- Center of Mass: $r_{cm} = \dfrac{\sum m_i r_i}{\sum m_i}$
- **Rocket Propulsion**: $F = v \cdot \dfrac{dm}{dt}$

Chapter 8 Objectives

- the momentum of a particle, and how the net force acting on a particle causes its momentum to change.
- the circumstances under which the total momentum of a system of particles is constant (conserved).
- **elastic**, **inelastic**, and **completely inelastic** collisions.
- what’s meant by the center of mass of a system, and what determines how the center of mass moves.
- how to analyze situations such as rocket propulsion in which the mass of an object changes as it moves.

Introduction

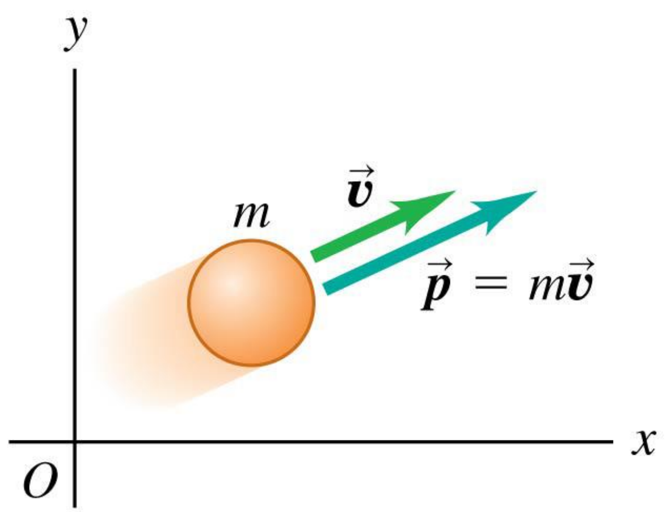
- In many situations, such as hailstones shattering a roof, we cannot use Newton's second law to solve problems because we know very little about the complicated forces involved.
- In this chapter, we shall introduce **momentum** and **impulse**, and the **conservation of momentum**, to solve such problems.

Momentum and Newton's Second Law

- The **momentum** (mass in motion) of a particle is the product of its mass and its velocity:

$$\vec{p} = m\vec{v}$$

- Newton's second law can be written in terms of momentum:



Momentum $\vec{p}$ is a vector quantity; a particle's momentum has the same direction as its velocity $\vec{v}$.

Newton's second law in terms of momentum: The net external force acting on a particle ...

$$\sum \vec{F} = \frac{d\vec{p}}{dt}$$

... equals the rate of change of the particle's momentum.

Impulse (1 of 3)

- The **impulse** [(kg·m)/s => mv => "momentum"] of a force is the product of the force and the time interval during which it acts:

Impulse of a **constant net external force**

$$\vec{J} = \sum \vec{F}(t_2 - t_1) = \sum \vec{F}\,\Delta t$$

Constant net external force

Time interval over which net external force acts

- On a graph of $\sum F_x$ versus time, the impulse is equal to the area under the curve:

Impulse of a **general net external force** (either constant or varying)

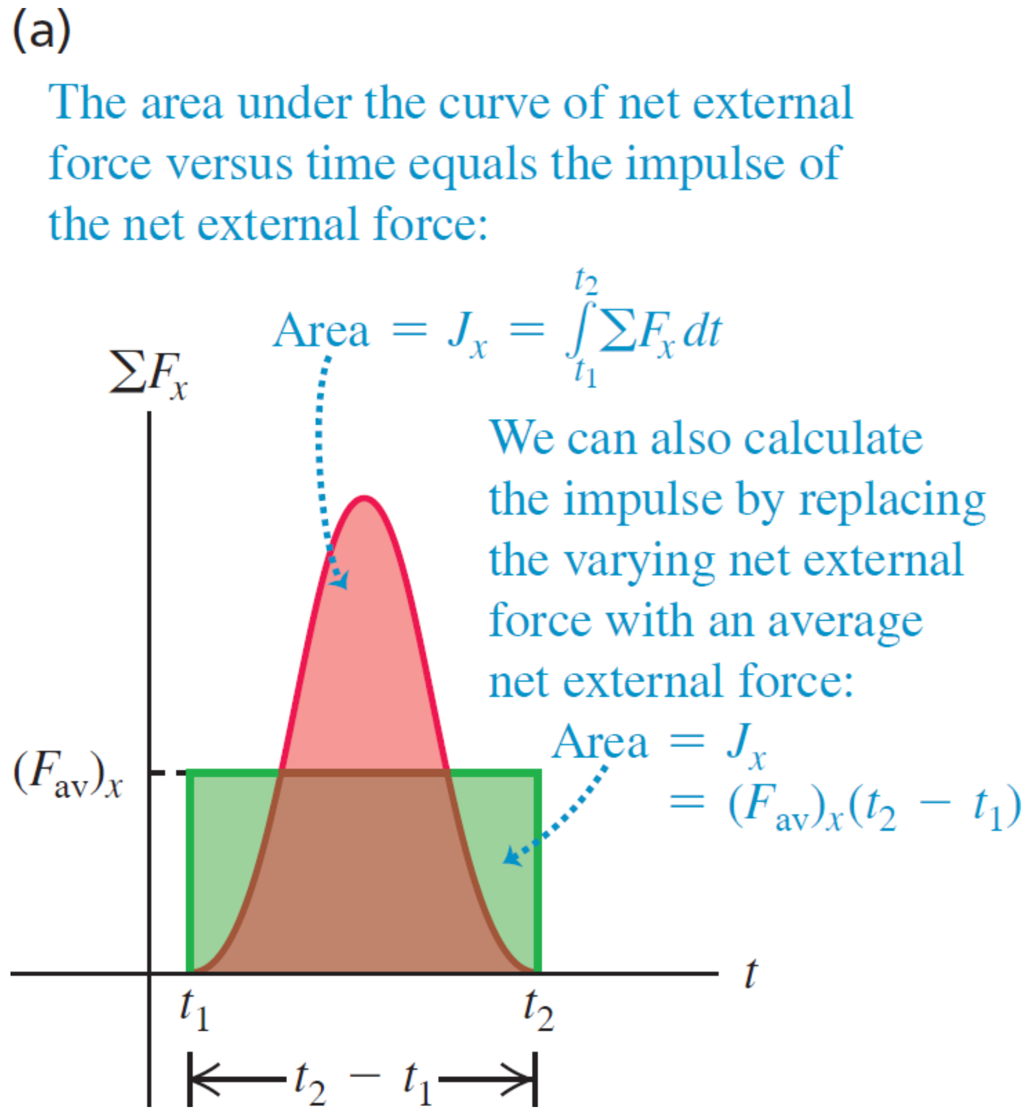
$$\vec{J} = \int_{t_1}^{t_2} \sum \vec{F}\,dt$$

Upper limit = final time

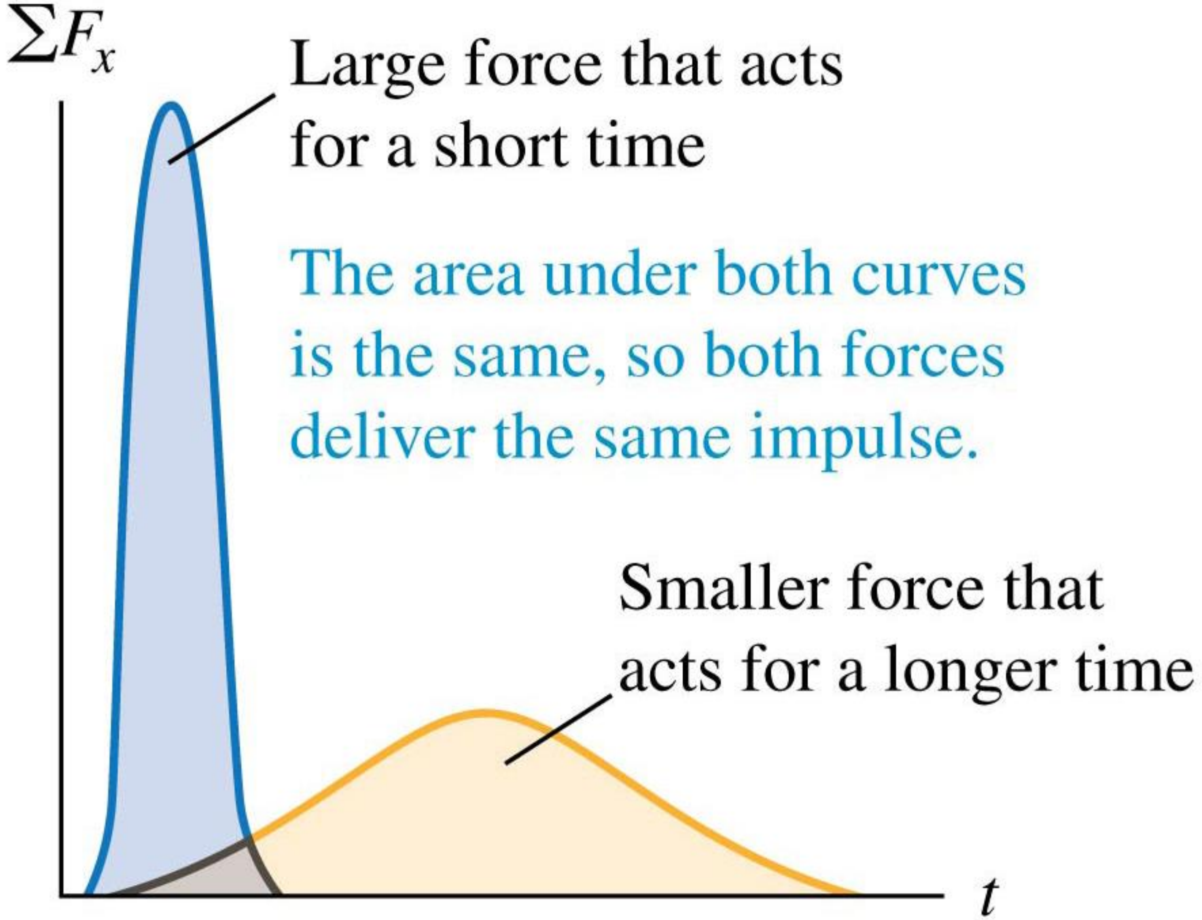
Time integral of net external force

Lower limit = initial time

Impulse (2 of 3)



Impulse (3 of 3)



A hypothetical experiment…

- At what velocity could a mango (m_m = 0.33 kg) make an oil tanker (m_t = 10^8 kg) traveling at a velocity of 1 m/s come to a complete stop?

v_m = ??

<u>Apply Conservation of Momentum</u>: $$m_m v_m - m_t v_t = 0$$

$$v_m = \frac{m_t}{m_m} v_t = \left(\frac{10^8\ kg}{0.33\ kg}\right)\left(1\frac{m}{s}\right) = \mathbf{3 \cdot 10^8}\ \boldsymbol{m/s}$$

- FYI: In reality, v_t is much less due to the relativistic mass of the mango is increasing as it's velocity approaches the speed of light…

$$m'_m = \frac{m_m}{\sqrt{1 - \left(\frac{v_m}{c}\right)}}$$

Impulse and Momentum

- **Impulse–momentum theorem**: The change in momentum of a particle during a time interval is equal to the impulse of the net force acting on the particle during that interval:

Impulse–momentum theorem: The impulse of the net external force on a particle during a time interval equals the change in momentum of that particle during that interval:

Impulse of net external force over a time interval

$$\vec{J} = \vec{p}_2 - \vec{p}_1 = \Delta\vec{p}$$

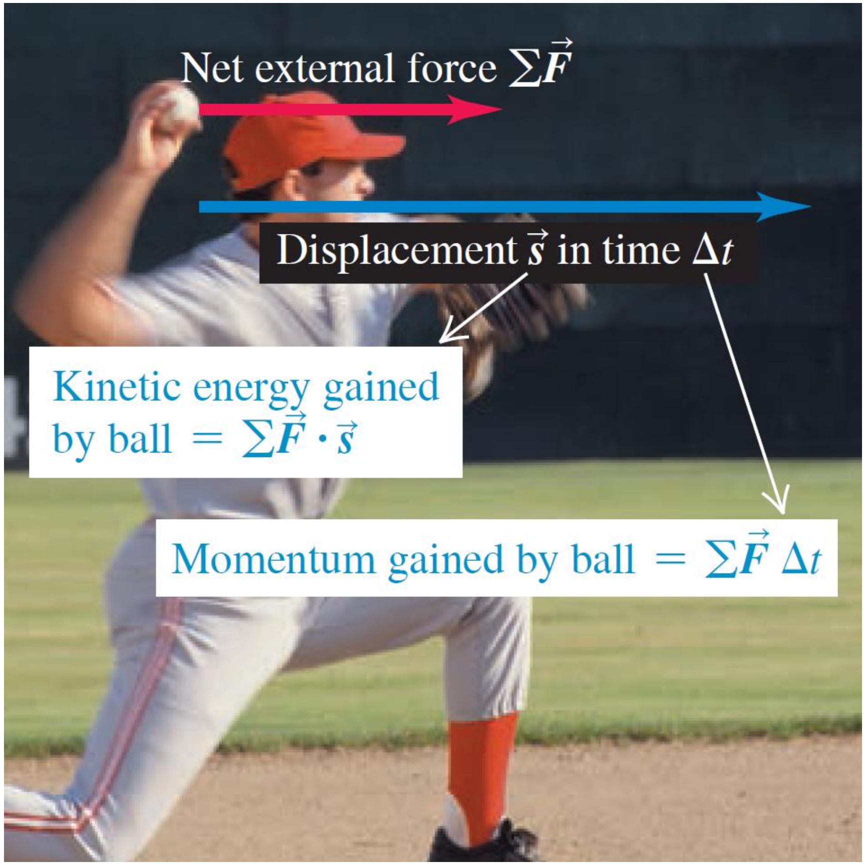
Change in momentum

Final momentum

Initial momentum

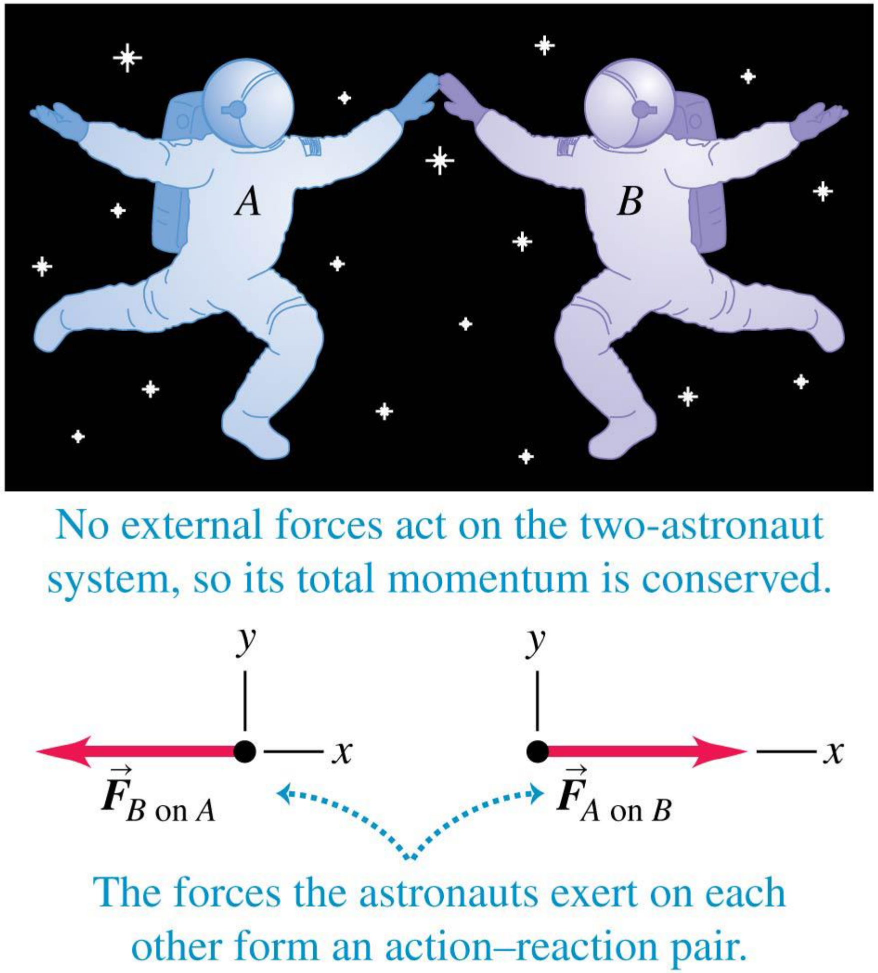
Compare Momentum and Kinetic Energy

- The kinetic energy of a pitched baseball is equal to the work the pitcher does on it (force multiplied by the distance the ball moves during the throw).
- The momentum of the ball is equal to the impulse the pitcher imparts to it (force multiplied by the time it took to bring the ball up to speed).



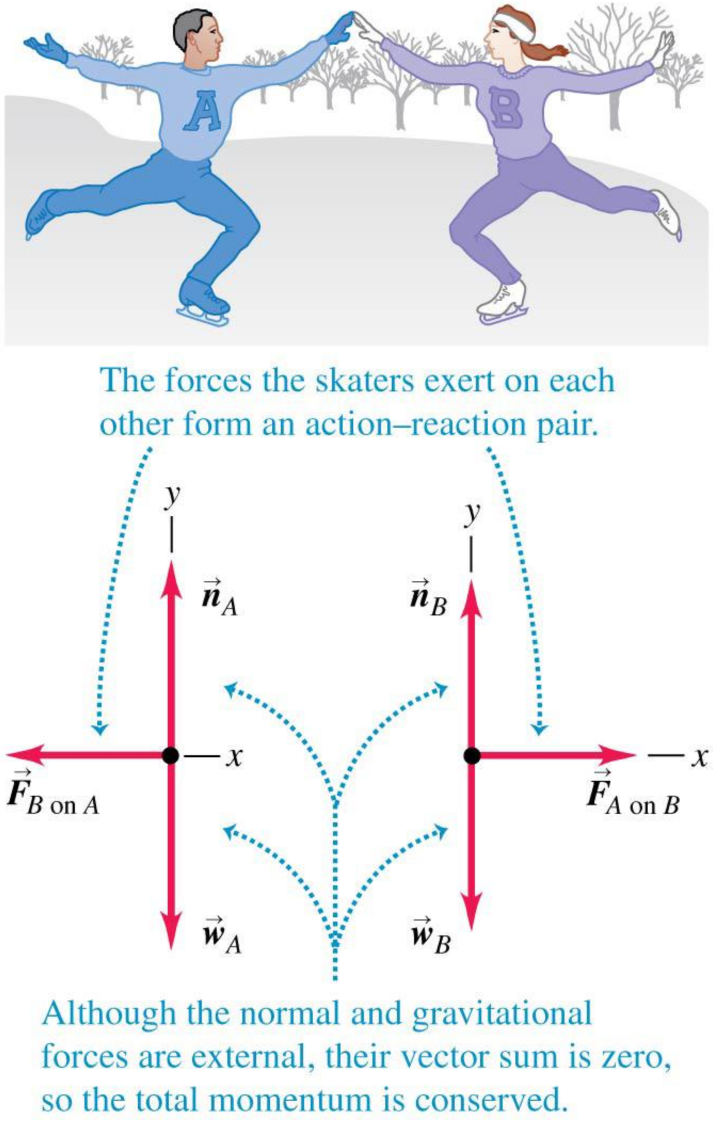
An Isolated System

- Two astronauts push each other as they float freely in the zero-gravity environment of space.
- There are **no** external forces; when this is the case, we have an **isolated system**.



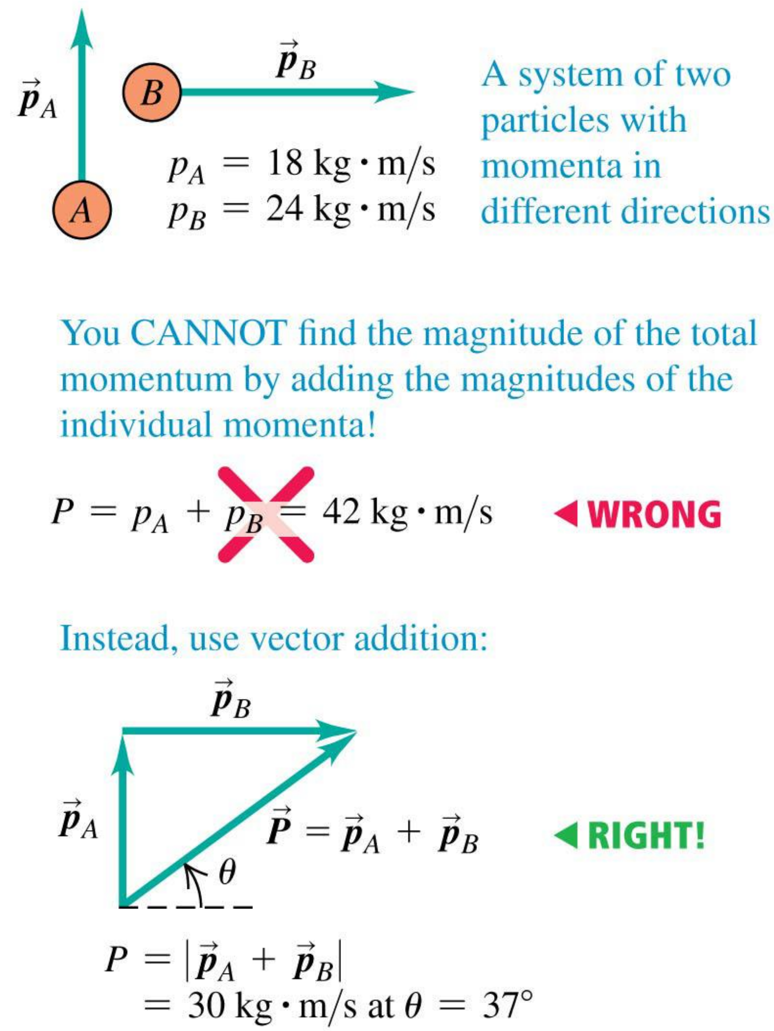
Conservation of Momentum

- External forces (the normal force and gravity) act on the skaters shown, but their vector sum is zero.
- Therefore the total momentum of the skaters is conserved.
- **Conservation of momentum**: If the vector sum of the external forces on a system is zero, the total momentum of the system is constant.



Remember that Momentum Is a Vector!

- When applying conservation of momentum, remember that momentum is a vector quantity.
- Use vector addition to add momenta, as shown at the right.



Example Problem

- (8.25) A hunter on frozen, frictionless pond uses a rifle that shoots 4.2 g bullets at 965 m/s. The mass of the hunter and his gun is 72.5 kg, for which the hunter holds the gun tight after firing it. Find the recoil velocity of the hunter if he fires the rifle (a) horizontally and (b) at 56^o above the horizon.

Given: v_B = 965 m/s, m_B = 0.0042 kg, m_H = 7.25 kg, v_H = ??

Apply Conservation of Momentum…

$$m_B v_B - m_H v_H = 0$$

$$v_H = -\frac{m_B}{m_H} v_B = -\left(\frac{4.2 \cdot 10^{-3}}{7.25}\right)\left(965 \frac{m}{s}\right) = \mathbf{-0.06}\ \boldsymbol{m/s}$$

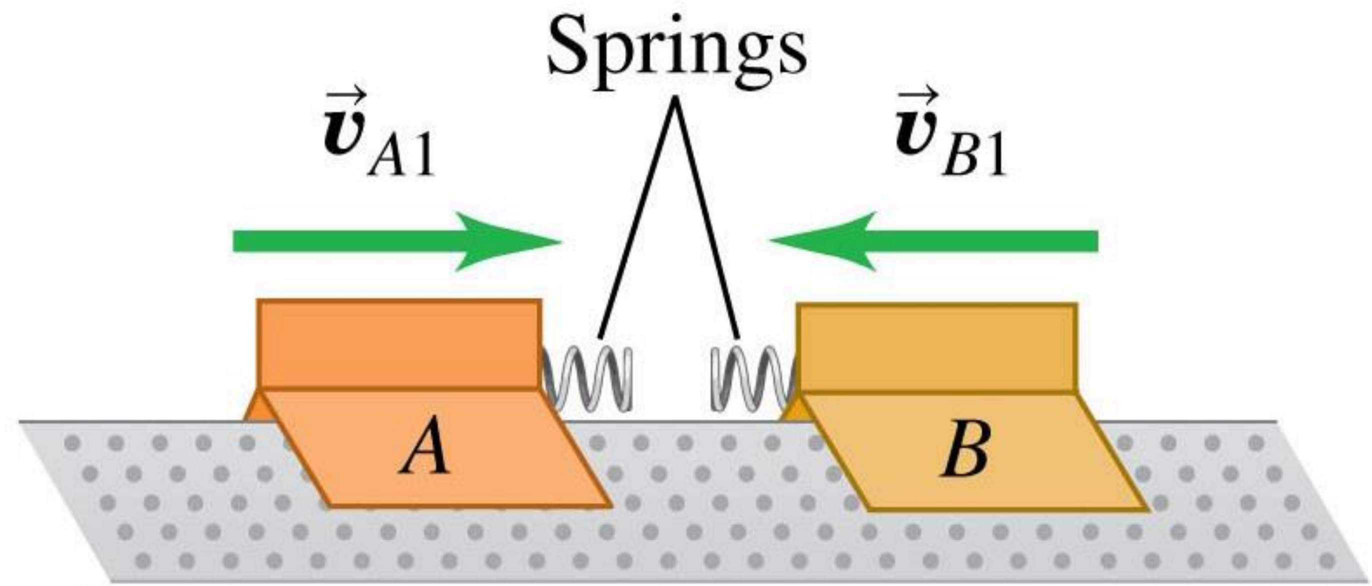
(b) At 56^o above horizon…

Horizontal velocity: $v_{2B} = v_{2B} \cos 56^o = 540\ m/s$

$$v_H = -\frac{m_B}{m_H} v_{2B} = \mathbf{-0.03}\ \boldsymbol{m/s}$$

Elastic Collisions: Before

- In an **elastic collision**, the total kinetic energy of the system is the same after the collision as before.



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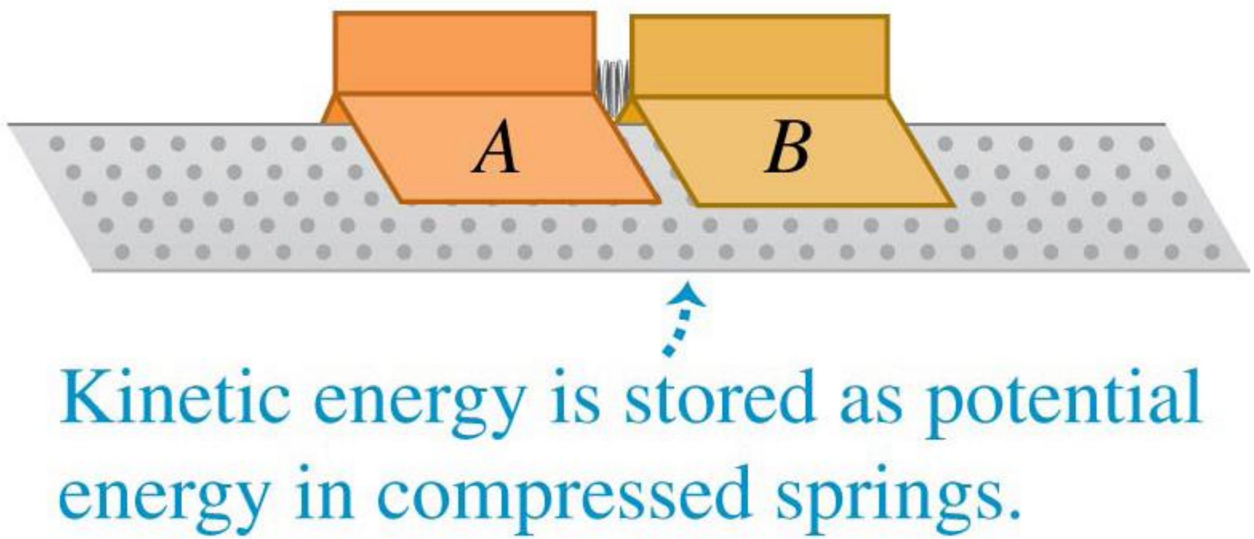

Elastic Collisions: After

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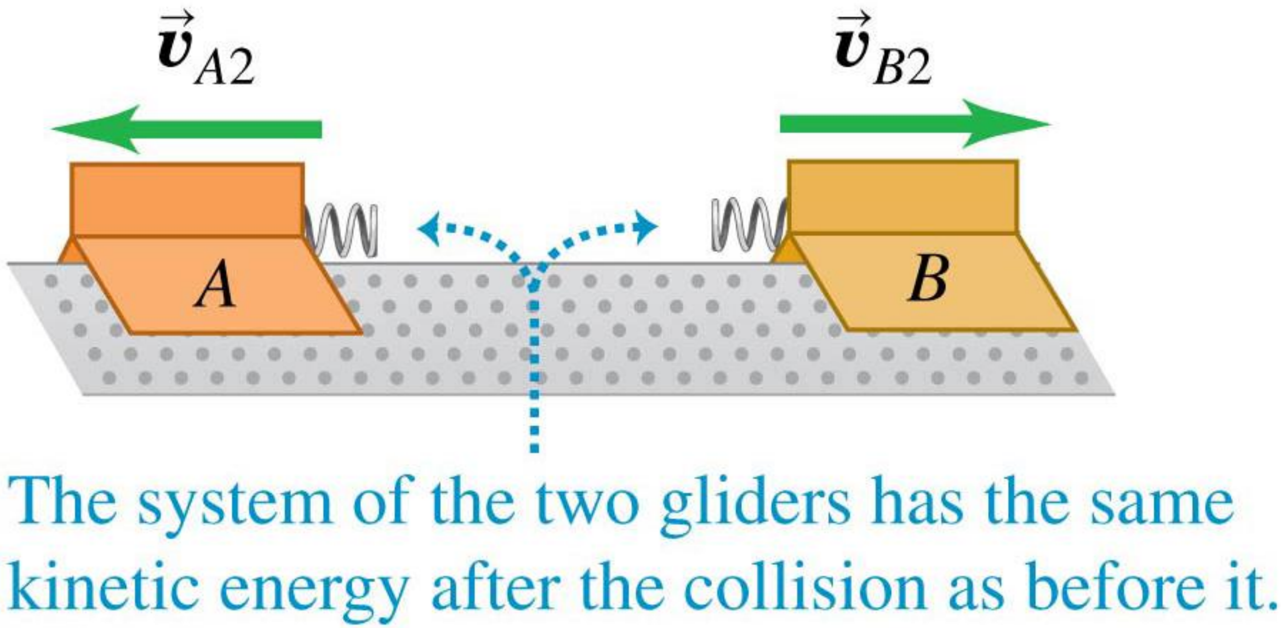
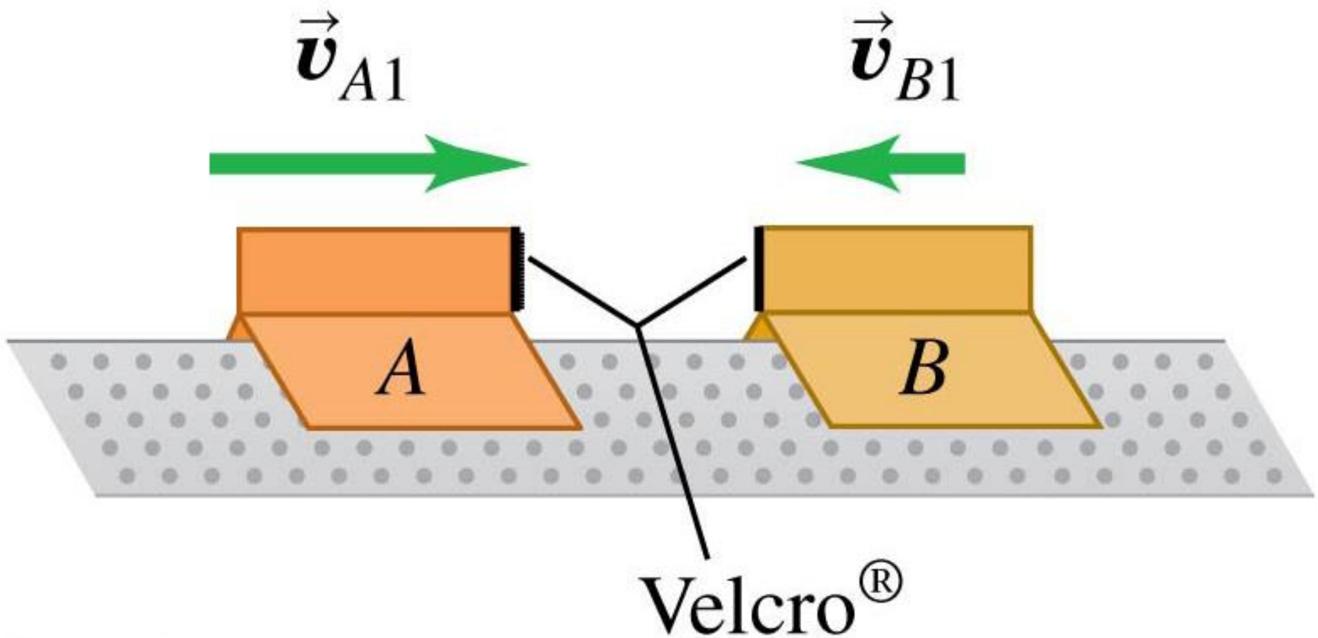


Completely Inelastic Collisions: Before

- In an **inelastic collision**, the total kinetic energy after the collision is less than before the collision.



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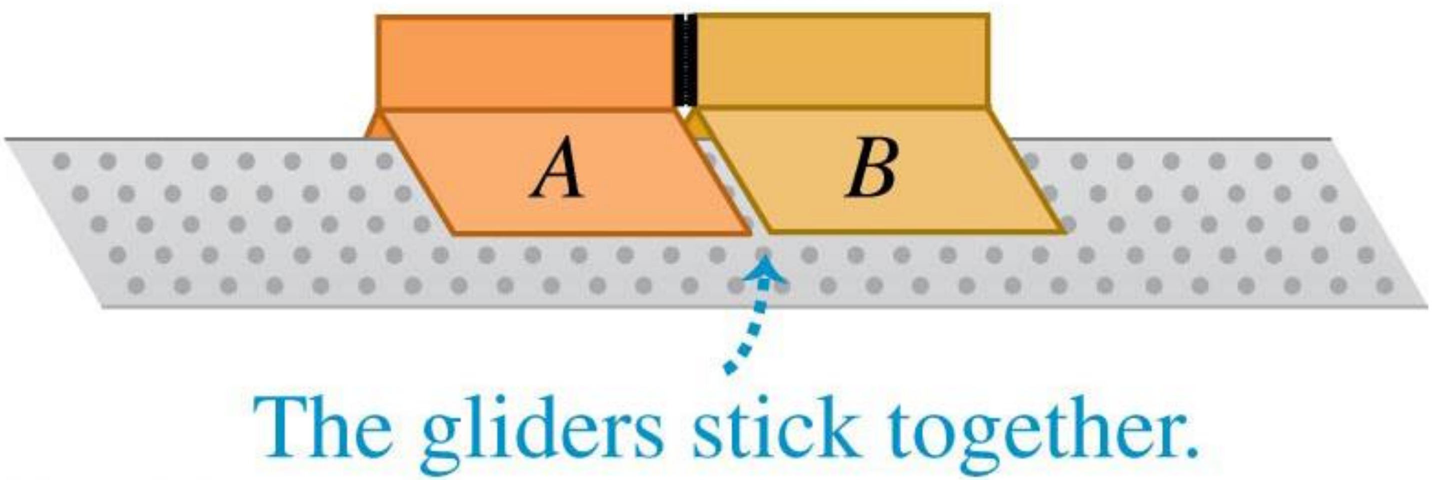

Completely Inelastic Collisions: After

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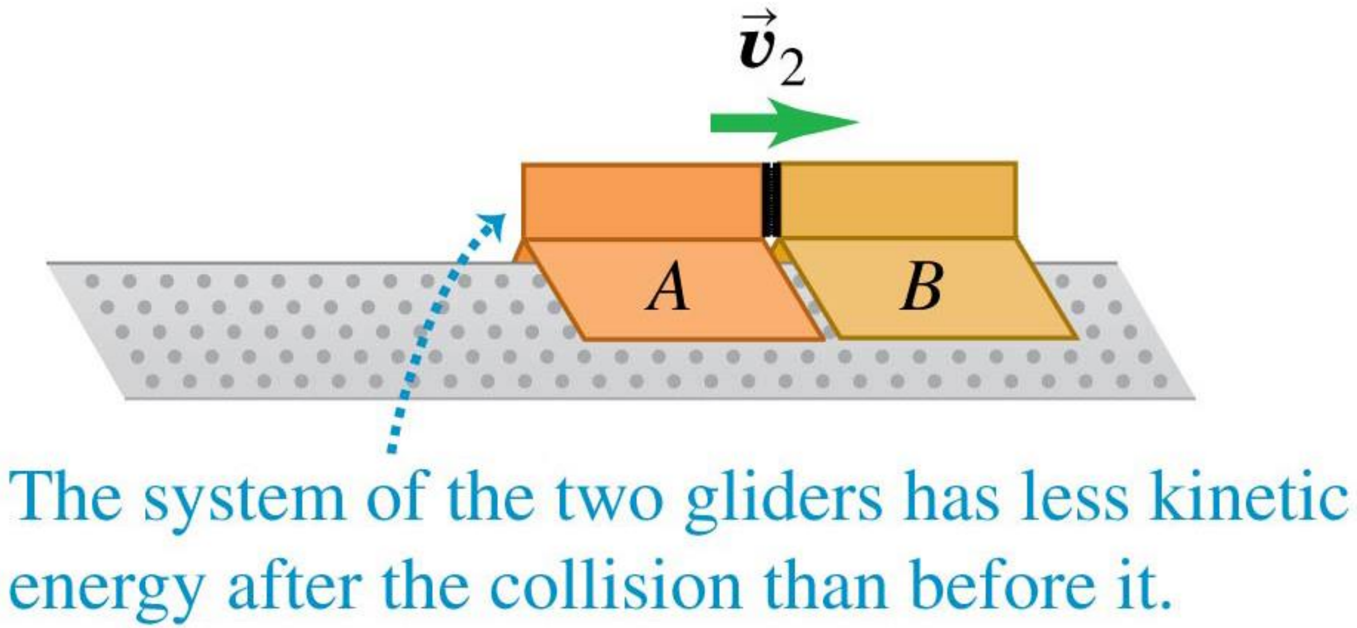


Collisions

- In an **inelastic collision**, the total kinetic energy after the collision is less than before the collision.
- A collision in which the objects stick together is called a **completely inelastic collision**.
- In **any** collision in which the external forces can be neglected, the total momentum is conserved.
- In elastic collisions **only**, the total kinetic energy before equals the total kinetic energy after.

Inelastic Collision Example

- Cars are designed so that collisions are inelastic—the structure of the car absorbs as much of the energy of the collision as possible.
- This absorbed energy cannot be recovered, since it goes into a permanent deformation of the car.

Elastic Collision Example

- Billiard balls deform very little when they collide, and they quickly spring back from any deformation they do undergo.
- Hence the force of interaction between the balls is almost perfectly conservative, and the collision is almost perfectly elastic.

Elastic Collisions in One Dimension

- Let's look at a one-dimensional elastic collision between two objects *A* and *B*, in which all the velocities lie along the same line.
- We will concentrate on the particular case in which object *B* is at rest before the collision.
- Conservation of kinetic energy and momentum give the result for the final velocities of *A* and *B*:

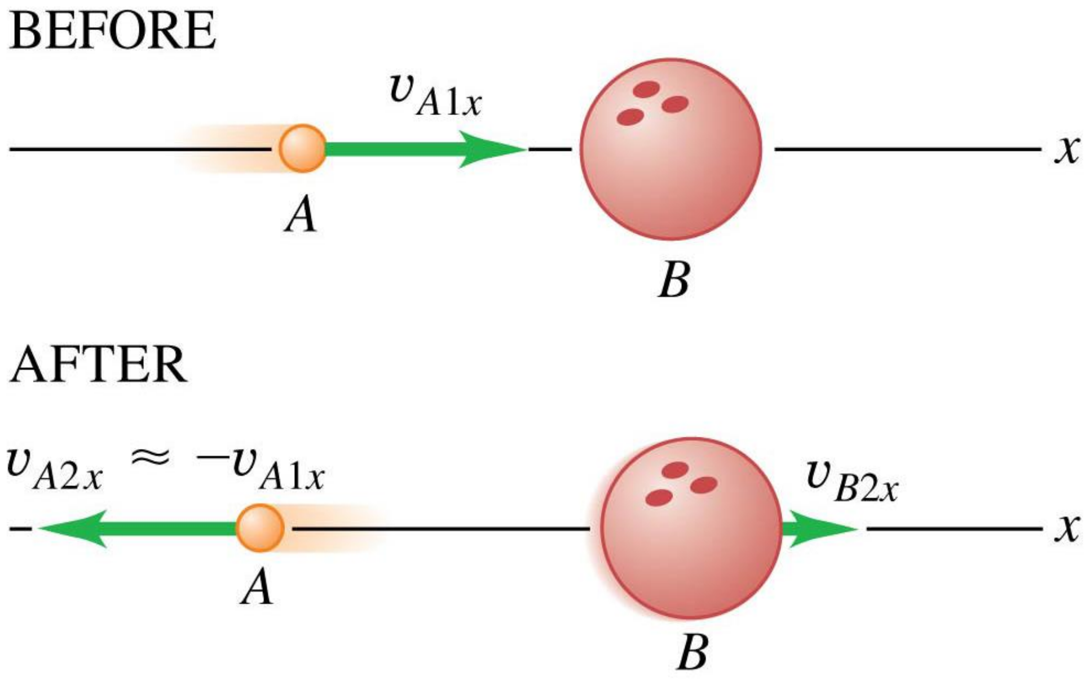
$$m_B(v_{A1x} + v_{A2x}) = m_A(v_{A1x} - v_{A2x})$$

$$v_{A2x} = \frac{m_A - m_B}{m_A + m_B} v_{A1x}$$

$$v_{B2x} = \frac{2m_A}{m_A + m_B} v_{A1x}$$

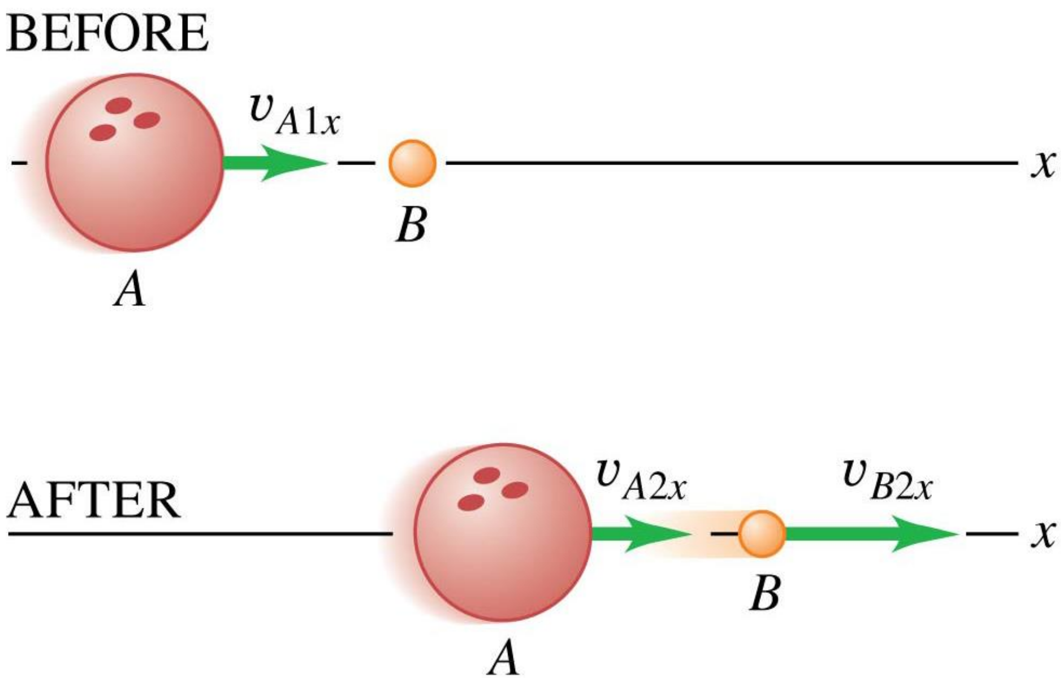
Elastic Collisions in One Dimension

- When B is much more massive than A, then A reverses its velocity direction, and B hardly moves.



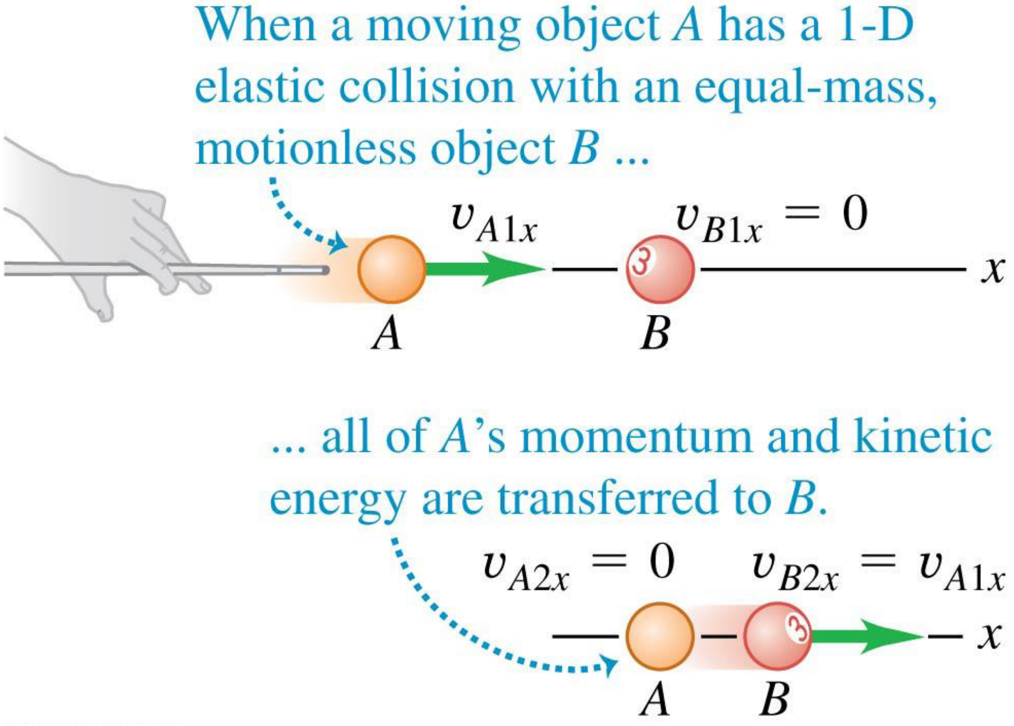
Elastic Collisions in One Dimension

- When *B* is much less massive than *A*, then *A* slows a little bit, while *B* picks up a velocity of about twice the original velocity of *A*.



Elastic Collisions in One Dimension

- When *A* and *B* have similar masses, then *A* stops after the collision and *B* moves with the original speed of *A*.



Center of Mass

- We can restate the principle of conservation of momentum in a useful way by using the concept of center of mass.
- Suppose we have several particles with masses m_1, m_2, and so on.
- We define the center of mass of the system as the point at the position given by:

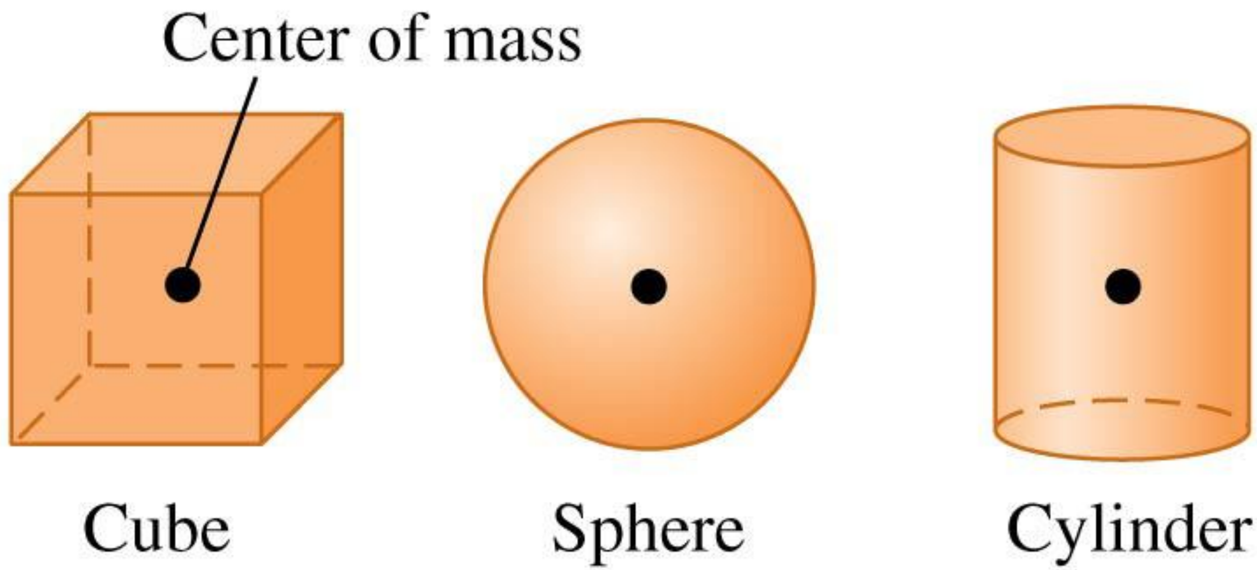
Position vectors of individual particles

Position vector of **center of mass** of a system of particles

$$\vec{r}_{\mathrm{cm}} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} = \frac{\sum_i m_i\vec{r}_i}{\sum_i m_i}$$

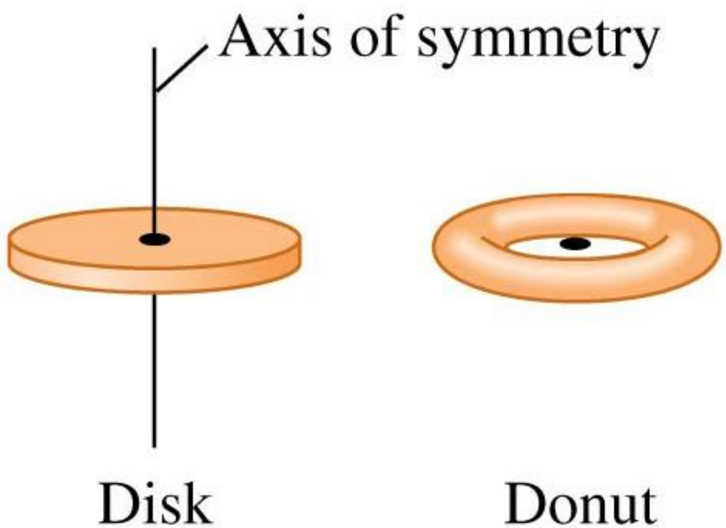
Masses of individual particles

Center of Mass of Symmetrical Objects
(1 of 2)



If a homogeneous object has a geometric center, that is where the center of mass is located.

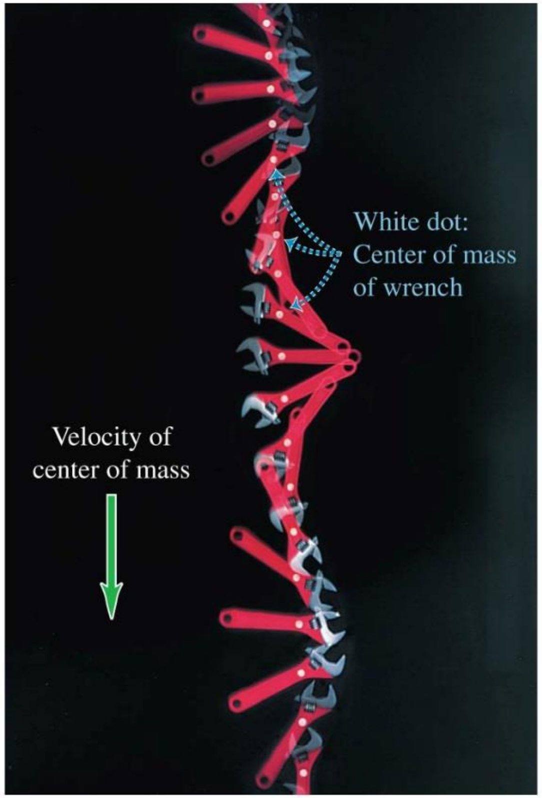
Center of Mass of Symmetrical Objects
(2 of 2)



If an object has an axis of symmetry, the center of mass lies along it. As in the case of the donut, the center of mass may not be within the object.

Motion of the Center of Mass

- The total momentum of a system is equal to the total mass times the velocity of the center of mass.
- The center of mass of the wrench at the right moves as though all the mass were concentrated there.



Total mass of a system of particles

Momenta of individual particles

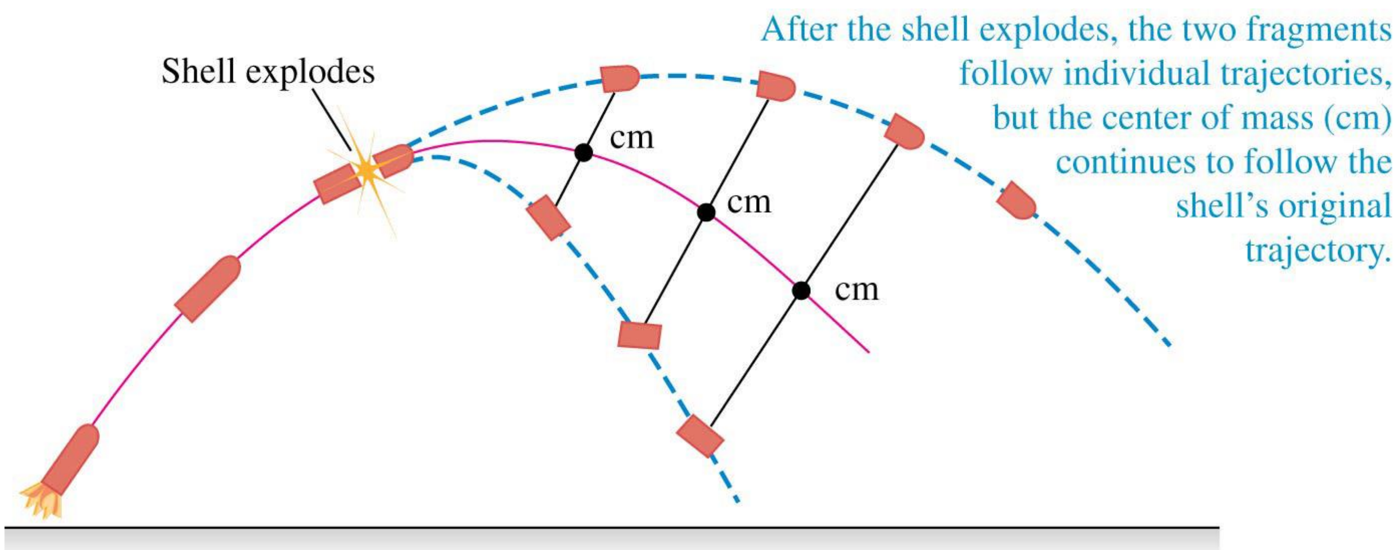
$$M\vec{\boldsymbol{v}}_{\mathrm{cm}} = m_1\vec{\boldsymbol{v}}_1 + m_2\vec{\boldsymbol{v}}_2 + m_3\vec{\boldsymbol{v}}_3 + \cdots = \vec{\boldsymbol{P}}$$

Velocity of center of mass

Total momentum of system

External Forces and Center-Of-Mass Motion

- When an object or a collection of particles is acted on by external forces, the center of mass moves as though all the mass were concentrated at that point and it were acted on by a net force equal to the sum of the external forces on the system.



Rocket Propulsion

- As a rocket burns fuel, its mass decreases.

$$F = v \cdot \frac{dm}{dt}$$

- To provide enough thrust to lift its payload into space, this **Atlas V** launch vehicle ejects more than 1000 kg of burned fuel per second at speeds of nearly 4000 m/s.

Example

- (8.63) ROCKET PROPULSION Assume that a rocket is fired from rest deep in space (no gravity). (a) If the rocket ejects gasses at a relative speed of 2000 m/s and you want the rockets to be $3{\cdot}10^5$ m/s, what fraction of the initial mass of the rocket is not fuel? (b) What is this fraction if the final speed is 3000 m/s?

<u>Given</u>: v_i = 2000 m/s, v_f = 30,000 m/s, m_i/m_f = ??

Via conservation of momentum…

$$m_f v_f \; - \; m_i v_i = 0$$

$$\frac{m_i}{m_f} = \frac{v_f}{v_i} = \frac{3{\cdot}10^5\ \text{m/s}}{2000\ \text{m/s}} = \mathbf{15}$$

(b) v_f = 3,000 m/s

$$\frac{m_i}{m_f} = \frac{v_f}{v_i} = \frac{3000\ \text{m/s}}{2000\ \text{m/s}} = \mathbf{1.5}$$