## CH3 EXERCISES:

#1 (3.1) GIVEN @ $t_1 = 0$ $\quad \vec{r}_1 = 1.1\hat{i} + 3.4\hat{j}$
@ $t_2 = 3s$ $\quad \vec{r}_2 = 5.3\hat{i} - 0.5\hat{j}$

(a) find components of the average velocity...

$$\Delta\vec{v} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1}$$

$$\Delta v_x = \frac{(5.3 - 1.1)}{(3 - 0)}\hat{i} = \boxed{1.4\ m/s}$$

$$\Delta v_y = \frac{(-0.5 - 3.4)}{(3 - 0)}\hat{j} = \boxed{-1.3\ m/s}$$

(b) what is magnitude and direction of the avg. velocity?

$$v_{ave} = \sqrt{(\Delta v_x)^2 + (\Delta v_y)^2} = \sqrt{(1.4)^2 + (-1.3)^2} = \boxed{1.9\ m/s}$$

$$\tan\theta = \frac{\Delta v_y}{\Delta v_x} \Rightarrow \theta = \tan^{-1}\left(\frac{-1.3}{1.4}\right) = \boxed{-42.9^\circ}$$

(or 42.9° South of East)

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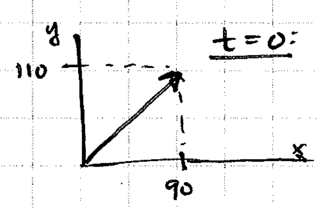
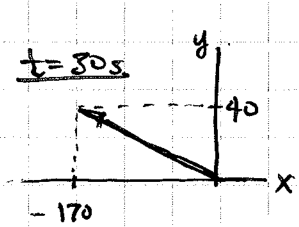
#2 (3.5) GIVEN: @ $t_1 = 0$ $\quad v_x = 90\ m/s,\ v_y = 110\ m/s$
@ $t_2 = 30s$ $\quad v_x = -170\ m/s,\ v_y = 40\ m/s$

(a) Sketch velocity vectors




(b) components for avg. acceleration:

$$\Delta a_x = \frac{\Delta v_x}{\Delta t} = \frac{(-170 - 90)}{(30 - 0)} = \frac{-260}{30} = \boxed{-8.67\ m/s^2}$$

$$\Delta a_y = \frac{\Delta v_y}{\Delta t} = \frac{(40 - 110)}{(30 - 0)} = \frac{-70}{30} = \boxed{-2.33\ m/s^2}$$

(c) magnitude: $a = \sqrt{\Delta a_x^2 + \Delta a_y^2} = \boxed{8.98\ m/s^2}$

direction: $\theta = \tan^{-1}(\Delta a_y / \Delta a_x) = \boxed{15^\circ \text{ South of WEST}}$

#3 (3.12) <u>Given</u>: $v_x = 20\ m/s$ , $v_y = 12\ m/s$

(a) when does football reach its highest point in trajectory?

<u>recall</u>: $v_y = v_0 + a_y t$ where $a_y = -g = -9.8\ m/s^2$

at highest point, $v_y = 0$ $\Rightarrow$ $t = \dfrac{-v_0}{a_y} = \dfrac{-(12\,m/s)}{-(9.8\,m/s^2)} = \boxed{1.22\ s}$

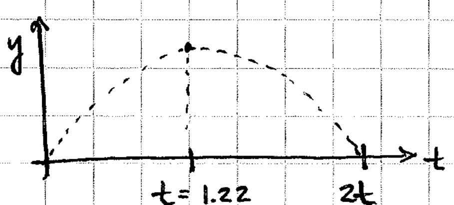
(b) How high does the ball reach?

<u>recall</u>: $(y - y_0) = \frac{1}{2}(v_0 + \cancelto{0}{v_y})t$

$$y - y_0 = \frac{1}{2} v_0 t = \frac{1}{2}(12\,m/s)(1.22\,s) = \boxed{7.32\ m}$$

(c) How much time does it take for ball to return to its original level?

By inspection




$2t = 2(1.22\,s) = \boxed{2.44\,s}$

or twice the time it took to reach highest point.

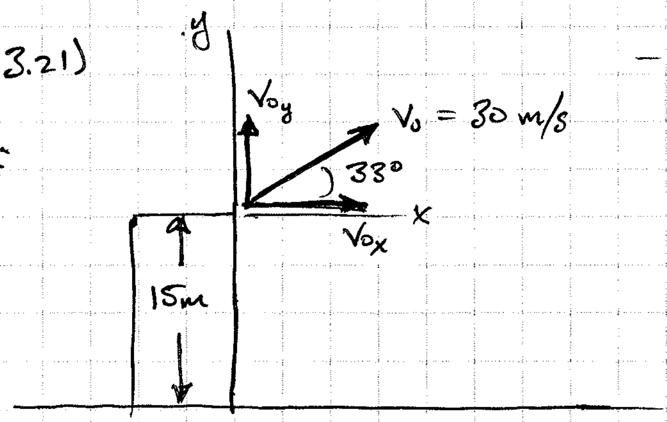
(d) How far horizontally does football travel?

<u>recall</u>: $x = x_0 + v_{0x} t + a_x t^2$ where $a_x = 0$

$$\Rightarrow (x - \overset{0}{x_0}) = v_{0x} t = (20\,m/s)(2.44\,s) = \boxed{48.8\ m}$$

\#4 (3.21)

GIVEN:




– First, solve for the components of initial velocity...

$$V_{0x} = V_0 \cos(33°) = 25.2 \text{ m/s}$$

$$V_{0y} = V_0 \sin(33°) = 16.3 \text{ m/s}$$

(a) What is maximum height above the roof does the rock reach?

recall Eqn of Motion: $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ where $a_y = -g$

at max height, $v_y = 0$ ...

$$\Rightarrow \quad y - y_0 = \frac{\cancel{v_y^2}^{\,0} - v_{0y}^2}{2a_y} = \frac{-(16.3 \text{ m/s})^2}{2(-9.8 \text{ m/s}^2)} = \boxed{13.6 \text{ m}}$$

(b) What is speed of rock just before it reaches ground?

We'll use $v = \sqrt{v_x^2 + v_y^2}$ where $v_x = 25.2$ m/s $= v_{0x}$

$v_y = ?? = -\sqrt{v_{0y}^2 + 2a_y(y - y_0)}$ where $v_{0y} = 16.3$ m/s, $a_y = -g$, $y - y_0 = -15$ m

Noted: the rock is below its initial position (hence, negative)

$$= -\sqrt{(16.3)^2 + 2(-9.8)(-15)}$$

$$= -\sqrt{(265.7) + 2(294)}$$

$= -23.7$ m/s $\Rightarrow$ rock is traveling downward, hence negative

$$\therefore \quad v = \sqrt{(25.2)^2 + (-23.7)^2} = \boxed{34.6 \text{ m/s}}$$

(c) Determine horizontal range from base of building...

How long is rock in the air? Use $v_y = v_{0y} + a_y t$

$$\Rightarrow \quad t = \frac{v_y - v_{0y}}{a_y} = \frac{(-23.7 \text{ m/s}) - (16.3 \text{ m/s})}{-9.8 \text{ m/s}^2} = \frac{40 \text{ m/s}}{9.8 \text{ m/s}^2} = \underline{4.08 \text{ s}}$$

$$\therefore \quad x(t) = \cancel{x_0}^{\,0} + v_{0x} t + \cancel{a_x}^{\,0} t^2 = (25.2 \text{ m/s})(4.08 \text{ s}) = \boxed{102.8 \text{ m}}$$

#5. (3.23) <u>Given</u> $R_E = 6380 \text{ km} = 6.4 \cdot 10^3 \text{ km} = 6.4 \cdot 10^6 \text{ m}$

$$T = 24 \text{ hrs} \times \left(\frac{60 \text{ min}}{\text{hr}}\right) \times \left(\frac{60 \text{ s}}{\text{min}}\right) = 8.64 \cdot 10^4 \text{ s}$$

(a) What is the radial acceleration of an object at the equator?

$$a_{rad} = \frac{v^2}{R} \qquad \text{where } v = \frac{2\pi R}{T}$$

$$= \frac{(2\pi R/T)^2}{R} = \frac{4\pi^2 R^{\cancel{2}}}{\cancel{R} \cdot T^2} = \frac{4\pi^2 R}{T^2}$$

$$= \frac{4\pi^2 (6.4 \cdot 10^6 \text{ m})}{(8.64 \cdot 10^4 \text{ s})^2} = \frac{2.53 \cdot 10^8 \text{ m}}{7.46 \cdot 10^9 \text{ s}} = \boxed{0.034 \text{ m/s}^2}$$

(b) What would period of Earth's rotation be for objects to fly off surface?

$$a_{rad} = \frac{4\pi^2 R}{T^2} = g$$

$$\Rightarrow T^2 = \frac{4\pi^2 R}{g} = \frac{4\pi^2 (6.4 \cdot 10^6 \text{ m})}{(9.8 \text{ m/s}^2)} = \frac{2.53 \cdot 10^8 \text{ m}}{9.8 \text{ m/s}^2}$$

$$\therefore T = \sqrt{2.58 \cdot 10^7} = 5.01 \cdot 10^3 \text{ s} \times \left(\frac{1 \text{ hr}}{3600 \text{ s}}\right) = \boxed{1.4 \text{ hrs}}$$

#6 (3.25) At what speed does a pilot black out given an acceleration $a_{rad} = 5.5g$ and radius of curvature $R = 280\,m$?

recall: $a_{rad} = \dfrac{v^2}{R}$ where $a_{rad} = 5.5g = 53.9\,m/s^2$, $R = 280\,m$

$$\Rightarrow v = \sqrt{a_{rad}(R)} = \sqrt{(53.9\,m/s^2)(280\,m)} = \boxed{122.8\,m/s}$$

$$= 122.8\,\frac{m}{s} \times \left(\frac{1\,\text{mile}}{1609\,m}\right) \times \left(\frac{3600\,s}{hr}\right) = \underline{274.8\,mph}$$

$\sim 270$ mph (2 sig figs)

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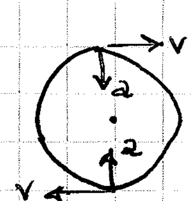
#7 (3.27) Given: $R = 14\,m$ and $v = 6\,m/s^2$

(a) what is acceleration at lowest point given uniform circular motion?

$$a_{rad} = \frac{v^2}{R} = \frac{(6\,m/s)^2}{14\,m} = \boxed{2.57\,m/s^2 \text{ upwards}}$$




(b) acceleration at highest point $\Rightarrow$ $\boxed{2.57\,m/s^2 \text{ downwards}}$

(c) what is the period of rotation? ($T$ = time to make one revolution)

recall: $v = \dfrac{2\pi R}{T} \Rightarrow T = \dfrac{2\pi R}{v} = \dfrac{2\pi(14\,m)}{6\,m/s} = \boxed{14.7\,s}$

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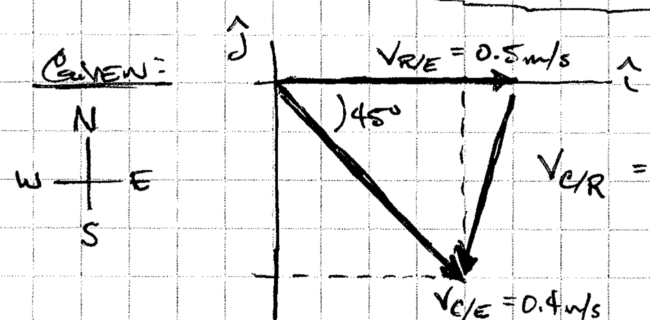
#8 (3.33) LET: $V_{C/E} = V_{C/R} + V_{R/E}$

where velocities...
- $V_{C/E} \Rightarrow$ canoe w.r.t Earth
- $V_{C/R} \Rightarrow$ canoe w.r.t River
- $V_{R/E} \Rightarrow$ River w.r.t Earth

Given:




$$V_{C/R} = V_{C/E} - V_{R/E}$$

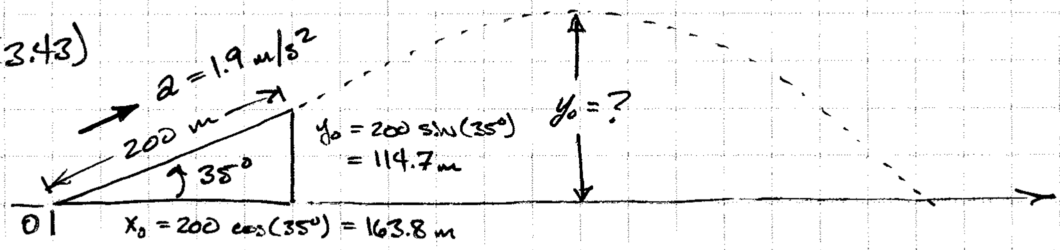
$$V_{C/E} = 0.4\cos(45^\circ)\hat{i} - 0.4\sin(45^\circ)\hat{j}$$
$$V_{R/E} = 0.5\,\hat{i}$$

$$V_{C/R} = (0.4\cos(45^\circ) - 0.5)\hat{i} - 0.4\cos(45^\circ)\hat{j} = \underset{\text{(west)}}{-0.22\hat{i}} \underset{\text{(South)}}{-0.28\hat{j}}$$

$$|V_{C/R}| = \sqrt{v_x^2 + v_y^2} = \sqrt{(-0.22)^2 + (0.28)^2} = \boxed{0.36\,m/s}$$

$$\theta = \tan^{-1}(v_y/v_x) = \tan^{-1}(0.28/0.22) = \boxed{52^\circ \text{ South of West}}$$

#9 (3.43)




Given: at instant rocket leaves incline, $a_x = 0$.

(a) what is maximum height rocket reaches?

– determine the velocity of rocket when it leaves the incline...

$$v_x^2 = \cancel{v_0^2} + 2a(x - x_0) \Rightarrow v_x = \sqrt{2a_x(x-x_0)} = \sqrt{2(1.9)(200)} = 27.6 \text{ m/s}$$

So, when projectile motion begins, the rocket has an initial velocity
$v_0 = 27.6$ m/s @ 35° above horizontal

$$\Rightarrow v_{0y} = 27.6 \text{ m/s} \sin(35°) = \underline{15.8 \text{ m/s}}$$

At max height, $v_y = 0$. Use: $\boxed{v_y^2 = v_0^2 + 2a_y(y - y_0)}$

$$\Rightarrow y - y_0 = \frac{\cancel{v_y^2} - v_{0y}^2}{2(a_y)} = \frac{-(15.8 \text{ m/s})^2}{2(-9.8 \text{ m/s}^2)} = \underline{12.8 \text{ m}}$$

Now, add the vertical displacement of the incline: $y_0 = 200 \sin(35°) = 114.7$

$$\therefore \quad y_{max} = 12.8 \text{ m} + 114.7 \text{ m} = \boxed{127.5 \text{ m}}$$

(b) the rocket's greatest range from bottom of incline?

Solve: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ where $y - y_0 = -114.7$, $v_{0y} = 15.8$ m/s, $a_y = -9.8$ m/s²

$$\Rightarrow 0 = (-114.7) + (15.8)t - (4.9)t^2$$

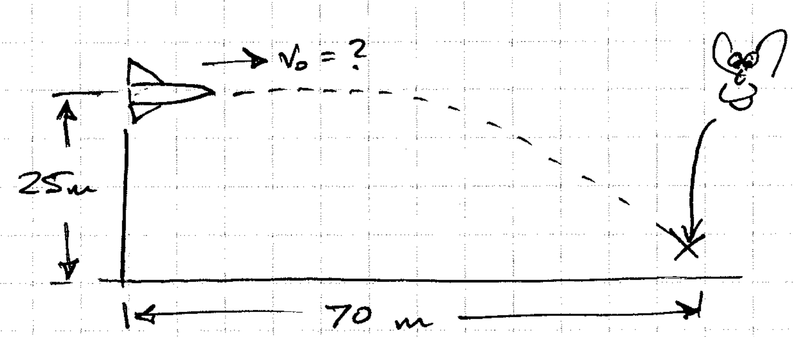
$$t = \frac{-15.8 \pm \sqrt{(15.8^2 - 4(-114.7)(4.9))}}{2(4.9)} = 6.7 \text{ s}$$

Use: $x - x_0 = v_0 t + \frac{1}{2}\cancel{a_x}t^2$ where $v_{0x} = (27.6 \text{ m/s})\cos(35°) = 22.6$ m/s

$$\Rightarrow x - x_0 = (22.6 \text{ m/s})(6.7 \text{ s}) = 151.5 \text{ m}$$

$$\therefore \quad x_{max} = x_0 + 151.5 \text{ m} = 163.8 \text{ m} + 151.5 \text{ m} = \boxed{315 \text{ m}}$$

#10 (3.45)




- what is the min velocity of dart to hit monkey before he reaches ground?

▶ Solve for time ($t$) it takes for monkey to reach ground...

$$y - y_0 = \cancel{v_0 t} + \tfrac{1}{2} a_y t^2 \Rightarrow 25m = \tfrac{1}{2}(9.8\,m/s^2)\,t^2$$

$$\Rightarrow t = \sqrt{2(25m)/(9.8\,m/s^2)} = 2.26\,s$$

Now, dart should reach monkey in same amount of time...

$$v = \frac{\Delta x}{\Delta t} = \frac{(70m)}{2.26\,s} = \boxed{31\,m/s}$$

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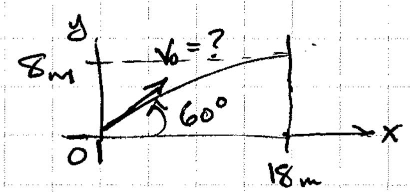
#11 (3.55)




(a) What is the magnitude and direction of ball's initial velocity $v_0 = ?$

Eqn ①: $x = v_0 \cos\theta \cdot t \Rightarrow t = \frac{x}{v_0 \cos\theta}$

Eqn ②: $y = \cancel{y_0} + v_0 \sin\theta \cdot t + \frac{1}{2} g t^2$

Sub Eqn ① into ②: $y = \cancel{v_0} \sin\theta \left(\frac{x}{\cancel{v_0}\cos\theta}\right) + \frac{1}{2} g \left(\frac{x}{v_0 \cos\theta}\right)^2$

$$\Rightarrow y - x\tan\theta = \frac{g x^2}{2 v_0^2 \cos^2\theta}$$

$$\Rightarrow v_0^2 = \frac{g x^2}{2(\cos^2\theta)(y - x\tan\theta)}$$ where $g = 9.8\,m/s^2$, $x = 18\,m$, $\theta = 60°$, $y = 8m$

$$v_0 = \sqrt{\frac{(9.8)(18)^2}{2\cos^2\theta\cdot(8 - (\tan 60°)\cdot(18))}} = \boxed{16.6\,m/s}$$

#11 cont...

(b) Find magnitude and direction of velocity of ball just before it strikes the building?

$v_x = v_0 \cos\theta = (16.6\ m/s)\cos(60°) = \underline{\underline{8.3\ m/s}}$

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ where $v_{0y} = (16.6)\sin 60° = 14.4\ m/s$

$a_y = -9.8\ m/s^2$

$y - y_0 = 8\ m$

$\Rightarrow v_y = \sqrt{v_{0y}^2 + 2a_y(y - y_0)}$

$= \sqrt{(14.4\ m/s)^2 + 2(-9.8\ m/s^2)(8\ m)} = \underline{\underline{-7.1\ m/s}}$

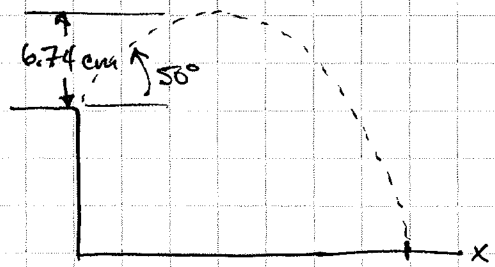
$\therefore |v| = \sqrt{v_x^2 + v_y^2}$

$= \sqrt{(8.3\ m/s)^2 + (-7.1\ m/s)} = \sqrt{119.3} = \boxed{10.9\ m/s}$

<u>direction:</u> $\tan\theta = \left(\frac{v_y}{v_x}\right) \Rightarrow \theta = \tan^{-1}\left(\frac{-7.1}{8.3}\right) = \boxed{-40.5°}$

#12 (3.57)

GIVEN:




(a) what is the initial speed of grasshopper?

USE: $v_y^2 = v_{oy}^2 + 2a_y (y - y_o)$ where $a_y = -9.8\ m/s$

at $v_y = 0$, $y - y_o = 0.0674\ m$ ...

$\Rightarrow v_{oy} = \sqrt{-2a_y(y-y_o)} = \sqrt{-2(-9.8\ m/s)(0.0674)} = \underline{1.15\ m/s}$

Noted: $v_{oy} = v_o \sin\theta \Rightarrow v_o = \frac{v_{oy}}{\sin(50°)} = \frac{1.15\ m/s}{\sin 50°} = \boxed{1.50\ m/s}$

(b) what is the height of the cliff?

Use: $\left\{ x = x_o + v_{ox} t + \frac{1}{2} \cancel{a_x} t^2 \right\}$ ~~0~~ — to solve for time grasshopper is airborne...

$\Rightarrow x - x_o = v_{ox} t$ where $x - x_o = 1.06\ m$

$v_{ox} = v_o \cos(50°) = \underline{0.96\ m/s}$

$\Rightarrow t = \frac{(x - x_o)}{v_{ox}} = \frac{1.06\ m}{0.96\ m/s} = \underline{1.10\ s}$

– Solve for the vertical displacement at $t = 1.1\ s$ ...

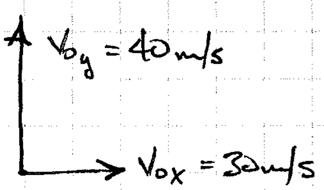
$$y - y_o = v_{oy} \cdot t + \frac{1}{2} a_y t^2$$

$$= (1.15\ m/s)(1.1\ s) + \frac{1}{2}(-9.8\ m/s^2)(1.1\ s)^2$$

$$= \underline{-4.66\ m} \Rightarrow \text{cliff is } \boxed{4.66\ m} \text{ in height}$$

#13 (3.67) A cart moves missile launcher horizontally 30 m/s. Missile launches vertically with velocity 40 m/s relative to cart.

(a) How high does rocket go?




Use: $v_y^2 = v_0^2 + 2a_y(y - y_0)$

when at max height, $v_y = 0$, $a_y = -9.8\,m/s^2$

$$\Rightarrow (y - y_0) = \frac{-v_0^2}{2a_y} = \frac{(40\,m/s)^2}{2(-9.8\,m/s^2)} = \boxed{81.6\,m}$$

(b) How far does does cart travel while rocket is in air?

Noted: at highest trajectory $v_y = 0 = v_{0y} + a_y t$

$$\Rightarrow t = \frac{-v_{0y}}{a_y} = \frac{-(40\,m/s)}{-9.8\,m/s^2} = 4.08\,s$$

So, by time rocket reaches ground... $2t = \underline{8.16\,s}$

Use: $x - x_0 = v_{0x}t + \cancel{a_x}^{0} t^2$ to solve for cart displacement

$$\Rightarrow x - x_0 = (30\,m/s)(8.16\,s) = \boxed{245\,m}$$

(c) where does rocket land relative to the cart?

Noted: both cart and rocket have $v_{0x} = 30\,m/s$

$\therefore$ $\boxed{\text{rocket will land in cart}}$

#14 (3.50) Given: $v(t) = (\alpha - \beta t^2)\hat{i} + \gamma t\hat{j}$

$\alpha = 2.4\ m/s$
$\beta = 1.6\ m/s^3$
$\gamma = 4.0\ m/s^2$

- position vector: $\vec{r} = \vec{r}_0 + \int_0^t \vec{v}(t)\,dt$

Let: $x_0 = y_0 = 0$ at $t = 0$

$$\vec{r} = \int_0^t (\alpha - \beta t^3)\,dt\,\hat{i} + \int_0^t \gamma t\,dt\,\hat{j}$$

$$= \left(\alpha t - \tfrac{1}{3}\beta t^3\right)\hat{i} + \left(\tfrac{1}{2}\gamma t^2\right)\hat{j}$$

- acceleration vector: $\vec{a} = \frac{d\vec{r}}{dt}$

$$\vec{a} = \frac{d}{dt}(\alpha - \beta t^2)\hat{i} + \frac{d}{dt}(\gamma t)\hat{j}$$

$$= 2\beta t\,\hat{i} + \gamma\hat{j}$$

(b) what is the bird's altitude (y-coordinate) as it flys over $x = 0$ at some time after $t = 0$...

Identify: $x(t) = 0 = \alpha \cancel{t} - \frac{1}{3}\beta t^{\cancel{3}\,2}$

Solve for $t$: $3\alpha = \beta t^2$

$$\Rightarrow \quad t^2 = \frac{3\alpha}{\beta} = \frac{3(2.4)}{1.6} = 4.5\ s^2$$

$t = 2.12\ s$

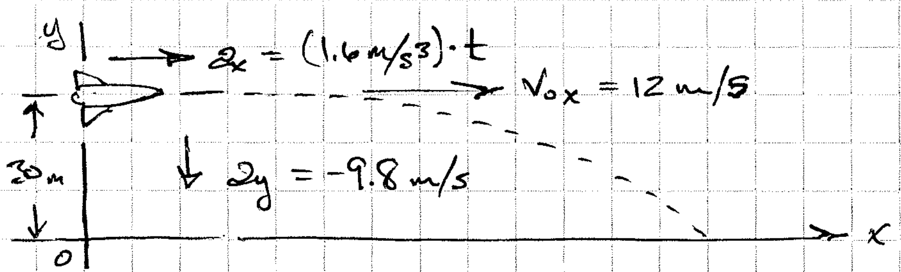
$$y(t) = \tfrac{1}{2}\gamma t^2 = \tfrac{3}{2}\frac{\gamma\alpha}{\beta}$$

$$= \tfrac{1}{2}(4.0)(4.5)$$

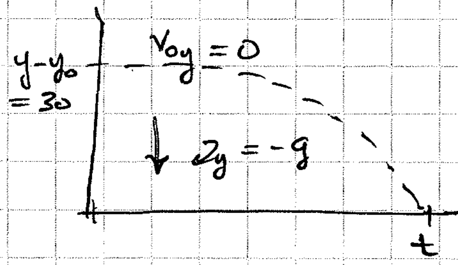
$$= \boxed{9\ m}$$

#15 (3.51)

Given:




Question: how long does it take for earth's gravity to pull rocket to ground?




Use: $$y - y_0 = \cancel{v_{0y}}^{\,0} \cdot t + \frac{1}{2} a_y t^2$$

$$\Rightarrow \quad t^2 = \frac{-2(y - y_0)}{a_y}$$

$$\Rightarrow \quad t = \sqrt{\frac{2(30m)}{(9.8\,m/s^2)}} = \underline{2.47\,s}$$

So what is the horizontal distance the rocket travels before it reaches ground?

Use: $$x(t) = \cancel{x_0}^{\,0} + v_{0x} \cdot t + \frac{1}{2} a_x t^2$$

we can't use simple equation of motion because acceleration is NOT constant. :)

$$x(2.47) = (12\,m/s)(2.47s) + \frac{1}{2}(1.6\,m/s^3)t^3$$

$$= (29.6\,m) + (12.1\,m)$$

$$= \boxed{41.6\,m}$$

wrong!

Noted: $$v_x(t) = v_{0x} + \int_0^t a_x \cdot t\,dt = v_{0x} + \frac{1}{2} a_x t^2$$

So... $$x(t) = x_0 + \int_0^t \left(v_{0x} + \frac{1}{2} a_x t^2\right) dt$$

$$= \cancel{x_0}^{\,0} + v_{0x} \cdot t + \frac{1}{6} a_x t^3$$

$$\therefore \quad x(2.47s) = (12\,m/s)(2.47s) + \frac{1}{6}(1.6\,m/s^3)(2.47)^3$$

$$= (29.7m) + (4.04\,m) = \boxed{33.7m}$$