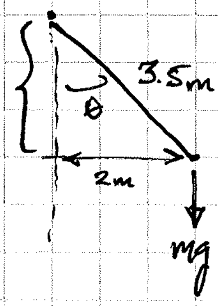
Ch7 Exercises:

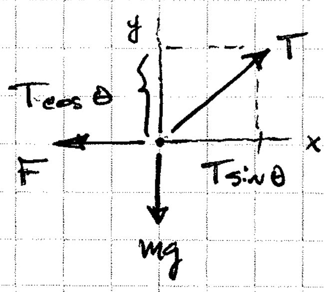
#1 Given: $m = 90\,kg$, $l = 3.5\,m$, $x_0 = 2\,m$, $F_x = ?$

(a)



where $\sin\theta = \left(\frac{2\,m}{3.5\,m}\right) \Rightarrow \theta = \sin^{-1}\left(\frac{2}{3.5}\right) = 34.8^\circ$

System is in equilibrium, so $\sum_i F_x = \sum_i F_y = 0$



▶ $\sum_i F_x = 0$

$\Rightarrow \quad F = T\sin\theta$: Eqn (1)

▶ $\sum_i F_y = 0$

$\Rightarrow \quad T\cos\theta = mg$

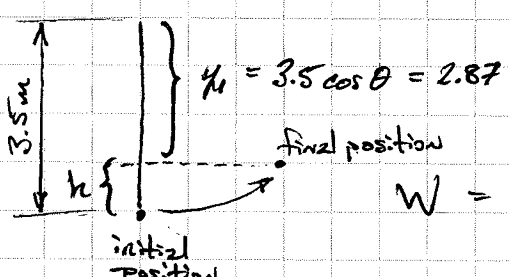
$\Rightarrow \quad T = \frac{mg}{\cos\theta}$: Eqn (2)

Substitute Eqn (2) in Eqn (1):

$$F = T\sin\theta = \left(\frac{mg}{\cos\theta}\right)\sin\theta = mg\tan\theta$$

$$= (90\,kg)(9.8\,m/s^2)\tan(34.8^\circ) = \boxed{614\,N}$$

(b) Work $= mgh = mg(y_2 - y_1)$ where $y_2 = 3.5\,m$, $y_1 = 2.87\,m$



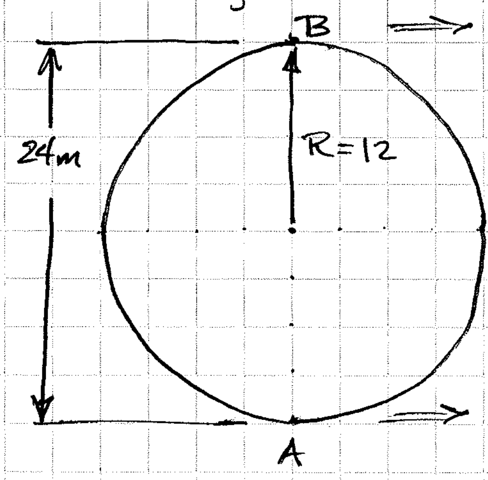
So, $(y_2 - y_1) = (3.5 - 2.87)\,m = 0.66\,m$

$$W = mgh = (90\,kg)(9.8\,m/s^2)(0.66\,m) = \boxed{552\,J}$$

(ii) Since the force applied by the worker varies during the motion of the box, it would be difficult to calculate the work done by the worker directly

#2 (7.11) Given: $m = 120\,kg$, $R = 12m$, $v_A = 25\,m/s$, $v_B = 8\,m/s$ ②

$W_f = ??$



$B \Longrightarrow \begin{cases} K_B = \frac{1}{2} m v_B^2 \\ U_B = mgh \end{cases}$

Via work-Energy relation...

$$K_A + U_A + W_f = K_B + U_B$$

$A \Longrightarrow \begin{cases} K_A = \frac{1}{2} m v_A^2 \\ U_{\cancel{B}A} = 0 \end{cases}$

$$\Rightarrow W_f = (K_B + U_B) - (K_A + \cancel{U_A}^{\,0})$$

$$= \left(\tfrac{1}{2} m v_B^2 + mgh\right) - \left(\tfrac{1}{2} m v_A^2\right)$$

$$= \left[\tfrac{1}{2}(120\,kg)(8\,m/s)^2 + (120\,kg)(9.8\,m/s^2)(24\,m)\right] - \tfrac{1}{2}(120\,kg)(25\,m/s)^2$$

$$= (3840\,J + 28{,}224\,J) - 37{,}500\,J$$

$$= \boxed{-5{,}436\,J}$$

#3 (7.15) Given: $F = 520\,N$, $\Delta x = 0.2\,m$

(a) $U_{elastic} = \frac{1}{2} k x^2 = \frac{1}{2} F \cdot x$ recall: $F_{spring} = kx$

$$= \tfrac{1}{2}(520\,N)(0.2\,m) = \boxed{52\,J}$$

(b) Notable: the potential energy is proportional to the square of the compression "or" extension.

Thus, by law of proportionality ☺...

$$\frac{(0.2\,m)^2}{52\,J} = \frac{(0.05)^2}{U_x} \Rightarrow U_x = \left(\frac{0.05}{0.2}\right)^2 \cdot (52\,J)$$

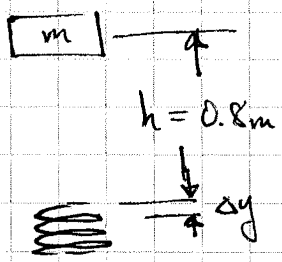
$$= \boxed{3.25\,J}$$

#4 (7.21) Given: $k = 1600\ N/m$

(a) $U_{el} = \frac{1}{2} k x^2 = 3.2\,J$

$$\Rightarrow \quad x = \sqrt{\frac{2 \cdot U_{el}}{k}} = \sqrt{\frac{2\,(3.2\,J)}{(1600\ N/m)}} = \sqrt{0.004\,m^2} = \boxed{0.063\,m}$$

(b)



where $m = 1.2\ kg$

Let: $U = mgh = \frac{1}{2} k x^2$

$$\Rightarrow \quad x^2 = \frac{2mgh}{k}$$

$$x = \frac{2\,(1.2\,kg)(9.8\,m/s^2)(0.8\,m)}{1600\ N/m} = \boxed{0.012\,m}$$

or 12 cm

#5 (7.25) Given: $m = 1160\ kg$, $v = 2.5\ m/s$, $a_{max} = 5g = 49\ m/s^2$

(a) $k = ?$ recall: $F = ma = kx \Rightarrow x = \frac{ma}{k}$

Via conservation of energy ... $\Delta E = \Delta K - \Delta U = 0$

$\Rightarrow \Delta K = \Delta U$ where $\Delta K = \frac{1}{2} m v^2$ and $\Delta U = \frac{1}{2} k x^2$

$$\cancel{\tfrac{1}{2}} m v^2 = \cancel{\tfrac{1}{2}} k x^2 = \cancel{\tfrac{1}{2}} k \left(\frac{ma}{k}\right)^2 = \frac{m^2 a^2}{k}$$

$$\Rightarrow \quad k = \frac{m^{\cancel{2}} a^2}{\cancel{m} v^2} = m \frac{a^2}{v^2} = (1160\,kg) \frac{(49\,m/s^2)^2}{(2.5\,m/s)^2} = \boxed{4.45 \cdot 10^5\ N/m}$$

$= \underline{4.46 \cdot 10^5\ N/m}$

(b) Spring compression: $x = \frac{ma}{k}$

$$x = \frac{(1160\,kg) \cdot 5g}{(4.46 \cdot 10^5\ N/m)} = \frac{(1160\,kg)(49\,m/s^2)}{4.46 \cdot 10^5\ N/m} = \boxed{0.128\,m}$$

6 (7.29) Given: $m = 62\ kg$, $v_1 = 6.5\ m/s$, rough patch $\Delta x = 4.2\ m$, $\mu_k = 0.3$
$h = 2.5\ m$

Apply conservation of Energy:

$$\Delta K_{final} = \Delta K_{initial} + \Delta U_{grav} - W_{friction}$$

where
$$\Delta K_{initial} = \frac{1}{2} m v_1^2$$
$$\Delta U_{grav} = mgh$$
$$W_{friction} = \mu_k \cdot (mg)\, \Delta x$$

thus,

$$\frac{1}{2} \cancel{m} v_2^2 = \frac{1}{2} \cancel{m} v_1^2 + \cancel{m} gh - \mu_k (\cancel{m} g)\, \Delta x$$

Solve for v_2 ...

$$v_2 = \sqrt{v_1^2 + 2gh - 2\mu_k g \cdot \Delta x}$$
$$= \sqrt{(6.5\,m/s)^2 + 2(9.8\,m/s^2)(2.5\,m) - 2(0.3)(9.8\,m/s^2)(4.2\,m)}$$
$$= \sqrt{66.55\ m^2/s^2}$$
$$= \boxed{8.15\ m/s}$$

(b) How much internal energy is generated crossing the rough patch?

$$W_f = \mu_k (mg)\, \Delta x$$
$$= (0.3)(62\,kg)(9.8\,m/s^2)(4.2\,m)$$
$$= \boxed{766\ J}$$

#7. (7.31) Given: $U(x) = \alpha x^4$ where $\alpha = 0.63\ J/m^4$

what is $F(x)$ at $x = 0.8m$?

recall: $F(x) = -\frac{dU}{dx} = -\frac{d}{dx}(\alpha x^4) = -4\alpha x^3$

$$F(0.8) = -4(0.63\ J/m^4)(0.8\ m)^3 = \boxed{1.29\ N}$$

#8. (7.33) Given: $m = 0.04\ kg$, $U(x,y) = \left(5.8\frac{J}{m^2}\right)x^2 - \left(3.6\frac{J}{m^3}\right)y^3$

what is the magnitude and direction of acceleration at $(x,y) = (.3, .6)$?

recall: $F_x = \frac{-\partial}{\partial x} U_x$ and $F_y = \frac{-\partial}{\partial y} U_y$

$$F_x = -\frac{\partial}{\partial x}\left(5.8\frac{J}{m^2}\right)x^2 = -2\left(5.8\frac{J}{m^2}\right)x = \left(-11.6\frac{J}{m^2}\right)(0.3\,m)$$
$$= \underline{-3.48\ N}$$

$$F_y = \frac{-\partial}{\partial y}\left(-3.6\frac{J}{m^3}\right)y^3 = 3\left(3.6\frac{J}{m^3}\right)y^2 = \left(10.8\frac{J}{m^3}\right)(0.6m)^2$$
$$= \underline{3.89\ N}$$

$F = ma$:

$$\Rightarrow \quad a_x = \frac{F_x}{m} = \frac{(-3.48\ N)}{(0.04\ kg)} = -87.0\ m/s^2$$

$$a_y = \frac{F_y}{m} = \frac{(3.89\ N)}{(0.04\ kg)} = 97.2\ m/s^2$$

magnitude:

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(-87)^2 + (97.2)^2} = \boxed{130\ m/s^2}$$

direction: $\tan\theta = \frac{97.2}{87.0} \Rightarrow \theta = \tan^{-1}\left(\frac{97.2}{87}\right) = \boxed{48.2^\circ}$

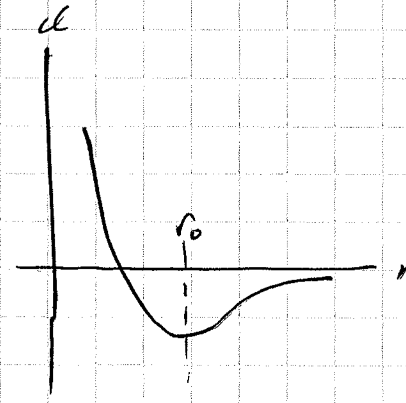
"or" 132° counterclockwise from the $+x$ axis.

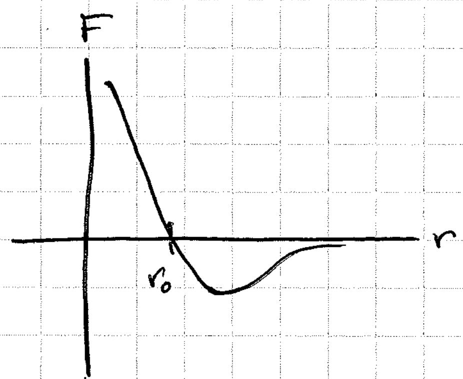
#9. (7.35) Given: $U(r) = (\alpha/r^{12}) - (\beta/r^{6})$

(a) what is $F(r) = ?$

$$F(r) = -\frac{d}{dr}U(r) = -\frac{d}{dr}\left(\alpha \cdot r^{-12} - \beta r^{-6}\right) = -\left(\frac{-12\alpha}{r^{13}} + \frac{6\beta}{r^{7}}\right)$$

$$= \boxed{\frac{12\alpha}{r^{13}} - \frac{6\beta}{r^{7}}}$$





(b) At Equilibrium, $F = dU/dr = 0$

$$\Rightarrow \frac{12\alpha}{r^{13}} = \frac{6\beta}{r^{7}} \Rightarrow 12\alpha r^{7} = 6\beta \cdot r^{13}$$

$$\Rightarrow 12\alpha = 6\beta\left(r^{13-7}\right) = 6\beta r^{6}$$

Thus, $r^{6} = \frac{12\alpha}{6\beta} = \frac{2\alpha}{\beta}$ $\therefore$ $\boxed{r_0 = \left(\frac{2\alpha}{\beta}\right)^{1/6}}$

(c) $$U(r_0) = \frac{\alpha}{\left[(2\alpha/\beta)^{1/6}\right]^{12}} - \frac{\beta}{\left[(2\alpha/\beta)^{1/6}\right]^{6}}$$

$$= \frac{\alpha}{(2\alpha/\beta)^{2}} - \frac{\beta}{(2\alpha/\beta)} = \frac{\alpha \cdot \beta^{2}}{4\alpha^{2}} - \frac{\beta^{2}}{2\alpha}$$

$$= \cancel{\frac{\beta^2}{4\alpha}} = -\frac{\beta^{2}}{4\alpha}$$

(d) Given: $r_0 = 1.13 \cdot 10^{-10}\,m = (2\alpha/\beta)^{1/6}$

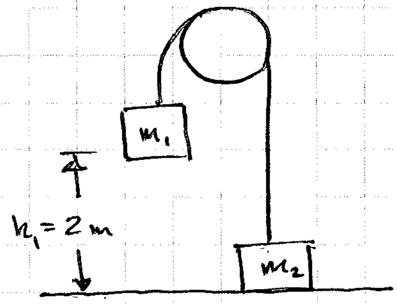
$U(r_0) = 1.54 \cdot 10^{-18}\,J = -\beta^{2}/4\alpha$

It can be shown that...

$\beta = 6.41 \cdot 10^{-78}\,J \cdot m^{6}$ and $\alpha = 6.67 \cdot 10^{-138}\,J \cdot m^{12}$

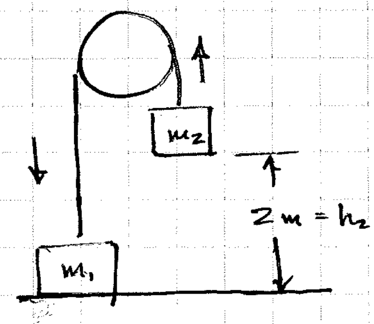
#11 (7.51) Given: $m_1 = 12\ kg$, $m_2 = 4\ kg$

Initial:



$h_1 = 2\ m$
$h_2 = 0$
$v_1 = v_2 = 0$

FINAL:



$h_1 = 0$
$h_2 = 2\ m$

via Conservation of Energy ...

$$\Delta E = E_{final} - E_{initial} = 0$$

$$\Rightarrow \quad (\Delta K_f + \Delta U_f) - (\Delta K_i + \Delta U_i + \cancel{W_f}^{\,0}) = 0 \quad \text{(no friction)}$$

where $\Delta K_f = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2}(m_1 + m_2) v^2$

noted: both buckets are connected by rope, thus $v_1 = v_2 = v$

$$\Delta U_f = m_2 g h = (4\,kg)(9.8\,m/s^2)(2\,m) = \underline{78.4\ J}$$

$$\Delta K_i = 0 \quad \text{(buckets start at rest, } (v_1 = v_2)_{initial} = 0\text{)}$$

$$\Delta U_i = m_1 g h = (12\,kg)(9.8\,m/s^2)(2\,m) = \underline{235.2\ J}$$

So ...

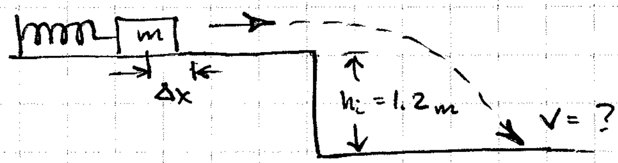
$$\Delta K_f = (\cancel{\Delta K_i}^{\,0} + \Delta U_i) - \Delta U_f$$

$$\Rightarrow \frac{1}{2}(m_1 + m_2) v^2 = (235.2\ J) - 78.4\ J = 156.8\ J$$

$$v^2 = \frac{2(156.8\ J)}{(12\,kg + 4\,kg)} = \frac{313.6\ J}{16\,kg}$$

$$v = \sqrt{19.6\ m^2/s^2} = \boxed{4.4\ m/s}$$

#12 (7.63) Given: $m = 0.15\ kg$, $h = 1.2\ m$, $k = 1900\ N/m$, $\Delta x = 0.045$



Use Conservation of Energy ...

$$\Delta E = E_{final} - E_{initial} = 0$$

$$\Rightarrow \quad (\Delta K_f + \Delta U_f) - (\Delta K_i + \Delta U_i + W_f) = 0$$

where ... $\Delta K_f = \frac{1}{2}mv^2$ – solve for "v"

$\Delta U_f = 0 \quad (h_f = 0)$

$\Delta K_i = \frac{1}{2}k\Delta x^2 = \frac{1}{2}(1900\ N/m)(0.045)^2 = 1.9\ J$

$\Delta U_i = mgh = (0.15\ kg)(9.8\ m/s^2)(1.2\ m) = 1.76\ J$

$W_f = 0 \quad$ (no friction)

thus;

$$\Delta K_f = (\Delta K_i + \Delta U_i) - \cancel{W_f}^{\,0}$$

$$\frac{1}{2}mv^2 = \frac{1}{2}k\Delta x^2 + mgh$$

$$v^2 = \frac{2(1.9\ J + 1.76\ J)}{(0.15\ kg)}$$

$$v = \sqrt{48.9\ m^2/s^2} = \boxed{7.0\ m/s}$$