PHYS 1310

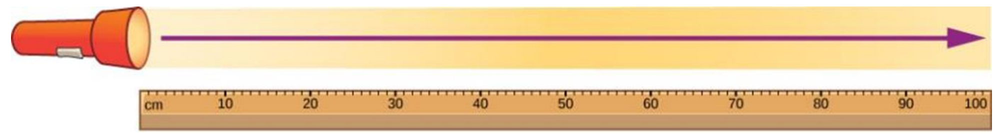
Chapter 1 –VECTORS!

Recap: Standards and Units

- Experiments require measurements.
- **Physical quantity** - any number that describes physical phenomenon quantitatively.
- Generally, we separate **fundamental** and **derived** physical quantities.
- Length, time, and mass are three **fundamental** quantities of physics.
- The **International System** (SI for **Système International**) is the most widely used system of units.
- In SI units, length is measured in **meters**, time in **seconds**, and mass in **kilograms**.

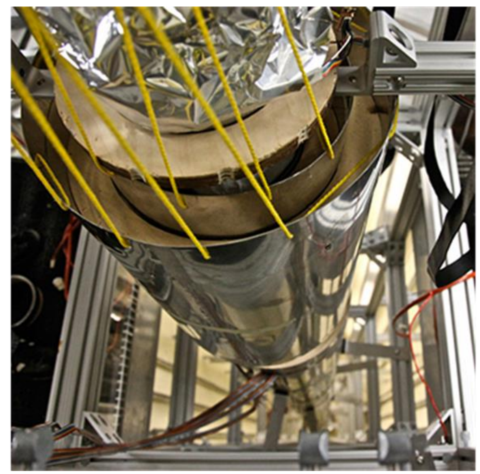
3 Fundamental Quantities in Physics

- Length (**meter**) –the distance light travels in 1/299,792,458 second.



Light travels a distance of 1 meter
in 1/299,792,458 of a second

- Time (**second**) –interval required for 9,192,631,770 atomic vibrations of cesium.



An atomic clock such as this one uses the vibrations of cesium atoms to keep time to a precision of better than a microsecond per year. The fundamental unit of time, the second, is based on such clocks. This image looks down from the top of an atomic fountain nearly 30 feet tall.

- Mass (**kilogram**) –defined in terms of Plank's constant…

  $h = 6.626 \cdot 10^{8}$ kg m^{2} s^{-1}.

The primary standard of mass for this country is United States Prototype Kilogram 20, which is a platinum-iridium cylinder kept at NIST. The kilogram, originally defined as the mass of one cubic decimeter of water at the temperature of maximum density, was known as the Kilogram of the Archives.

Lecture 2: Overview

- Scalars “vs” Vectors
- Vector Components
- Vector Addition
- Unit Vectors
- Scalar Product
- Vector Product

Scalar Quantities

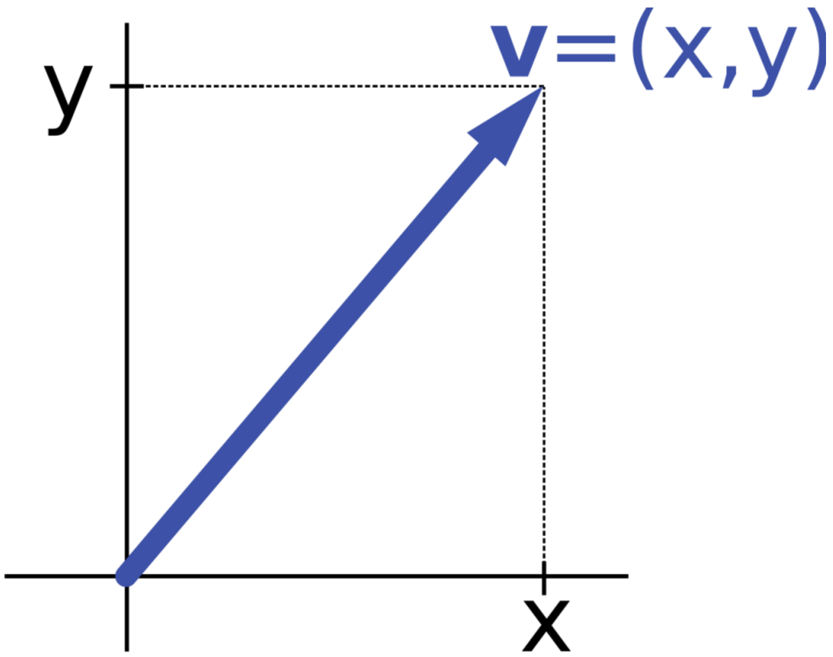
- Physical quantities described completely by a single number with a unit.

- Scalar quantities a include:
  - Time
  - Temperature
  - Mass
  - Density

Why Vectors?

- Scalar quantities only tell us something about a numerical magnitude.

- Vectors quantities tell us something about both **magnitude** and **direction**.

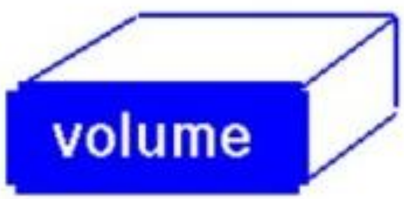


Scalars “vs” Vectors

A scalar quantity has only magnitude.
A vector quantity has both magnitude and direction.

Scalar Quantities

length, area, volume
speed
mass, density
pressure
temperature
energy, entropy
work, power

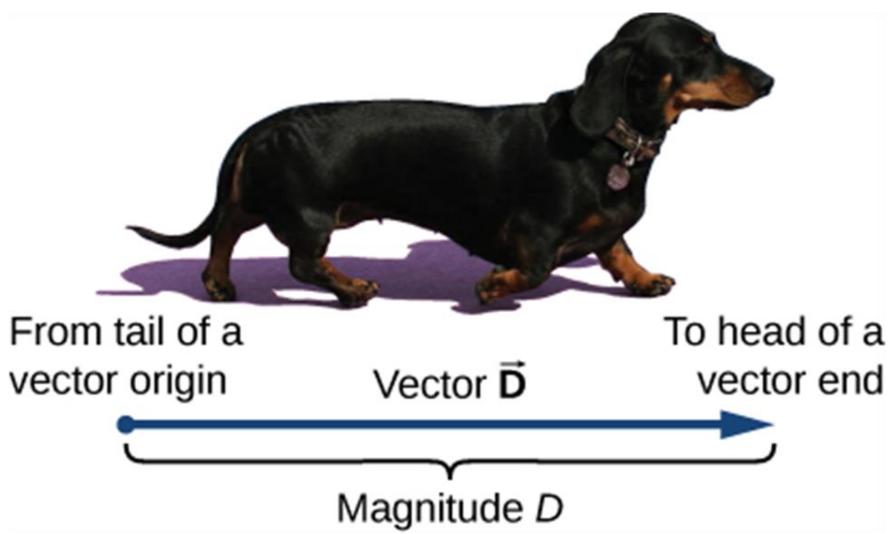


Vector Quantities

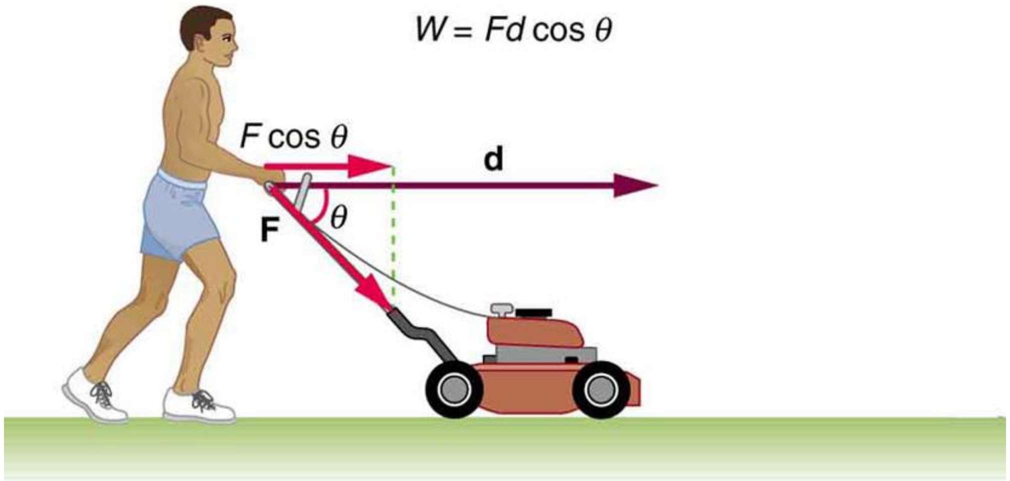
displacement
velocity
acceleration
momentum
force
lift , drag , thrust
weight

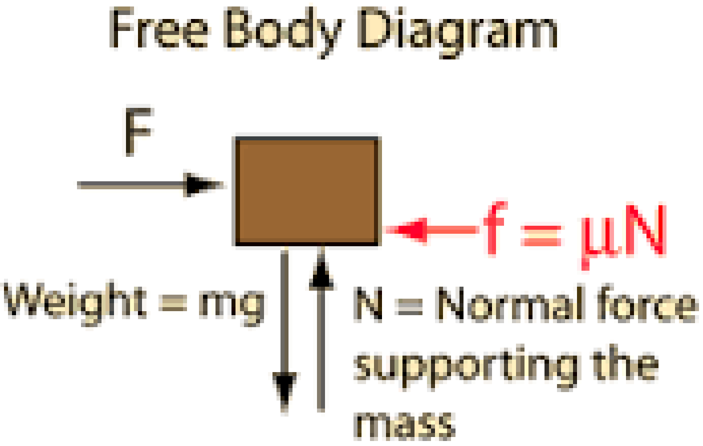
Drawing Vectors

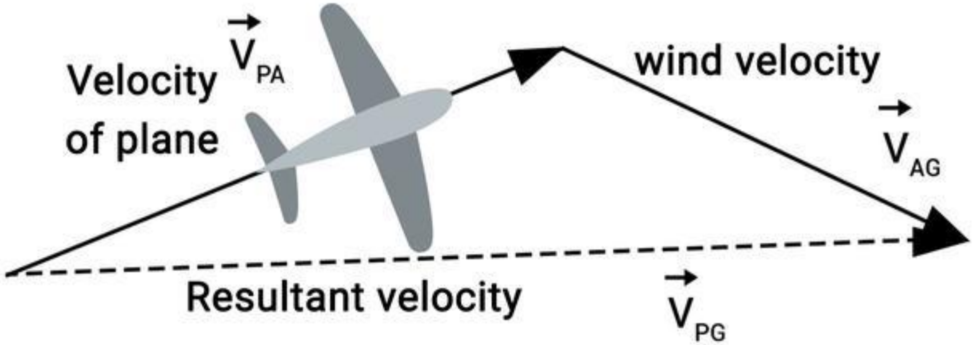
- We draw a vector from the initial point or origin (called the "tail" of a vector) to the end or terminal point (called the "head" of a vector), marked by an arrowhead. Magnitude is the length of a vector and is always a positive scalar quantity.

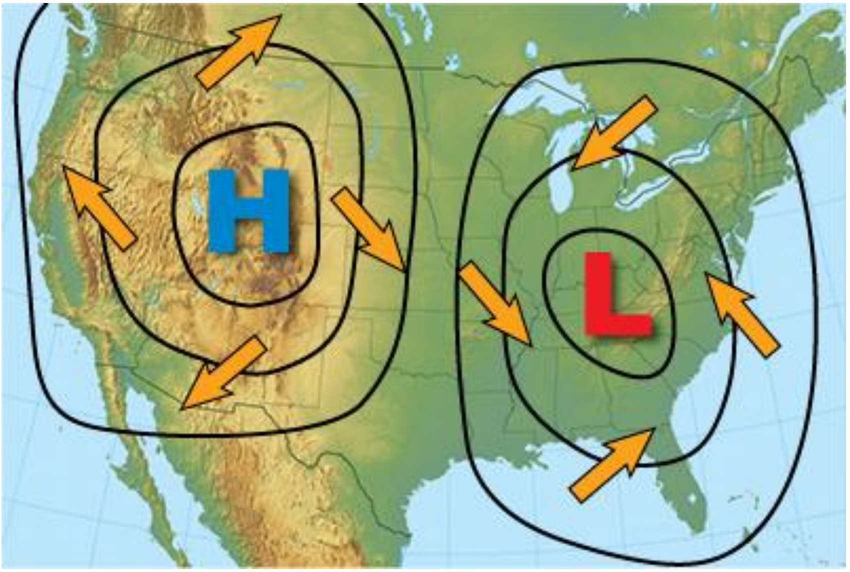


Vector Diagrams









Vector lingo…

(Various relations between two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$)

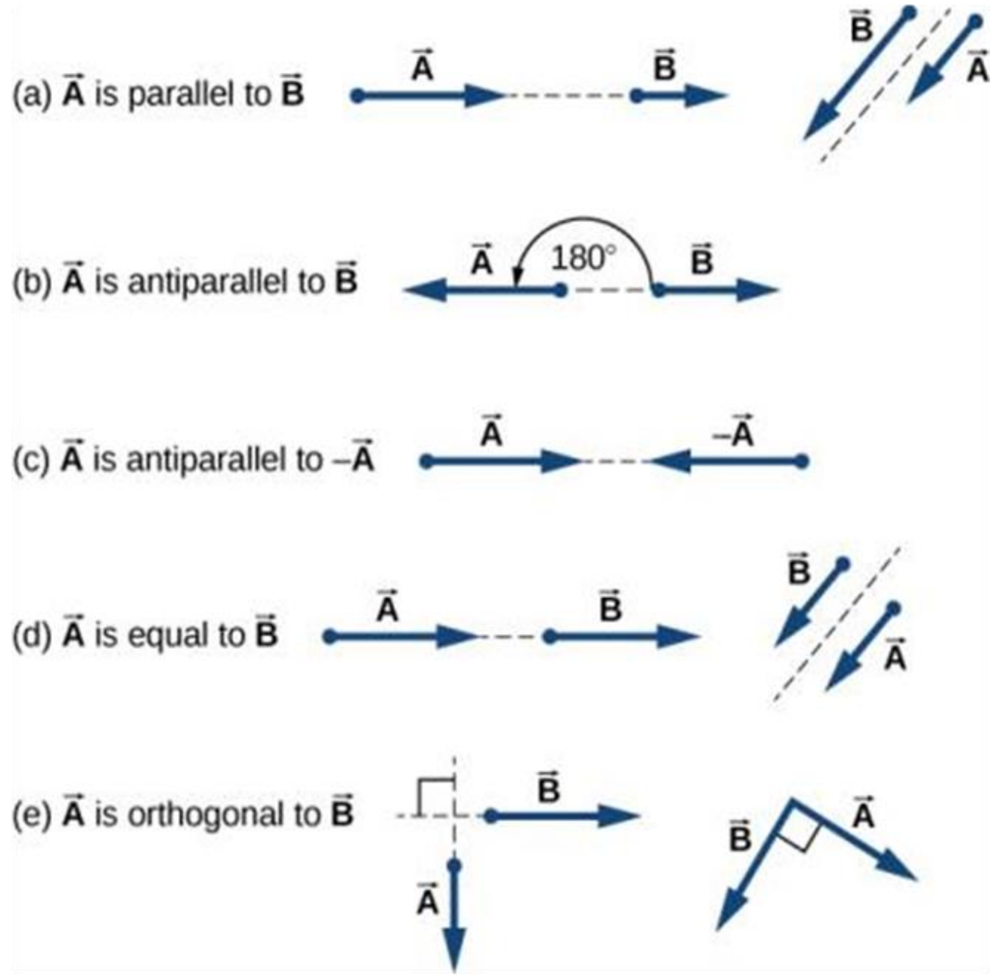
(a) $\vec{\mathbf{A}} \neq \vec{\mathbf{B}}$ because $A \neq B$.

(b) $\vec{\mathbf{A}} \neq \vec{\mathbf{B}}$ because they are not parallel and $A \neq B$.

(c) $\vec{\mathbf{A}} \neq -\vec{\mathbf{A}}$ because they have different directions (even though $\vec{\mathbf{A}} = \overrightarrow{-\mathbf{A}} = \boldsymbol{A}$).

(d) $\vec{\mathbf{A}} = \vec{\mathbf{B}}$ because they are parallel and have identical magnitudes $A = B$.

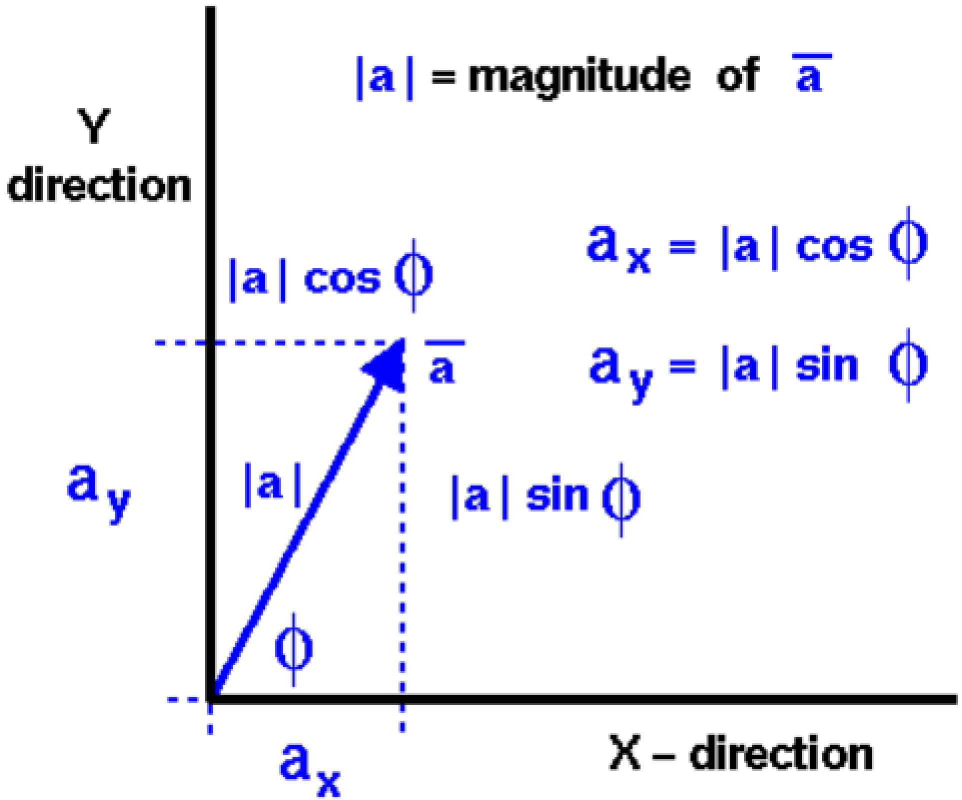
(e) $\vec{\mathbf{A}} \neq \vec{\mathbf{B}}$ because they have different directions (are not parallel); here, their directions differ by 90°—meaning, they are orthogonal.



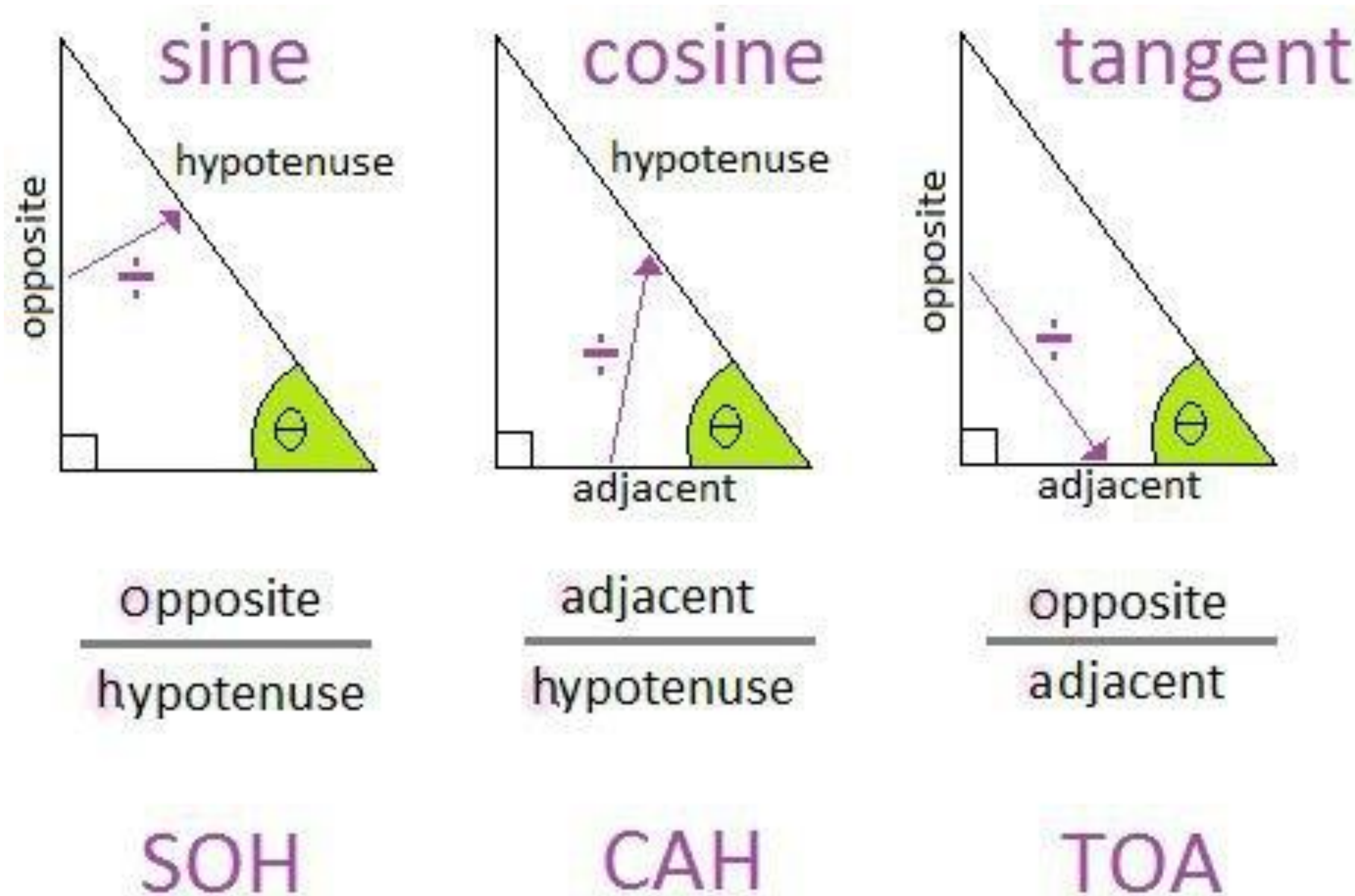
Vector Components

A vector quantity has both magnitude and direction.

components are scalars

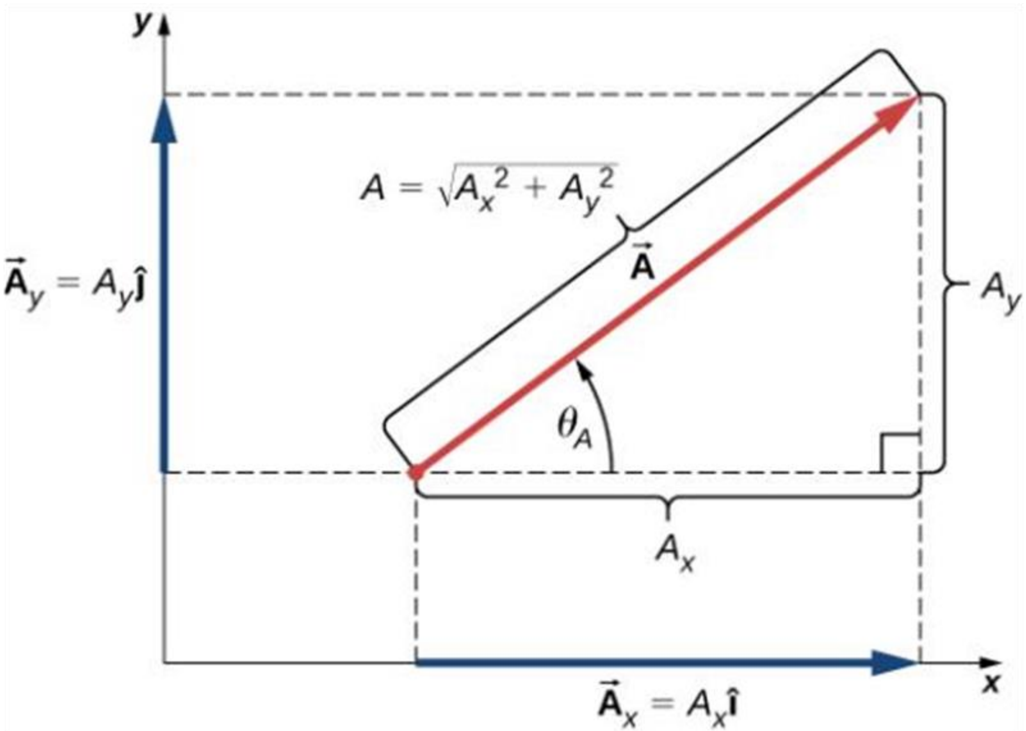


<u>Recall</u>: Chief "SOH CAH TOA"



Finding the Vector Components

- We can calculate the components (Ax, Ay) of a vector (**A**) from its magnitude and direction…

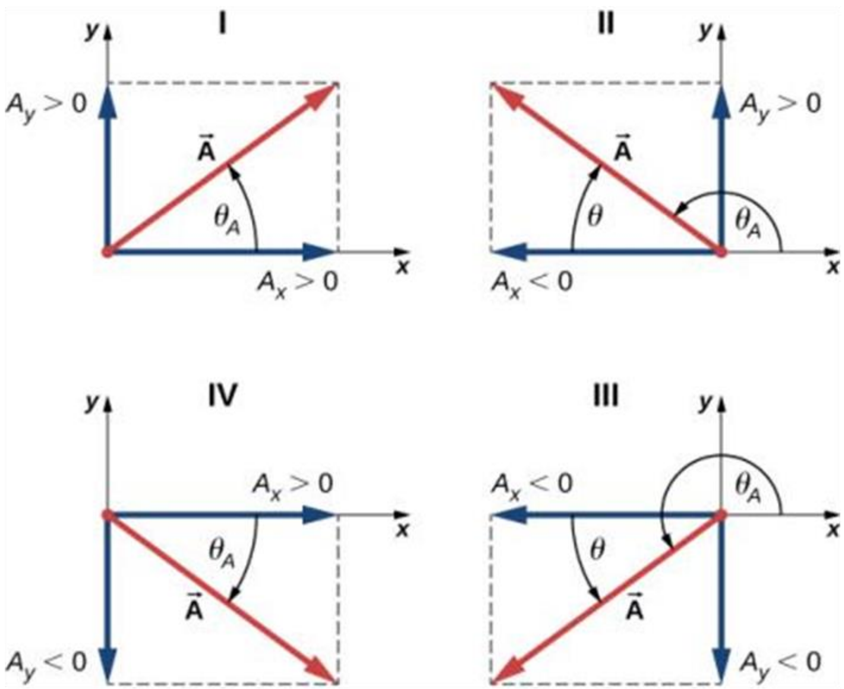


$$A_x = A\cos(\theta_A)$$
$$A_y = A\sin(\theta_A)$$

For vector $\vec{\mathbf{A}}$, its magnitude A and its direction angle θ_A are related to the magnitudes of its scalar components because A, A_x, and A_y form a right triangle.

Positive and Negative Components

- The components of a vector may be positive or negative numbers, as shown below.



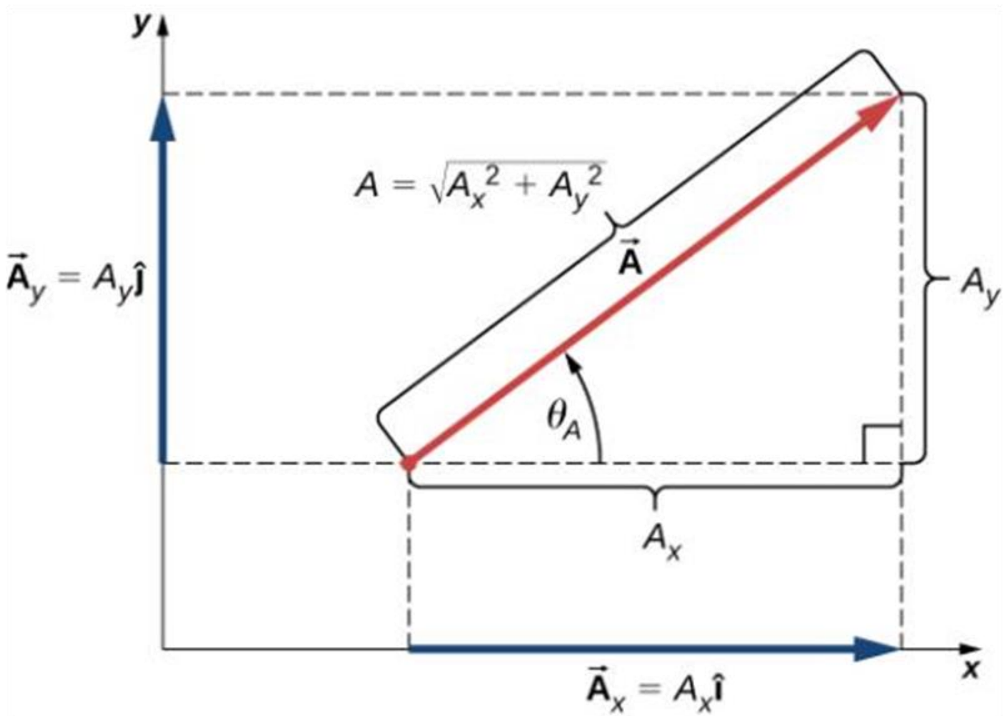
Vectors in the first quadrant (I) have both scalar components positive and vectors in the third quadrant have both scalar components negative. For vectors in quadrants II and III, the direction angle of a vector is $\theta_A = \theta + 180°$.

Calculations Using Components

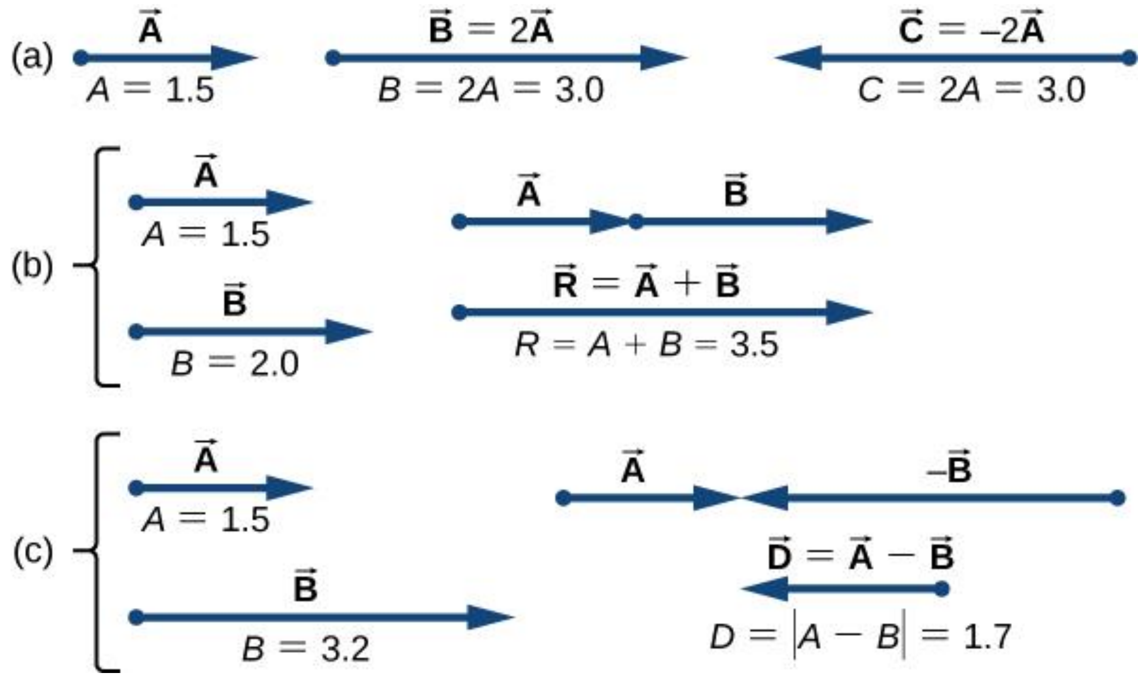
- The components (Ax, Ay) of vector A can be used to find its **magnitude** and **direction**:

$$A = \sqrt{A_x^2 + A_y^2}$$

$$\theta_A = tan^{-1}(\frac{A_x}{A_y})$$



Vector Algebra in One Dimension

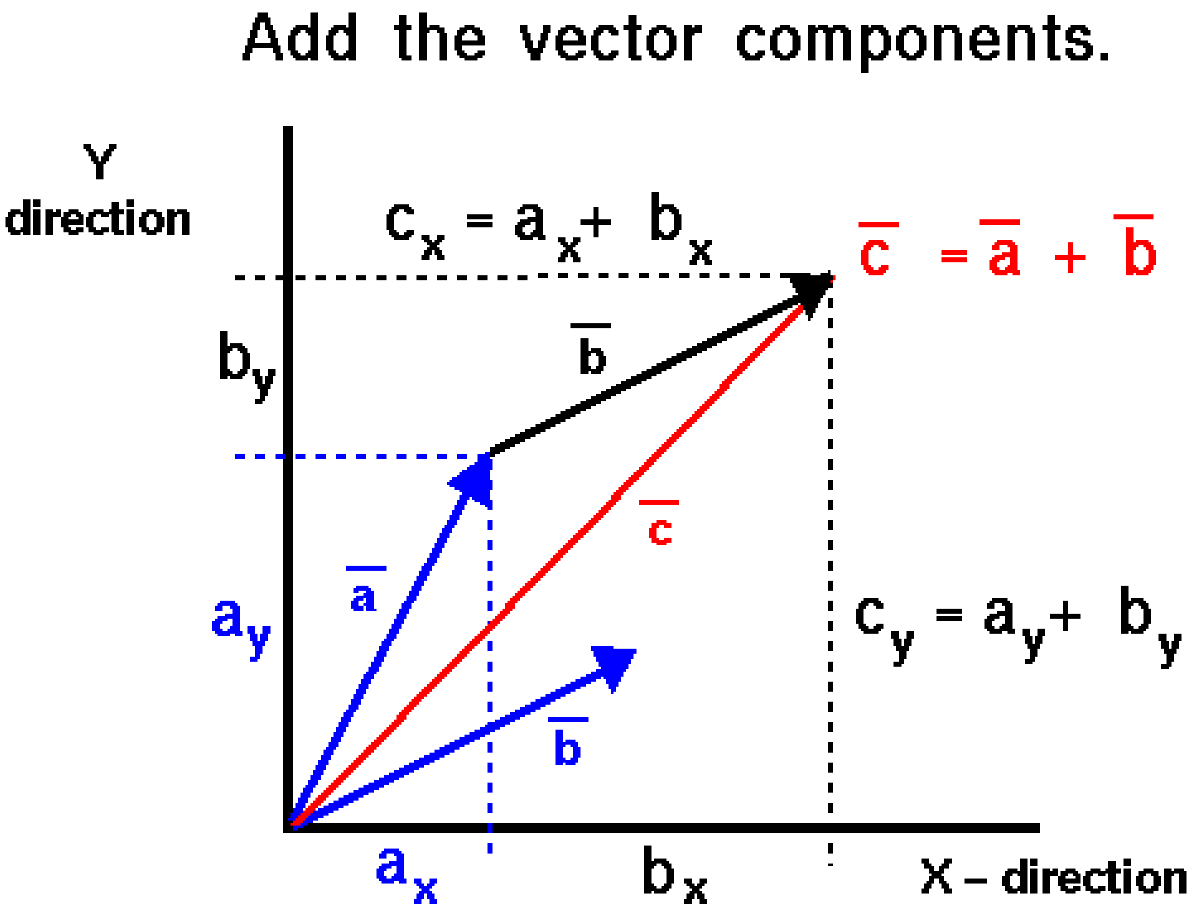


(a) Multiplication by a scalar.

(b) Addition of two vectors ($\vec{\mathbf{R}}$ is called the *resultant* of vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$).

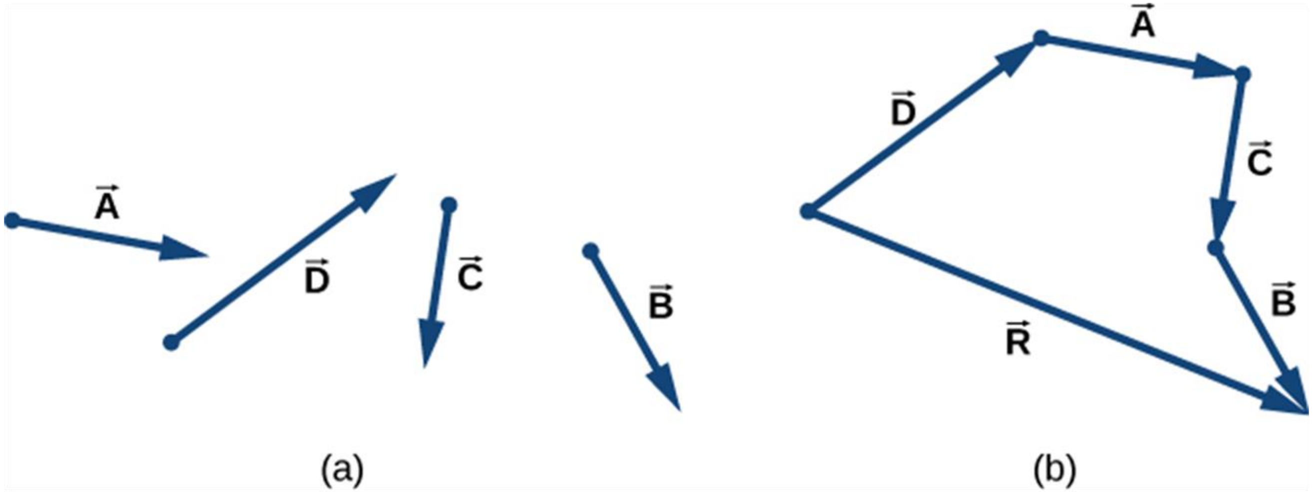
(c) Subtraction of two vectors ($\vec{\mathbf{D}}$ is the difference of vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$).

Vector Addition (Tail to Tail method)



Adding Vectors (Tail to Head Method)

- Tail-to-head method for drawing the resultant vector $\overrightarrow{\mathbf{R}} = \overrightarrow{\mathbf{A}} + \overrightarrow{\mathbf{B}} + \overrightarrow{\mathbf{C}} + \overrightarrow{\mathbf{D}}$.

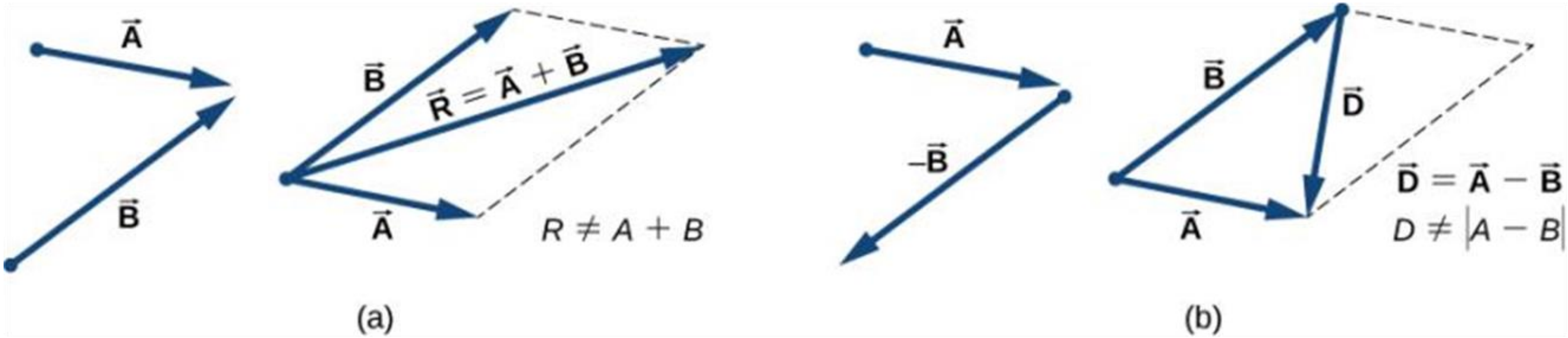


(a) Four vectors of different magnitudes and directions.

(b) Vectors in (a) are translated to new positions where the origin ("tail") of one vector is at the end ("head") of another vector. The resultant vector is drawn from the origin ("tail") of the first vector to the end ("head") of the last vector in this arrangement.

Adding/Subtracting Two Vectors Graphically (Parallelogram Method)

The parallelogram rule for the addition of two vectors. Make the parallel translation of each vector to a point where their origins (marked by the dot) coincide and construct a parallelogram with two sides on the vectors and the other two sides (indicated by dashed lines) parallel to the vectors.

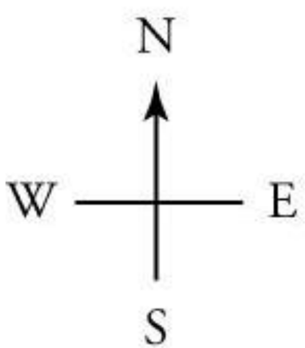


a. Draw the resultant vector $\vec{\mathbf{R}}$ along the diagonal of the parallelogram from the common point to the opposite corner. Length $\vec{\mathbf{R}}$ of the resultant vector is not equal to the sum of the magnitudes of the two vectors.

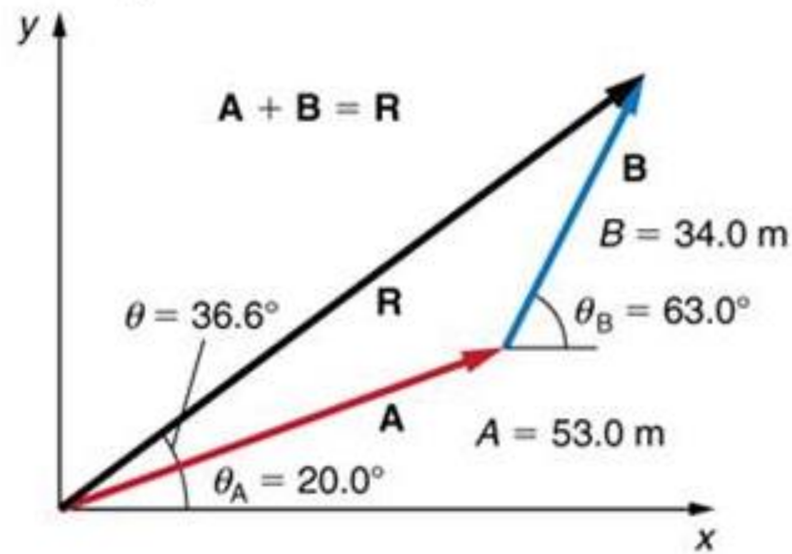
b. Draw the difference vector $\vec{\mathbf{D}} = \vec{\mathbf{A}} - \vec{\mathbf{B}}$ along the diagonal connecting the ends of the vectors. Place the origin of vector $\vec{\mathbf{D}}$ at the end of vector $\vec{\mathbf{B}}$ and the end (arrowhead) of vector $\vec{\mathbf{D}}$ at the end of vector $\vec{\mathbf{A}}$. Length D of the difference vector is not equal to the difference of magnitudes of the two vectors.

Example Problem

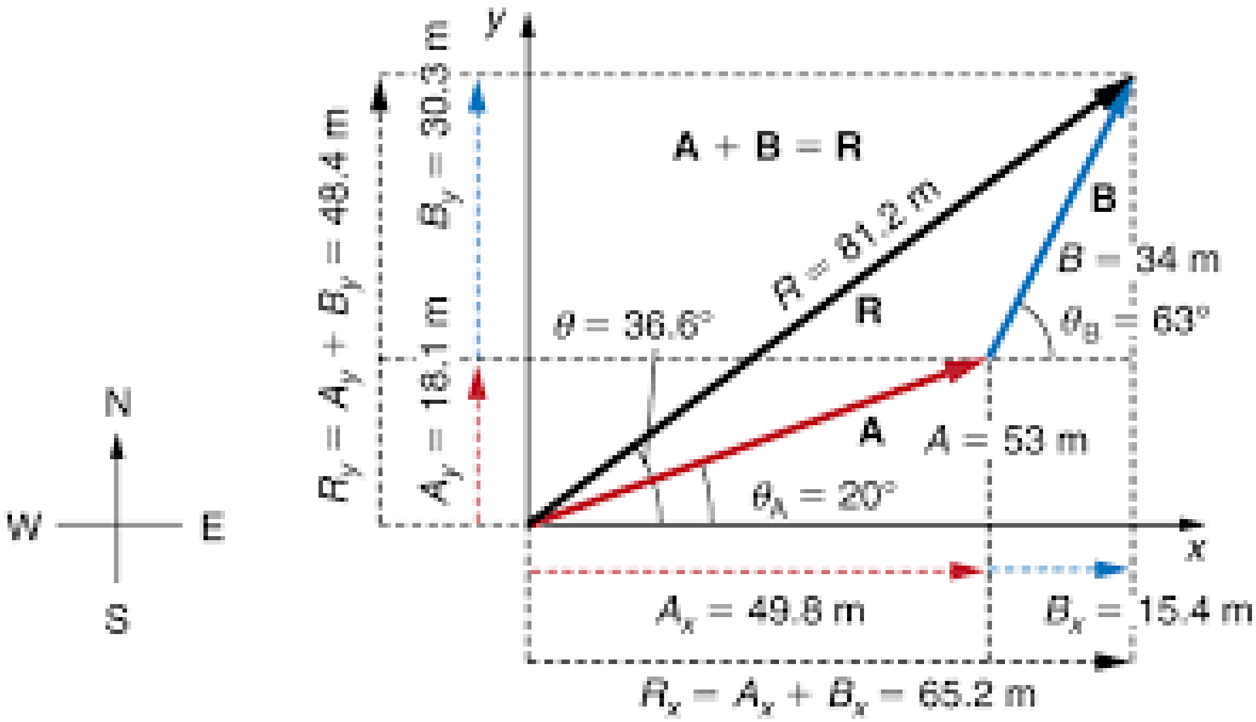
A person sets out on walk 20º North of West for 53 m. He then turns another 63º degrees North for 34 m. What is his distance from the point of origin?



The magnitude of the resultant of vectors **A** and **B** shown below is



Example cont...



$A_x = 53cos(20^o) = 49.8$

$B_x = 34cos(63^o) = 14.4$

$R_x = A_x + B_x = 65.2\ m$

$A_y = 53sin(20^o) = 18.1$

$B_y = 34sin(63^o) = 30.3$

$R_y = A_y + B_y = 48.4\ m$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(65.2)^2 + (48.4)^2} = \mathbf{81.2\ m}$$

Unit Vectors

- Are used to describe a direction in space. They are of magnitude of 1, have no units, with sole purpose to only "point."

- Given:

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$

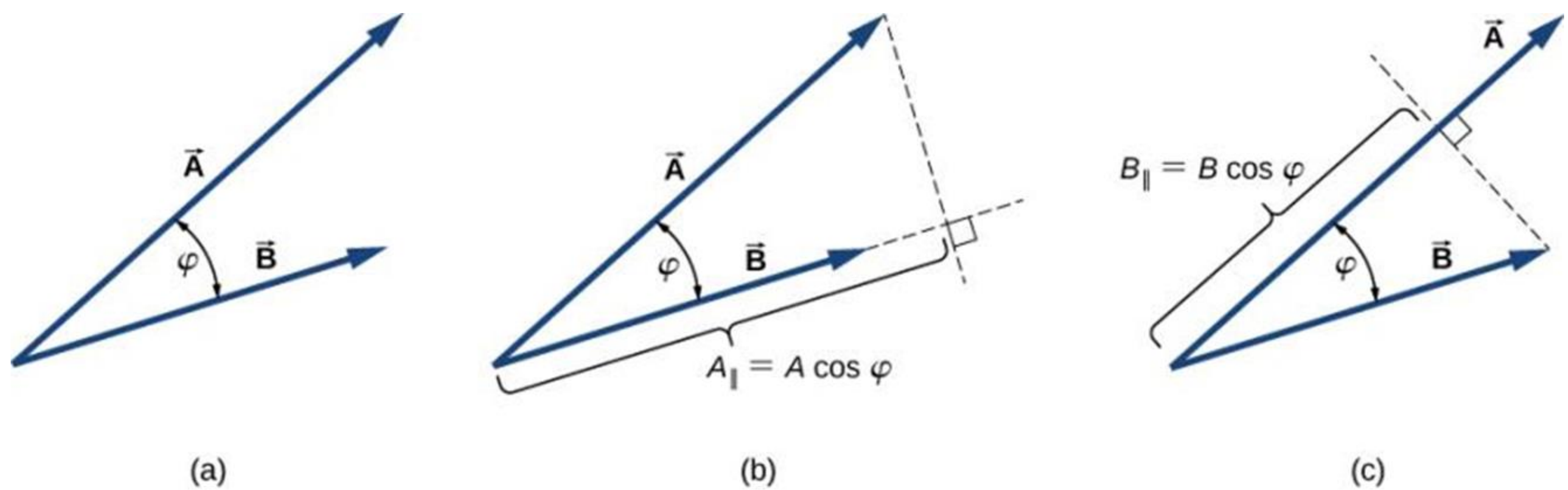
$$\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + C_z\hat{k}$$

$$\vec{R} = (A_x + B_x)\hat{\imath} + \left(A_y + B_y\right)\hat{\jmath} + (A_z + B_z)\hat{k}$$

Scalar "dot" Product Defined

- To define the scalar product $\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}}$ we draw the two vectors $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$ tail to tail.

$$\vec{A} \cdot \vec{B} = ABcos\varphi$$



The scalar product of two vectors.

(a) The angle between the two vectors.

(b) The orthogonal projection $A_{||}$ of vector $\vec{\mathbf{A}}$ onto the direction of vector $\vec{\mathbf{B}}$.

(c) The orthogonal projection $B_{||}$ of vector $\vec{\mathbf{B}}$ onto the direction of vector $\vec{\mathbf{A}}$.

Scalar "dot" Product via Components

- The scalar product of two vectors is the sum of the products of their respective components.

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

- Keep in mind:

$$\hat{\boldsymbol{\imath}} \cdot \hat{\boldsymbol{\imath}} = \hat{\boldsymbol{\jmath}} \cdot \hat{\boldsymbol{\jmath}} = \widehat{\boldsymbol{k}} \cdot \widehat{\boldsymbol{k}} = \cos 0^o = 1$$

$$\hat{\boldsymbol{\imath}} \cdot \hat{\boldsymbol{\jmath}} = \widehat{\boldsymbol{\imath}} \cdot \widehat{\boldsymbol{k}} = \hat{\boldsymbol{\jmath}} \cdot \widehat{\boldsymbol{k}} = \cos 90^o = 0$$

Vector “cross” Product

- The magnitude of the vector cross product of $\vec{A} \times \vec{B}$:

$$\vec{A}\ X\ \vec{B} = AB \sin \emptyset$$

- The components of the vector product can be calculated using the determinate method:

$$\vec{A} \times \vec{B} = \begin{bmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{bmatrix} = \hat{\imath} \begin{bmatrix} A_y & A_z \\ B_y & B_z \end{bmatrix} - \hat{\jmath} \begin{bmatrix} A_x & A_z \\ B_x & B_z \end{bmatrix} + \hat{k} \begin{bmatrix} A_x & A_y \\ B_x & B_y \end{bmatrix}$$

$$\vec{A} \times \vec{B} = \left(A_y B_z - A_z B_y\right)\hat{\imath} - (A_x B_z - A_z B_x)\hat{\jmath} + \left(A_x B_y - A_y B_x\right)\hat{k}$$

$$\hat{\imath} = \hat{\jmath} \times \hat{k} \qquad \hat{\jmath} = \hat{\imath} \times \hat{k} \qquad \hat{k} = \hat{\imath} \times \hat{\jmath}$$

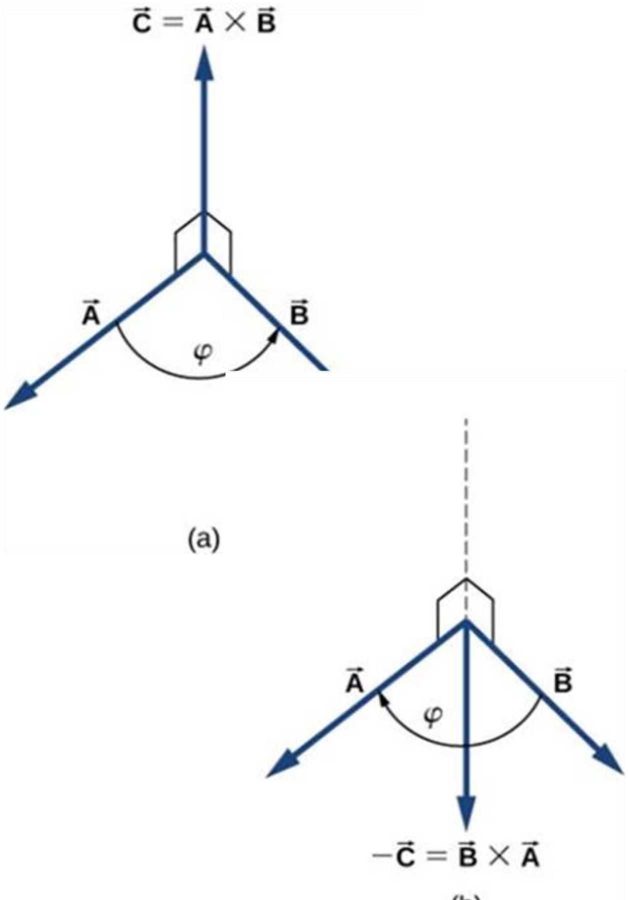
Vector "cross" Product

- If the vector product ("cross product") of two vectors is $\vec{C} = \vec{A} \times \vec{B}$, then…

(a) The vector product $\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}}$ is a vector perpendicular to the plane that contains vectors $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$. Small squares drawn in perspective mark right angles between $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{C}}$, and between $\vec{\boldsymbol{B}}$ and $\vec{\boldsymbol{C}}$ so that if $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$ lie on the floor, vector $\vec{\boldsymbol{C}}$ points vertically upward to the ceiling.

(b) The vector product $\vec{\boldsymbol{B}} \times \vec{\boldsymbol{A}}$ is a vector antiparallel to vector $\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}}$.

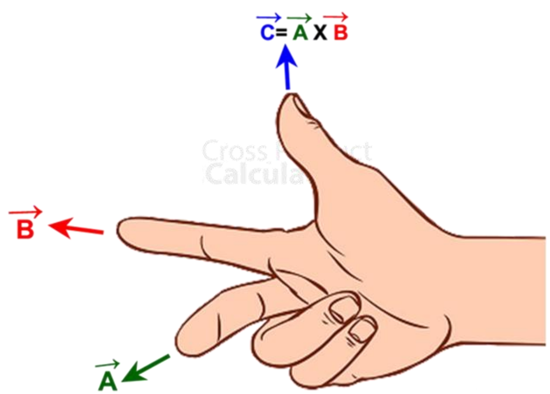
(c) The corkscrew right-hand rule can be used to determine the direction of the cross product $\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}}$.

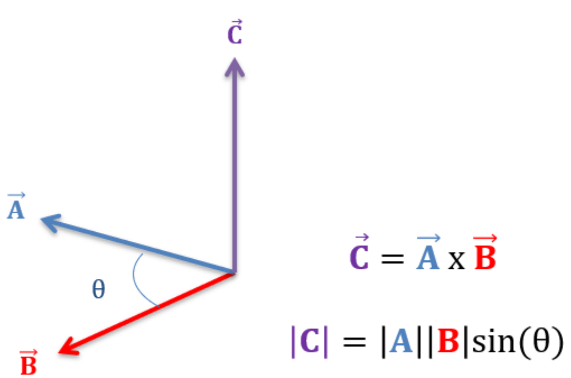


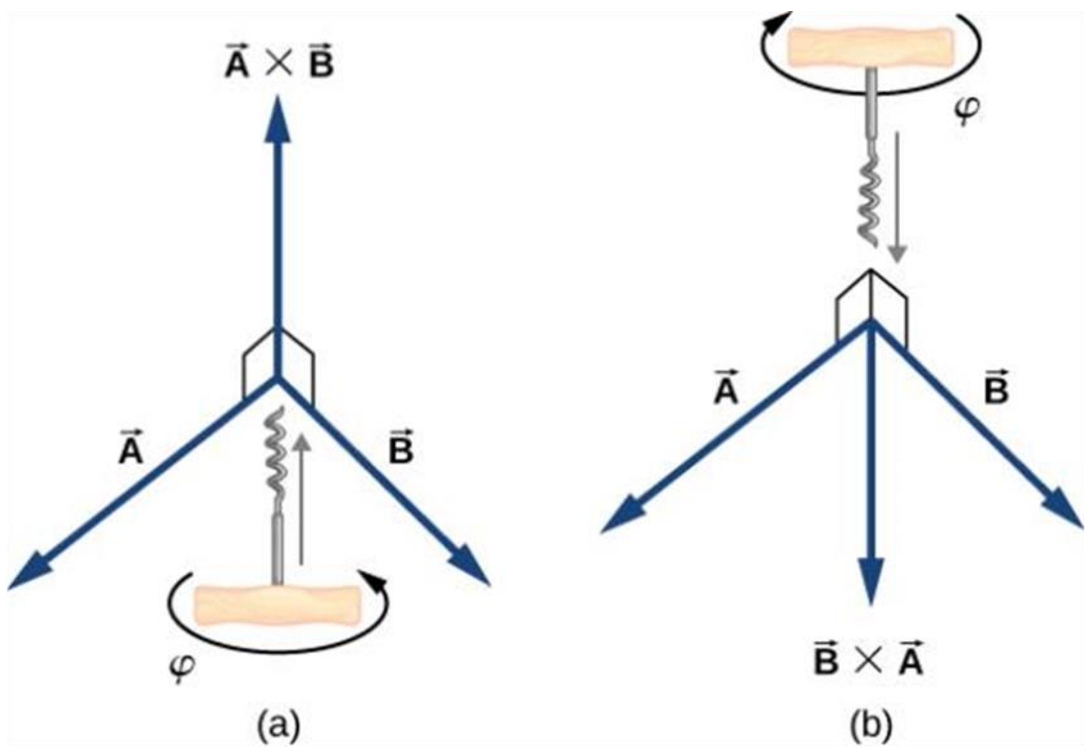
(a)
(b)

Right Hand Rule

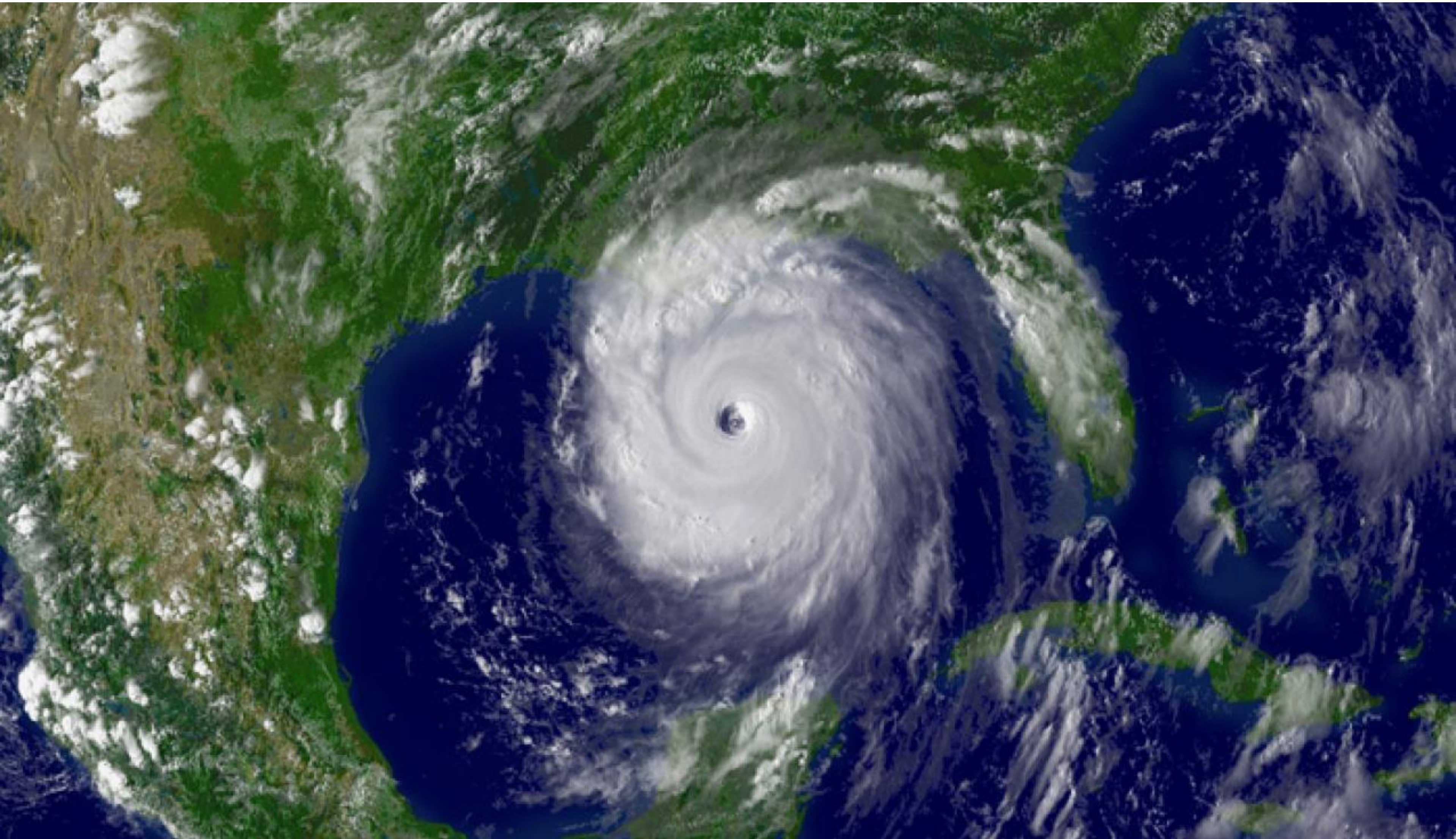
- Place a corkscrew in the direction perpendicular to the plane that contains vectors $\vec{A}$ and $\vec{B}$, and turn it in the direction from the first to the second vector in the product. The direction of the cross product is given by the progression of the corkscrew.







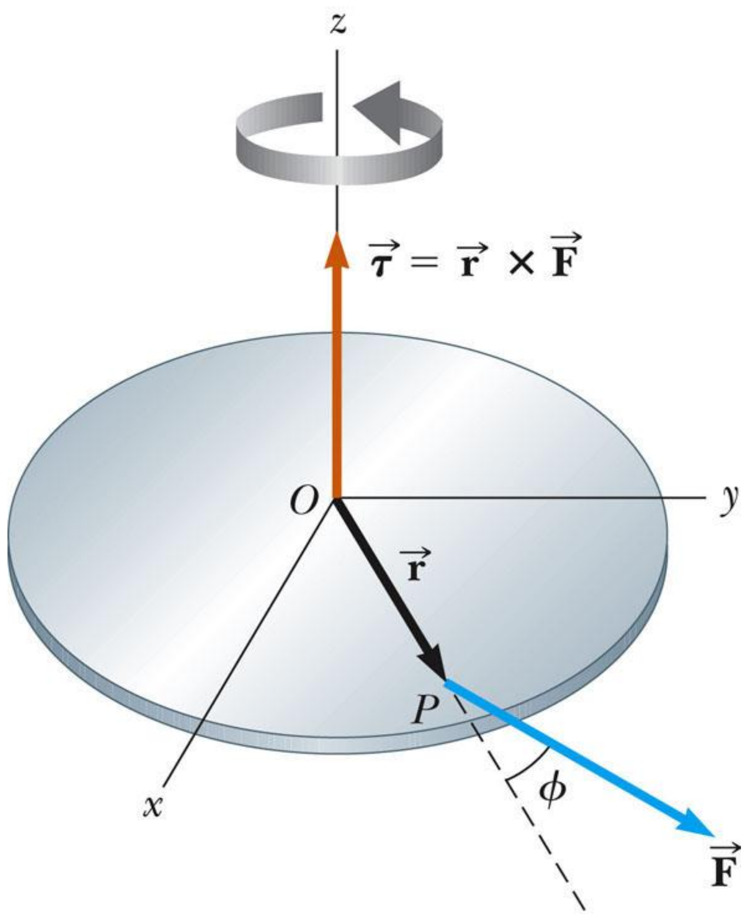
Using RHR, is this hurricane a “high” or “low” pressure region??



Application of the Cross Product

- Calculating torque:

$$\vec{\tau} = \vec{r} \times \vec{F}$$



The torque is the cross product of a force vector with the position vector to its point of application

The torque vector is perpendicular to the plane formed by the position vector and the force vector (e.g., imagine drawing them tail-to-tail)

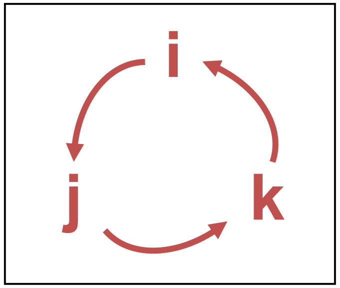
Right Hand Rule: curl fingers from r to F, thumb points along torque.

Direction of $\boldsymbol{\tau}_{\mathbf{net}}$ is angular acceleration axis

Examples of Calculating Cross Product

Find: $\vec{A}\times\vec{B}$ **Where:** $\vec{A}=2\hat{i}+3\hat{j}$ $\vec{B}=-\hat{i}+2\hat{j}$

Solution:
$$\vec{A}\times\vec{B}=(2\hat{i}+3\hat{j})\times(-\hat{i}+2\hat{j})$$
$$=2\hat{i}\times(-\hat{i})+2\hat{i}\times2\hat{j}+3\hat{j}\times(-\hat{i})+3\hat{j}\times2\hat{j}$$
$$=0+4\hat{i}\times\hat{j}-3\hat{j}\times\hat{i}+0=4\hat{k}+3\hat{k}=7\hat{k}$$



Calculate torque given a force and its location

$$\vec{F}=(2\hat{i}+3\hat{j})N \qquad \vec{r}=(4\hat{i}+5\hat{j})m$$

Solution:
$$\vec{\tau}=\vec{r}\times\vec{F}=(4\hat{i}+5\hat{j})\times(2\hat{i}+3\hat{j})$$
$$=4\hat{i}\times2\hat{i}+4\hat{i}\times3\hat{j}+5\hat{j}\times2\hat{i}+5\hat{j}\times3\hat{j}$$
$$=0+4\hat{i}\times3\hat{j}+5\hat{j}\times2\hat{i}+0=12\hat{k}-10\hat{k}=2\hat{k}\ \text{(Nm)}$$

Solving problems in Physics

- Typically, try to follow these steps:

- ➢ **Identify** the relevant concepts, target variables, and known quantities, as stated or implied in the problem.
- ➢ **Set Up** the problem: Choose the equations that you'll use to solve the problem, and draw a sketch of the situation.
- ➢ **Execute** the solution: This is where you "do the math."
- ➢ **Evaluate** your answer: Compare your answer with your estimates, and reconsider things if there's a discrepancy.

- Finally, ask yourself: "Does it make sense?"

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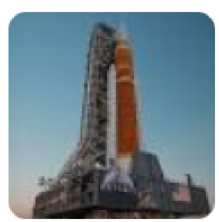

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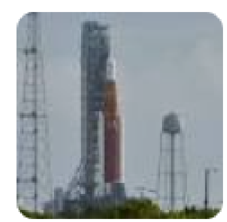

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Why NASA is returning to the moon 50 years later with Artemis I

(CNN) It's time to go back to the moon!

Almost 50 years after the last Apollo mission ventured to the lunar surface, NASA has established a program that promises to land humans on unexplored lunar regions and eventually the surface of Mars -- and it all begins with Artemis I.

Goals of the Artemis program include landing diverse crews of astronauts on the moon and exploring the shadowy lunar south pole for the first time. The ambitious effort also aims to establish a sustained presence on the moon and create reusable systems that can enable human exploration of Mars and perhaps beyond.

Artemis… the twin sister to Apollo.