CH2 EXERCISES:

#1. (2.3) You normally drive at a average speed of 105 km/h, which takes 1 hr 50 min. Traffic slows down average speed to 70 km/h. How much longer will your trip take?

We know $v_{ave} = \frac{\Delta x}{\Delta t} \Rightarrow \Delta x = v_{ave} \cdot \Delta t$ and $\Delta t = \frac{\Delta x}{v_{ave}}$

Given: $v_{ave} = 105\ (km/h)$ and $\Delta t = 1\ hr\ 50\ min = 1.83\ hr$

Solve for Δx: $\Delta x = 105\left(\frac{km}{hr}\right)\cdot(1.83\ hr) = 192.5\ km$

Now, solve for Δt given heavy traffic where $v_{ave} = 70\ km/hr$

$$\Delta t = \frac{\Delta x}{v_{ave}} = \frac{(192.5\ km)}{(70\ km/hr)} = 2.75\ hr = 2\ hr\ 45\ minutes$$

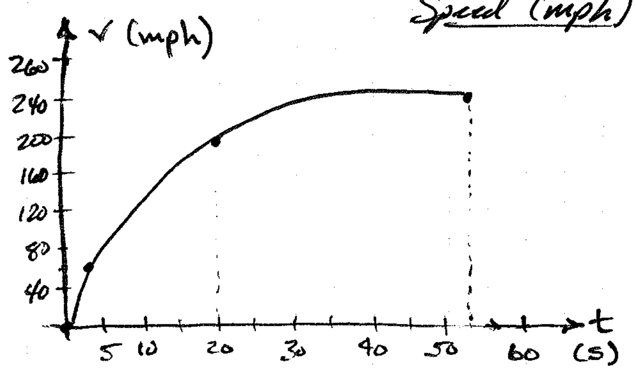
the additional time it takes $\Rightarrow (2.75 - 1.83)\ hr = 0.92\ hr \times \left(\frac{60\ min}{hr}\right)$

$= \boxed{55.2\ minutes}$

#2 (2.13) Given:

Time (s):	0	2.1	20.0	53
Speed (mph):	0	60	200	253

(a)



(b) Calculate the car's average acceleration (m/s^2) between...

$\Delta t_1 = 2.1 - 0$ where $v_{ave_1} = 60\ mph$

$$a_{ave} = \frac{v_{ave_1}}{\Delta t_1} = \frac{60\ mph}{(2.1\ s)} \times \left(\frac{1609\ m}{mi}\right) \times \left(\frac{1\ hr}{3600\ s}\right)$$

$$= \frac{96{,}540\ m}{7{,}560\ s^2} = \boxed{12.8\ m/s^2}$$

$\Delta t_2 = 20 - 2.1 = 17.9\ s$, $v_{ave_2} = 140\ mph$

$$a_{ave} = \frac{v_{ave2}}{\Delta t_2} = \frac{140\ mph}{(17.9\ s)} \times \left(\frac{1609\ m}{mi}\right) \times \left(\frac{1\ hr}{3600\ s}\right) = \frac{225{,}260\ m}{64{,}440\ s} = \boxed{3.50\ m/s^2}$$

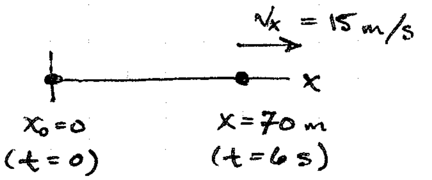
$\Delta t_3 = 53 - 20 = 33\ s$, $v_{ave3} = 253 - 200 = 53\ mph$

$$a_{ave} = \frac{v_{ave3}}{\Delta t_3} = \frac{53\ mph}{(33\ s)} \times \left(\frac{1609\ m}{mi}\right) \times \left(\frac{1\ hr}{3600\ s}\right) = \boxed{0.718\ m/s^2}$$

#3 (2.19) An antelope moving with constant acceleration covers the distance between two points 70 m in 6 s. Its speed as it passes the second point is 15 m/s. What are:

(a) Its speed at the first point

GIVEN:



$$x - x_0 = 70\,m$$
$$t = 6\,s$$
$$v_x = 15\,m/s$$
$$v_0 = ??$$

Eqn of Motion (iv): $x - x_0 = \frac{1}{2}(v_0 + v_x)t$

$$\Rightarrow \quad v_0 = \frac{2(x-x_0)}{t} - v_x = \frac{2(70m)}{6s} - v_x = \frac{140m}{6s} - 15\,m/s = \boxed{8.33\ m/s}$$

(b) It's acceleration

Eqn of Motion (ii): $v_x = v_0 + a_x t \Rightarrow a_x = \frac{(v_x - v_0)}{t}$

$$\Rightarrow \quad a_x = \frac{(v_x - v_0)}{t} = \frac{(15 - 8.33)}{6\,s} = \frac{6.6\,m/s}{6} = \boxed{1.11\ m/s^2}$$

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4 (2.21) The fastest measured pitch in baseball left the pitcher's hand at a speed of 45 m/s. If the pitcher was in contact with the ball over a distance 1.5 m and produced constant acceleration,

(a) What acceleration did he give the ball?

GIVEN:

$$x - x_0 = 1.5\,m$$
$$v_x = 45\,m/s$$
$$v_0 = 0$$

} that is, we are assuming the ball starts from rest and moves in the positive x-direction...

Eqn of Motion (iii): $v_x^2 = v_0^2 + 2a_x(x - x_0)$

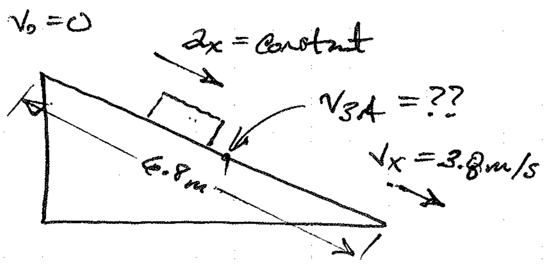
$$\Rightarrow \quad a_x = \frac{v_x^2 - v_0^2}{2(x-x_0)} = \frac{(45\,m/s)^2}{2(1.5\,m)} = \frac{2025\,m^2/s^2}{3\,m} = \boxed{675\ m/s^2}\ !!$$

(b) How much time did it take him to pitch it?

Eqn of Motion (ii): $v_x = v_0 + a_x t$

$$\Rightarrow \quad t = \frac{(v_x - v_0)}{a_x} = \frac{45\,m/s}{675\,m/s^2} = \boxed{0.067\ s}$$

#5 (2.33) A small block has constant acceleration as it slides down a frictionless incline. The block is released from rest at the top of incline, and its speed after it has traveled 6.80 m to the bottom is 3.80 m/s. What is its speed when it is 3.40 m from the top of incline?



GIVEN: $v_0 = 0$

$x - x_0 = 6.8$ m

$v_x = 3.8$ m/s

what is $a_x = ?$

Eqn of Motion (iii): $v_x^2 = v_0^2 + 2a_x(x - x_0)$

$$\Rightarrow a_x = \frac{v_x^2 - \cancel{v_0^2}}{2(x-x_0)} = \frac{(3.8\ m/s)^2}{2(6.8)m} = 1.06\ m/s^2$$

Now, at $x - x_0 = 3.4$ m, solve for v_x ...

$$v_x^2 = \cancel{v_0^2} + 2a_x(3.4m) = 2(1.06)(3.4) = 7.22\ m^2/s^2$$

$$\therefore v_x = \sqrt{7.22\ m^2/s^2} = \boxed{2.69\ m/s}$$

#6. (2.35) A flea jumps to height 0.44 m, what is its initial speed when it leaves the ground? How long is it airborne?

GIVEN: $v_y = 0$

$y - y_0 = 0.44$ m

$a_y = -9.8\ m/s^2$

$v_0 = ??$

Use Eqn of Motion (iii): $\cancel{v_y^2}^{\,0} = v_0^2 + 2a_y(y - y_0)$

$$\Rightarrow v_0^2 = -2a_y(y - y_0)$$

$$v_0 = \sqrt{-2(-9.81\ m/s^2)(0.44m)} = \sqrt{(8.63\ m^2/s^2)} = \boxed{2.94\ m/s}$$

(b) Solve for t (time airborn) Eqn of Motion (i): $y = y_0 + v_0 t + \frac{1}{2} a_y t^2$

GIVEN: $y - y_0 = 0$

$v_0 = 2.94$ m/s

$a_y = -9.8\ m/s^2$

$$\Rightarrow y - y_0 = v_0 t + \frac{1}{2} a_y t^2$$

$$\Rightarrow 0 = v_0 \cancel{t} + \frac{1}{2} a_y t^{\cancel{2}}$$

$$\Rightarrow v_0 = \frac{1}{2} a_y t$$

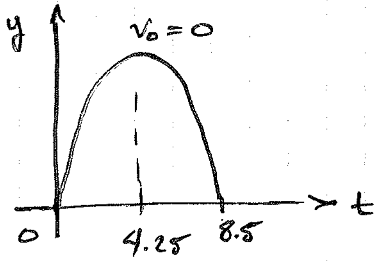
$$\Rightarrow \frac{2v_0}{a_y} = t$$

$$\therefore t = \frac{2v_0}{a_y} = \frac{2(2.94\ m/s)}{9.8\ (m/s^2)} = \frac{(5.88\ m/s)}{9.8\ m/s^2} = \boxed{0.60\ s}$$

#7. (2.39) A tennis ball on Mars ($a_m = 3.72\ m/s^2$) is hit upwards and returns to surface 8.5 s later.

(a) How high did it travel?

<u>Given:</u> at max height, $v_0 = 0$ and $t = \frac{1}{2}(8.5s) = 4.25s$

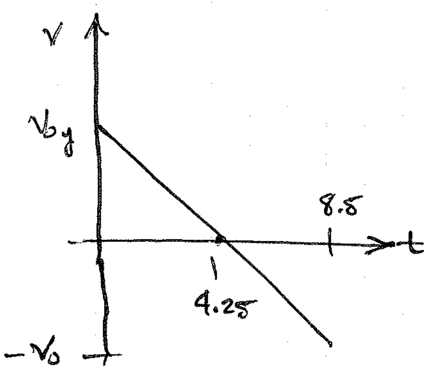


<u>Eqn of Motion (i):</u> $y = y_0 + v_0 t + \frac{1}{2} a_y t^2$

$$\Rightarrow y = 0 + (0)t + \frac{1}{2} a_y t^2$$

$$\therefore y = \frac{1}{2}(-3.72\ m/s^2)(4.25s)^2 = \boxed{33.6\ m}$$

(b) How fast was it moving just after it was hit?



<u>Eqn of Motion (ii):</u> $v_y = v_0 + a_y t$

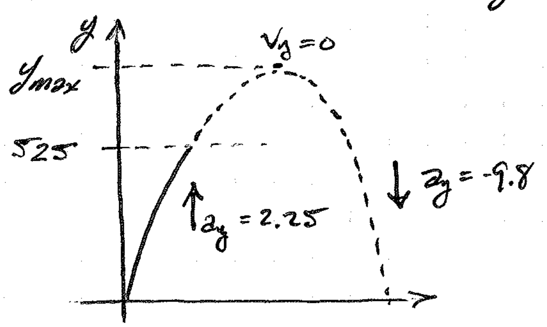
- consider the motion from just after it was hit to maximum height ($v_y = 0$, $t = 4.25s$)

Solve for v_0 ...

$$\Rightarrow v_0 = -(-3.72 m/s^2)(4.25s) = \boxed{15.8\ m/s}$$

#8 (2.43) A rocket blasts upwards with constant acceleration of $2.25\ m/s^2$. When it reaches a height of 525 m, its engines fail, for which only force acting on it is gravity.

(a) What is maximum height the rocket will reach above launch pad?



<u>NOTED:</u> there are 2 interval where constant acceleration apply ($a_y = +2.25$, $a_y = -9.8\ m/s^2$)

<u>Eqn of Motion (iii):</u> $v_y^2 = v_0^2 + 2a_y(y - y_0)$

where $y - y_0 = 525$, $v_0 = 0$, $a_y = 2.25\ m/s^2$

Solve for the velocity of rocket (v_y) when $y - y_0 = 525m$ ...

$$\Rightarrow v_y = (2(2.25\ m/s^2)(525\ m))^{1/2} = \underline{48.6\ m/s}$$

Now at max height (y_{max}): $y_0 = 525$, $v_0 = 48.6\ m/s$, $v_y = 0$ ... $y = ??$

$$\Rightarrow 2a_y(y_{max} - y_0) = v_y^2 - v_0^2$$

8 (a) continued...

$$y_{max} - y_0 = \frac{v_y^2 - v_0^2}{2a_y} \qquad \text{where } \begin{cases} y_0 = 525 \text{ m} \\ v_0 = 48.6 \text{ m/s} \\ v_y = 0 \\ a_y = -9.8 \text{ m/s}^2 \end{cases}$$

$$\Rightarrow \quad y_{max} = \frac{(0)^2 - (48.6 \text{ m/s})^2}{2(-9.8 \text{ m/s}^2)} + y_0$$

$$= \frac{-(2362 \text{ m}^2/\text{s}^2)}{-19.6 \text{ m/s}^2} + 525 \text{ m} = 121 \text{ m} + 525 = \boxed{646 \text{ m}}$$

(b) How much time will elapse <u>after</u> <u>engine failure</u> before the rocket crashes?

<u>recall</u>: <u>Eqn of Motion (ii)</u>: $\{ v_y = v_0 + a_y t \}$ — to solve for "t" we need to know the velocity of rocket just before it reaches ground...

<u>Given</u>:
$y - y_0 = -525$ m
$a_y = -9.8$ m/s²
$v_0 = 48.6$ m/s
$v_y = ??$

<u>Use Eqn (iii)</u>:

$$v_y^2 = v_0^2 + 2a_y(y - y_0)$$

$$\Rightarrow \quad v_y = -\sqrt{(48.6)^2 + 2(-9.81)(-525)} = \sqrt{(2362) + (15{,}645)} = \boxed{-112 \text{ m/s}}$$

(downwards) !!

Solve for t where $a_y t = v_y - v_0$

$$\Rightarrow \quad t = \frac{v_y - v_0}{a_y} = \frac{(-112 \text{ m/s}) - (48.6 \text{ m/s})}{-9.8 \text{ m/s}^2} = \frac{-161 \text{ m/s}}{-9.8 \text{ m/s}^2} = \boxed{16.4 \text{ s}}$$

(c) What is the rocket's velocity when it strikes the launch pad?

(d) How much time will elapse before engine failure?

<u>Given</u>: $y - y_0 = 525$, $v_0 = 0$, $a_y = +2.25$ m/s²

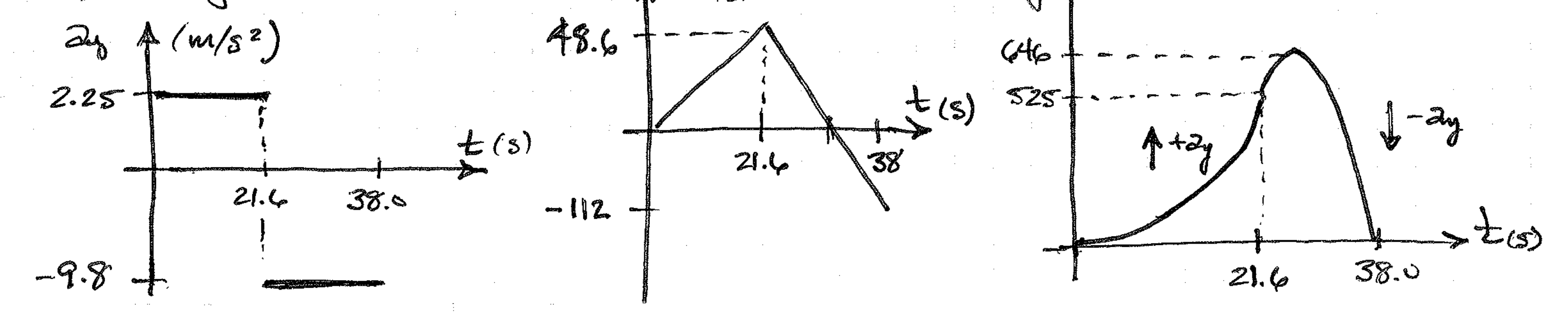
<u>Eqn of Motion (i)</u>: $\{ y = y_0 + v_0 t + \frac{1}{2} a_y t^2 \} \Rightarrow y - y_0 = \cancel{v_0 t}^{\,0} + \frac{1}{2} a_y t^2$

$$\Rightarrow \quad t^2 = \frac{2(y - y_0)}{a_y} = \frac{2(525) \text{ m}}{2.25 \text{ m/s}^2} = 467 \text{ s}^2$$

$$t = \sqrt{467} = \boxed{21.6 \text{ s}}$$

$\therefore$ total time rocket is airborne $= 21.6 + 16.4 = \underline{38 \text{ s}}$.

<u>Graphically</u>:



#9 (2.47) A 15 kg rock is dropped from rest and reaches ground at 1.75 s. When it is dropped on another planet (moon) it takes 18.6 s to reach ground. What is the acceleration due to gravity on different moon?

On Earth given:

$v_0 = 0$
$a_y = -9.8\ m/s^2$
$t = 1.75\ s$
$y = ??$

Eqn of Motion (i): θ^0

$$y - y_0 = \cancel{v_0 t} + \frac{1}{2} a_y t^2$$

Noted: we know the height dropped $(y-y_0)$ is constant in both scenario's ....

Let: $a_{Earth} = -g$ and $a_{En} \Rightarrow$ acceleration due to gravity on Enceladus

Eqn #1: $(y-y_0) = \frac{1}{2} a_E t_E^2$ and Eqn #2: $(y-y_0) = \frac{1}{2} a_{En} t_{En}^2$

Set Eqn 1 = Eqn 2: $\cancel{\frac{1}{2}} a_E t_E^2 = \cancel{\frac{1}{2}} a_{En} t_{En}^2$ when $t_{En} = 18.6\ s$

Solve for a_{En} ... $\therefore\ a_{En} = a_E \left(\frac{t_E}{t_{En}}\right)^2 = (9.8\ m/s^2)\left(\frac{1.75}{18.6}\right)^2$

$= (9.8)(0.009) = \boxed{0.087\ m/s^2}$

#10 (2.58) A brick is dropped from a tall building ... and after a few seconds it falls 40 m in a 1 s interval. What is its displacement in the next 1.0 sec interval?

- assume "after a few seconds" $v_0 = 40\ m/s$

Apply Eqn of Motion (ii): $v_y = v_0 + a_y \cdot t$

So, in the next 1 s interval ...

$v_y(1) = 40\ m/s + (9.81\ m/s^2)(1\ s) = \underline{49.8\ m/s}$

Now, apply Eqn of Motion (iv): $\Delta y = (y - y_0) = \frac{1}{2}(v_0 + v_y)\, t$

$\therefore\ \Delta y = \frac{1}{2}(40\ m/s + 49.8\ m/s)(1\ s)$

$= \frac{1}{2}(89.8\ m/s)(1\ s)$

$= \boxed{44.9\ m}$

#11 (2.56) Given: $y(t) = y_0 - v_0 t + a_m t^2$ where.. $y_0 = 800\ m$, $v_0 = 60\ m/s$, $a_m = 1.05\ m/s^2$

(a) what is the initial velocity at $t=0$?

$$v_y = \frac{dy}{dt} = \frac{d}{dt}\left(y_0 - v_0 t - a_m t^2\right) = -v_0 + 2a_m t$$

$$v_y(0) = -v_0 + 2(1.05)(0) = \boxed{-60\ m/s}$$

(b) what is the velocity just before the lander reaches the surface?

$$y(t) = 0 \Rightarrow y_0 - v_0 t + a_m t^2 = 0 \Rightarrow \underline{t = 21.2\ s}$$

$(28.57\ s \pm 7.38\ s)$

Eqn of Motion (ii): $v_y = v_0 + 2a_m t$

note: we have a factor of "2" this time!!

$$v_y = -60\ m/s + 2(1.05\ m/s^2)(21.2\ s)$$
$$= -60\ m/s + 44.5\ m/s = \boxed{-15.5\ m/s}$$

#12 (2.69) The acceleration of a particle is given by $a_x(t) = -2\ m/s^2 + 3\frac{m}{s^3}t$

(a) find the initial velocity v_0 such that the particle will have the same x-coordinate at $t = 4.0\ s$ as it had at $t=0$?

WET: $a_x(t) = \alpha + \beta t$ where $\alpha = -2\ m/s^2$ and $\beta = 3\ m/s^3$

$$v_x = v_0 + \int_0^t a_x\, dt = v_0 + \int_0^t (\alpha + \beta t)\, dt = v_0 + \alpha t + \frac{1}{2}\beta t^2$$

$$x = x_0 + \int_0^t v_x\, dt = x_0 + \int_0^t \left(v_0 + \alpha t + \frac{1}{2}\beta t^2\right) dt$$

$$= x_0 + v_0 t + \frac{1}{2}\alpha t^2 + \frac{1}{6}\beta t^3$$

for $\underline{x = x_0}$ at $\underline{t_1 = 4.0\ s}$ requires that...

$$v_0 \cancel{t_1} + \frac{1}{2}\alpha t_1^{\cancel{2}} + \frac{1}{6}\beta t_1^{\cancel{3}} = 0$$

Thus,

$$v_0 = -\frac{1}{2}\alpha t_1 - \frac{1}{6}\beta t_1^2 = -\frac{1}{2}\underset{(m/s^2)}{(-2)}(4) - \frac{1}{6}\underset{(m/s^3)}{(3)}(4)^2$$

$$= 4\ m/s - 8\ m/s = \boxed{-4\ m/s}$$

(b) what is its velocity at $t = 4\ s$?

$$v_x = v_0 + \alpha t + \frac{1}{2}\beta t^2 = (-4\ m/s) + (-2\ m/s^2)(4\ s) + \frac{1}{2}(3\ m/s^3)(4\ s)^2$$

$$= \boxed{+12\ m/s}$$

#13 (2.81) An object is moving along x-axis at $t=0$, $v_x = 20$ m/s. Starting at time $t=0$ it has an acceleration $a_x = -Ct$ ($C \to$ m/s³).

(a) what is the value of C if the object stops in 8 s?

Noted: in this problem the acceleration is NOT constant, so we must use calculus instead of the standard kinematic formulas...

Given: $a_x(t) = -Ct$, $v_0 = 20$ m/s, $v_x(8s) = 0$

velocity: $v_x(t) = v_0 + \int_0^t a_x \, dt = v_0 + \int_0^t (-Ct)\, dt = v_0 - \frac{1}{2}Ct^2$

So, $v_x(8) = 0 = 20 \text{ m/s} - \frac{C(8s)^2}{2}$

$$\Rightarrow \quad C = \frac{2(20)}{64 s^2} = \boxed{0.625 \text{ m/s}^3}$$

(b) For the calculated value of C, how far does the object travel in 8 seconds?

Displacement: $x(t) = x_0 + \int_0^t v_x(t)\, dt$

$$= x_0 + \int_0^t \left(v_0 - \tfrac{1}{2}Ct^2\right) dt$$

$$= x_0 + v_0 t - \tfrac{1}{6}Ct^3$$

∴ at $t = 8$ s

$$x(8) = \cancel{x_0}^{\,0} + (20 \text{ m/s})(8s) - \tfrac{1}{6}(0.625 \text{ m/s}^3)(8s)^3$$

$$= 160 \text{ m} - 53 \text{ m}$$

$$= \boxed{107 \text{ m}}$$

(c) At what time(s) is the distance from A to B neither increasing or decreasing?

IDENTIFY: distance from A to B $\Rightarrow$ $x_B - x_A$, so rate of displacement

$$\Rightarrow \frac{d}{dt}(x_B - x_A) = 0 \quad \Rightarrow \quad v_{Bx} - v_{Ax} = 0$$

"or" $v_{Bx} = v_{Ax}$ — solve for t for this to be true!!

#14 (2.83) Cars A and B travel in a straight line. The displacement of A from the starting point is given by

$$x_A(t) = \alpha t + \beta t^2 \qquad \text{where} \quad \alpha = 2.6\ m/s,\ \beta = 1.2\ m/s^2$$
$$x_B(t) = \gamma t^2 + \delta t^3 \qquad\qquad \gamma = 2.8\ m/s^2,\ \delta = 0.2\ m/s^3$$

(a) which car is ~~first~~ ahead just after they leave the starting point?

NOTED: the car that moves ahead of the other will have larger v_0...

So... $v_{Ax} = \frac{d}{dt} x_A = \frac{d}{dt}(\alpha t + \beta t^2) = \alpha + 2\beta t$

at $t=0$: $v_{Ax}(0) = \alpha + 2\beta(0) = \alpha$

$v_{Bx} = \frac{d}{dt} x_B = \frac{d}{dt}(\gamma t^2 + \delta t^3) = 2\gamma t + 3\delta t^2$

at $t=0$: $v_{Bx}(0) = 2\gamma(0) + 3\delta(0)^2 = 0$ $\therefore$ $\boxed{\text{car } \underline{A} \text{ has greater } v_0}$

(b) At what time(s) are the cars at the same point?

$\Rightarrow$ $x_A(t) = x_B(t)$

$\Rightarrow$ $\alpha t + \beta t = \gamma t^2 + \delta t^3 \Rightarrow \alpha + \beta t = \gamma t + \delta t^2$

LET: $0 = \alpha + (\beta - \gamma)t + \delta t^2$

$$t = \frac{1}{2\delta}\left(-(\beta-\gamma) \pm \sqrt{(\beta-\gamma)^2 - 4\delta\alpha}\right)$$

$$= \frac{1}{0.4}\left(+1.6 \pm \sqrt{(1.6)^2 - 4(0.2)(2.6)}\right)$$

$$= 4.00 \pm 1.73\ s$$

$\therefore$ $x_A = x_B$ at times $\boxed{t = 0,\ 2.27 \text{ and } 5.73\ s}$

(c) At what times is the distance from A to B neither increasing nor decreasing?

IDENTIFY: distance from A to B is $x_B - x_A$

rate of change $\frac{d}{dt}(x_B - x_A) = v_{Bx} - v_{Ax} = 0$ (not changing)

$\therefore$ $v_{Bx} = v_{Ax}$

$v_{Ax} = \alpha + 2\beta t = 2\gamma t + 3\delta t^2$

So, $3\delta t^2 + (2\beta - 2\gamma)t + \alpha = 0$

#14 (c) Continued...

$$3\delta t^2 + 2(\beta - \gamma)t + \alpha = 0$$

recall: $\begin{cases} \alpha = 2.6 \text{ m/s} \\ \beta = 1.2 \text{ m/s}^2 \\ \gamma = 2.8 \text{ m/s}^2 \\ \delta = 0.2 \text{ m/s}^3 \end{cases}$

$$\Rightarrow t = \frac{1}{6\delta}\left(-2(\beta-\gamma) \pm \sqrt{4(\beta-\gamma)^2 - 12\delta\alpha}\right)$$

$$= \frac{1}{6(0.2)}\left(-2(1.2-2.8) \pm \sqrt{4(1.2-2.8)^2 - 12(0.2)(2.6)}\right)$$

$$= \frac{1}{1.2}\left(3.2 \pm \sqrt{4(1.6)^2 - 12(0.2)(2.6)}\right)$$

$$= 2.667 \pm 1.667 \text{ s}$$

$\therefore$ $\boxed{t = 1.0 \text{ and } 4.33 \text{ s}}$ when $v_{Ax} = v_{Bx}$

(d) At what times do A and B have the same acceleration?

$$a_{Ax} = \frac{d}{dt} v_{Ax} = \frac{d}{dt}(\alpha + 2\beta t) = 2\beta$$

$$a_{Bx} = \frac{d}{dt} v_{Bx} = \frac{d}{dt}(2\gamma t - 3\delta t^2) = 2\gamma - 6\delta t$$

$\therefore$ when $a_{Ax} = a_{Bx}$ $\Rightarrow 2\beta = 2\gamma - 6\delta t$

$$\Rightarrow 6\delta t = 2\gamma - 2\beta$$

$$t = \frac{2(\gamma - \beta)}{6\delta}$$

$$= 2\,\frac{(2.8 - 1.2)}{6(0.2)}$$

$$= \frac{2(1.6)}{1.2}$$

$$= \frac{3.2}{1.2}$$

$\therefore$ $\boxed{t = 2.67 \text{ s}}$