Interweavings of Alan Turing's Mathematics and Sociology of Knowledge

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Abstract. We start from the analysis of how Alan Turing proceeded to build the notion of computability in his famous 1936 text 'On computable numbers, with an application to the Entscheidungsproblem'. Looking in detail at his stepwise construction, which starts from the materialities to achieve a satisfactory level of abstraction, we will demonstrate how his way of doing mathematics was one that constructs mathematical knowledge by evading a definite separation between matter and form; in this way, making the world and language come together. Following the same line of reasoning, the abstract and the concrete, the deduction and the induction, the technical and the social as well as the objective and the subjective are unthinkable as pure entities. By considering the controversies and discussions from the mid-nineteenth century until now, we can indicate local (social) elements that necessarily participate in what is usually considered 'technical content' or 'objectivity'. While Alan Turing was a precursor of what today might be said to be an 'anthropological approach to mathematical culture', unveiling and reviving approaches that enable the axis of authority for mathematics, logic and computing to be shifted, he also opened different paths for the construction of a variety of mathematical knowledge as well.

Keywords: Computability, Alan Turing, Knowledge Construction

1 Introduction

The practice in the field of mathematics often attributes the soundness and completeness of results to purely deductive reasoning, and the criterion of truthfulness as well as the applicability of results are usually dependent upon confidence in proof. Hence, in the so called 'objective discourse', inductive reasoning, tests and empirical approaches are often rejected. Computer science suggests that this should be open to discussion since abstract (formal) knowledge becomes directly embodied in computer programs, and therefore would seem to make 'immediate' (that is, without mediation) contact with the 'only real world, the one that is actually given through perception, that is ever experienced and experienceable - our every-day life-world' (Husserl, 1954:48-49). For Alan Turing, the empirical question and inductive thought seemed to have been clear from the start. Turing's way of doing mathematics contrasts with that of those who, at this time in the 1930s, were engaged in questions regarding computability. The prevailing view was that mathematical deduction was the guarantee of correct thinking. Hence, a study of Turing's way of working draws attention to approaches that consider diverse factors of diverse natures, these being on the same scale or order as what is usually indicated as 'objective factors' in the construction of the 'objective' facts of science. These approaches may shed new light on questions about the neutrality and universality of mathematical knowledge.

2 Turing: An Ethnographer

In 1936, the mathematician Alan Turing designed a machine, and proposed it as a formal counterpart to the intuitive notion of mechanical procedure (algorithm). His proposition was readily accepted by mathematicians. 'What are the possible processes which can be carried out in computing a number?' was the starting question of Turing (1936). He made the (imagined) experiment of following a human actor in the process of calculating. In order to provide a convincing argument for what is understood by the term 'mechanical' Turing held closely to the material processes observed in the act of calculating and defined an extremely simple device based on the materiality of the assemblage (human + pencil + paper). This approach reenacts the process that a human actant, equipped with pencil and paper, enacts to perform a calculation. Turing's approach relates directly to the ethnographic approaches developed by anthropologists to identify relevant information, this commonly being disguised behind everyday practices. Ethnographers collect and record information to yield analysis. '[T]he good ethnographer is capable not only of good description but of recognizing what elements most warrant attention when ethnography (...) is the intended outcome' (Wolcott, 1987:39). To make explicit the tacit knowledge ethnographers 'are rightly accused of making the obvious obvious' (Wolcott, 1987: 41-42). In order to achieve this, the observer and observed remain so close that it is sometimes difficult to identify who is who.

As an ethnographer who follows traces and behaviour, Turing took into account the details of the activity of the computer: 'We may now construct a machine to do the work of this computer' (Turing, 1936) The word 'computer' was the term adopted by him to describe the assemblage (human + pencil + paper) in the act of calculation. The 1936 paper shows how Turing held obsessively close to materiality as he observed and traced each step in the process of a calculation. He literally stated that the abstract machine he had conceived possessed all the materiality that corresponded with the materiality of the calculating activity of a person with a pencil and paper: 'We may compare a man in the process of computing a real number to a machine which is only capable of a finite number of conditions' (Turing, 1936). This correspondence is accurate enough to consider situations where the man takes a break, so interrupting calculations to resume them later on: 'It is always possible for the computer to break off from his work, to go away and forget all about it, and later to come back and go on with it. If he does this, he must leave a note of instructions (written in some standard form) explaining how the work is to be continued. This note is the counterpart of the "state of mind" '. (Turing, 1936). This process resulted in an 'abstract' conception (apparently devoid of materiality) which is, nevertheless, clearly embodied in the computer machines which emerged shortly afterwards. Although Turing never mentioned this anthropological technique, his approach to the issues of mathematics, and particularly the question of what is 'calculable' or 'computable' is precisely ethnographic.

Twenty years later, Turing reinforced the ethnographic character of his work. In 'Computing Machinery and Intelligence' (Turing, 1950), he proposed a game where a man and a machine have their roles confused as players: 'What will happen when a machine takes the part of A in this game? Will the interrogator decide wrongly as often when the game is played like this as he does when the game is played between a man and a woman?' The ethnographic nature of this approach is explained by (Cohen & Court, 2003): '[T]he ultimate test of an ethnographer would be for a naive member of a specific culture to ask both an ethnographer and a member of that culture to respond to specific questions and perform certain actions. If the judge failed to determine who is the genuine member and who is the "imposter" ethnographer, this would indicate that the ethnographer has fully identified the characteristics of the group'. Turing's commitment to the notion of materiality is also visible in the 1950 paper where he takes into account new elements for a new time: 'He has also an unlimited supply of paper on which he does his calculations. He may also do his multiplications and additions on a "desk machine" ' . For Turing, more than just a style of writing, adherence to empirical facts, inductive reasoning and local conjunctures were a way of thinking.

3 Turing: An Empiricist

In the mid-nineteenth century, John Stuart Mill argued that knowledge initiates with experience. For Mill, an empiricist, the foundations of all of sciences come from experience and observation, and thus, all sciences are inductive, including mathematics (Mill, 1848: 148). A few years later, the argument of Mill was severely criticized by philosophers and mathematicians, for example, Gottlob Frege (1950). When looking for sound foundations for mathematics in the late nineteenth and early twentieth century, these critics attributed supreme power to mathematical truth through an attempt at a purification process. This sought to eliminate from deductive sciences any trace of subjectivity: 'mathematics has become a court of arbitration, a supreme tribunal to decide fundamental questions on a concrete basis on which everyone can agree and where every statement can be controlled' (Hilbert, 1925). In this context, philosophers and mathematicians took any evidence of experience and observation as a threat to the authority of science or even, as a mere joke: 'all Münchhausen's tales are empirical too; for certainly all sorts of observations must have been made before they could be invented'(Frege, 1950:12).

Despite the criticisms, the empirical ideal resisted throughout the twentieth century, with new contributions and questions being posed. About one hundred years after Mill's 'A System of Logics', the empiricist Quine rejected dichotomies in the understanding of science, indicating the need to consider science as a single body, where truth values depend on experience only as border conditions, and are therefore constantly subject to revisions (Quine,1951:39-40). He also rejected 'the belief that each meaningful statement is equivalent to some logical construct upon terms which refer to immediate experience'. As for Quine (1951:20), experiences were peripheral conditions and not attached to statements.

'This is in accordance with experience' wrote Turing (1936) when concluding that the machine should take into account one symbol at a time, since a human would not be able to decide at a glance if the two sequences 9999999999999999 and 999999999999999 are the same. It is clearly evident here that even before Quine, Turing seems to have realized that mathematics and a kind of immediate experience overlap only in a limited way: the inequality between 99 and 999, for example, would be 'immediately' (i.e. without mediation) perceived.

The Turing machine is perhaps the most famous characterization of a formal counterpart to the informal notion of algorithm. The equivalence between the Turing machine and several other characterizations that have been proposed suggests that mathematicians were in agreement about what they thought was calculability. However, as Rogers (1967:20) observes: 'The claim that each of the standard formal characterizations provides satisfactory counterparts to the informal notions of algorithm and algorithmic function cannot be proved. It must be accepted or rejected on grounds that are, in large part, empirical'. By adopting an empirical attitude, Turing faced the problem of formalizing an intuitive notion: 'All arguments which can be given are bound to be, fundamentally, appeals to intuition, and for this reason rather unsatisfactory mathematically. (...) The arguments which I shall use are of three kinds. a. A direct appeal to intuition. b. (...) c. Giving examples of large classes of numbers which are computable. (Turing, 1936). What emerged in Turing's approach was the empiricist nature of knowledge construction, which he reached through ethnographic research.

4 The Mathematics of Turing and Models of Knowledge Construction

Occasionally in the process of knowledge construction, the proximity between the material (matter, thing, object) and the form is no longer so evident. Once the links to the materialities are lost, there is a sense of the 'abstract', of ideas that come out of 'no things'. The pragmatist William James explains this process of purification that produces an epistemological gap between matter (thing/object) and form (idea): 'For we first empty idea, object and intermediaries of all their particularities, in order to retain only a general scheme, and then we consider the latter only in its function of giving a result, and not in its character of being a process. In this treatment, the intermediaries shrivel into the form of a mere space of separation, while the idea and object retain only the logical distinctness of being the end-terms that are separated' (James, 1907:VI).

James defines his approach to knowledge construction as *ambulatory*, in the sense that it considers the intermediate steps that characterize knowledge in every real case, as opposed to the *saltatory* approach which describes results that are only abstractly attained. By highlighting the bridge between object and idea, he undoes the separation between the concrete and the abstract, and brings back to the scene all the particularities and local contingencies that the process of abstraction eradicates.

According to Latour (1999:69), knowledge is constructed by means of a chain of short reversible steps; he named these the 'circulating reference'. Here, Latour's conception of reference differs from Frege's (1892), for whom the reference is a pointer to something in the exterior world. For Latour, each intermediate step in the chain of knowledge construction is only a small gap between matter (thing) and form, and what serves as matter (thing, world) in one step becomes form in the next step. In this chain, what remains invariant when going forward or backward is the reference. Thus, the world and the language are not isolated domains that are linked by the reference. Instead, the chain is a continuum where thing and form are very close: 'If the chain is interrupted in any point, it ceases to transport truth –ceases, that is, to produce, to construct, to trace, and to conduct it. *The word "reference" designates the quality of the chain in its entirety*, and no longer *adequatio rei et intellectus*. The truth values *circulates* here like electricity through a wire, so long as this circuit is not interrupted' (Latour,1999:69).

Turing (1936) enables one to circulate the small steps in the chain *(human + pencil + paper)* $\leftrightarrow$ *state of mind* $\leftrightarrow$ *note of instructions* $\leftrightarrow$ *instruction table* back and forth. What is originally thing (matter) is also idea (form) when a (small) step is taken in the chain of knowledge construction. Thus, the materiality of the amalgamation (human + pencil + paper) becomes the form of the state of mind, which then becomes as concrete as a materia to inspire the form of a note of instructions. As Turing explains: 'If he does this [the computer breaks off] he must leave a note of instructions (written in some standard form) explaining how the work is to be continued. This note is the counterpart of the "state of mind".' Ultimately, the note of instructions took the form of a table with a shape closer to a program for the computing machines which came about a few years later: 'q1 S0 PS1, R q2 q2 S0 PS0, R q3 q3 S0 PS0, R q4 q4 S0 PS0, R q1'. Again, the form serves as thing in the current step. Along this chain from the (human + pencil + paper) computer to the computing machine, something remains unchanged; this makes a tracing back and forward possible, which occurs step by step, and from the most abstract representation of computability to the original materiality. What then remains unchanged here despite the transformations is the reference.

James's approach is resumed in Latour's concept of circulating reference, and can also be seem in Alan Turing's work. Here we will not pursue the kinds of models of construction put forward by authors such as James and Latour, but these models will lead to an understanding of any knowledge, including mathematical knowledge, as historical and local constructions, as 'situated knowledge'.

5 A Sociology of Mathematics

If it is recognized that mathematical knowledge, like any other form of knowledge, is locally produced and kept attachable (by circulating reference) to the conditions of its construction, it begins to make sense to talk about the sociology of mathematics, which could 'explain the logical necessity of a step in reasoning or why a proof is in fact a proof' (Bloor, 1976:84). Such an approach helps to bring out asymmetric power relations which are manifested and strengthened in mathematics. These asymmetries are performative, that is to say, they *determine* mathematical configurations which are often presented and justified by mathematicians as purely technical options. Hence, 'the argument is: although mathematical shapes are historically acquired and learned, they are subsequently naturalized, apprehended, and widely used in enacting the real world' (Marques, 2004). Controversies about the use of formal specifications of software for ensuring correctness exemplify how the authority of what is said to be 'objective', coupled with its naturalized attributes of universality and neutrality, serves as an ultimate guarantee of correctness, so preventing further questions.
Computer systems certification companies adopt standards that ensure reliability in a broad spectrum of applications. The vast majority of computer professionals are not able to understand the complicated proofs of correctness of systems and there are anecdotal evidences that it is easier to understand the programs themselves. However, in practice, if the software is 'certified', it is generally accepted that it is ready to be used even in extremely high risk situations.

In 1999, an arrangement of international organizations called Common Criteria (CC) created a basis for evaluating the security of information technology products. The CC defined seven levels of assurance, establishing a degree of trust directly proportional to an adherence to formal methods. To earn certification the developer *chooses* and formalizes the properties *he considers* indispensable for safety, provides a formal specification of the parts of the software *he considers* critical and proof that *the chosen* properties meet the specification. The last step is then to prove that the program is indeed a refinement (an implementation) of the given specification, and thus meets the properties proved at the formal level. These documents are then analysed by the 'evaluation authority' –a team of specialists whose name reveals the sense of authority provided by mathematics.

A closer examination of the process[3] will reveal, however, that arbitrariness, convention, and hence 'subjectivity' are inevitabily present in the initial stages, when several choices are made by the developer. Being unable to eliminate subjectivity, the formal method, stealthily propagates it throughout the

[3] For an example, see the approach of Heitmeyer (2008) to support a CC evaluation of the separation kernel of an embedded software system. See also the report CCMB-2009-07-003:229 available on the CC web page.

entire process. Therefore, what prevails here is not purely deductive thinking where mathematical entities are said to be built, but the strength of the discourse of true and universal knowledge and a 'centered reality'. This authorizes statements such as 'IT products and protection profiles which earn a Common Criteria certificate can be procured or used without the need for further evaluation' (http://www.commoncriteriaportal.org/ccra/).

The myth of security arising out of the mathematization of software that remains until today had its origins in the discussions sponsored by the North American Department of Defense in the seventies (Makenzie,1996). However, even then the confidence in formal methods was not a consensus: '[L]et us suppose that the programmer gets the message 'VERIFIED.' (...) What does the programmer know? He knows that his program is formally, logically, provably, certifiably correct. He does not know, however, to what extent it is reliable, dependable, trustworthy, safe; he does not know within what limits it will work; he does not know what happens when it exceeds those limits. And yet he has that mystical stamp of approval: "VERIFIED." ' (DeMillo, et al.,1979). Hence, subjectivity was clearly pointed out, but was insufficient to shake the confidence that rested solely on formal methods, and even applies today•

A sociology of mathematics allows for explanations of mathematical facts (such as proved theorems) which distance themselves from explanations of a more absolutist flavor prevailing among the majority of mathematicians. For the Social Studies of Science and Technology, where the universality of knowledge is understood as a mechanism to ensure authority and science is viewed as a local phenomenon, objectivity is addressed in its interweaving with the social; this makes it possible that other elements besides those considered as 'technical' come into play in the composition of the facts regarded as 'mathematical'. An intertwined approach could possibly have placed more light on the *exposure of software to collective processes* as an additional mechanism to ensure correctness; this was raised by researchers, but was not considered by their peers.

Social processes for software correctness: 'What elements could contribute to making programming more like engineering and mathematics? One mechanism that can be exploited is the creation of general structures whose specific instances become more reliable as the reliability of the general structure increases. This notion has appeared in several incarnations, of which Knuth's insistence on creating and understanding generally useful algorithms is one of the most important and encouraging. Baker's team-programming methodology is an explicit attempt to expose software to social processes. If reusability becomes a criterion for effective design, a wider and wider community will examine the most common programming tools.'(DeMillo, et al., 1979). Researchers cited *generally useful algorithms* that took the form of the present design patterns, *team programming methodologies* that nowadays have been improved by the collaborative capabilities introduced through the internet and *reusabily*, which is a key issue in the conception of modern program environments. Finally, there is currently a proliferation of software development methodologies which rely on social collaboration for secure software development. This starts from the assumption that the collective creation, negotiation, discussion and review by multiple agents, among other mechanisms of participation, tend to maximize the chances of success in building a product, especially software•

6 An Open Path to a New Mathematics

The discussions on the issues of the mathematical foundations at the end of nineteenth and early twentieth centuries provided space for an alternative form of mathematics that would be able to survive incompleteness. This trend was reinforced by the materialization of formal systems on computers that appeared in the second half of the twentieth century. Now computers are present in almost all of the everyday life. Computing machines therefore require constant negotiation with mathematics to meet the new conformations of material requirements and to highlight the reconfigurations resulting from the coming together of what would be taken as theory and things of life. An example of this are the controversies giving rise to the IEEE Standard for Floating-Point Arithmetic:
The attempts to define computer arithmetic in the 1970s failed miserably as regards the 'objectivity of arithmetic in solving life-world problems. The core issue was the confrontation of the infinite expansion of certain real numbers and the finite size of computational representation, which certainly requires some form of truncation. Different algorithms were used by different companies (IBM, Digital, HP, Intel, Texas) which generated different results for the same purpose. A comparison between them showed that there were *many decisions to be taken*: 'One specialist cite[d] a compound-interest problem producing four different answers when done on calculators of four different types: $331,667.00, $293,539.16, $334,858.18 and $331,559.38. He identifie[d] machines on which a/1 is not equal to a (as, in human arithmetic, it always should be) and $e\pi - \pi e$ is not zero.' (MacKenzie, 1996:168) Hence, negotiating arithmetic proved to be a long process. A committee began to work in 1977 but IEEE 754, Numbers Fractional Binary Arithmetic, was not adopted until 1985. The crucial point here, is that the stable, consensual human arithmetic against which computer arithmetic could be judged was insufficient to determine the best form of computer arithmetic. (Mackenzie, 1996:182) Following this, extra-mathematical factors emerged and the negotiation of an *agreement* overlapped with the authority of mathematics, resulting in the arithmetic of overwhelmingly prevalent use now.•

These 'negotiations' and 'reconfigurations' resonate with the hypotheses of James (1907: VI) concerning the 'vulgar fallacy of opposing abstractions to the concretes from which they are abstracted', causing a nuisance to those who seek in mathematics a field uncertainties that would be able to provide a rigorous foundation for other fields of pure science.Life runs its course and in its flux requires mathematics to be able to act in uncertainty, to abide creativity, and not to be paralyzed when facing incompleteness. We therefore claim that in some ways Turing opened a path to this 'new' mathematics; as early as 1939, he introduced the notion of an oracle attached to a computation and inaugurated a mathematical intimacy between of what is formal and what cannot be formalized: ‘Let us suppose that we are supplied with some unspecified means of solving number-theoretic problems; a kind of oracle as it were. We shall not go any further into the nature of this oracle apart from saying that it cannot be a machine. With the help of the oracle we could form a new kind of machine (...), having as one of its fundamental processes that of solving a given number-theoretic problem.’(Turing 1939:172-173). Later, in 1960, Computer Science showed a case where the mathematical accuracy and requirements of life were actually mediated by an oracle:

Errors among algorithm certitude: In 1960, the computer scientist Michael Rabin, worked with probabilistic automata, employing coin tosses in order to decide which state transition to take. As a result, he achieved an exponential reduction in the number of states of automata. In 1975, he again used the same idea for adapting a primality test algorithm which, although deterministic, depended on an unproven assumption: ‘With the idea of using probability and allowing the possibility of error, I took [this] test and made it into what is now called a randomized algorithm’ (Rabin, 2010) Even replacing an unproved statement, the possibility of error caused discussions among mathematicians. Rabin showed that the number $2^{400} - 593$ passes his test and hence is a prime ‘for all practical purposes’ (Kolata,1976). Disputes about algorithmic techniques declined in terms of the definition of what constituted ‘practical purposes’. ‘Of Rabin’s contention that probabilistic methods of proof are necessary, Weinberger answers, “I’m willing to be convinced. Just show me one substantial example.” ’ (Kolata,1976) Despite criticisms and rejections, Rabin asserted that research in Randomized Algorithms was growing as an important branch in Computer Science (Shasha, 2010). In the life-world, the mathematics fitted the new realities and became useful for ‘practical purposes’•

Given that it is ‘unspecified, the use of an ‘oracle means recognizing that the ‘objectivity of mathematics cannot control everything. Moreover, the harmonic coexistence of unruliness and formalized mathematics, as it happens in Turing work, indicates a possibility of mathematical action with incomplete knowledge, allowing mathematics to reach out to other domains.

The Metabiology proposed by Chaitin (2011) provides a description of a biologic organism in its essential features, making DNA into a programming language that computes the fitness of an organism. In order to model life, and considering all the possible configurations of biological organisms, metabiology employs the space of all possible algorithms, these being written in a fixed programming language. In the analogy of life as an evolving software, what accounts for biological creativity is randomness, which represents the generation of organism mutations that, being eventually fitter, replaces the previous organisms. ‘[A]nd that is sufficiently simple to permit rigorous proofs or at least heuristic arguments as convincing as those that are employed in theoretical physics.’(Chaitin 2011, p.51). Here, metabiologia moves away from a ‘realistic’ fidelity to biological processes, and thus simultaneously deprives and releases mathematics of its role as a spokesman of a pregiven world. Employing mathematical apparatus over an *inspiration* in biology, metabiology undermines the dichotomy ‘subjective'/'objective' and enriches both biology and mathematics in an interweaving of knowledge. •

The authority of the 'objectivity', contrasted with its powerlessness in solving particular problems in everyday life, had to give way to new forms of mathematics which closely reflect local requirements. The negotiation between the consensual deductive reasoning and the new conformations imposed by local constraints thereby gave rise to new "objectivities". Although not always widely recognized this emergent and negotiated mathematics remains in use, as in the world of life, it is what makes things working.

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