3 << 2: Dai-di Analysis

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Abstract

Chor Dai Di is a popular local card game played usually with four players. Unlike other popular card games like Bridge or Poker, Big Two, as it is also known as, has little formal study made on it. An attempt is therefore made here to employ a systematic approach, not unlike that for the other card games, to formalise the workings of the game, as well as provide an insight to more advance play. With the compilation of a set of useful probabilistic data of the game, heuristics for playing can then be verified and new ones created.

Introduction

Amongst local card players, Chor Dai Di has to rank as one of the most popular, as the game is easy to get started on, though deceptively difficult to master. For some, the appeal of the game lies in its reliance on skill and judgment, though one can of course throw the decisions of game play to chance. As such, we believe that since it is possible to master the game, it must be possible to devise some guidelines for better play.

To better appreciate the game, we had hoped to find out its origins. Unfortunately, the exact history of the game is unavailable, as we have yet to come across any book or documentation of the game. We did find a website on the game, but it had only an outline on the rules. Perhaps, it is then fair for us to make an inference on this apparent lack of information. The game is believed to have originated from Hong Kong, and is predominantly an Asian game, with a close relative being 'President', which in contrast is a Western game. Any way, the game is believed to have begun only this century.

1. Rules of the game

1.1 Objectives

4 players, each dealt a hand of 13 cards from a shuffled poker deck play the game. To win, a player must play out all the cards in his hand. The other non-winning players will attempt to minimize their losses by playing as many cards from their hand as possible.

1.2 Order Of Card Plays

The cards can be cleared from the hand either as single cards, pairs or as 5-card poker plays. The cards are ordered first in terms of their rank, and then their suits. The order of ranks is as follows:

$$2 > A > K > Q > J > T > 9 > 8 > 7 > 6 > 5 > 4 > 3$$

The order of suits is as follows:

♠ > ♥ > ♣ > ♦

Thus, the highest single card is 2♠, followed by 2♥ and so forth, and the lowest card is 3♦. When single cards are played, the next player in turn can play a single card that is of higher order than the previous card played. For example, if the previous player played a 7♦, then the next player can place down a 7♣ or higher card. When pairs of cards are played, the next player can place down a pair of a higher rank, or if a pair of the same rank is to be played, the pair played later must contain the ♠ card of that rank.

When a poker-pattern of 5 cards are played, they are ranked according to the following:

Straight Flush > Four Of a Kind > Full House > Flush > Straight

A Straight is made up of five cards with cards in successive ranks. The highest card in the Straight would determine its order. Thus a Straight led by 8♠ would beat one led by 8♥. Note that the highest Straight is the one led by an Ace and not by a 2, parallel to the order of cards in the game of poker. Thus, AKQJT would beat 65432, which would in turn beat

5432A. A play that contains the A, 2 and K together would not be considered a valid Straight. The highest Straight would be A♠KQJT and the lowest would be 5♦432A.

A Flush is made up of five cards of the same suit. The highest card in the Flush would determine the order of the Flush. A flush led by a 2♠ would beat any other Flush.

A Full House is made up of a triple of one rank and a pair of any other rank. The order of the Full House is determined by the rank of the triple. The highest Full House would be made up of three 2s and any other pair.

A Four Of A Kind is made up of a quadruple of one rank and any other single card. The order of the Four Of A Kind is determined by the rank of the quadruple. The highest Four Of A Kind would be made up of four 2s and one other single card.

A Straight Flush, as the name suggests, is a Straight whereby all the cards are of the same suit. As per the play for Straights, the highest Straight Flush would be AKQJT of ♠. This is known as the Royal Flush of ♠.

1.3 Gameplay

The player with 3♦ gets to start the game, and the first combination played must consist of that card. The card may be played alone, as part of a pair, or as a 5-card play. Thereafter, the player in the clockwise direction may elect to play the same number of cards as before, but of a higher order. He may also elect to pass. This continues until 3 players pass in succession, whereby the player to last put the card may begin with a single card, pair or five-card pattern of his choice. The game ends when one of the players successfully plays out all his cards in hand.

1.4 Stakes

At the end of the game, the non-winners now count the number of cards that they have remaining in their hand and are awarded a point penalty as follows:

- If they have below 9 cards in hand, then they are awarded 1 point for each card.
- If they have 10 to 12 cards in hand, then they are awarded 2 points for each card in hand.
- If they have 13 cards in hand, then they are awarded a total of 39 points.

These points are accumulated in each player's account. The total accumulated points after a fix number of games (maybe 30 games or 50 games) will show the winnings of the players, with the overall winner having accumulated the least points and the overall loser having accumulated the most points.

Thus, besides aiming to win, players must also aim to minimize their losses when they are dealt with a non-winning hand. Players who are dealt a possible winning hand must aim to catch the other players with as many cards-in-hand as possible at the end of the game.

1.5 Game Variations

There are several variations to the game rules described above. Some of the rules that may change locally are as follows:

- *Triplets* or *Three-of-a-kind* plays are allowed in some variations of the game. In this case, three cards of the same rank are played. The order of the play is parallel to that of the Full House.
- In certain variations, the ranking of the 5 card play differs such that for the Straights and the Straight-Flushes, The play with A2345 > 23456 > AKQJT > KQJT9 > … > 76543. There are also certain variations where 23456> A2345 > AKQJT > KQJT9 > … > 76543.

- Certain variations do not allow the 5-card play of Flush. Instead, the next highest class of 5 cards after the Full House is the Straight.

2. Before the first card

2.1 Arranging the cards

Now that you have gotten your hand, there are many things to do before you play out your first card. I find it natural to check whether you have any of the four deuces, and next for 3♦, to see if you are starting first.

For convenience, it would be best to arrange your cards in order of their individual rank, so as to make it easier for you to study them. Of course, this may leave you vulnerable to card-sharks who would study the way you arrange your cards so as to guess your holding, but as I feel that this will open a lot of room for personal interpretations, I will ignore this possibility. Intuitively, one would check for possible five card combinations next, followed by pairs. Upon identifying the possible permutations by which to play the cards, the trick now is to decide which play will maximise your chances of winning.

2.2 Counting controls and stragglers

To begin studying your hand, you have to look at *controls* and *stragglers*. The definition of a control is a card, which upon its being played, will result in the other three players passing consecutively. This in turn means that you will get to decide what category of combinations to be played next, be it singles, pairs or 5-cards, and hence the name. Note that controls can be a single, pair or a 5-carder. For each category, there can only be one highest-ranking combination, namely 2 , 2 2 , A KQJT, it is not significant to be restrict the definition to find 100% controls. Thus we also look out for cards that have a good chance of acting as a control, in that the other three players will pass on it.

Stragglers will then somewhat be the opposite of controls. These are single cards, which have little or no chance to be played, and thus rely on controls for them to be played out. If two stragglers are of the same face value, then you have a *straggler pair*. It may seem trivial to define a straggler pair, but the manner in which it is played can be very different. A straight can usually be classified as a straggler initially, since to have a lower straight played towards it is not very likely.

We use the terms controls and stragglers, but even as they are so defined, controls are not restricted to 2♠ or Royal Flush of spades only, as that would make our definition rather useless.

In the case of 2♣ with 2♠, the smaller deuce makes an effective control, as more often than not, the player with 2♥ will not play it out to cover the 2♣. We call this type of control as a *promoted control*. Another examples are King pairs supported above by an Ace pair. The earlier combination of deuces guarantees one control (assuming single cards are played) and a possibility of two. Taking the chances of either to be roughly equal, the expected number of controls from it will be 1½ control. A holding of 2♥ and 2♣ only guarantees 1 control, even though a 2♥ singly is worth only ½ control, and a 2♣ by itself is worth less than ½ control. This is because 2♠ can only cover either one of the two deuces only, not both. Note that 2♥ by itself can be promoted to be worth a full control once the Big two is played out, but more often than not the 2♥ will need to be supported below, perhaps by Aces, which then is more likely to force the 2♠ out before the hearts deuce.

As for an Ace full house, we will expect it to be worth one control, even though we cannot be certain that it will not be beaten by a four-of-a-kind or a straight flush. This confidence stems from the small probability of any of the higher five carders being held. This is a *probabilistic control*. The probability of this being a control is easily affected by the cards we have in hand. For example, a set of four-of-a-kind tens with a 3 with a 5 full 6 may reduce the number of possible straight flushes to zero, and thus the only five-cards that can beat the four tens will be four deuces, aces, kings, queens or jacks. In this sense, the four tens is promoted.

The third type is a *pre-emptive control*. This is usually applicable only to single cards. This occurs when a card which is sufficiently high ranking, but is not sufficiently supported by higher cards or is very likely to be covered by some higher ranking card (it may be supported below though). The best way to gain a control out of this card is to play it out early, thus taking up playing space. The effectiveness of this form of control arises from the likelihood that the other players have other cards of that category that they need to play out. Thus if they have insufficient controls or promoted controls with which to accommodate the pre-emptive card, they are likely to pass and wait for a more favourable lead, especially so if they have definite controls that they wish to hang on to till they can clear out the lower ranking cards of that category. Note that pre-emptive controls should only be played to clear out a straggler, or a likely straggler, which should be of a different combination from the current one.

2.3 Choosing game plans

Although 5-carders may allow you to play more cards at one time, it may be done at the expense of pairs or some highly ranked single cards. This is often the decision that you have to make while deciding on a game plan, what permutation or combination to play. A good game plan should maximise the chances of you playing out all the cards, as well as leave your opponents with as many cards as possible (will come at a later stage).

One must realise that although it is possible to find a very good game plan, it may not always turn out to be the best as the game goes on. This is because the cards which you had expected to be controls may not function as so, or the combination seemingly of intermediate rank may well become a straggler. Then one will have to think up of new ways to play in face of these situations.

Sometimes, the decision to not play a particular combination may not be due to the hope of promoting it. When there is much uncertainty about the opponents' holding, it may be necessary to hold back in order to preserve the flexibility of the hand.

For instance, one may withhold playing his four aces as part of a five-carder if he needs to guard against the possibility of an opponent running pairs. It must be noted that in order to employ such hold up play, the combination that it held must either consist of sufficiently high single cards, or there must be another good control that can allow the combination to be played. Else, the cards may be stranded in the hand, thus chalking up avoidable points.

This is one of the key skills that differentiate between a good Dai-Di player and a poor player: the ability to size up the situation and play it to his advantage. Therefore a good game plan must always be backed up by as many feasible contingency plans as one can think up of, looking out for the one with the optimal rate of returns.

3. Hand Evaluation

3.1 Mentality behind the approach

Before we begin to decide whether a particular hand is good or bad, we need to define what we mean by goodness. And for that to be possible, we will like to go back to the objectives of the game to begin our analysis.

The aim of the game is to minimize the total score in the long run, which is given by the number of cards that one has left at the end of each deal. This is subjected to doubling for 10 – 12 cards and tripling for 13 cards. To achieve this, one should try to minimize the number of cards in the hand if not all the cards are likely to be played out, and attempt to win if the chance is available. Of course, such judgment is usually uncertain till the play progresses but this does not render our subsequent evaluation useless.

3.2 **Ranking each combination**

For a meaningful reference to combinations in terms of being controls or stragglers, we have chosen to use the 4-classes method of classification. The concept is nothing mysterious. Class A is used to describe combinations, which are absolute or 100% controls at the point in time, as described earlier. For purposes of 5-carders, we usually consider any straight flush to belong to Class A too, as the probability of a higher straight flush being played over it is almost negligible. Class B, in the next lower level of dominance, contains the combinations which have a very high chance of being played out (by dominating another combination) or could serve as a control in one way or another (see three types of controls). Cards that are sufficiently likely to be played out, but with little chance of being a control, will come under Class C. All these are propped up by the Class D, consisting basically of the stragglers or combinations that are unlikely to be played without a control prior to it.

3.3 **Assessing the total strength of the hand**

With an idea of how to rate individual combinations, we can proceed to assess the strength of the hand. In Section 7, we will illustrate our attempts in find a straightforward manner in which to calculate the total strength of the hand. It follows naturally that a hand that has mainly Class A's & B's will be good, and that laden with C's and D's is less than desired. In our approach to assessment of the hand, we essentially wish to look to clear out all of cards. As such, Class D's are of the greatest concern, and the initial plan should aim to play it out. However, when A + B < D, it is futile to attempt to clear all the D's. Game plans will then have to discount some D's in order to clear an optimal number of cards. Below are some useful figures that can help decide how dominant some cards are.

No. of cards of the value in hand	Probability of		
	Pair	Triplet	Quadruplet
0	156 / 703 (two)	143 / 4218	55 / 2019
1	234 / 703	66 / 703	0
2	6 / 19	0	0

4. Evaluation: A Recursive Process

4.1 Possibilities of Promotion

As play progresses and the number of cards, of which the disposition still remains unknown, decreases, the player will have to constantly reevaluate their hands. In the course of the game, opponents will have to play high single cards, pairs or 5-cards in order to either retain control of the play, or to gain control so that they can switch the form of play. After these cards are played, players holding smaller hands will find that the combinations that they are holding, which were of lesser value at the start of the game, have been promoted to become controls. In the previous section, the 4-class method of classifying plays was introduced. The topic being discussed in this section will be how to make use of the classification system to 'discover' Class B plays, and how to play Class B combinations to the fullest advantage.

For example, consider the scenario of a player having the following hand:

The highest 5-card play that the player has is an 8-led Full House. His hand does not contain any Ace, King and 10, 5 or 3, thus allowing the possibility that an opponent may hold a 4-of-a-kind of these ranks. There are also sufficient cards outside his hand to allow for a full house led by a 2, Ace, King, Queen, or 10, all of which will beat his 8-led Full House. Of course, there are also the Straight Flush Combinations, which will also defeat his Full House. With so many possible plays that could exist to defeat his highest 5-card play, the 8-led Full House can be safely considered a class B play and not a class A play. However, in the absolute ranking of 5-card plays, this particular 5-card play is within the top 20 percentile of the ranking list (15.8% to be exact). Thus, it has a high chance of being promoted to becoming a control. The player has few other possible controls. His highest single card is a 2 of Diamonds, 4th in line for a control in that category and his next highest single is the Queen of Diamonds, making it difficult for him to force out the other 3 2s in order to promote his own

card. His highest pair is a Jack pair, and he only holds 2 cards of a higher rank than Jack. Thus, he has no controls in the pair combination either. Thus, we can establish that the player has no class A plays in any of the combinations and only one class B play, the 8-led Full House. This is the only play that has possibility of being promoted to being a control.

The concept of control has already been discussed two sections earlier and obtaining control is vital to winning the game. With only one possible control available from this hand, it is important that the play be played as a control, and not be beaten by other plays. He can choose not to play his class B combination when faced with an option to do so in the hope that his opponents draw out each other's high ranking plays, leaving his own play as the control. For example, if the game starts and the player's left begins with a small flush, the next player plays a larger flush and the player's right plays a 4-led Full House, the player can choose to wait and hope that one of his opponents is still holding on to a 5-card lower than his that can be played, and hope that the 4-led Full House can draw out any higher 5-card plays, thus promoting his 8-led Full House to a control.

The player must however be careful not to hold his 5-card play for too long. Each player can only play a maximum of 2 5-card plays, and in an overwhelming majority of games, the number of 5-cards played per game ranges from 2 to 4. If the player holds his cards for too long, while waiting for his play to be 'certainly' promoted, he risks the chance of ending the game with the play still in hand. If 4 or more 5-cards have already been played when his turn arrives, it is best to play the combination and not risk getting caught with 5 more cards in hand when the game ends. To this end, the player must not make the mistake of classifying high class Cs as class B plays and end up getting caught with the cards at the end of the game.

4.2 Cards To Look Out For

In order to keep track of card promotion possibilities and to look out for controls in opponents' hands, it would be ideal to be able to remember each and every one of the cards that have been played. However, as much as people would love to be able to this, it is beyond

the ability of all but a few memory prodigies. Thus, it would be best to be able to keep track of the most likely controls of each combination that usually occur in games. These recommendations of what to keep track of would be most useful during the early and intermediate phases of the game. During the endgame phase, most of the higher-ranked cards would have been played out, allowing some surprisingly low plays such as straights to become controls.

Keeping track of which 2s and which Aces that have been played is important not only for the single-card combination, but also for the pair play as well. King pairs frequently become control cards during the course of the game. In some rare cases where either the higher cards are very evenly distributed among the 4 players, or they have been used in Full Houses or 4-of-a-kind plays, Queen pairs can become control cards.

For 5-card play, it is important for the player to look at his cards at the start and note down which 4 of a kind-plays are still available to his opponents after going through his hand and to note down which have been played. The higher ranked play of a straight flush is extremely rare (There is only a 0.015 probability of getting such a combination within the 13 card hand.) and it is not worth it to take the effort to compute the probability of the existence of such a combination, when the limited mental resources can be directed toward more significant measures such as remembering the 2s and Aces that have been played. After that, a player with a low Full House or a 2-led flush can watch out for the Full Houses that have been played and thus keep track of the promotion of his 5-card.

4.3 Memorizing The Game Play

As mentioned earlier, remembering all the plays is an extremely difficult task for the average card player. Thus, besides knowing what cards to look out for, it is also useful to learn and make use of some techniques that might be useful in helping to remember cards.

Some players use the elimination method to keep track of the promotion of the combinations in their hand. Players will look at their highest play in each combination and

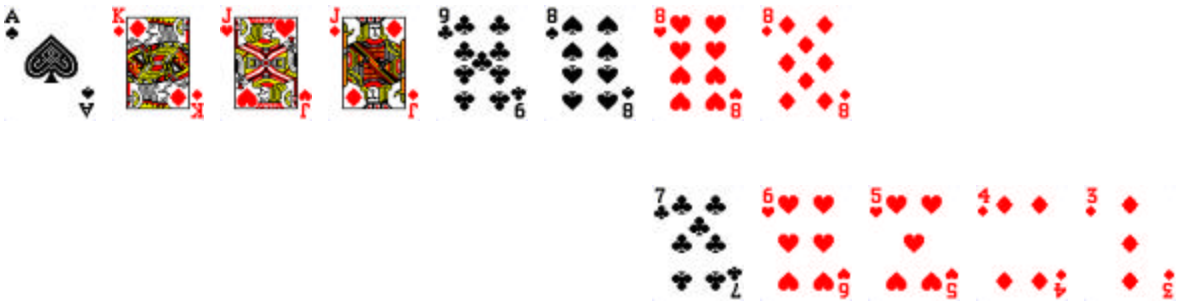
make a mental note of the plays that might beat it, which is extremely limited in number, especially with regards to pairs and single-card plays. They would then eliminate each play as cards are played that remove the possibility of those plays, and thus know when their combination has, with certainty, become a class A play.

5 Know Your Opponent

5.1 Guessing His Cards

After a person gets his hand, the first thing he does is to examine his cards and formulate his own game plan. However, the formulating of the game plan also depends on the hands of the opponents. A simple case will be if one does not have deuce in his hand, he will know for certain that one of his opponents has a deuce pair.♠♥♦♣

Case Study



Looking at the above hand of cards, one can see few ways of playing the cards. One of the ways is to play a 8♠-led Full House, with a 7♣-led straight with A♠, K♦ and 9♣ as single cards. From the previous section, it has been discussed about the possible promotions of Class B plays. Here, it can be seen that the 8♠-led Full House can probably be a promoted control. Also, A♠ may be a control if introduced into the game soon enough, although that is not hopeful since one of the opponents has a deuce pair. Now, what is needed is to see what type of hands the opponents may have, from the way they play their hands, so as to make a smart guess whether the controls in one's hands can be promoted.

Known for certain:

1) there exists at least a 2 pair.
2) there exists at least a Q pair.
3) there exists at least a 10 pair.
4) the only straight flushes possible is
    i) A♣-led royal flush
    ii) 6♣-led or 5♣ straight flush
    iii) K♠-led, 7♠-led or 6♠-led straight flush

To find out as plays goes on:

1) there may be 4 of a kind of 2, Q and 10.
2) there may be Full-Houses led by 2, K, Q, 10 and 9.
3) there may be other Full-Houses led by other numbers that are smaller than 8 and other flushes and straights that are bigger than 7♣-led straight.

Basically, what have been pointed out above are the possible hands that the opponents may have. It must be noted that only the 5-cards combinations are considered here. This is because the promoted control one hopes to get is the 8♠-led Full House and one needs to consider the other 5-cards combinations that are larger than this. As play progresses, if one is observant enough, one may find out what are actually the controls that one actually has.

For example, as play progresses, one notices that when a pair was played, the Q pair was not played out. One may then suspect that a Q-led Full House is present in one of the opponents' hands. Also, with the memorizing techniques introduced in the previous section, one can make a smart guess to what cards are still out there in the opponents' cards and the possible combinations.

Hence, armed with the above information, one may then go for a maximum winnings hand or a minimum losing hand. Also, one may then get an inference of the opponents' game plan.

5.2 Inference of His Game Plan

From the above, after one gets an idea of how the opponents' hands are like. From the progress of play, one can see the playing patterns of the opponents. For example, if one of them is constantly playing pair and subsequently gaining control with him still playing another pair, then it may be time to split one's 5 cards to play pairs to minimize losses.

Using the above example, if the trend is playing of pairs, one may need to split the 8♠-led Full House into a 8 pair which may gain control if sufficient pairs have been played out. This frees up a K♦ flush, which can help to minimize losses.

Hence, with the above inference of the opponents' game plans, one may then decide on the best game plan to proceed with.

6. Checking the Tempo

6.1 Passing and Running

During a game of Daidi, it is not just about playing out the cards on your hand whenever you can. It is extremely important to be able to learn how to hold back your cards as required in order to achieve your objectives, whether it is to minimize your losses or maximize your winnings. Holding back your cards at the appropriate juncture of the game may turn the whole game to your advantage significantly, even if you cannot win the game still at the end. Similarly, playing out your cards too hastily may result in giving away an advantageous game, which you may have won. In this section, we shall look into some of the details about holding back of your cards, such as the kind of situations to exercise it.

Generally, combinations of low ranking, whether it is a single card, a pair or a five-card, should be discarded as soon as possible. Such combinations have very slim chance to be promoted to become a control, therefore it is only wise to discard them whenever possible. As

for the middle-range and absolute control combinations, holding them back in appropriate situations may yield much greater advantages in the later stages of a game. Let us look at the three cases of playing with single cards, pairs and five-carders.

Case 1: Single cards:

It is half way through a game and all 4 players are still holding on to about six to seven cards each. Not all the 2's are being played out yet. The A♥ is being played. You are next to play and you have the A♠, you highest ranked card at hand. If you are thinking of discarding your A♠, think twice. We advise you not to discard it and here are the reasons. A♠ is the next immediate card to A♥, so if the next player after you is going to discard a card, whether you discard or hold back your A♠, it does not affect his play. If you choose to discard it, then your A♠ will seem to be "wasted" because it does not serve much a purpose at all. It is only in the middle of the game and there are still some 2's that can over-play your A♠. Even if none of the remaining 2's are being discarded, you do not lose out still, although you could have gotten a control. The player who gets the control is just immediately before you, so you are second to play after him. By holding back now, the A♠ may serve a better or more critical control at a later stage of the game, say when all the 2's are out.

Case 2: Pairs:

More than half of the total number of cards is being discarded and every player has more than 2 cards remaining. The last played combination is a Jack pair. The next higher pair is an Ace pair consisting A♠ and A♥ and you are the one holding on to it, the highest ranked cards you have at hand. You remembered that only 2♣ is still not discarded from the set of four 2's. You are also very sure that you will only have a single strangler card to clear in order to win. At this juncture, our advice is that you hold on to your pair of aces. You can use one of them to "lure" out the 2♣ and this will give you one absolute control to get rid of your strangler.

Case 3: Five-carders:

The game has just started. Player 1 discards a straight. Player 2 discards a 8-full house. Player 3 passes. You have 2 combinations of five-carders, namely a straight and a A-full house. You should not discard your A-full house yet, although it will most likely give you a control to discard the straight. This is because you will most likely to lose the control again after you discard you straight since it is one of the lowest ranked five-carders. Remember: Player 3 passes earlier on. He may have a flush that will out play your straight. Only at the second round of play, then you discard you're A-full house, followed by your straight. With this, you can then even play out at least one of your 3 remaining cards, instead of just the 2 five-carders.

6.2 Looking for a Favourable Return

The game of Daidi is not just about assessing and playing your own set of cards. You have to take note of all your opponents and the cards that they are holding too. The position of the leading player of a cycle relative to you is important in a game. If the player immediately after you discards a high card and the next two players both pass, in most cases, you should out play that player if you are able to. This is because if he has the control and after he starts a new cycle, you will be the last to discard and this is obviously disadvantageous to you, especially when you have lowly ranked cards. On the other hand, if the next player who is going to have a control is the one immediately before you, then when he starts the next cycle, you will be the second player to discard, which is more advantageous to you than the former case. However, this is only in the general case.

There are exceptional cases when after some play into the game, you are able to deduce the strength and combinations of the hands of your opponents. If the player that is going to have the control next is not the one immediately before you, but if you can deduce from his previous plays the kind of combination that he is going to next and that is the combination that you have a strong one, then it is more advisable to let him have the control. Similarly, if the player immediately before you is quite likely to have the next control, but you deem that he is not going to play in your favour, then you should out play his cards when you are able to do so.

6.3 Pre-emptive Control

By pre-emptive control, we refer to those cards or combinations that are not the absolute control types, but whose ranks are high enough. Such cards, when discarded early enough in a game, are sufficient to earn you a control. This concerns playing with single cards generally and we shall use two examples to illustrate this point.

Case 1: Highest card is 2♦:

You are being dealt with a hand that has 2♦ as the highest ranked card. You have a lowly ranked five-carder to get rid of. The game started with single card play. The best strategy here now is to discard your 2♦ to try to gain a control. Because it is still in the very early stages of the game, your opponents will most likely need to withhold their other 2's in order to clear their stranglers in the later stages. As such, your 2♦ has become a control that serves a great purpose, although it is originally not an absolute control.

Case 2: Highest cards are 2♦ and 2♠

You have a hand that consists of 2♦ and 2♠ as the top two cards. As long as it is not in the final stages of the game with 2♥ and 2♣ not discarded, your 2♦ will most likely provide you with a control. This is due to the fact that 2♠ is with you and thus, your opponent may not want to out play your 2♦ since they do not have the 2♠ and also do not know whom it is with. Hence, 2♦ has given you another control.

7. Search for the Secret Formula

7.1 An indicator

From the beginning, we aimed at writing an intelligent program to play Chor Dai Di, as computer players on the market just didn't seem good enough. We did discover along the way that there was plenty of analysis and groundwork to be done before we can even hope to embark on it, as we were not yet sure exactly how we decide whether a play is better or worse. Nevertheless, we wanted to devise a closed formula using the numerical equivalent of each of the 13 out of 52 cards as input. When we are armed with such a computable indicator, we can then easily have the program assess the hands, aiding subsequent heuristics that we can teach it to employ.

7.2 The 'Experiment'

To achieve this, we needed to think up of a suitable formula for the indicator. This however, is unlikely to be provable analytically. Therefore it necessitates us to test it against a reliable control, and at the point in time, what can serve us more accurately than our own experience and judgment.

Using MatLab, we first randomly generate 120 hands in plain text, which is then converted to PowerPoint slides as shown later in Fig 1. The syntax used for the generation of the hands are included in Appendix A. A total of six people ranked these hands from 1 through to 120, as they deem the fit while following the mentality that was stipulated.

Taking the average ranking attributed to each hand, we used this as a good estimate of the valuation of the hands.

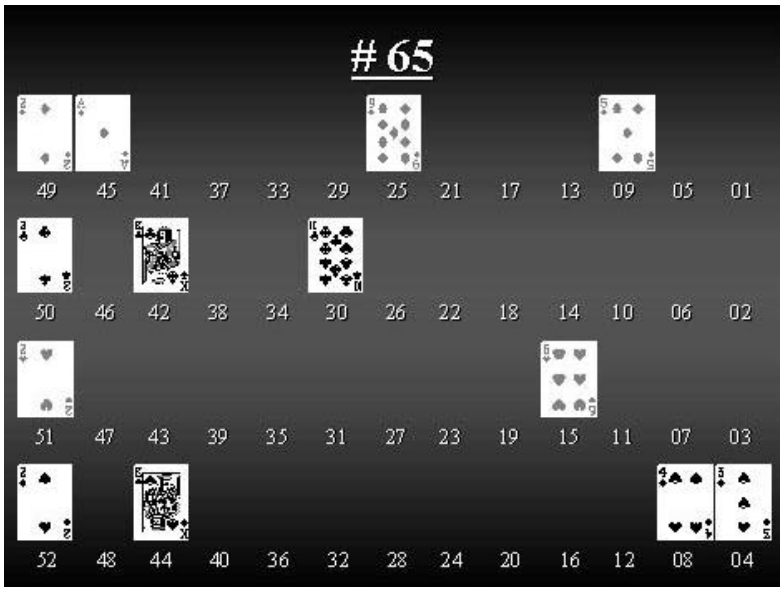


Fig1. Snapshot of Hand #65 out of 120

7.3 Shortlist of Candidates

In the search for the function, we begin by examining a few factors that apparently have a positive correlation with the valuation of its strength. The details of the hands, and the individuals' rankings are documented. Intuitively, we have examined the mean, median and standard deviation of the hand, as well as the highest/lowest cards in the hand.

HCP or high card points is a term borrowed from Bridge, and in our context it denotes the 'average single card' of all the thirteen cards in any single hand. When HCP is high, we naturally expect to see many high-ranking single cards. For example, in #41, HCP = 38.6, which is equivalent of a Q . It is then not surprising to see three Kings and three Deuces in the hand, which of course makes it rather desirable! This hand is incidentally ranked 5th overall. The reasoning we have for using the median is similar.

We expect variance to be positively correlated with the rank, in that a lower variance could mean that the cards are less spread, thus improving chances for straights, full houses and four-of-a-kinds.

The next step for us is to compound these factors together, and to get a function that correlates to our average ranking with a coefficient of at least 0.70. This value may seem too low for implementation, but at the moment it is a reasonable for us to hope to achieve. To give an idea of how to 'compound' factors, we first define $Skew = HCP - Median$. $Skew > 0$ indicates that there are more cards greater than the mean, but this means that these cards above the mean will have a low spread, with a lower likelihood of high-ranking single cards. The cards lower than the mean in order to 'balance out' the cards greater than the mean, will have to be more extreme.

7.4 Findings

As of yet, due to the difficulties we had in ranking the hundred and twenty hands, we have yet to obtain an equation that can give a correlation higher than that given by HCP, which is 0.530. Even with the situation as such, we have discovered that variance correlates positively with its valuation, with a coefficient of –0.2, contrary to our expectation. The low coefficient also tells us that the variance could have very little or no bearing on our valuation of the hand. It was observed that whenever variance was compounded with another factor, the correlation coefficient falls.

From observing our rankings o f the hands, we observed a great range of given ranks for certain hands. As such, we have decided to leave out the highest and lowest rank that were given, hopefully to eliminate much of the human factor that arose from the tedious job of sorting the hands. Given our tight schedule, we were unable to extend the ranking exercise to more hands, much as we would hope it to be, as well as to request for more volunteers to participate in it.

Conclusion

From the beginning, we had undertaken the project knowing fully that it is probably the only one that does not relate to energy, as well as being something very unstructured. Nevertheless, we went ahead, with the blessings from our mentors, in the hope of achieving

some meaningful results that could inspire others to study the game more closely. Perhaps some others with better background in statistics could take the time to work out relevant statistics, which may shed some light on the key to mastering the game. We plan to continue tinkering with our data in hope of presenting some additional information during the presentation.

Acknowledgements

We would hereby firstly like to thank our mentor, Dr Chan Onn, who shared the same interest as us, and had provided us with numerous insights and suggestions into the topic, which we unfortunately were not always able to follow up on. Next, we would have to thank our mentor Khoy Yan and pseudo-mentor Chu Wee for having helped us accomplish the ranking of the hundred and twenty hands, sharing the same late hours as us. Last but not the least to all who had stroked the flame to fire our enthusiasm for the game, through the practical sessions that they had accompanied us through.

Appendix A1

Matlab Program to Generate the 120 Hands Needed for the Experiment.

```
for s=1:5
   A=rand(52,1);[B,I]=sort(A);
   D=0;
   for i=1:4
      for j=1:13
         D(i,j)=I(i+4*(j-1));
      end
   end
   W=0;X=0;Y=0;Z=0;
   for b=1:13
      W(1,b)=b;W(mod(D(1,b)+3,4)+2,ceil(D(1,b)/4))=D(1,b);
      X(1,b)=b;X(mod(D(2,b)+3,4)+2,ceil(D(2,b)/4))=D(2,b);
      Y(1,b)=b;Y(mod(D(3,b)+3,4)+2,ceil(D(3,b)/4))=D(3,b);
      Z(1,b)=b;Z(mod(D(4,b)+3,4)+2,ceil(D(4,b)/4))=D(4,b);
   end
   disp(W');disp('----------------------------------------')
   disp(X');disp('----------------------------------------')
   disp(Y');disp('----------------------------------------')
   disp(Z');disp('----------------------------------------')
end
```

Appendix A2

Matlab Program to Redistribute the Remaining 3 Hands after taking out the 4th.

```
A=rand(52,1);[B,C]=sort(A);D=0;
for r=1:4
   for s=1:13
      D(r,s)=C(r+4*(s-1));
   end
end
E=sort(D')
F=[D(1,:) D(2,:) D(3,:)];
FF=F';
sss=0;fff=0;hhh=0;qqq=0;
for x=1:100
   G=rand(39,1);[H,K]=sort(G);L=0;
   for t=1:3
      for u=1:13
         L(t,u)=K(t+3*(u-1));
      end
   end
   M=0;
   for v=1:3
      for w=1:13
         M(v,w)=FF(L(v,w));
      end
   end
   for i=1:3
      Dm(i)=0;Cb(i)=0;Ht(i)=0;Sp(i)=0;

Th(i)=0;Fo(i)=0;Fi(i)=0;Sx(i)=0;Sv(i)=0;Eg(i)=0;Nn(i)=0;Tn(i)=0;Jk(i)=0;Qn(i)=0;Kg(
i)=0;Ac(i)=0;Di(i)=0;
      for j=1:13
         if mod(M(i,j),4)==1 Dm(i)=Dm(i)+1;
         elseif mod(M(i,j),4)==2 Cb(i)=Cb(i)+1;
         elseif mod(M(i,j),4)==3 Ht(i)=Ht(i)+1;
         elseif mod(M(i,j),4)==0 Sp(i)=Sp(i)+1;
         end
         if ceil(D(i,j)/4)==1 Th(i)=Th(i)+1;
         elseif ceil(M(i,j)/4)==2 Fo(i)=Fo(i)+1;
         elseif ceil(M(i,j)/4)==3 Fi(i)=Fi(i)+1;
         elseif ceil(M(i,j)/4)==4 Sx(i)=Sx(i)+1;
         elseif ceil(M(i,j)/4)==5 Sv(i)=Sv(i)+1;
         elseif ceil(M(i,j)/4)==6 Eg(i)=Eg(i)+1;
         elseif ceil(M(i,j)/4)==7 Nn(i)=Nn(i)+1;
         elseif ceil(M(i,j)/4)==8 Tn(i)=Tn(i)+1;
         elseif ceil(M(i,j)/4)==9 Jk(i)=Jk(i)+1;
         elseif ceil(M(i,j)/4)==10 Qn(i)=Qn(i)+1;
         elseif ceil(M(i,j)/4)==11 Kg(i)=Kg(i)+1;
         elseif ceil(M(i,j)/4)==12 Ac(i)=Ac(i)+1;
         elseif ceil(M(i,j)/4)==13 Di(i)=Di(i)+1;
         end
      end
   end
   ss=0;ff=0;hh=0;qq=0;
   for k=1:3
      st(k)=0;
      if Ac(k)*Di(k)*Th(k)*Fo(k)*Fi(k)>0
st(k)=st(k)+Ac(k)*Di(k)*Th(k)*Fo(k)*Fi(k);end
      if Di(k)*Th(k)*Fo(k)*Fi(k)*Sx(k)>0
st(k)=st(k)+Di(k)*Th(k)*Fo(k)*Fi(k)*Sx(k);end
      if Th(k)*Fo(k)*Fi(k)*Sx(k)*Sv(k)>0
st(k)=st(k)+Th(k)*Fo(k)*Fi(k)*Sx(k)*Sv(k);end
```

```
      if Fo(k)*Fi(k)*Sx(k)*Sv(k)*Eg(k)>0
st(k)=st(k)+Fo(k)*Fi(k)*Sx(k)*Sv(k)*Eg(k);end
      if Fi(k)*Sx(k)*Sv(k)*Eg(k)*Nn(k)>0
st(k)=st(k)+Fi(k)*Sx(k)*Sv(k)*Eg(k)*Nn(k);end
      if Sx(k)*Sv(k)*Eg(k)*Nn(k)*Tn(k)>0
st(k)=st(k)+Sx(k)*Sv(k)*Eg(k)*Nn(k)*Tn(k);end
      if Sv(k)*Eg(k)*Nn(k)*Tn(k)*Jk(k)>0
st(k)=st(k)+Sv(k)*Eg(k)*Nn(k)*Tn(k)*Jk(k);end
      if Eg(k)*Nn(k)*Tn(k)*Jk(k)*Qn(k)>0
st(k)=st(k)+Eg(k)*Nn(k)*Tn(k)*Jk(k)*Qn(k);end
      if Nn(k)*Tn(k)*Jk(k)*Qn(k)*Kg(k)>0
st(k)=st(k)+Nn(k)*Tn(k)*Jk(k)*Qn(k)*Kg(k);end
      if Tn(k)*Jk(k)*Qn(k)*Kg(k)*Ac(k)>0
st(k)=st(k)+Tn(k)*Jk(k)*Qn(k)*Kg(k)*Ac(k);end
      ss=ss+st(k);
      fl(k)=0;
      if Dm(k)>4 fl(k)=fl(k)+ nchoosek(Dm(k),5);end
      if Cb(k)>4 fl(k)=fl(k)+ nchoosek(Cb(k),5);end
      if Ht(k)>4 fl(k)=fl(k)+ nchoosek(Ht(k),5);end
      if Sp(k)>4 fl(k)=fl(k)+ nchoosek(Sp(k),5);end
      ff=ff+fl(k);
      Pr(k)=0;Pa(k)=0;Tp(k)=0;Tr(k)=0;Qd(k)=0;
      if Th(k)==2 Pr(k)=Pr(k)+1;elseif Th(k)==3 Tp(k)=Tp(k)+1;elseif Th(k)==4
Qd(k)=Qd(k)+1;end
      if Fo(k)==2 Pr(k)=Pr(k)+1;elseif Fo(k)==3 Tp(k)=Tp(k)+1;elseif Fo(k)==4
Qd(k)=Qd(k)+1;end
      if Fi(k)==2 Pr(k)=Pr(k)+1;elseif Fi(k)==3 Tp(k)=Tp(k)+1;elseif Fi(k)==4
Qd(k)=Qd(k)+1;end
      if Sx(k)==2 Pr(k)=Pr(k)+1;elseif Sx(k)==3 Tp(k)=Tp(k)+1;elseif Sx(k)==4
Qd(k)=Qd(k)+1;end
   if Sv(k)==2 Pr(k)=Pr(k)+1;elseif Sv(k)==3 Tp(k)=Tp(k)+1;elseif Sv(k)==4 Qd(k)=Qd(k)+1;end
      if Eg(k)==2 Pr(k)=Pr(k)+1;elseif Eg(k)==3 Tp(k)=Tp(k)+1;elseif Eg(k)==4
Qd(k)=Qd(k)+1;end
      if Nn(k)==2 Pr(k)=Pr(k)+1;elseif Nn(k)==3 Tp(k)=Tp(k)+1;elseif Nn(k)==4
Qd(k)=Qd(k)+1;end
      if Tn(k)==2 Pr(k)=Pr(k)+1;elseif Tn(k)==3 Tp(k)=Tp(k)+1;elseif Tn(k)==4
Qd(k)=Qd(k)+1;end
      if Jk(k)==2 Pr(k)=Pr(k)+1;elseif Jk(k)==3 Tp(k)=Tp(k)+1;elseif Jk(k)==4
Qd(k)=Qd(k)+1;end
      if Qn(k)==2 Pr(k)=Pr(k)+1;elseif Qn(k)==3 Tp(k)=Tp(k)+1;elseif Qn(k)==4
Qd(k)=Qd(k)+1;end
      if Kg(k)==2 Pr(k)=Pr(k)+1;elseif Kg(k)==3 Tp(k)=Tp(k)+1;elseif Kg(k)==4
Qd(k)=Qd(k)+1;end
      if Ac(k)==2 Pr(k)=Pr(k)+1;elseif Ac(k)==3 Tp(k)=Tp(k)+1;elseif Ac(k)==4
Qd(k)=Qd(k)+1;end
      if Di(k)==2 Pr(k)=Pr(k)+1;elseif Di(k)==3 Tp(k)=Tp(k)+1;elseif Di(k)==4
Qd(k)=Qd(k)+1;end
      if Pr(k)>0 Pa(k)=Pa(k)+Pr(k);end;if Tp(k)>0 Pa(k)=Pa(k)+3*Tp(k);end;if
Qd(k)>0 Pa(k)=Pa(k)+6*Qd(k);end
      if Tp(k)>0 Tr(k)=Tr(k)+Tp(k);end;if Qd>0 Tr(k)=Tr(k)+4*Qd(k);end
      fh(k)=0;
      if Pr(k)*Tp(k)>0 fh(k)=fh(k)+Pr(k)*Tp(k);end
      if Pr(k)*Qd(k)>0 fh(k)=fh(k)+Pr(k)*4*Qd(k);end
      if Tp(k)*Qd(k)>0 fh(k)=fh(k)+3*Tp(k)*4*Qd(k)+6*Qd(k)*Tp(k);end
      hh=hh+fh(k);
      qq=qq+Qd(k);
   end
   sss=sss+ss; fff=fff+ff; hhh=hhh+hh; qqq=qqq+qq;
end
disp('Total Straights =');disp(sss)
disp('Total Flushes =');disp(fff)
disp('Total Full Houses =');disp(hhh)
disp('Total 4 of a Kind =');disp(qqq)
```

Appendix B

Table of 120 Hands with Individuals' Rankings

No.	The Cards													KM	RK	JA	WC	KX	ZW	Ave
1	11	12	15	18	19	20	26	27	34	44	46	47	51	60	31	32	54	73	27	44.3
2	5	6	8	14	16	22	33	35	38	41	46	51	52	40	8	10	50	19	9	19.5
3	2	9	17	22	23	26	28	34	45	46	47	48	49	69	32	57	64	61	22	53.5
4	1	4	14	15	16	19	25	27	30	31	34	39	46	104	74	53	86	78	113	85.5
5	5	9	25	29	30	31	35	37	39	40	42	45	52	7	6	2	9	13	6	7
6	1	2	7	9	10	18	29	30	36	43	45	47	49	88	60	98	111	105	88	94.8
7	1	3	6	8	13	14	16	18	19	24	27	36	43	111	111	62	101	120	117	110
8	2	10	20	24	28	32	35	36	38	45	49	51	52	36	2	6	2	1	2	3
9	2	3	4	6	7	10	13	16	17	33	48	49	50	42	40	41	61	34	67	46
10	2	3	11	12	14	16	21	26	29	35	42	44	47	120	118	87	115	117	118	117
11	11	12	15	29	30	31	33	34	38	40	42	51	52	38	12	9	6	14	4	10.3
12	1	6	8	11	12	16	18	20	23	29	30	31	42	109	116	86	120	110	95	108
13	1	8	14	21	22	23	24	28	32	36	38	41	43	3	33	33	5	6	112	19.3
14	8	10	13	15	17	22	24	25	28	32	46	50	51	43	47	67	75	55	59	57
15	4	5	7	10	20	21	25	32	37	39	41	44	50	85	95	112	117	42	100	98
16	9	10	13	19	21	22	24	25	27	41	45	46	49	20	36	50	55	40	42	42
17	11	12	13	15	17	19	21	23	26	28	31	40	48	34	96	106	76	32	115	78
18	1	5	6	20	30	31	33	36	38	41	43	49	52	41	49	51	13	37	38	41.3
19	3	5	7	8	9	11	13	17	41	43	44	47	50	18	55	23	8	29	25	23.8
20	2	3	5	7	17	26	32	33	35	36	38	39	52	59	46	45	65	18	45	48.8
21	3	4	20	24	25	27	32	34	37	39	42	44	50	84	84	100	93	67	39	82
22	4	7	9	18	19	23	27	34	37	39	40	45	48	96	92	54	69	107	114	91
23	6	12	18	21	22	23	26	29	33	37	40	42	48	100	103	111	74	89	110	101
24	4	14	15	28	34	37	40	43	44	47	48	50	51	45	26	36	12	52	10	29.8
25	3	4	5	11	12	13	14	20	30	38	47	49	51	44	59	107	106	90	70	81.3
26	1	6	9	13	14	22	23	24	30	31	34	45	48	98	68	56	70	76	107	78
27	7	9	10	16	18	19	20	21	22	33	34	35	52	5	10	4	3	5	12	6
28	3	10	13	14	17	20	22	32	34	36	38	40	43	116	113	116	112	108	116	114
29	1	6	9	15	17	24	28	34	35	37	39	43	45	113	66	110	104	104	69	96.8
30	2	3	5	10	11	17	18	19	21	25	33	41	47	15	56	24	26	30	109	34
31	2	3	4	6	13	14	30	36	38	39	40	46	49	14	29	3	21	36	55	25
32	4	6	12	25	28	29	31	37	42	45	50	51	52	56	14	29	43	9	7	23.8
33	2	8	18	21	26	29	31	32	33	36	40	41	50	71	91	102	78	75	82	81.5
34	6	12	14	15	20	21	23	37	43	47	49	50	52	57	3	38	24	27	8	24.3
35	1	5	15	17	26	29	32	37	41	44	45	50	51	49	43	35	71	46	29	43.3
36	1	2	3	4	11	18	28	31	32	37	38	46	47	94	44	58	20	33	21	39
37	7	10	16	19	22	23	25	27	42	44	46	48	52	72	15	11	66	96	13	41.5
38	2	17	18	25	26	27	29	30	35	36	38	42	45	16	38	91	34	82	86	60
39	8	11	12	23	24	25	27	28	31	42	43	47	48	27	62	15	36	57	48	42
40	8	10	16	19	22	24	29	30	33	34	35	49	50	11	13	25	15	17	15	15

No.	The Cards													KM	RK	JA	WC	KX	ZW	Ave
41	20	27	28	29	35	37	42	43	44	46	49	50	52	2	5	14	18	4	3	6.5
42	1	3	4	7	8	9	10	11	19	32	41	46	51	65	86	48	82	51	89	71
43	2	7	8	9	16	19	27	30	39	44	46	47	49	86	61	84	92	94	68	82.5
44	5	7	9	12	17	25	26	27	36	43	48	51	52	39	16	13	31	44	17	25.8
45	4	7	8	12	15	16	26	32	36	38	39	40	51	9	23	21	28	22	14	20
46	5	13	16	22	24	28	31	33	34	39	40	44	48	33	88	108	35	116	97	82
47	1	5	11	15	18	21	23	24	26	3	35	41	48	21	22	12	33	25	46	25.3
48	6	13	14	15	20	21	23	39	40	41	42	44	45	6	18	34	19	10	64	20.3
49	1	3	4	5	10	11	26	35	38	41	42	46	48	101	65	60	57	84	76	71.3
50	1	2	3	10	12	14	17	28	29	32	36	37	47	107	97	61	118	87	111	101
51	4	19	20	21	25	26	32	34	38	40	43	47	51	77	69	72	119	99	54	79.3
52	1	5	9	10	12	20	27	30	34	35	37	39	45	105	90	78	108	83	92	92.5
53	18	19	21	22	23	27	36	37	39	43	47	49	51	13	21	16	17	47	20	18.5
54	6	7	16	18	19	22	23	26	34	35	39	46	48	26	78	17	29	21	87	38.5
55	1	3	8	10	18	23	24	27	28	31	41	45	48	119	106	104	107	118	106	109
56	4	6	17	24	26	32	36	40	44	48	47	50	52	53	9	27	37	16	19	24.8
57	8	12	13	15	20	28	30	31	32	34	40	44	52	8	11	7	4	23	11	9.25
58	1	2	5	11	13	16	17	20	21	24	26	41	42	35	102	114	38	31	99	68.5
59	5	6	9	11	12	13	16	17	29	37	39	49	52	12	24	8	14	15	18	14.8
60	3	10	11	13	17	30	32	34	36	37	38	47	49	90	93	90	109	100	77	93.3
61	2	6	7	9	14	16	17	24	25	29	33	45	50	81	94	118	98	69	90	90.8
62	6	7	10	19	27	30	31	37	43	46	47	49	50	24	37	19	63	56	30	36.8
63	2	7	14	15	22	30	33	35	36	42	44	46	50	10	25	18	62	39	43	31.3
64	4	12	14	15	16	19	21	22	27	29	45	48	52	30	45	28	56	49	60	45
65	4	8	9	15	25	30	42	44	45	49	50	51	52	1	1	5	1	2	1	1.25
66	3	12	15	18	32	36	38	39	40	45	48	51	52	48	19	31	59	26	28	33.3
67	2	3	8	11	16	18	21	23	28	31	38	42	49	93	117	109	91	115	105	106
68	1	2	5	6	7	9	28	31	35	41	42	44	46	102	82	52	30	85	75	73.5
69	5	11	13	20	21	24	27	31	33	38	40	41	43	115	107	115	77	50	96	98.8
70	4	8	9	14	22	23	25	28	29	33	34	35	44	110	115	105	116	86	94	106
71	7	13	14	15	19	22	25	29	33	41	43	46	51	17	41	26	45	41	66	38.3
72	8	18	20	23	24	25	26	33	39	40	43	50	51	47	51	69	47	68	65	57.8
73	5	13	15	21	22	25	32	34	36	37	40	42	52	63	52	85	67	64	33	61.5
74	1	4	6	12	15	18	20	24	26	27	33	35	48	112	98	103	87	111	103	104
75	5	7	8	9	16	19	32	34	38	40	43	47	52	64	48	65	96	48	56	58.3
76	3	6	12	16	20	28	30	35	36	37	40	48	50	82	71	68	68	66	51	68.3
77	1	3	4	7	8	9	11	12	20	26	39	43	50	78	75	70	83	63	93	76.5
78	2	8	9	11	14	23	29	32	36	37	39	40	43	108	112	97	102	88	104	103
79	1	3	10	13	17	18	31	33	41	42	44	48	50	67	76	44	90	38	79	66.5
80	4	7	9	14	18	22	24	25	26	31	43	46	51	74	83	101	105	98	62	89
81	2	6	10	14	16	18	19	28	30	41	47	48	51	70	27	47	32	7	26	33
82	7	18	19	23	24	25	28	31	39	40	42	50	52	46	42	59	52	53	34	48.3
83	6	14	15	20	21	23	24	26	27	35	36	37	49	73	77	63	49	81	83	73.5
84	13	15	21	28	29	31	32	34	35	42	46	50	52	37	30	20	44	43	16	32.5
85	17	23	24	27	29	31	33	35	38	44	45	46	49	32	72	81	58	97	36	61.8

No.	The Cards													KM	RK	JA	WC	KX	ZW	Ave
86	11	16	21	22	26	36	37	43	44	45	47	48	51	66	58	39	89	35	35	49.5
87	2	4	11	12	22	25	28	29	30	39	45	46	51	80	87	73	110	54	52	73.5
88	2	16	17	18	20	23	24	25	36	38	44	48	51	62	81	42	81	72	58	68.3
89	5	7	13	16	19	21	22	25	30	34	41	42	46	29	110	93	95	102	74	91
90	1	2	4	5	9	10	13	14	27	29	34	38	41	28	99	92	48	93	102	83
91	1	5	11	13	21	27	32	33	38	39	41	44	52	31	39	43	88	28	23	35.3
92	3	4	5	7	8	9	12	14	27	37	41	43	47	106	100	46	72	109	91	92.3
93	3	10	17	28	31	38	44	45	47	49	50	51	52	4	4	1	16	3	5	4
94	3	6	8	12	15	17	20	30	32	33	35	46	49	91	105	80	80	114	84	90
95	2	8	10	15	17	19	23	29	34	42	45	47	49	92	80	74	103	103	61	87.3
96	1	6	10	11	19	22	26	30	33	39	40	45	49	87	79	77	85	65	85	81.5
97	1	2	4	6	10	13	21	24	32	36	44	45	51	68	89	75	79	62	72	73.5
98	4	6	7	9	10	13	15	17	26	34	35	37	44	118	119	120	113	119	119	119
99	2	5	6	13	15	18	19	22	23	28	29	33	40	117	120	118	114	113	120	117
100	6	7	13	16	18	20	21	22	28	34	43	49	52	51	70	79	39	71	53	61.3
101	14	18	20	22	23	27	33	38	40	41	43	50	52	50	50	49	10	60	24	43.3
102	3	8	14	20	22	25	28	29	30	31	39	45	48	99	101	88	97	106	73	96.3
103	1	4	10	14	16	21	24	38	41	43	45	48	51	79	64	83	60	92	50	71.5
104	4	8	10	14	17	33	36	37	38	39	41	46	48	22	7	22	7	20	47	17.8
105	5	8	9	12	16	26	29	34	35	39	46	47	48	95	85	71	84	74	81	81
106	2	5	20	21	30	31	39	40	44	48	49	50	52	54	17	30	25	24	37	29
107	12	17	20	25	27	30	31	32	35	39	42	46	47	97	67	82	53	79	78	76.5
108	1	20	21	23	25	26	28	30	34	35	39	49	51	52	20	40	22	45	40	36.8
109	3	7	11	15	17	19	25	28	30	31	37	42	49	83	54	55	41	8	44	48.5
110	1	7	9	10	11	17	18	26	36	45	46	47	51	61	57	37	42	59	63	54.8
111	3	7	8	9	11	26	34	36	37	44	49	50	52	58	34	66	11	11	31	33.5
112	3	8	9	10	13	15	19	22	24	29	36	46	50	89	104	113	100	101	80	98.5
113	1	5	11	12	21	23	32	40	41	43	49	50	51	55	28	64	23	12	32	34.5
114	3	4	13	22	23	24	28	29	32	33	37	38	43	23	108	95	46	77	98	79
115	1	5	9	19	23	25	29	30	31	42	45	47	51	19	63	96	40	80	41	56
116	2	6	16	18	31	33	38	40	43	44	47	48	52	75	35	89	94	91	49	76
117	2	16	18	19	24	27	33	36	38	42	46	47	52	76	53	76	73	70	57	69
118	6	8	12	14	15	16	19	25	27	34	35	41	42	103	109	94	51	95	101	98.3
119	2	3	11	12	15	24	26	27	32	35	40	44	50	25	73	99	27	58	71	57.3
120	4	5	7	11	12	14	17	27	32	37	41	42	45	114	114	119	99	112	108	112