

Do Diversified Portfolios Effectively Hedge Inflation and Volatility Risks?

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Abstract

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Resumo

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1. **Introduction**

Inflation and Market Volatility are at the core of every investment decision-making process. Inflation will diminish the purchasing-power of your income and the real value of your portfolio’s returns, while market volatility will introduce uncertainty and risk that you may be unwilling to take. Shocks in any of these macroeconomic variables are a major concern for money managers and retail investors. [Bodie (1976)](#Bodie1), defines the effectiveness of common stocks as inflation hedge, as the extent to which they can be used to diminish or eliminate the risk of the portfolio’s real return in what concerns the future levels of the consumer price index (CPI). In this paper we use the same approach for inflation and, as well, for market volatility. Additionally, not only for stocks is this methodology employed, but for all asset classes and portfolios. In that sense, the effectiveness of these instruments as a hedge, will be measured to the extent they can reduce or eliminate the risk that the portfolio’s real returns, will be eroded by uncertainty in future changes in the CPI and in the market’s volatility.

Over the past century, the world has experienced periods of high inflation, such as the German hyperinflation post-World War I, the “Great Inflation” of the 1970s and the post-COVID inflation surge. Volatility has also played a major role in recent financial history. Periods, such as, the Great Depression of 1929, Black Monday on October 1987, the Dot-com crash, the Global Financial Crisis and the latest, the COVID-19 crash, being the most remarkable ones. However, some of these periods are marked for both phenomena being observed, namely the 1970s Stagflation and the post-COVID period. The latest so-called reciprocal tariffs announced by the US highlight the relevance of the studying the topic in this paper.

Many studies have been conducted regarding the effectiveness of certain asset classes in hedging inflation. [Bodie (1976)](#Bodie1), [Nelson (1976)](#Nelson), [Jaffe and Mandelker (1976)](#Jaffe_Mandelker), [Gultekin (1983)](#Gultekin) and more recently, [Boudoukh and Richardson (1993)](#Boudoukh_Richardson_Long_term) and [Anari and Kolari (2001)](#Kolari_Anari), all studied common stocks’ characteristic as inflation hedges. [Jaffe (1989)](#Jaffe_gold), [Baur and McDermott (2010)](#Baur_McDermott) and [Ghosh et al. (2004)](#Ghosh_gold) examined Gold and Gold’s characteristics as a hedge and as a safe-haven asset. Fixed-income securities have also been subject to the study of their hedging ability in [Fama (1976)](#Fama_Bonds), [Fama and Schwert (1977)](#FamaSchwert) and [Campbell et al. (2009)](#Campbelletall). Other fixed-income assets, designed with the purpose of protecting the returns against inflation, commonly referred as TIPS (Treasury Inflation-Protected Securities) were studied in [Bodie (1988)](#Bodie_TIPS) and [Kothari and Shanken (2004)](#Kothari_Shanken). Real Estate studies as inflation hedges were pioneered by [Fama and Schwert (1977)](#FamaSchwert), and maintained their momentum throughout the years, as in [Huang and Hudson-Wilson (2007)](#Huang_Wilson). Concerning the study of market volatility hedges the literature is not so diverse, and most of it like, [Brenner, Ou, and Zhang (2006)](#Zhang_Ou_Brenner), primarily aims at short-term hedging, and [Dew-Becker, Giglio, and Kelly (2021)](#Giglio_Kelly_Becker) and [Bakshi and Kapadia (2001](#Bakshi_Kapandia)) focus on the use of complex and structured derivative products, such as options on straddles and delta hedging. Yet, there are some studies that try to find assets that may be good hedges against equity market risk, such as [Campbell, Serfaty‐De Medeiros, and Viceira (2010)](#GlobalCurrencyHedging) for Foreign Currencies in the case of world equity markets.

The contribution of this paper to the literature resides in the creation of an optimal hedging portfolio that includes different asset classes within different geographies. In other words, there are assets that are effective hedges against inflation, effective hedges against market volatility and effective hedges against both variables. Here we look for the optimal combination of these assets that can increase the effectiveness of these assets as hedges. For that rationale, based on [Markowitz (1952)](#Markowitz) diversification concept, we create an efficient frontier of portfolios that reduces the degree of risk associated to an individual security, without changing the expected return that an investor may get for their investment. Let’s say that the investor has considered to allocate their money into an asset that is a great inflation hedge, but this asset is subject to an idiosyncratic risk that a portfolio of assets that are effective inflation hedges may not be. This asset may also be a good hedge in periods where inflation is, above the central bank’s inflation target, but, overall, at a moderate level. However, when inflation reaches unforeseen levels, the asset fails to maintain the investor’s purchasing power levels. The same logic can be applied to assets that are effectively hedging inflation innovations or even shocks but perform poorly in stable periods. Let us now consider that there is another asset that hedges stock market volatility risk throughout the whole sample period, especially when it carries “bad” uncertainty. Nonetheless, this asset is not an inflation hedging asset, per se. And, if inflation and volatility are two sources of risk that are not perfectly correlated, as we see in this paper, the investor surely wants to find the optimal combination of these assets that allows them to keep returns at the same level while reducing the expected level of risk of the individual assets. As Modern Portfolio Theory (MPT) indicates, it is not the security’s own risk that is important but the impact it makes on the portfolio’s overall variance.

These portfolios will allow to trace an efficient frontier that is built on the shoulders of [Markowitz (1952)](#Markowitz) own. While MPT plots the optimal portfolio is designed in a two-dimensional expected returns-variance (E-V) space. We introduce the three-dimensional plane, adding inflation in the third axis. Bringing this contribution the paper presents this novelty to the finance and portfolio theory literature. For Inflation Hedging literature, this study contributes by studying geographically disperse assets, stock indices, foreign currencies, and comparing the hedging ability of TIPS to multiple asset classes. To the stock market risk hedging literature, the main contributions lie on the study of diverse assets, not including options or swaps, as volatility hedges while focusing on long-term hedging.

Investors care about maintaining purchasing-power, not only regarding the level of risk that affects their portfolio. The marginal increase in adding an inflation and volatility hedging asset can then be measured in the framework we propose. The efficient surface built allows for investors to spot portfolios that provide the same level of return while hedging against another source of risk that is not perfectly correlated with volatility risk.

To build these portfolios, we use strategic asset allocation with a short-selling constraint. The investor buys and holds the same assets at the same weights throughout the whole investment horizon. Nonetheless, for the hypotheses in this paper to hold, it is strictly necessary that inflation and volatility exhibit some degree of orthogonality. Section 2.3 discusses this in a much more detailed manner, but, in a nutshell, it seems that the orthogonality appears to hold for both variables, supporting the variable split and the testing of the proposed hypotheses, even though there is some level of ambiguity in Granger-causality.

Overcoming the most immediate problem with the explanatory variables, the risk of them being highly correlated, we can then present the different questions addressed in this paper.

The main goal of the paper is to find the optimal combination of assets that effectively hedge inflation and volatility consistently, not only during inflationary and/or volatile periods, and that produce a good trade-off between expected return and risk, i.e., that lie on the efficient surface created by the three axes. This allows the investor to make an optimal choice between earning higher returns for the same level of risk or lower risk for the same degree of returns. Additionally, this study builds on the literature of decomposition of macroeconomic variables into anticipated and unanticipated component, to check whether these portfolios are able to effectively hedge innovations in both inflation and volatility. It is almost impossible to talk about inflation and not mentioning the Fisher Hypothesis. Introduced by [Irving Fisher (1930)](#Fisher), poses that nominal interest is, approximately, the sum of the real interest with the expected inflation rate, or, in other words, only the unexpected/unanticipated inflation rate can influence real returns, and with this, laying the foundation for further studies on the impact of the unexpected inflation rate. [Bodie (1976)](#Bodie1), [Fama and Schwert (1977)](#FamaSchwert), and [Boudoukh and Richardson (1993)](#Boudoukh_Richardson_Long_term), and all relevant literature on inflation hedging ability of certain assets, have all studied with the decomposition of expected and unexpected inflation. In volatility literature, past studies have also focused on this split, [French, Schwert and Stambaugh (1987)](#French_Schwert_Stambaugh). Furthermore, comparison tests between commonly regarded inflation-hedging, volatility-hedging and safe-haven assets with the built portfolios have been conducted, with the intention of providing further robustness in the results. These tests also focus on comparison with benchmark portfolios (e.g. 60/40 portfolios) and on the possibility of the portfolios to generate alpha, i.e., an abnormal rate of return, beyond standard factor models (FF3, Carhart, FF5). Finally, we study the effectiveness of diversified portfolios - both across asset classes and geographies – in outperforming the hedging effectiveness and return generating ability of concentrated portfolios.

The paper is organized in the following manner. On Section 2, we describe our sample data and provide an overview of its statistical properties. Complimentarily, we study the relationship between inflation and volatility. On Section 3 we document the empirical methodology employed in our study. Sections 4 provides the results and the analysis with a detailed economic analysis and validation of the hypotheses tested. Lastly, Section 5 concludes the study.

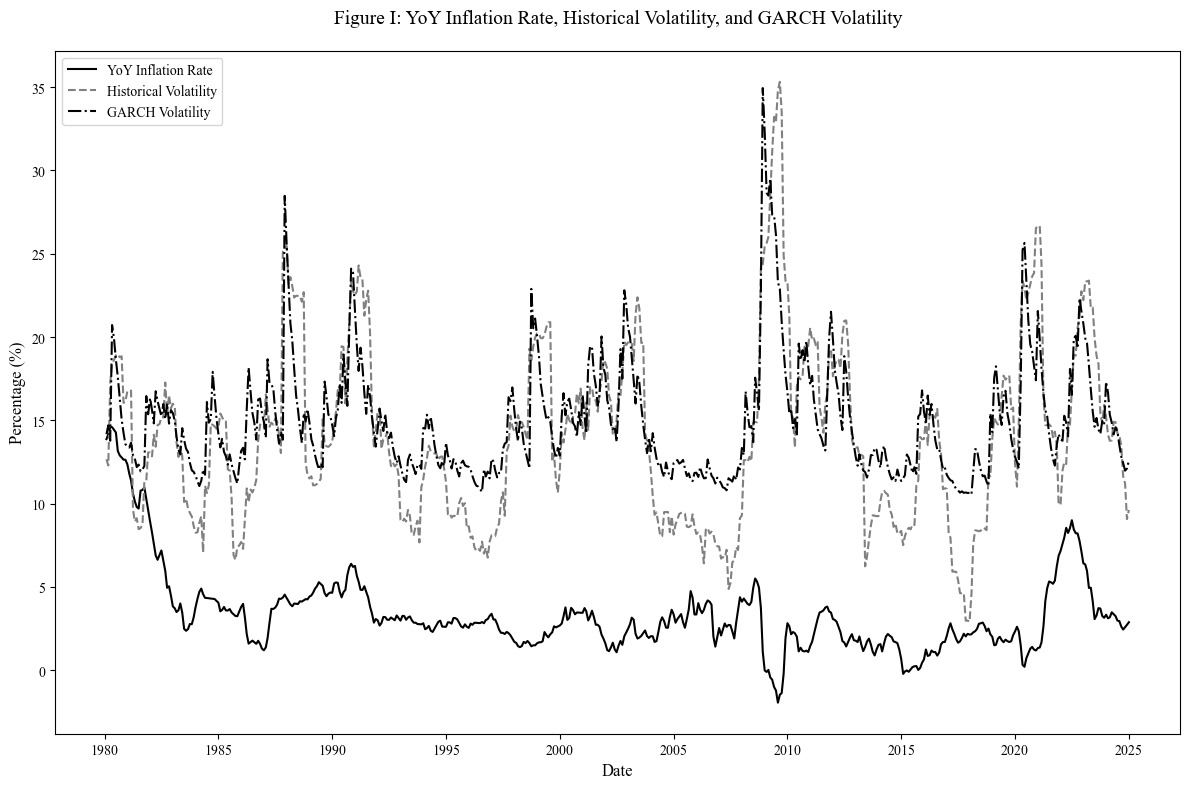
**What the paper finds**. -🡪 Put comparison with literature on the discussion of results

**Extensions/heterogeneity analyses/mechanisms (“extensions+”)**

1. **Data**

**2.1 Data Description**

The period that serves as input is the start of 1980 until the last trading day of December 2024, and all data is analysed and aggregated on a monthly frequency. Inflation rates are estimated using the log difference of monthly values in the US Consumer Price Index for All Urban Consumers from FRED, either in a one-month interval or a twelve-month interval. Throughout the thesis, inflation is reported on a year-over-year basis. Its evolution from 1980 to 2024 is shown in Figure I. Volatility data is calculated from the monthly returns on MSCI World Index, obtained from LSEG. The returns for this series are calculated as the log differential from the monthly closing prices of the index. Two volatility estimates were calculated, an Historical (realized) Volatility Variable, computed from a rolling twelve-month standard deviation of the index continuously compounded monthly returns, and a GARCH (conditional) Volatility variable, using a GARCH (1,1) model, meaning, an order one autoregressive term and an order one moving average term. A GARCH (1,1) model is used to estimate monthly volatility in MSCI World Index returns, capturing the expected volatility clustering. This variable will be mainly used for robustness. Figure I exhibits the small difference between the two volatility measures and highlights the GARCH model’s impact on return tails.

To build the diversified investment portfolios, and efficient surface, several assets were selected. The rationale behind the choices resides in the fact that most of the individual assets have previously been studied in inflation or volatility hedging literature. This selection was, also, done to have as much diversification as possible, on the one hand to have a wide exposure to different asset classes, on the other hand, so that this diversification also happens geographically. For all assets, continuously compounded returns were calculated, allowing an easier computation of cumulative returns, through its additive properties, whilst its statistical properties allow greater symmetry and its approximation to normal distribution, catalysing an increasingly reliable regression analysis and hypotheses testing. The equity indexes chosen are the S&P 500 Index, the Russell 2000 Index, the tech heavy NASDAQ Composite Index, the FTSE 100 Index, the DAX 40 Index, the Euro Stoxx 50 Index, and the Nikkei 225 Index. The reasoning for the choice of these specific assets lies in a mix between most observable data available and market capitalization level, with geographical diversification in mind. These indices’ data was obtained mainly from LSEG Workspace, whilst the S&P 500 data was extracted from CRSP and the Nikkei 225 data was retrieved from Compustat in WRDS. For Real Estate Investment Trusts (REITs), the selection was the FTSE Nareit U.S. Real Estate Index Series, from National Association of Real Estate Investment Trusts database.

Gold, commonly regarded by the average investor as the obvious hedge against inflation risk, is one of the key assets in this paper, its spot price in US Dollars is also extracted from LSEG Workspace. For Oil, the most traded commodity worldwide, this paper will study the NYMEX Light Sweet Crude Oil (WTI) Electronic Energy Future Continuation 1 (CLc1). On Foreign Exchange pair territory, the EUR/USD, JPY/USD, CHF/USD spot rates were selected for the portfolios construction. The reader may argue that the EURO was only launched in 1999, that’s why Refinitiv uses a synthetic rate, following the European Central Bank’s approach, of a weighted average of the currencies of the eleven countries that originally constituted the Eurozone. Due to limited access to Treasury Inflation-Protected Securities data, as a proxy, for the total return index, the chosen asset was the iShares TIPS Bond ETF which tracks the results of an index composed by Treasury Inflation-Protected Securities bonds with different maturities. To conclude the disclosure of the sources for the universe of assets, for the 10-Year Treasury Bond, we selected the data from CRSP on the Total Return Index and the Risk-Free Rate is extracted from Kenneth French’s Data Library.

Data for Expected Inflation is obtained from the University of Michigan Inflation Expectations, a consumer survey that measures the market sentiment for the changes on prices over the next 12 months, the data provider is FRED. Due to the lack of measure of implied volatility for the MSCI World Index, Expected Volatility is derived from the VIX Index, as a proxy, which is retrieved from FRED. The computation is based on the rule of 16, further details available on Section 3.2. It is worth mentioning that the VIX Index data is only available from January 1990, which contrasts with other inflation and volatility measures’ data sources used in this paper. This limitation is taken into account in the relevant empirical results analysis.

Finally, to control for systematic risk exposures, the study uses factor returns from Kenneth French’s data library. This includes the Fama-French three-factor model (Market, SMB, HML), the Carhart four-factor model (adding Momentum), and Fama-French five-factor model, which includes the market risk premium, size (SMB), value (HML), profitability (RMW), and investment (CMA) factors. The data are retrieved from Kenneth French’s online data library.

* 1. **Summary Statistics**

Table I presents the summary statistics for all the assets selected for portfolio construction and for the two macroeconomic variables this paper studies. The study is conducted on 540 months of data and most assets have observable data on all these months, in fact, 15 of the 16 assets have data on more than 82% of the period’s months.

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| Table I | | | | | | | | | | | |
| Panel A: Summary Statistics of Each Asset's Continuously Compounded Monthly Returns (1980-2024) | | | | | | | | | | | |
| **Variable** | **Observations** | **Mean (%)** | **Std Dev (%)** | **Min (%)** | **25%** | **50% (Median)** | **75%** | **Max (%)** | **Annualized Return (%)** | **Skewness** | **Kurtosis** |
| **S&P 500** | 540 | 0.740 | 4.420 | -24.540 | -1.750 | 1.190 | 3.520 | 12.380 | 9.260 | -0.850 | 2.730 |
| **Russell 2000** | 540 | 0.680 | 5.820 | -36.770 | -2.680 | 1.540 | 4.150 | 16.790 | 8.520 | -1.080 | 4.220 |
| **NASDAQ** | 537 | 0.930 | 6.110 | -31.790 | -2.190 | 1.710 | 4.480 | 19.870 | 11.750 | -0.880 | 2.960 |
| **FTSE** | 491 | 0.420 | 4.330 | -30.170 | -1.850 | 0.870 | 3.010 | 13.480 | 5.100 | -1.080 | 5.220 |
| **DAX** | 444 | 0.670 | 5.880 | -29.330 | -2.340 | 1.320 | 4.340 | 19.370 | 8.380 | -0.840 | 2.770 |
| **STOXX** | 456 | 0.370 | 5.270 | -24.190 | -2.450 | 1.110 | 3.760 | 16.600 | 4.550 | -0.760 | 1.980 |
| **Nikkei** | 468 | 0.240 | 5.870 | -27.220 | -2.880 | 0.690 | 4.310 | 18.290 | 2.900 | -0.560 | 1.250 |
| **Gold** | 540 | 0.300 | 4.910 | -25.220 | -2.460 | -0.070 | 3.040 | 24.330 | 3.690 | 0.020 | 3.440 |
| **Oil** | 501 | 0.180 | 10.400 | -78.190 | -5.480 | 0.920 | 6.090 | 63.330 | 2.170 | -0.570 | 9.570 |
| **10Y Treasury Note** | 540 | 0.530 | 2.310 | -6.920 | -0.810 | 0.460 | 1.870 | 9.530 | 6.520 | 0.220 | 0.820 |
| **TIPS ETF** | 252 | 0.020 | 1.750 | -9.230 | -0.800 | 0.180 | 0.940 | 6.300 | 0.220 | -0.970 | 5.460 |
| **NAREIT** | 540 | 0.790 | 5.010 | -35.990 | -1.500 | 1.160 | 3.640 | 24.670 | 9.840 | -1.420 | 8.870 |
| **EUR/USD** | 540 | 0.070 | 2.920 | -9.630 | -1.770 | 0.090 | 1.810 | 10.500 | 0.840 | 0.170 | 0.770 |
| **JPY/USD** | 540 | 0.080 | 3.190 | -9.710 | -1.960 | -0.070 | 1.890 | 16.390 | 0.950 | 0.470 | 1.540 |
| **CHF/USD** | 540 | 0.100 | 3.210 | -11.940 | -2.030 | 0.100 | 2.050 | 12.870 | 1.260 | 0.140 | 0.950 |
| Panel B: Summary Statistics of Inflation Rate and Market Volatility (1980-2024) | | | | | | | | | | | |
| **Monthly Inflation Rate** | 540 | 0.263 | 0.298 | -1.771 | 0.124 | 0.245 | 0.386 | 1.430 |  | -0.140 | 6.300 |
| **YoY Inflation Rate** | 540 | 3.337 | 2.489 | -1.959 | 1.910 | 2.837 | 3.974 | 14.592 |  | 2.100 | 5.920 |
| **Historical Volatility** | 540 | 14.062 | 5.406 | 2.903 | 9.371 | 13.783 | 16.913 | 35.325 |  | 0.760 | 0.770 |
| **GARCH (1,1) Volatility** | 540 | 14.917 | 3.454 | 10.545 | 12.359 | 14.235 | 16.388 | 34.951 |  | 1.810 | 4.970 |
| Note: Volatility data is computed as rolling 12-month standard deviation. The annualized return in the summary statistics table represents the compounded average yearly return of an asset based on its monthly returns. | | | | | | | | | | | |

The Table shows that, based on annualized nominal returns from 1980 to 2024, the best-performing assets were the Equity Indexes from the United States and Germany, along with the

NAREIT Index. Whilst the ones that exhibited the lowest level of returns were the currencies and the iShares TIPS Bond ETF. In what concerns volatility of these assets, one can immediately observe that the instruments with lowest performance throughout this period are also the ones with the lowest volatility.

Table II displays the estimation of the first thirteen autocorrelations of the CPI year-over-year Inflation Rate, estimates of the rolling 12-month standard deviation of returns of the MSCI World Index and the monthly nominal rates of return of the different assets. An analysis of this table shows that autocorrelations at all lags are low and near zero for most assets – except for oil at the first lag – supporting the efficient market hypothesis for equity indexes. Small-cap index Russell 2000 exhibits some short-term mean reversion, since its first order autocorrelation lags are mildly negative. Commodities on the other hand, vary slightly, whilst Oil, has positive and high (when compared to other asset classes) first order autocorrelation, which indicates that its returns gain from momentum from supply or demand shocks. Gold has negative autocorrelation, which may suggest a slight tendency towards mean-reversion over short horizons. The analysis for the 10-year treasury note demonstrates that there is a modest positive autocorrelation, which tends to decay over 1 or 2 lags, meaning that yields usually change gradually and its ability to reflect policy changes. Currencies experience a random walk throughout the 13 order lags reflecting its rather inherent unpredictable behaviour. Finaly, the results for the NAREIT Index show some cyclicality throughout the period under analysis. In any case, the results hold little significance, as all values are very close to 0.

However, the results for Inflation and Volatility are much more interesting to analyse. Monthly Inflation Rate and YoY Inflation rate have very high and significantly different from zero sample autocorrelations, particularly in the short-term ones, suggesting that there is a very significant degree of persistence, with slow movements and small fluctuations, a clear argument that supports the idea that, in the short-term, current inflation influences next month’s inflation, and, in the longer-term and strong memory. The same can be inferred for Historical and GARCH Volatility, high or low levels of volatility tend to stay in that way for months.

Following [Fama and Schwert (1977)](#FamaSchwert) approach, Table III, contrasts the average annualized continuously compounded nominal rates of return with the average inflation and volatility rates in different subperiods, which, provides a general picture of these assets’ ability to hedge against inflation and market volatility.

From this table, we can immediately conclude that all Equity Indices fail to consistently hedge

inflation and/or volatility. The returns exhibited vary substantially in the different sub-periods and suffer when there are highly inflationary and volatile episodes (e.g. 2000-09, 2020-24), indicates the expected relationship between stocks and inflation and/or volatility. Gold, however, provides protection against both macroeconomic shocks, performing remarkably in periods of high inflation, such as the period 2000-09. This reinforces its status as a safe-haven asset, in the way that [Baur and McDermott (2010)](#Baur_McDermott), have defined it. For Oil, though the relationships are less clear, but there is a positive correlation with both inflation and volatility – supporting our expectation that oil is a poor inflation hedge and reflecting its commodity nature, where supply-driven price increases raise production costs and fuel further inflation. Additionally, as we saw in Table I, Oil is the most volatile asset in this universe. The Fixed-income securities in our universe, are expected to have different behaviours during inflationary times, due to the increase in interest rates, whilst Treasury Inflation-Protected Securities (TIPS) are designed to naturally hedge inflation.

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| Table III | | | | | | | | | |
| Average Annualized Nominal Returns of the Different Instruments per 5-year Interval | | | | | | | | | |
| **Asset** | **1980-84** | **1985-89** | **1990-94** | **1995-99** | **2000-04** | **2005-09** | **2010-14** | **2015-19** | **2020-24** |
| **S&P 500** | 9.12 | 16.03 | 5.37 | 25.90 | -3.78 | -1.65 | 12.98 | 9.39 | 12.66 |
| **Russell 2000** | 12.60 | 10.75 | 8.24 | 14.96 | 5.23 | -0.82 | 13.93 | 6.71 | 5.96 |
| **NASDAQ** | 14.23 | 12.89 | 10.53 | 39.52 | -11.83 | 0.85 | 15.75 | 13.56 | 16.45 |
| **FTSE** | 17.36 | 14.39 | 4.81 | 17.59 | -7.05 | 2.37 | 3.93 | 2.81 | 1.62 |
| **DAX** |  | 32.56 | 3.35 | 26.80 | -9.40 | 6.94 | 10.43 | 6.19 | 8.46 |
| **STOXX** |  | 6.82 | 3.75 | 29.64 | -9.70 | 0.09 | 1.19 | 3.54 | 5.49 |
| **Nikkei** |  | 30.93 | -12.78 | -0.81 | -9.55 | -1.70 | 10.55 | 6.26 | 10.97 |
| **Gold** | -9.69 | 5.48 | -1.02 | -5.57 | 8.76 | 19.96 | 1.55 | 5.08 | 11.53 |
| **Oil** | -5.72 | -3.75 | -4.04 | 7.56 | 11.11 | 12.74 | -7.69 | 2.76 | 3.27 |
| **10Y Treasury Note** | 11.81 | 12.90 | 7.82 | 7.58 | 7.76 | 5.07 | 5.87 | 2.51 | -1.92 |
| **TIPS ETF** |  |  |  |  | 4.02 | -0.36 | 1.51 | 0.80 | -1.78 |
| **NAREIT** | 21.20 | 4.27 | 8.47 | 7.68 | 22.33 | -0.92 | 16.51 | 8.42 | 2.78 |
| **EUR/USD** | 16.11 | -9.84 | -0.58 | 4.01 | -5.79 | -1.08 | 3.42 | 1.53 | 1.60 |
| **JPY/USD** | -0.92 | 11.77 | 7.58 | -0.50 | -0.05 | 1.97 | -4.95 | 1.96 | -7.15 |
| **CHF/USD** | -9.34 | 11.00 | 3.31 | -3.83 | 6.90 | 1.90 | 0.82 | 0.54 | 1.30 |
| **Monthly Inflation Rate** | 0.53 | 0.30 | 0.29 | 0.20 | 0.21 | 0.21 | 0.14 | 0.15 | 0.34 |
| **YoY Inflation Rate** | 7.53 | 3.59 | 3.65 | 2.36 | 2.55 | 2.60 | 1.99 | 1.55 | 4.21 |
| **Historical Volatility** | 13.24 | 14.50 | 14.70 | 12.31 | 15.12 | 13.96 | 14.52 | 10.80 | 17.40 |
| **GARCH Volatility** | 14.43 | 15.26 | 14.93 | 13.75 | 15.72 | 15.67 | 14.79 | 13.20 | 16.50 |

Furthermore, during volatile periods, fixed-income securities are considered natural safe-havens and hedges. Real Estate, commonly regarded as an inflation hedge, like shown by [Fama and Schwert (1977)](#FamaSchwert) for the period 1953-71, exhibits good and sustained returns in all sub-sample periods except for the turbulent period of the subprime crisis, as expected. Finally, currencies, display inconsistent behaviours during volatile and inflationary periods, albeit the Swiss Franc which is commonly regarded as a safe-haven asset, can be described in the same manner during these periods, matching the results of [Campbell, Serfaty‐De Medeiros, and Viceira (2010)](#GlobalCurrencyHedging). These inconsistencies may be related to some idiosyncratic and/or country specific risks that are beyond the scope of this paper.

* 1. **Inflation and Volatility: Independent Forces?**

A natural next step is to address the relationship of inflation and market volatility. To do so, we must question whether they evolve independently, and are orthogonal, or if they are correlated or exhibit some degree of Granger-causality. I perform a correlation analysis and a Granger causality test. These two approaches are complimentary. For instance, two variables can have a significantly low correlation or even be totally uncorrelated, and Granger-causality can still detect the predictive relationship between the variables that correlation fails to capture.

The correlation analysis was done to assess the power and path of the linear relationship between inflation rate and volatility rate. This would test if changes in the rate of inflation are associated with simultaneous changes in the volatility rate, i.e., if the variables exhibit a positive correlation, moving together; negative correlation, opposite movements; or independent movements with no significant correlation. This test helps validate the strategy and overall hypothesis in this paper. For this to happen, the explanatory variables need to be weakly correlated or weakly uncorrelated. If the variables are strongly correlated, treating them as separate risk sources may introduce multicollinearity issues to our model. If, conversely, they are uncorrelated or negatively correlated, it provides validity to our approach of separating the two variables into distinct sources of risk in the hedging framework.

The actual result of the correlation test carries practical relevance for this validation. For instance, if the correlation estimate is close to zero, i.e., orthogonal, this implies that inflation and volatility shocks are distinct and occur independently from one another. In this situation, the optimal solution for an investor may be to allocate into assets that hedge each source of risk separately, as we study in this paper. If, however, the correlation coefficient is a strong positive, it indicates that both variables rise together. In this case, the investor should build a portfolio composed by assets that effectively hedge both sources of risk simultaneously. Finally, if the correlation is significantly negative, it suggests that inflation and volatility risks can offset each other. In such a scenario, a 50/50 split between an inflation-hedging asset and a volatility-hedging asset would be the optimal allocation for the investor.

In summary, this study fills the gap in the literature that is the study of this macroeconomic risk sources simultaneously. While many studies have studied volatility independently, , [Brenner, Ou, and Zhang (2006)](#Zhang_Ou_Brenner), [Dew-Becker, Giglio, and Kelly (2021)](#Giglio_Kelly_Becker), [Bakshi and Kapadia (2001](#Bakshi_Kapandia)) and [Campbell, Serfaty‐De Medeiros, and Viceira (2010)](#GlobalCurrencyHedging); and others have studied inflation, [Bodie (1976)](#Bodie1), [Nelson (1976)](#Nelson), [Jaffe and Mandelker (1976)](#Jaffe_Mandelker), [Gultekin (1983)](#Gultekin), and [Boudoukh and Richardson (1993)](#Boudoukh_Richardson_Long_term); none has studied both variables at the same time.

A graph of a graph

AI-generated content may be incorrect.The assessment was done to the variable YoY Inflation Rate with both measurements of volatility. For Historical Volatility the result was -0.0063, whilst for GARCH Volatility the result was -0.0287. Actually, the number of months that exhibit positive rolling 12-month correlation, on average for both volatility estimates, is 225, whilst the number of the ones with negative is 304. The pattern is quite clear, a negligible and very weak negative relationship between the inflation rate and volatility rates. Figure 2 helps us visualize this. It tells us that the correlation levels fluctuate significantly over time, oscillating between strong positive and negative values. Which may indicate some degree of influence from exogenous factors. Even though, there is not a perfect alignment between both measures of volatility, they tend to move in similar directions, capturing similar dynamics of correlation. Essentially, changes in the Inflation Rate do not have a meaningful impact on volatility, which highlights the fact that they are pretty much independent from each other on our data sample. We can therefore conclude that these variables are orthogonal.

It is now required to address the causality that can be exhibited by the variables, in other words, whether one of the variables leads or is able to predict the behaviour of the other. To achieve this, the Granger Causality test assesses the potential of past values of one of the time series to provide statistically significant information to predict future values of another time series. Before proceeding, it is worth noting that Granger-Causality tests are dependent on the fact that both Inflation and Volatility Data are stationary. This stationarity of our variables was verified through an Augmented Dickey-Fuller (ADF) test. The null hypotheses tested here is if any of the variables contains a unit root, i.e., is non-stationary. The p-values for all variables tests are all smaller than 0.05, meaning that we reject the null hypothesis, and confirms that all variables are, therefore, stationary. [Granger (1969)](#Granger_Causality) proposed a statistical framework to test whether a time series past values may be able to predict another time series’ future values. In this study, the test was conducted on both directions of our variables, that is, if inflation’s past values can predict future values of volatility and vice-versa. The application consisted of performing the Granger-causality test at four lags and through all of them, the p-values are high, and well above the 0.05 threshold, confirming that we fail to reject the null hypotheses of “inflation does not Granger-cause volatility”. Meaning that there is no statistically significant evidence that sustains the null hypothesis, i.e., no statistical proof for past values of inflation improving the forecasting of future values of volatility. This applies to both measurements of volatility. It is worth mentioning that the p-values decrease as the lags increase, even though they remain statistically insignificant. This further validates the correlation analysis results, indicating that inflation and volatility are largely independent. More perplexing are the results for the other hypothesis tested. While we would expect some degree of forecasting in inflation data towards volatility, it is actually the other way around on our sample data, as can be seen on Table IV, which displays the results of the conducted tests. It does appear that on longer lags, historical volatility, is Granger-causing inflation, and with past volatility data, one may be able to predict, in the long-term, the future levels of inflation. However, the exact opposite is verified for GARCH Volatility, which indicates that on the short-term lag it rejects the null hypothesis but on the long-term lags it fails to reject it. This suggests that conditional volatility may only contain short-term foretelling power over future levels of inflation, whereas realized volatility appears to provide a more persistent predictive power over longer horizons.

This inconsistency between both “Volatilities”, makes the study of our hypotheses even more interesting. It suggests that different measures of volatility capture different properties of market risk, emphasizing the need to test the robustness of the optimal portfolios hedging different volatility frameworks. The Granger-causality test results indicate that inflation and volatility, depending on the measure, behave as independent variables. Their weak correlation and lack of consistent Granger-causality underscore that they should be addressed as two separate sources of risk, validating the analysis done in this paper. Nevertheless, for investors, however, it does not matter if one can predict the other, or even if they are totally independent from one another. If they are independent, so be it, the goal is finding the optimal portfolio that can effectively hedge them separately or together.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Table IV | | | | | |
| Granger Causality Test Results for Different Lags | | | | | |
| Causality Direction | Null Hypothesis | Lag | F-Statistic | P-Value | Decision |
| Inflation -> Volatility (Hist) | Inflation does not Granger-cause Volatility | 1 | 0.9335 | 0.3344 | Fail to Reject |
| Inflation -> Volatility (Hist) | Inflation does not Granger-cause Volatility | 2 | 1.4190 | 0.2429 | Fail to Reject |
| Inflation -> Volatility (Hist) | Inflation does not Granger-cause Volatility | 3 | 1.5102 | 0.2109 | Fail to Reject |
| Inflation -> Volatility (Hist) | Inflation does not Granger-cause Volatility | 4 | 1.7926 | 0.1289 | Fail to Reject |
| Volatility (Hist) -> Inflation | Volatility does not Granger-cause Inflation | 1 | 3.3763 | 0.0667 | Fail to Reject |
| Volatility (Hist) -> Inflation | Volatility does not Granger-cause Inflation | 2 | 17.0269 | 0.0000 | Reject |
| Volatility (Hist) -> Inflation | Volatility does not Granger-cause Inflation | 3 | 11.8282 | 0.0000 | Reject |
| Volatility (Hist) -> Inflation | Volatility does not Granger-cause Inflation | 4 | 9.4804 | 0.0000 | Reject |
| Inflation -> Volatility (GARCH) | Inflation does not Granger-cause GARCH Volatility | 1 | 2.0662 | 0.1512 | Fail to Reject |
| Inflation -> Volatility (GARCH) | Inflation does not Granger-cause GARCH Volatility | 2 | 1.8816 | 0.1534 | Fail to Reject |
| Inflation -> Volatility (GARCH) | Inflation does not Granger-cause GARCH Volatility | 3 | 1.3434 | 0.2594 | Fail to Reject |
| Inflation -> Volatility (GARCH) | Inflation does not Granger-cause GARCH Volatility | 4 | 1.7756 | 0.1323 | Fail to Reject |
| Volatility (GARCH) -> Inflation | GARCH Volatility does not Granger-cause Inflation | 1 | 5.3869 | 0.0207 | Reject |
| Volatility (GARCH) -> Inflation | GARCH Volatility does not Granger-cause Inflation | 2 | 0.4426 | 0.6426 | Fail to Reject |
| Volatility (GARCH) -> Inflation | GARCH Volatility does not Granger-cause Inflation | 3 | 0.4614 | 0.7094 | Fail to Reject |
| Volatility (GARCH) -> Inflation | GARCH Volatility does not Granger-cause Inflation | 4 | 0.6086 | 0.6566 | Fail to Reject |

1. **Methodology**

**3.1 Measuring a Hedge**

As initially highlighted in the introduction of the paper, we followed [Bodie (1976)](#Bodie1) approach

to measure an effective hedge. While [Bodie (1976)](#Bodie1) focus solely on inflation, I extend the method to measure the effectiveness of the diversified portfolio or asset against volatility. The definition indicates that for our portfolios or individual assets to hedge inflation and / or volatility risks their goal is to minimize the influence of the risk factors associated with these variables on the excess returns of the diversified portfolios or the individual assets used on our hypothesis.

Empirically, our methodology will be primarily based on the equation in [(1)](#Equation1) and will, upon the course of the analysis, evolve into different equations with additional independent variables or incorporate the lagged data around YoY Inflation Rate and both the realized and conditional estimates of volatility. The baseline equation aims to quantify the sensitivity of the assets or portfolios to contemporaneous levels of inflation and volatility. These coefficients will be estimated through an OLS (Ordinary Least Squares) regression. On the left-hand side of equation [(1)](#Equation1), is the nominal rate of return of the portfolio or asset *j,* at time *t*, whereas represents the risk-free rate at time *t*. On the right-hand side of the equation, stands for the coefficient of the YoY Inflation Rate at time *t*, or, in other words, the sensitivity of the excess returns of the asset/portfolio *j* to changes in the inflation rate. Similarly, represents the coefficient of the Historical (realized) Volatility at time *t*, meaning, the sensitivity of the excess returns of asset/portfolio *j* to changes in the volatility. , the intercept, denotes the average excess returns of asset/portfolio *j* after accounting for the impact of inflation and volatility. Finally, , represents, the error term, which captures the variations of the excess returns of asset/portfolio *i* that are not captured by the model. This methodology helps answering the main goal of the paper, to assess whether any of the diversified investment portfolios constructed effectively hedges inflation and volatility. It also fosters the testing of the remaining hypotheses proposed. Questioning whether assets commonly regarded as inflation hedges, safe-haven assets, or volatility hedges exhibit these qualities better than any of the optimal portfolios built. Additionally, if these newly created efficient portfolios overperform while hedging volatility and inflation, more traditional portfolios, such as the 60/40 portfolio.

The “hedge” is measured, as seen, by the degree of sensitivity of the excess returns of portfolios, to either inflation and/or volatility. It is imperative that for the instrument to effectively hedge both variables, the coefficients of and need to be close 1 for inflation beta, meaning that the nominal return of the portfolio increases in line with inflation, hence preserving the purchasing-power of the investor; the volatility beta, however, must be close to 0, implying that return of the portfolio will be independent from stock market volatility shocks. The primary objective of this study is to identify portfolios that simultaneously hedge against both inflation and volatility, while delivering returns comparable to those of traditional inflation-hedging or safe-haven assets, but with reduced exposure to inflationary erosion and volatility-induced risk. Moreover, the presence of a statistically significant and positive 𝛼 in these portfolios would further validate the robustness and reliability of the results.

* 1. **Inflation and Volatility Decomposition**

To assess the implications of the unexpected inflation rate on the return of the assets, it is primarily important to determine the market’s expectations for inflation, i.e., the expected inflation rate. [Fama (1975)](#Fama1) developed a framework for the use of short-term interest rates on US Treasury Bills to predict the inflation rate of the subsequent months, on which he finds a market efficiency, where nominal interest rates contain all the information pertaining to future inflation rates. [Hafer and Hein (1985)](#HaferHein) studied this and other forecasting procedures and concluded that surveying forecasts are the most accurate inflation predictors. It is with this conclusion in mind that on our split of the expected and unexpected component of the inflation rate happens. Expected inflation rate is obtained from the *University of Michigan Inflation Expectations,* a median expected price change of the 12 subsequent months, representing the consumer sentiment regarding future inflation levels. The unexpected component is then derived from the difference between the actual Year-over-year inflation rate and the expected inflation rate. The rationale is that investors may have already priced in expected inflation, whereas unexpected inflation represents a surprise to the markets and thus carries greater explanatory power for abnormal returns.

Innovations in volatility are model exactly in the same sense, the residual between realized or conditional volatility and the expected volatility rate. In the case of volatility, though, the expected component is fetched from the VIX Index, which captures the market participants’ expectations on the next 30 days equity markets volatility. According to some literature, namely [Christensen and Prabhala (1998)](#Christensen_Prabhala), implied volatility, outperforms past volatility in forecasting the future rates of volatility. To calculate the daily implied volatility, we have applied the Rule of 16, which is a simple way to convert an annualized implied volatility measure like the VIX into daily moves. The number 16 comes from the fact that, there are 252 trading days in a month, so to de-annualize the daily moves we should divide the value of the index by the square root of 252, which is, approximately, 16. We then calculate the monthly expected volatility by scaling the daily reads, or in other words, we multiply it by the square root of the number of trading days in a month, approximately, 21.

By isolating these effects, this study aims to better assess how diversified portfolios respond to both anticipated and innovations in inflation and volatility.

* 1. **Avoiding Simultaneity Bias and Endogeneity**

In Section 3.1, equation [(1)](#Equation1) we laid the foundations for the methodology that allows us to measure the sensitivity of the assets or portfolios to inflation and volatility. The exposition of the equation in this manner is strictly because it allows to clearly understand the conceptual model built. However, this contemporaneity, will lead to simultaneity bias and could introduce endogeneity. The regressions performed in this paper will, in fact, rely on lagged explanatory variables, extending up to 5 lags, to mitigate the issues discussed above. In addressing, the simultaneity bias, as [Schwert (1981)](#Schwert_simultaneity) puts it, for instance, there is a delay of more than a month between the time it takes for the data collectors at the Bureau of Labor Statistics and the time when CPI data is published and the time it takes for investors to adjust their portfolios to the news. Similarly, and again emphasizing the importance of the decomposition in expected and unexpected components, volatility data, historical or GARCH-based estimates, are inherently calculated based on prior return observations and are not immediately in the realm of the investor’s current information set. Moreover, even forward-looking measures such as the VIX, which reflects market expectations of future volatility, may not capture immediate investor reactions due to behavioural biases and information processing delays. Therefore, using lagged variables will help to make endogeneity free regressions, that may have been caused by contemporaneous feedback effects from volatility and / or inflation.

Endogeneity and robustness would not be fully addressed if we did not induce further explanatory power to our models. To that end, in this paper we will base it on the common practice in empirical finance, that is, to use asset pricing factor models in the control variables, to fully capture the anomalies in the portfolios’ excess returns. The methodology of the paper will then employ the following models, the Fama-French 3 (FF3) Factor Model from [Fama and French (1993)](#Fama_French_3) which includes systematic sources of risk such as market risk premium, size and value, that intends to capture the outperformance of small-sized companies compared to large-cap companies and the premium exhibited by the firms with high book-to-market ratio when compared with the ones with a smaller ratio. The Carhart four-factor model [Carhart (1997)](#Carhart) which builds on the FF3 Model and introduces the Momentum factor, which aims to translate the tendency of substantial asset returns to be followed by a continuation of the observed outperformance. Lastly, [Fama and French (2015)](#Fama_French) Five-factor model (FF5), which also builds on the FF3 but, adds two different dimensions than the one proposed by Carhart, namely, profitability, that seizes the amplitude between firms with robust and weak operating profitability, and investment patterns, gauging the investment strategy of different firms, comparing firms with aggressive and conservative strategies. It is very clear that failing to control for these variables, i.e. omitting them would lead to biased estimates and thwart the true hedging ability of our portfolios. In summary, these factors will help to isolate the true influence that inflation and volatility as return drivers.

After the above discussions around inflation and volatility decompositions, lagged variables and factor models, all the pieces are now set on the board, and it presents the perfect timing to present the equations on which our model will be based.

To avoid duplication of information, the explanation around the terms in the equations above will only focus on the terms that were not present in [(1)](#Equation1) and/or in the prior equations.

Equation [(2)](#Equation2) presents the result of section 3.2. Where, and represents, respectively, the expected inflation rate and the unexpected inflation rate for time *t*. Likewise, the same approach is done for volatility with and , standing for the expected volatility rate and the unexpected volatility rate for time *t*.

Equation [(3)](#Equation3) showcases the much-needed implementation of lagged control variables, of up to 5 lags, and, in that same rationale, and are the lagged Year-over-year inflation and the lagged rolling 12 month standard deviation of returns, correspondingly.

For equation [(4)](#Equation4), the only innovation not seen in the previous equations concerns the , the coefficient of factor *k* at time *t*, and , the representation of each of the *k* factors at time *t*.

* 1. **The Benchmark Asset Allocation Models**

Idiosyncratic risk is the main reason behind investor’s decision to allocate their funds into portfolios instead of into a single asset. The reasoning is quite intuitive, if we think about our day-to-day life. Individual risk is very difficult to forecast, however the frequency of these risks in a larger pool of events is easier to predict. Let’s say we have decided to allocate one hundred percent of our funds into a single stock. The company’s stock we just bought, has performed quite well over the past years, and is in the business of selling combustion engine vehicles. However, some weeks after we bought the stock, the company announces a plan to pursue a new venture that will expand its production of combustion engines. A few months later, a competitor announces a groundbreaking technology that makes the use of combustion engines inefficient, and the price of the stock of the company we are exposed to, immediately drops. This was not foreseeable for us, as an investor, and for the company whose stock we just bought. This is an example of how idiosyncratic risk can affect our investments. The solution would be to invest in many companies, not just this one. The intuition of our portfolios is the same. In finance, the investors require a certain amount of return for the risk they are willing to take. So, if, according to the aforementioned logic, we increase the number of assets owned and, in turn, the number of asset classes and geographies, the greater the probability of elimination of the idiosyncratic risk. Nevertheless, correlation has a word here. [Haim and Sarnat (1970)](#Haim_Sarnat) argue that if returns are not correlated, diversification can eliminate the idiosyncratic risk, however, if there is correlation between the assets’ returns, no degree of correlation can change risk.

Then, forming portfolios with assets that are uncorrelated and can effectively hedge inflation and market volatility’s risk, while improving the hedging asset’s risk-return profile, should be the best asset allocation strategy possible.

Complimentarily, to the hedging assets and the hedging portfolios comparison, we use different benchmark portfolios based on common strategic investment strategies. All portfolios are built in-sample and tested out-of-sample, are subject to full investment and no short-selling. Rebalancing happens quarterly and its returns are adjusted to reflect the transaction costs when rebalancing. The weights are, within every rebalance period, allowed to drift according to the realized asset returns. If during the portfolios construction, for any asset, the returns data is missing, weights are redistributed, in proportion, to the remaining assets. In the next subsection each portfolio is enumerated and detailed. Below is a remainder of the portfolio’s constraints.

It is worthwhile mentioning that we did not build value-weighted portfolios because of data limitations, and, in some cases, there is not a well-defined measure of market capitalization.

The choice of strategic asset allocation to the detriment of more dynamic allocation, lies on the fact that dynamic allocations, usually require further adjustments in comparison to the strategic allocation models. Dynamic allocations also bring other sets of problems, like misspecification and overfitting, that strategic allocations are less susceptible to. Finally, it aligns with the objectives of the paper, allowing to focus on long-term hedging rather than on adjusting our positions based on short-term fluctuations.

**3.4.1 E qual-Weighted Portfolios / Naïve Portfolio (EW)**

The Equal-Weighted Portfolio or Naïve Portfolio, hereinafter designated as “ew” is constructed to provide maximum simplicity by allocating the same weight to each asset in our investment universe, using the 1/n approach. [DeMiguel, Garlappi, and Uppal (2009)](#DeMiguel) concluded that out-of-sample this allocation strategy is not consistently outperformed by the 14 models analysed in their study, including the optimal portfolios. And the reasoning behind resides on the fact that this approach avoids the estimation error that other models are subject to. It mainly serves as a basilne for evaluating the more sophisticated allocation strategies.

The weights are then given by:

Where corresponds to the weight of asset *j*, and N is the total number of assets in the portfolio.

Besides the portfolio that includes all the assets in the investment universe. We have also built portfolios per nature of their asset class. In that sense, a portfolio that includes only stock market indices (Stocks); one portfolio composed by alternative assets only, i.e., Gold, Oil and the Real Estate Index (Alts); another built by using only Bonds (Bonds); and another built by using only foreign exchanges pairs (FX).

**3.4.2 60/40 Portfolio**

Secondly, we chose one of the most mainstream strategic allocation strategies. A 60/40 portfolio, or a portfolio that allocates 60% into equites and the remaining 40% into government bonds. In our case, the 60% will be equally allocated between the equity indeces () and the remaining 40% ( between the TIPS ETF and the 10 Year Treasury Yield. The weights are given below:

**3.4.3 Minimum Variance Portfolio (MVP)**

The third benchmark portfolio created was one that followed [Markowitz (1952)](#Markowitz) approach, optimizing the trade-off between the mean and variance of portfolio returns. The goal is then to minimize the total portfolio risk. The optimization problem is defined as:

Where w is the vector of portfolio weights and stands for the covariance matix of asset returns. The objective fucntions represent the portfolio’s variance, which, as we have said, we seek to minimize. Solving this problem, provides us the Minimum Variance Porfolio (MVP). That is the portfolio that achieves the lowest level possible of risk for a given set of asset covariances.

**3.4.4 Tangency Portfolio (TP)**

A Tangency portfolio, which maximizes the Sharpe Ratio by optimizing the trade-off between expected excess returns and risk, was our fourth choice. This portfolio represents the point on the efficient frontier with the highest risk-return trade-off, i.e., the highest excess return per unit of risk and is based on the work in [Sharpe (1966)](#sharpe).

*w* and are, once again, the vector of the portfolio weights and the the covariance matix of asset returns, respectively. stands for the vector of the expected excess returns (estimated in-sample). The objective function is a mathematical representation of the Sharpe Ratio. Maximizing the ratio provides the tangency portfolio, which lies at the tangency point of the Capital Market ine and the efficient frontier of risky assets. This tangency ensures that the portfolio offers the highest Sharpe Ratio among the universe of portfolios.

**3.4.5 Hierarchical Risk Parity (HRP) Portfolio**

The fifth portfolio built followed the [Lopez de Prado (2016)](#LopezdePrado) HRP approach. Hierarchical Risk Parity allocates weights to the assets by clustering these assets based on their correlation, then it recursively allocates the risk across the clusters to minimize risk concentrations.

It aims at diversifying risk without relying on the direct inversion of the covariance matrix. Instead it calculates the matrix and transforms it into a distance matrix, which quantifies pairwise similarity. Using this distance matrix, the clustering is then done based on the similarities between the assets. The covariance matrix is then rearranged to reflect the clustering. The reordered list is recursevely separated into two clusters at each step. At each split, the risk in the portfolio is allocated between the two clusters in inverse proportion to their variances. This process continues until each cluster is reduced to the number of assets in our investment universe.

**3.4.6 Minimum Correlation Portfolio (MCP)**

To follow a moderner approach to asset allocation, we have also tested the Minimum Correlation Portfolio. A less conventional approach introduced in [Varadi et al. (2012)](#varadi). In this model, the weights are allocated to each asset by minimizing the average pairwise correlation among those assets in our investment universe. This helps us diversify our portfolio by avoiding the selection of assets that move together, ultimately reducing the overall risk of the portfolio.

Its optimization problem is given in:

The correlation coefficient is presented by . The numerator of the function represents the weighted sum of all pairwise correlations in the portfolio, subtracting the variances, in order to dedicate the focus to on cross-asset movements. The denominator serves as a normalization factor based on the Herfindahl Index. This ensures that we penalize concentration and the proper scaling of the correlations measured.

**3.4.7 Maximum Diversification Ratio Portfolio (MDR)**

The next portfolio built focuses on maximizing the diversification ratio, defined as the ratio of the weighted sum of individual assets’ volatilities to the portfolio volatility. Once again, the goal is to achieve diversification. The optimization will encourage the portfolio to allocate more to assets that are less correlated with the remaining assets, i.e., allocating more weight to the assets that contribute the most to diversification. This model was based on [Choueifaty and Coignard (2008)](#maximum_diversification). Its formulated as:

The volatility of asset *j* is represented by . The numerator calculates the weighted sum of the individual asset volatility. The denominator corresponds to the overall portfolio volatiity. Maximizing this ratio guarantees a portfolio allocation in which risk is more uncorrelated, enhancing diversification.

**3.4.8 Risk Parity Portfolio (RPP)**

Our eighth portfolio is based on the [Maillard, Roncalli and Teiletche (2010)](#RiskParity) equally weighted risk contributions. We call it Risk Parity, and its optimization is based on the sum of the squared differences between each asset’s risk contribution and the average risk of the securities universe. This approach is designed to avoid concentration in a single asset, balancing risk contributions instead of capital allocations.

The marginal risk contribution of asset *j* to the portfolio is given by . The term represents the average risk contribution of the assets in the portfolio.

**3.4.9 Minimum Semivariance Portoflio (MSVP)**

[Estrada (2008)](#estrada) proposed a model that followed Markowitz’s mean-variance approach. It is not so well-known that Markowitz also considered another measure of risk. According to Estrada, semivariance has an entire chapter on semivariance on his 1959 book Portfolio Selection: Efficient Diversification of Investments. We follow [Estrada (2008)](#estrada) and apply a similar methodology to minimize semivariance. Semivariance measures the average squared deviation of the nominal excess returns below a target threshold, in our case, 0. Variance penalizes both “good” and “bad” volatility, while semivariance will only focus on negative deviations.

Here, the optimization question is formulated as:

The vector of portfolio weights is . While is the vector of asset returns at time *t*. represents the target return threshold, as mentioned, in our case it is set to 0. Finally T, is the total number of returns observations. This objective function calculates the average squared shortfall of portfolio returns to 0. Considering only the periods in which the asset underperformed this threshold.

**3.4.10 Treynor-Black Portfolio (TB)**

In tenth place, we developed the classical Treynor-Black Model that resulted from [Treynor and Black (1973)](#TreynorBlack). This model combines an actively managed portfolio, which is built using pre-estimated alphas and the variance of the residuals, with a passive market portfolio, in our case the MSCI World Index. The final portfolio is a mixture of both, the weights are determined by the Information Ratio (IR) of the active portfolio and the Sharpe Ratio (SR) of the market.

For each asset *j,* we estimated its alpha and variance by using a CAPM regression. This regression was used for regressing the excess returns of each asset on the excess returns of the market portfolio. Then the weights for the active portfolio were computed by the following formula: . We then computed the IR for the active portfolio and the market’s SR. The optimal mix is given by .

**3.4.11 Maximum “Sharpe” per Beta Sensitivity (SBS)**

Finally, based on the Tangency Portfolio, we built a portfolio that aims at maximizing returns per unit of sensitivity to inflation and volatility. The sensitivity is based on the first lags of the year-over-year inflation rate and conditional volatility. The optimization process is then, on maximizing this Sharpe-like ratio. The optimization problem stands like this:

The first term in the denominator is the penalization given for deviations from a unit of sensitivity to inflation. The second term penalizes any exposure to conditional volatility. The final term is a L2 regularization term to discourage over-concentration.

**3.4.12 Inflation-Hedging (IHP) and Safe-Haven Portfolio (SHP)**

The last two benchmark portfolios built are directly related to the existent literature or their natures. We build two portfolios, one for assets that have been found to be inflation hedges or, due to their nature, should be natural inflation-hedges. And another that is comprised of assets that have been found to be safe-haven and inflation-hedging assets.

In that sense, IHP includes: Gold, following the results of [Ghosh et al. (2004)](#Ghosh_gold) and the Real Estate Index after [Fama and Schwert (1977)](#FamaSchwert). To these assets we had the Treasury Inflation-Protected Securities (TIPS) and oil because of its nature and the fact that it is included in the CPI calculation.

[Baele et al. (2020)](#Baele) provides evidence of the CHF being a flight so safety currency, and is thus included in SHP. This portfolio is also comprised by TIPS due to [Bodie (1988)](#Bodie_TIPS), the Real Estate Index and the 10 Year Treasury Bond based on [Fama and Schwert (1977)](#FamaSchwert).

* 1. **Optimal Portfolio with Inflation Constraint**

This section describes the methodology used to build the efficient portfolios that account for market volatility risk and inflation risk. Building on [Markowitz (1952)](#Markowitz), mean-variance optimization framework, this study introduces two nuances into the asset allocation problem. First, instead of minimizing only the volatility of the portfolios, its goal is to minimize the sensitivity of portfolio returns to global market volatility and, at the same time, having a beta to inflation that is close to 1 since we are dealing with nominal excess returns. In visualization terms, excess returns, as in the mean-variance context are represented in the y axis, whilst market volatility risk is represented in the x axis. Finally, inflation risk is plotted on the z axis. The efficient surface allows the investor, based on their risk aversion profile, to choose a portfolio that achieves an optimal trade-off between return, market volatility and inflation sensitivity. The efficient frontier that emerges from this study is not a curve, as in the mean-variance optimization, but instead a convex surface in a three-dimensional space as we saw earlier. The solution obtained, a Pareto-like, is a set of portfolios, and in each point lies a portfolio that cannot be improved in all three dimensions simultaneously. The surface introduces new trade-offs and interactions between asset returns and macroeconomic variables that are overlooked in a two-dimensional framework. Investors with a high degree risk aversion to both inflation and stock market volatility risks, may select portfolios closer to the origin. Whereas investors with higher risk tolerance for inflation, may place themselves in allocations where inflation’s beta is closer to one. The innovation of this framework, as already highlighted, resides not only on the analytical evaluation but also on its practical relevance. Over the past years, inflation has been elected, by both consumers in the US and Europe, as their main concern. Having proved that inflation and volatility pose different risks to the investors, establishing this trade-off scenario is quite valuable. The optimization problem is defined in:

Subject to:

This expression describes our multi-objective optimization, where the investor maximizes the expected returns while minimizing the sensitivity to the two independent risk sources in inflation and volatility. stands for the vector of portfolio weights, one for each asset in our universe. represents the vector of the expected nominal excess returns for each asset. The combination of these two terms, while transposing the vector of the weights, represents a matrix multiplication that has as output the portfolio’s expected return. and , are the vectors of each assets’ beta with respect to volatility and inflation, respectively. and are the investor’s risk aversion coefficient to volatility and inflation. A higher value in each of these coefficients means that the investor has less risk tolerance for the risk source, penalizing the corresponding variable strongly in the optimization problem. The absolute value in the volatility array ensures that both positive and negative exposures to volatility are considered unwanted. A similar reasoning is applied to the inflation array, it punishes the squared deviation from the optimal inflation beta of 1. This optimization is subject to some constraints, similarly to the ones imposed in the previous section. Meaning that there is a short-selling restriction and rebalancing once per quarter. In that sense, the optimization involves a constrained search for efficient weights that minimize the impact of inflation and volatility risk on the portfolio’s returns. A volatility-inflation-return relationship. Portfolios that occupy the efficient region in the three-dimensional surface achieve the desired hedging properties that our paper looks in this asset universe. The betas used in this model, are combination of the betas resulting from regressing the expected and unexpected components of our macroeconomic variables, on the assets’ excess returns. The last portfolio, Efficient Surface Portfolio (ESP), of our set will be the one that has the best efficiency score in the surface. This score penalizes deviations from the target betas and lower expected returns, ensuring that the portfolio’s betas are as close to 1 for inflation as possible and to 0 for volatility.

Ultimately, the goal is to evaluate if diversified portfolios can successfully hedge inflation and stock market volatility without compromising the target returns.

For investors, this extended framework, builds long-term resilience in periods of rising inflation and considerable market volatility.

All portfolios’ effectiveness will be measured out-of-sample using portfolio performance measurement metrics such as alpha, Sharpe Ratio, Certainty Equivalent and the hedging assessment tools, betas.

1. **Empirical Results**

This section aims at presenting the empirical findings of the methodology employed by our study. It begins by exhibiting the regression results of the single assets and how they respond to inflation and volatility risks. Starting with the estimation of the betas of each asset, in the investment universe, individually, through time-series regressions. These regressions form the backbone of our portfolio optimization methodology. In quantifying the changes of the asset returns in response to changes in the macroeconomic variables studied, we can capture the hedging ability of the securities against purchasing power erosion and / or stock market uncertainty. This sensitivity estimates (betas) allows for an informed decision upon the construction of the diversified portfolios with the goal of an effective and consistent hedging of inflation and volatility risk. Each equation enumerated on section 3.3 will have a dedicated sub-section illustrating the regression results together with its economic interpretation.

* 1. **Estimating Single Asset Betas**
     1. **Expected and Unexpected Components Regression**

Here we report the regression results for the regression equation in [(2)](#Equation2) for the induvial assets in the investment universe.

Table V presents the results of the regressions where the expected and unexpected components of inflation and realized volatility rates are the explanatory variables. The betas for the expected inflation and volatility rates measure how much the asset’s excess returns change, on average, in response to a percentage point change in the expected inflation rate and in the monthly expected market volatility rate, respectively. In its turn, the beta coefficients for the unexpected components, quantify how much, on average, inflation and volatility innovations impact excess returns. For the assets to effectively hedge inflation, their beta coefficients need to be close to

one, whereas for volatility the hedge is effective for assets with betas close to zero. The R-squared column indicates how much of the variation of the excess returns of each asset is explained by our model. A higher value means a higher explanatory power. The column named “N” provides the number of observations considered for each regression performed in this model. The number of observations in this model is considerably smaller than the number of observations in other models due to the limitation of data for the VIX index. RMSE (Root Mean Squared Error) measures the average significance of the residuals between the observed value and the predicted values in our regression model. A lower RMSE indicates that the predicted values are close to the actual data. The second to last column informs regarding the Standard Error (SE) of the constant, *α*, measuring its precision. A lower SE reveals a more precise estimate of the constant. Finally, the last columned denominated DW, displays the Durbin-Watson statistic test of autocorrelation in the residuals of the regression models, which ranges from zero to four. If the amount of the statistic is approximately two, there is no evidence of autocorrelation in the residuals. However, if the amount is smaller than two, the residuals are positively correlated over time. Conversely, if the amount is higher than two, the residuals are negatively correlated over time. In the case of amounts below 1.5 or above the 2.5 threshold, the statistic indicates that there may be some problems with the model, such as omitted variables or misspecification.

Having provided the intuition behind every coefficient and estimate, an economic inference is needed. Thus, from this table we can conclude that for all equity indices, the coefficient of expected inflation is negative and highly significant. None of these assets seems to be an effective hedge against expected inflation, as expected. The interpretation for these stock market indices in the unexpected inflation context is quite similar. However, the significant coefficients can only be found for the Nikkei Index. Expected volatility has a strongly negative and highly significant effect on the stock market indices returns. This leads us to conclude that when market participants expect higher volatility, they demand a higher risk premium, resulting in lower excess returns. Unexpected volatility coefficients are positive and highly significant, which means that after controlling for what is already priced in, sudden spikes in realized volatility are associated with higher returns. One possible explanation for this phenomenon is that the markets overreact to volatility innovations, creating room for less risk averse investors to earn risk premia.

The NAREIT Index exhibits negative sensitivity to expected and unexpected inflation. Indicating that Real Estate is not, in this model, an effective inflation hedge, even if the results are not statistically significant for the unexpected component. Volatility coefficients experience different results, both highly significant, while anticipated volatility affects the returns negatively, unanticipated volatility, again, generates positive excess returns.

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| Table V | | | | | | | | | | |
| **Regressions of Nominal Excess Returns on Expected and Unexpected Rates of Inflation and Realized Volatility** | | | | | | | | | | |
| **Asset** | **α** | **βei** | **βui** | **βev** | **βuv** | **R²** | **N** | **RMSE** | **SE\_α** | **DW** |
| **S&P 500** | 0.0781\*\*\* (0.000) | -1.2127\*\*\* (0.005) | -0.3779 (0.157) | -1.0048\*\*\* (0.000) | 0.2481\*\*\* (0.000) | 0.271 | 252 | 0.0381 | 0.0153 | 1.945 |
| **Russell 2000** | 0.1032\*\*\* (0.000) | -1.7679\*\*\* (0.002) | -0.1395 (0.687) | -1.2845\*\*\* (0.000) | 0.3618\*\*\* (0.000) | 0.275 | 252 | 0.0495 | 0.0199 | 1.789 |
| **NASDAQ** | 0.1108\*\*\* (0.000) | -2.0224\*\*\* (0.005) | -0.3705 (0.409) | -1.2520\*\*\* (0.000) | 0.3428\*\*\* (0.000) | 0.183 | 252 | 0.0643 | 0.0258 | 1.771 |
| **FTSE** | 0.0686\*\*\* (0.000) | -1.0768\*\* (0.011) | -0.3487 (0.188) | -0.9060\*\*\* (0.000) | 0.2277\*\*\* (0.000) | 0.235 | 252 | 0.0379 | 0.0152 | 1.979 |
| **DAX** | 0.1095\*\*\* (0.000) | -1.6026\*\* (0.014) | -0.5884 (0.149) | -1.2731\*\*\* (0.000) | 0.1978\*\* (0.017) | 0.192 | 252 | 0.0583 | 0.0234 | 1.998 |
| **STOXX** | 0.1003\*\*\* (0.000) | -1.5713\*\*\* (0.004) | -0.4971 (0.148) | -1.1533\*\*\* (0.000) | 0.1958\*\*\* (0.005) | 0.220 | 252 | 0.0491 | 0.0197 | 1.877 |
| **Nikkei** | 0.0905\*\*\* (0.000) | -1.6341\*\* (0.015) | -0.8723\*\* (0.038) | -1.1766\*\*\* (0.000) | 0.2327\*\*\* (0.007) | 0.175 | 252 | 0.0601 | 0.0241 | 2.141 |
| **Gold** | 0.0281\* (0.097) | -0.5793 (0.218) | -0.3827 (0.193) | -0.0738 (0.560) | -0.0304 (0.609) | 0.017 | 252 | 0.0421 | 0.0169 | 2.206 |
| **Oil** | 0.0457 (0.229) | -0.0437 (0.967) | -1.3182\*\* (0.047) | -0.8387\*\*\* (0.003) | 0.0815 (0.542) | 0.043 | 252 | 0.0946 | 0.0379 | 1.732 |
| **10 Year Treasury Note** | 0.0047 (0.559) | -0.0710 (0.753) | 0.0860 (0.542) | 0.1261\*\* (0.039) | -0.0502\* (0.080) | 0.025 | 252 | 0.0202 | 0.0081 | 1.885 |
| **TIPS ETF** | 0.0384\*\*\* (0.004) | -1.0241\*\*\* (0.005) | -0.0695 (0.809) | -0.0947 (0.271) | -0.0027 (0.963) | 0.117 | 84 | 0.0202 | 0.0131 | 2.179 |
| **NAREIT** | 0.0879\*\*\* (0.000) | -1.2251\*\* (0.025) | -0.3091 (0.363) | -1.2065\*\*\* (0.000) | 0.3125\*\*\* (0.000) | 0.243 | 252 | 0.0487 | 0.0195 | 1.911 |
| **EUR/USD** | -0.0163 (0.181) | 0.4029 (0.236) | 0.1904 (0.370) | 0.0991 (0.279) | -0.0194 (0.652) | 0.016 | 252 | 0.0304 | 0.0122 | 1.813 |
| **JPY/USD** | -0.0170 (0.190) | 0.2967 (0.409) | -0.1636 (0.467) | 0.1390 (0.152) | 0.0202 (0.657) | 0.018 | 252 | 0.0322 | 0.0129 | 1.984 |
| **CHF/USD** | 0.0078 (0.550) | -0.2610 (0.472) | -0.1961 (0.388) | 0.0084 (0.932) | 0.0103 (0.823) | 0.009 | 252 | 0.0325 | 0.0130 | 1.891 |
| Notes: p-values are reported in parentheses below each coefficient. \*\*\*, \*\*, and \* indicate statistical significance at the 1%, 5%, and 10% levels, respectively. | | | | | | | | | | |
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Gold’s returns, similarly, to the already analysed assets, react negatively to expected and unexpected inflation. However, it does provide an effective hedge against both components of realized volatility, as expected. Oil does not break the trend, and its excess returns also suffer with changes in ex-ante inflation. Results for unexpected inflation, confirm this surprising incapability of hedging inflation, since the returns suffer negatively with inflation innovations. Similarly, oil’s returns experience decreases when the market anticipates high stock market volatility. However, and in line with the remaining assets, it’s positively sensitive to unexpected volatility. Additionally, due to the low-beta amount it can be considered an effective hedge against unexpected volatility. Fixed-income securities fail to effectively hedge both elements of inflation, surprisingly. But prove to be two effective hedges against the unexpected component of volatility. Although for the ETF that tracks TIPS, this hedging capability is registered for both anticipated and unanticipated elements of volatility, conclusions should be taken with caution due to limited number of months in this analysis. Lastly, foreign exchange pairs exhibit diverse results. EUR/USD pair is the closest thing to an inflation hedge in our investment universe. It’s the only asset that has positive sensitivity to both inflation components. Nevertheless, an increase of a percentage point in the rate of inflation is not accompanied by a percentage point increase in the nominal excess returns of this currency pair. Meaning that its real return will actually decrease when inflation increases. It can, nonetheless, be considered a volatility hedge on both components. JPY/USD pair can only provide an effective hedge against unexpected volatility, the CHF/USD pair effectively hedges both anticipated and unanticipated volatility.

In summary, none of the assets in our investment universe can effectively hedge any of the inflation elements. However, there are some strong candidates as volatility hedges, the strongest being Gold, TIPS ETF, EUR/US and CHF/USD.

All indices analysed, stock market and Real Estate, have moderate R-squared values, meaning that inflation and volatility explain a meaningful but not high portion of the return’s variation. For other assets, the amount of variation explained by our model is quite small. The other conducted tests, indicate that, in general, the model’s predicted values are very close to actual values; the Durbin Watson test estimates lead us to conclude that there is no statistical evidence of autocorrelation in the residuals of our model. Table VI provides the results when analysing the same regression methodology, but for conditional volatility. These estimates strongly back the results obtained with historical volatility. This provides robustness to our results and allows us to strongly believe that if the assets, such as the currency pairs, effectively hedge both estimates of volatility, they may play a central role in our efficient portfolios.

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| Table VI | | | | | | | | | | |
| **Regressions of Nominal Excess Returns on Expected and Unexpected Rates of Inflation and Conditional Volatility** | | | | | | | | | | |
| **Asset** | **α** | **βei** | **βui** | **βev** | **βuv** | **R²** | **N** | **RMSE** | **SE\_α** | **DW** |
| **S&P 500** | 0.0437\*\*\* (0.007) | -1.2229\*\*\* (0.003) | -0.3067 (0.227) | -1.0046\*\*\* (0.000) | 0.5998\*\*\* (0.000) | 0.336 | 252 | 0.0364 | 0.0160 | 1.814 |
| **Russell 2000** | 0.0589\*\*\* (0.005) | -1.7957\*\*\* (0.001) | -0.0688 (0.836) | -1.2689\*\*\* (0.000) | 0.8053\*\*\* (0.000) | 0.332 | 252 | 0.0476 | 0.0209 | 1.669 |
| **NASDAQ** | 0.0573\*\* (0.035) | -2.0231\*\*\* (0.004) | -0.2381 (0.579) | -1.2672\*\*\* (0.000) | 0.9000\*\*\* (0.000) | 0.249 | 252 | 0.0616 | 0.0271 | 1.651 |
| **FTSE** | 0.0388\*\* (0.017) | -1.0902\*\*\* (0.008) | -0.2936 (0.251) | -0.9010\*\*\* (0.000) | 0.5289\*\*\* (0.000) | 0.284 | 252 | 0.0367 | 0.0161 | 1.918 |
| **DAX** | 0.0744\*\*\* (0.003) | -1.5938\*\* (0.013) | -0.4885 (0.219) | -1.2927\*\*\* (0.000) | 0.5687\*\*\* (0.000) | 0.228 | 252 | 0.0570 | 0.0250 | 1.919 |
| **STOXX** | 0.0677\*\*\* (0.001) | -1.5674\*\*\* (0.004) | -0.4105 (0.219) | -1.1670\*\*\* (0.000) | 0.5372\*\*\* (0.000) | 0.261 | 252 | 0.0478 | 0.0210 | 1.789 |
| **Nikkei** | 0.0546\*\* (0.036) | -1.6357\*\* (0.013) | -0.7853\* (0.056) | -1.1856\*\*\* (0.000) | 0.6049\*\*\* (0.000) | 0.209 | 252 | 0.0588 | 0.0258 | 2.125 |
| **Gold** | 0.0171 (0.354) | -0.5446 (0.245) | -0.3059 (0.296) | -0.1130 (0.364) | 0.1059 (0.292) | 0.021 | 252 | 0.0420 | 0.0184 | 2.207 |
| **Oil** | 0.0508 (0.223) | -0.0834 (0.937) | -1.3875\*\* (0.036) | -0.7962\*\*\* (0.005) | 0.0024 (0.991) | 0.041 | 252 | 0.0947 | 0.0416 | 1.728 |
| **10 Year Treasury Note** | 0.0007 (0.939) | -0.0445 (0.844) | 0.1339 (0.345) | 0.0975 (0.106) | 0.0093 (0.849) | 0.013 | 252 | 0.0203 | 0.0089 | 1.871 |
| **TIPS ETF** | 0.0196 (0.222) | -0.8271\*\* (0.025) | 0.2101 (0.395) | -0.1449\* (0.097) | 0.1827\* (0.064) | 0.155 | 84 | 0.0198 | 0.0159 | 2.156 |
| **NAREIT** | 0.0662\*\*\* (0.002) | -1.2858\*\* (0.019) | -0.3416 (0.317) | -1.1502\*\*\* (0.000) | 0.4991\*\*\* (0.000) | 0.236 | 252 | 0.0489 | 0.0215 | 1.842 |
| **EUR/USD** | -0.0087 (0.513) | 0.3928 (0.245) | 0.1570 (0.457) | 0.1119 (0.213) | -0.1052 (0.147) | 0.023 | 252 | 0.0303 | 0.0133 | 1.817 |
| **JPY/USD** | -0.0152 (0.283) | 0.2858 (0.426) | -0.1835 (0.414) | 0.1508 (0.114) | -0.0051 (0.947) | 0.017 | 252 | 0.0322 | 0.0141 | 1.984 |
| **CHF/USD** | 0.0055 (0.701) | -0.2594 (0.474) | -0.1882 (0.406) | 0.0061 (0.949) | 0.0352 (0.650) | 0.009 | 252 | 0.0325 | 0.0143 | 1.896 |
| Notes: p-values are reported in parentheses below each coefficient. \*\*\*, \*\*, and \* indicate statistical significance at the 1%, 5%, and 10% levels, respectively. | | | | | | | | | | |
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* + 1. **Lagged Variables Regression**

Tables VII and VIII display the results of the regression model in [(3)](#Equation3).

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| Table VII | | | | | | | | | | | | | | | | | | |
| Regressions of Nominal Excess Returns on Lagged Rates of Inflation and Realized Volatility | | | | | | | | | | | | | | | | | | |
| **Asset** | **α** | **βi1** | **βi2** | **βi3** | **βi4** | **βi5** | **∑ βi** | **βv1** | **βv2** | **βv3** | **βv4** | **βv5** | **∑ βv** | **R²** | **N** | **RMSE** | **SE\_α** | **DW** |
| **S&P 500** | 0.0052 (0.526) | -0.5431 (0.425) | 0.2091 (0.861) | 1.1143 (0.379) | -1.6318 (0.167) | 0.7622 (0.238) | -0.0892 | -0.0624 (0.661) | 0.2225 (0.279) | -0.1725 (0.404) | 0.0404 (0.844) | 0.0019 (0.989) | 0.0298 | 0.015 | 367 | 0.0456 | 0.0081 | 1.853 |
| **Russell 2000** | -0.0072 (0.487) | -1.1692 (0.179) | 0.4961 (0.744) | 1.6433 (0.310) | -1.7507 (0.246) | 0.7759 (0.347) | -0.0046 | 0.0395 (0.828) | 0.0340 (0.897) | -0.1389 (0.599) | 0.0845 (0.747) | 0.0796 (0.666) | 0.0987 | 0.023 | 367 | 0.0582 | 0.0104 | 1.677 |
| **NASDAQ** | -0.0046 (0.701) | -0.7278 (0.463) | -0.8032 (0.643) | 2.4551 (0.183) | -2.1020 (0.222) | 1.1025 (0.241) | -0.0753 | 0.0363 (0.861) | 0.1300 (0.664) | -0.2473 (0.412) | 0.0096 (0.974) | 0.1716 (0.414) | 0.1002 | 0.023 | 367 | 0.0663 | 0.0118 | 1.715 |
| **FTSE** | 0.0139 (0.219) | -0.6767 (0.387) | 0.7528 (0.582) | 0.1454 (0.921) | -1.0294 (0.447) | 0.5273 (0.488) | -0.2806 | 0.1056 (0.500) | -0.0327 (0.884) | -0.2174 (0.339) | 0.1675 (0.461) | -0.0266 (0.869) | -0.0035 | 0.015 | 323 | 0.0477 | 0.0113 | 1.908 |
| **DAX** | 0.0202 (0.208) | -0.8709 (0.436) | 0.0647 (0.973) | 0.9728 (0.629) | -1.4686 (0.435) | 1.0450 (0.329) | -0.2570 | -0.0533 (0.825) | -0.0645 (0.855) | -0.1709 (0.617) | 0.1462 (0.649) | 0.1003 (0.659) | -0.0422 | 0.016 | 276 | 0.0642 | 0.0160 | 1.926 |
| **STOXX** | 0.0194 (0.159) | -0.8688 (0.360) | 0.2917 (0.859) | 0.9276 (0.596) | -1.6600 (0.308) | 0.9699 (0.292) | -0.3397 | -0.1200 (0.530) | 0.0361 (0.896) | -0.0958 (0.731) | 0.1266 (0.650) | 0.0135 (0.945) | -0.0396 | 0.018 | 288 | 0.0561 | 0.0138 | 1.764 |
| **Nikkei** | 0.0122 (0.422) | -2.4317\*\* (0.022) | 3.4372\* (0.065) | -2.1148 (0.284) | 0.5060 (0.782) | 0.1075 (0.917) | -0.4958 | -0.1809 (0.402) | 0.1409 (0.652) | -0.0211 (0.946) | -0.0037 (0.991) | 0.0729 (0.743) | 0.0080 | 0.028 | 300 | 0.0638 | 0.0152 | 1.911 |
| **Gold** | 0.0134 (0.109) | -0.4947 (0.480) | -0.7924 (0.517) | 2.9090\*\* (0.026) | -3.1744\*\*\* (0.009) | 1.2749\* (0.056) | -0.2776 | -0.2522\* (0.085) | 0.2548 (0.228) | 0.3449 (0.105) | -0.4094\* (0.053) | 0.0518 (0.727) | -0.0102 | 0.067 | 367 | 0.0468 | 0.0084 | 2.131 |
| **Oil** | 0.0459\*\* (0.041) | -0.2596 (0.865) | -3.6724 (0.172) | 5.6201\* (0.050) | -4.2817 (0.107) | 1.2304 (0.407) | -1.3632 | -0.5752\* (0.064) | 0.2802 (0.530) | 0.0580 (0.898) | 0.1523 (0.735) | 0.0698 (0.827) | -0.0149 | 0.061 | 333 | 0.0952 | 0.0224 | 1.743 |
| **10 Year Treasury Note** | 0.0100\*\* (0.018) | -0.8488\*\* (0.016) | 1.5503\*\* (0.012) | -0.9815 (0.132) | 0.2819 (0.643) | 0.0029 (0.993) | 0.0049 | 0.1467\*\* (0.046) | -0.0965 (0.361) | -0.1311 (0.218) | 0.0914 (0.386) | -0.0360 (0.628) | -0.0255 | 0.045 | 367 | 0.0235 | 0.0042 | 1.902 |
| **TIPS ETF** | 0.0174 (0.171) | -0.4588 (0.364) | 0.4247 (0.616) | -0.7280 (0.415) | 0.0232 (0.977) | 0.3237 (0.469) | -0.4154 | -0.1728 (0.214) | 0.3905\* (0.082) | -0.2550 (0.276) | -0.2021 (0.371) | 0.1906 (0.189) | -0.0488 | 0.155 | 84 | 0.0206 | 0.0125 | 2.099 |
| **NAREIT** | 0.0007 (0.937) | -1.1691 (0.117) | 1.9183 (0.141) | -0.4622 (0.738) | -0.9727 (0.451) | 0.6558 (0.353) | -0.0299 | -0.0458 (0.768) | 0.1114 (0.620) | 0.1071 (0.636) | -0.2923 (0.192) | 0.1792 (0.256) | 0.0595 | 0.022 | 367 | 0.0498 | 0.0089 | 1.676 |
| **EUR/USD** | -0.0115\*\* (0.039) | 0.4417 (0.344) | -0.2759 (0.735) | -0.4208 (0.628) | 0.9475 (0.242) | -0.4659 (0.293) | 0.2266 | 0.0328 (0.737) | -0.1998 (0.156) | 0.1776 (0.211) | 0.0009 (0.995) | 0.0143 (0.885) | 0.0257 | 0.045 | 367 | 0.0312 | 0.0056 | 1.907 |
| **JPY/USD** | 0.0005 (0.935) | -0.0319 (0.949) | -0.5914 (0.498) | 1.4327 (0.123) | -1.0861 (0.210) | 0.2001 (0.673) | -0.0766 | -0.0949 (0.363) | 0.3767\*\* (0.013) | -0.4498\*\*\* (0.003) | 0.1661 (0.269) | 0.0365 (0.730) | 0.0346 | 0.041 | 367 | 0.0334 | 0.0060 | 1.913 |
| **CHF/USD** | 0.0110\* (0.071) | -0.4187 (0.412) | 0.1521 (0.865) | 0.7548 (0.426) | -1.0451 (0.238) | 0.3699 (0.445) | -0.1872 | -0.0442 (0.679) | 0.2153 (0.162) | -0.1385 (0.372) | -0.0969 (0.528) | 0.0424 (0.695) | -0.0219 | 0.034 | 367 | 0.0342 | 0.0061 | 1.909 |
| Notes: p-values are reported in parentheses below each coefficient. \*\*\*, \*\*, and \* indicate statistical significance at the 1%, 5%, and 10% levels, respectively. | | | | | | | | | | | | | | | | | | |
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| Table VIII | | | | | | | | | | | | | | | | | | |
| Regressions of Nominal Excess Returns on Lagged Rates of Inflation and Conditional Volatility | | | | | | | | | | | | | | | | | | |
| **Asset** | **α** | **βi1** | **βi2** | **βi3** | **βi4** | **βi5** | **∑ βi** | **βv1** | **βv2** | **βv3** | **βv4** | **βv5** | **∑ βv** | **R²** | **N** | **RMSE** | **SE\_α** | **DW** |
| **S&P 500** | -0.0026 (0.839) | -0.4351 (0.521) | -0.0708 (0.952) | 1.3017 (0.297) | -1.6843 (0.151) | 0.8131 (0.214) | -0.0755 | 0.2910\*\* (0.043) | -0.4445\*\* (0.018) | 0.1100 (0.553) | 0.0357 (0.847) | 0.0843 (0.541) | 0.0765 | 0.031 | 367 | 0.0452 | 0.0128 | 1.799 |
| **Russell 2000** | -0.0183 (0.262) | -0.9874 (0.255) | 0.1188 (0.937) | 1.9356 (0.225) | -1.8372 (0.220) | 0.7904 (0.344) | 0.0202 | 0.2562 (0.162) | -0.5044\*\* (0.036) | 0.1689 (0.476) | 0.0929 (0.695) | 0.1475 (0.403) | 0.1611 | 0.038 | 367 | 0.0577 | 0.0163 | 1.655 |
| **NASDAQ** | -0.0122 (0.516) | -0.5107 (0.606) | -1.1832 (0.492) | 2.9276 (0.109) | -2.4355 (0.156) | 1.1442 (0.232) | -0.0576 | 0.3364 (0.109) | -0.5488\*\* (0.046) | 0.2573 (0.343) | 0.1053 (0.698) | -0.0091 (0.964) | 0.1411 | 0.030 | 367 | 0.0661 | 0.0187 | 1.689 |
| **FTSE** | 0.0100 (0.536) | -0.5519 (0.480) | 0.5722 (0.675) | 0.1881 (0.897) | -0.9705 (0.474) | 0.5089 (0.510) | -0.2532 | 0.1934 (0.208) | -0.2948 (0.149) | 0.0001 (1.000) | 0.1667 (0.414) | -0.0479 (0.754) | 0.0174 | 0.020 | 323 | 0.0476 | 0.0161 | 1.911 |
| **DAX** | 0.0226 (0.308) | -0.7258 (0.511) | -0.4700 (0.803) | 1.4309 (0.471) | -1.7737 (0.341) | 1.3712 (0.203) | -0.1674 | 0.1848 (0.453) | -0.7816\*\* (0.011) | 0.3780 (0.186) | -0.1447 (0.612) | 0.2914 (0.172) | -0.0721 | 0.048 | 276 | 0.0632 | 0.0221 | 1.912 |
| **STOXX** | 0.0214 (0.267) | -0.7457 (0.429) | -0.0737 (0.964) | 1.2068 (0.488) | -1.8354 (0.259) | 1.1408 (0.220) | -0.3072 | 0.1086 (0.558) | -0.4748\* (0.055) | 0.2408 (0.330) | -0.0637 (0.796) | 0.1324 (0.472) | -0.0567 | 0.031 | 288 | 0.0557 | 0.0192 | 1.724 |
| **Nikkei** | 0.0082 (0.705) | -2.1554\*\* (0.041) | 3.0225 (0.101) | -2.0770 (0.290) | 0.9527 (0.602) | -0.2281 (0.827) | -0.4853 | 0.0524 (0.802) | -0.1478 (0.594) | -0.2310 (0.405) | 0.6027\*\* (0.030) | -0.2450 (0.238) | 0.0314 | 0.042 | 300 | 0.0633 | 0.0216 | 1.878 |
| **Gold** | 0.0099 (0.456) | -0.4182 (0.554) | -0.9966 (0.417) | 2.9677\*\* (0.023) | -3.0426\*\* (0.013) | 1.2120\* (0.076) | -0.2776 | -0.0027 (0.986) | 0.2468 (0.208) | -0.3574\* (0.065) | 0.2225 (0.250) | -0.0955 (0.507) | 0.0137 | 0.056 | 367 | 0.0471 | 0.0133 | 2.119 |
| **Oil** | 0.0582\* (0.067) | -0.2912 (0.848) | -4.1644 (0.116) | 6.2975\*\* (0.026) | -3.9770 (0.131) | 0.7625 (0.609) | -1.3725 | -0.4385 (0.147) | -0.4603 (0.252) | 0.6413 (0.111) | 0.6021 (0.135) | -0.4385 (0.146) | -0.0938 | 0.084 | 333 | 0.0940 | 0.0316 | 1.739 |
| **10 Year Treasury Note** | 0.0160\*\* (0.015) | -0.7003\*\* (0.045) | 1.3025\*\* (0.032) | -0.8701 (0.175) | 0.2372 (0.694) | 0.0292 (0.931) | -0.0014 | 0.1904\*\*\* (0.010) | -0.1939\*\* (0.045) | -0.0584 (0.540) | 0.1426 (0.135) | -0.1436\*\* (0.043) | -0.0629 | 0.063 | 367 | 0.0232 | 0.0066 | 1.901 |
| **TIPS ETF** | 0.0458\*\*\* (0.003) | -0.2651 (0.557) | 0.0518 (0.946) | -0.7018 (0.397) | 0.1786 (0.814) | -0.1881 (0.666) | -0.9246 | 0.3919\*\*\* (0.003) | -0.1640 (0.357) | -0.4476\*\* (0.014) | 0.4820\*\*\* (0.007) | -0.4115\*\*\* (0.003) | -0.1492 | 0.327 | 84 | 0.0184 | 0.0151 | 2.072 |
| **NAREIT** | -0.0139 (0.312) | -1.1770 (0.107) | 1.8505 (0.145) | -0.7194 (0.592) | -0.5208 (0.680) | 0.5612 (0.425) | -0.0054 | 0.1008 (0.513) | -0.0188 (0.926) | -0.5622\*\*\* (0.005) | 0.3276 (0.101) | 0.2997\*\* (0.044) | 0.1470 | 0.068 | 367 | 0.0486 | 0.0138 | 1.654 |
| **EUR/USD** | -0.0129 (0.137) | 0.2641 (0.565) | 0.1633 (0.838) | -0.7649 (0.366) | 1.0258 (0.197) | -0.4612 (0.298) | 0.2271 | -0.3148\*\*\* (0.001) | 0.4332\*\*\* (0.001) | -0.0558 (0.657) | -0.2270\* (0.071) | 0.1975\*\* (0.035) | 0.0330 | 0.082 | 367 | 0.0306 | 0.0087 | 1.871 |
| **JPY/USD** | 0.0010 (0.918) | 0.2055 (0.676) | -0.8719 (0.307) | 1.4823 (0.102) | -1.1848 (0.164) | 0.2946 (0.534) | -0.0743 | 0.4934\*\*\* (0.000) | -0.5790\*\*\* (0.000) | 0.0614 (0.648) | 0.1187 (0.377) | -0.0653 (0.514) | 0.0291 | 0.077 | 367 | 0.0328 | 0.0093 | 1.914 |
| **CHF/USD** | 0.0120 (0.209) | -0.2540 (0.616) | -0.2143 (0.807) | 1.0303 (0.269) | -1.1542 (0.188) | 0.4040 (0.408) | -0.1881 | 0.3130\*\*\* (0.004) | -0.3371\*\* (0.017) | -0.0539 (0.697) | 0.2023 (0.144) | -0.1508 (0.144) | -0.0266 | 0.057 | 367 | 0.0337 | 0.0095 | 1.907 |
| Notes: p-values are reported in parentheses below each coefficient. \*\*\*, \*\*, and \* indicate statistical significance at the 1%, 5%, and 10% levels, respectively. | | | | | | | | | | |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |

Which means, the regression results evaluating the sensitivity of nominal excess returns to different lags of the rate of inflation and realized volatility. Much of these coefficients are aligned with the ones from the previous regressions, i.e., no asset is an effective inflation hedge and most of them cannot hedge volatility consistently, even though in the medium term all volatility impact on the excess returns is dissipated.

Equity indices exhibit a negative sensitivity to inflation in the first lag. Interestingly, all rebound in the second lag and react positively to changes in the inflation rate, except the NASDAQ. Many even make-up for the negative changes of the first lag. However, analysing lag by lag is not as conclusive as analysing the sum of all the coefficients. This cumulative result validates the conclusions in the previous section, stock market returns underperform when inflation rises. This model adds that, in the shorter and medium term, all indices fail to hedge inflation. Regarding sensitivity to volatility, the results are mixed and always statistically insignificant. Most indices have small and fluctuating coefficients, and the cumulative returns are very close to zero. Which may be an indication that in the medium-run, equities’ returns react on a greater degree to inflation than to volatility. The majority of the indices are very close to perfectly hedge volatility risk over the five lags.

Gold’s results are quite surprising, the inflation profile for this asset is mostly negative and so is its cumulative result. Similarly, to what was analysed in the past model, gold’s nominal excess returns suffer significantly negatively with positive changes in the rate of inflation. Volatility sensitivity is in line with the remainder asset classes and it exhibits volatility hedging capabilities, if we look at the cumulative results. Still in commodity universe, oil’s coefficients are mostly aligned with gold’s results, even though the magnitude of its sensitivity to inflation is much higher. The 10 Year Treasury Note overall inflation coefficient is very close to zero and thus, failing to hedge inflation in this model. While on volatility it seems to validate its volatility hedging profile exhibited in the previous regression. The TIPS ETF coefficients are also very surprising here, since these are instruments designed to hedge inflation, but failing to sustain that hypothesis in this model. The cumulative betas for volatility are also very small. Results for this particular asset are, again, to be interpreted with caution because of the limited data when compared to the remaining assets. The NAREIT Index also has a cumulative beta result very close to zero and behaves similarly to the equity indices, rebounding in the second lag, but failing to hedge inflation over the five lags. Again, in this model, it can also be considered a volatility hedging asset.

For the FX pairs, the majority of the coefficients is not statistically significant, and the results are pretty much in line with the rest of the assets in the investment universe. EUR/USD confirms to be the asset, in this investment universe, on which the nominal excess returns are less sensitive to inflation changes. However, still far away from hedging inflation perfectly. All pairs are also, over the five lags, volatility hedging assets.

Due to the very small amounts of the R-squared in this model, we cannot infer many things from these regressions. It is evident that the model, in which the decomposition of inflation and volatility into anticipated and unanticipated is done, is superior. In that sense, the betas used to build the efficient surface will be the ones estimated on those models.

Table VIII which makes the same analysis but using conditional volatility, validates most of the results from the model with realized volatility. However, many assets are now further way from perfectly hedging volatility, like the NAREIT Index and the TIPS ETF.

* + 1. **Lagged Variables Regression with Factor Models**

This section focuses on studying the model on equation [(4)](#Equation4). In this model, we aim at understanding the real impact of inflation and volatility risks. To do so, we augment the list of regressors by including the factors in the models FF3, Carhart and FF5. Each model is regressed separately together with the five lags of inflation and volatility rate.

As expected, these models corroborate the results in the last section and the conclusions to be taken are very similar. However, and against our preconceived expectations, these factors do not increase substantially the explanatory power of our models. In that sense, and to avoid repetitiveness in our paper, the results of this section are made available in the Appendix.

* 1. **Diversified Portfolios Performance Measurement**

To evaluate the robustness of our portfolios build in-sample, i.e., the period ranging from 1980 until the end of 2010, we test their effectiveness on a different test period. This testing period, starts in 2011 and extends to the end of 2024. The portoflios, were only determined based on the train period. We then apply the obtained results, either for the optimizations or the equal-weighted portfolios, in the test data. Table XV presents some summary statistics and performance measurement metrics.

The Portfolio producing the highest Sharpe Ratio is the SBS, the portfolio built to maximize returns per unit of sensitivity to inflation and volatility, while penalizing deviations from the effective hedge measure. It is alsom one of the two portfolios, producing the highest annualzied return.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Table XV | | | | | |
| Portfolio Performance Measurement OOS | | | | | |
| **Portfolio** | **Annualized Return** | **Annualized Volatility** | **Sharpe Ratio** | **CEQ (λ=3)** | **Maximum Drawdown** |
| **EW** | 0.04 | 0.09 | 0.46 | 0.03 | -0.19 |
| **60/40** | 0.05 | 0.09 | 0.56 | 0.04 | -0.21 |
| **MVP** | 0.01 | 0.03 | 0.47 | 0.01 | -0.07 |
| **TP** | 0.03 | 0.04 | 0.69 | 0.02 | -0.08 |
| **HRP** | 0.02 | 0.05 | 0.45 | 0.02 | -0.12 |
| **MCP** | 0.02 | 0.06 | 0.31 | 0.01 | -0.12 |
| **MDR** | 0.02 | 0.03 | 0.64 | 0.02 | -0.04 |
| **RPP** | 0.06 | 0.09 | 0.66 | 0.05 | -0.16 |
| **MSVP** | 0.02 | 0.04 | 0.43 | 0.01 | -0.08 |
| **TB** | 0.04 | 0.07 | 0.60 | 0.03 | -0.17 |
| **Stocks** | 0.08 | 0.14 | 0.56 | 0.05 | -0.24 |
| **Alts** | 0.03 | 0.18 | 0.19 | -0.01 | -0.43 |
| **Bonds** | 0.01 | 0.06 | 0.13 | 0.00 | -0.23 |
| **FX** | -0.01 | 0.04 | -0.13 | -0.01 | -0.14 |
| **IHP** | 0.02 | 0.14 | 0.18 | 0.00 | -0.33 |
| **SHP** | 0.02 | 0.07 | 0.32 | 0.02 | -0.20 |
| **ESP** | 0.01 | 0.05 | 0.15 | 0.00 | -0.13 |
| **SBS** | 0.08 | 0.11 | 0.74 | 0.06 | -0.25 |
| **Average** | 0.03 | 0.08 | 0.41 | 0.02 | -0.18 |

However, this comes with a moderatly-high degree of risk, its annualized volatility is 11% and significant drawdowns. The Portfolio that achieves the highest Sharpe Ratio with the lowest volatility is, as expected, the Tangency Portfolio.Minimum Variance Portfolio (MVP) and Maxmium Diversificaton Ratio (MDR), offer the lowest volatility but also their returns are below average. The Risk Parity Portfolio (RPP) also performs well. It achieves a 6% annualized return with a Sharpe Ratio of 0.66. Its certain equivalent is 0.05, reflecting good utility and could be one of the most attractive for risk averse investors. The tradtional approaches, EW and 60/40, offer very moderate returns, with above average Sharpe Ratios and annualized returns. However, they are outperformed in the risk-return relationship by more sophisticated strategies.

The most efficient portfolio resulting from the surface does produce a below average annualized volatility, but its return is also below average, with a very low Sharpe Ratio. In summary, our two signature methodologies have produced very distinct results. Furthermore, no portfolio in the listed set dominates across all metrics.

* 1. **Portfolios Hedging Profile**

**4.3.1. Expected and Unexpected Components Regression**

Table XVI presents the results of regression [(2)](#Equation2). This table will help us understand better how the set of portfolios reacts to changes in the antecipated and unanticipated elements of inflation and volatility. The benchmark portfolios, EW and 60/40, fail to produce statistically significant inflation betas. Both are also very far way from being effective inflation hedges. The only positive beta is the expected inflation beta in EW. But for every percentage point change in the expected year-over-year inflation rate the excess returns of this portfolio only increases 0.14 percentage points. Additionally, both, fail to hedge both components of volatility, significantly.

**Discussion**

**• Interpretation of Results: Explain the economic intuition and implications of the**

**findings.**

**• Comparison with Literature: Compare your results with previous studies and discuss**

**any discrepancies.**

**• Critical Assessment: Discuss weaknesses of your approach, possible alternative**

**explanations, endogeneity problems and other shortcomings of your thesis.**

**Acknowledge any limitations or weaknesses in the study. No study is perfect, be honest**

**about it and discuss how you would ideally improve on the shortcomings if you could.**

1. **Conclusion**

**Conclusion**

**• Summary of Findings: Recap the main findings of the paper.**

**• Contributions: Reiterate the paper’s contributions to the field**

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