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The bad student paradox: A theoretically rational approach to pupil dynamics

Rationality is a widespread concept often resorted to in the context of social science in an attempt to achieve an accurate description of human behavior. The precision with which formal social models operate usually face a trade-off with the theoretical parsimony such that the introduction of an excessive amount of model parameters will threaten the predictive behavior of additional individuals (overfitting in social science). In an everyday context, decisions both at an individual (micro) and aggregate (macro) level can be analyzed from the scope of rational choice. This can be understood as the underlying relationship between means and ends. Agents maximize their associated utility or outcome derived from performing a given set of actions subject to the opportunity they are exposed to and the available information they hold. On the other hand, game theory provides a mathematical basis to understand how rational interaction among agents leads to contrasting scenarios conditional to the assumptions we make on individuals, the consequences and, frequently, the probability distribution of the outcomes. The proposed exercise will study pupil conducts under three ordinary scenarios faced by every student throughout their academic fulfilment by modelling their performance through the lens of rational choice and game theory. We will explore the consequences of strategy-selecting in each of these contexts and construct a predictive framework based on real life empirical observations.

We will center our opening example in a classroom in which a lecture is taking place and we will assume that the instructor had previously notified students about an assigned compulsory reading they had to work on in order to follow the current course topic. For the sake of simplicity, our point of departure will focus on a two-student classroom, and we will later extend our reasoning to n people. Let´s consider that a question is being asked by the professor which enables he/she to infer whether the readings have actually been executed or not by the students such that this lecturer expects some interaction from the side of the pupils. One potential outcome is that no student is motivated to raise their hand, thus the teacher will be displeased with the class performance and set lower grades as a whole. An additional scenario is the one which both students decide to participate in the lesson (both hands raised) and the teacher randomly chooses one of them to answer, hence bringing up the grades of the class due to the satisfaction triggered on the teacher. We will make the assumption that any answer provided by an individual is satisfactory enough to motivate pleasure on the teacher.

We will proceed to elaborate our rational model and outcomes from the point of view of student interaction by recalling one of the main requirements of game theory analysis: preference ordering. One could notice that the payoff attained by an agent diverges depending on whether he/she had to make the effort of taking the initiative or not such that we will introduce some social cost of raising the hand. This cost can be interpreted as some risk aversion that an agent has when attempting to answer a teacher´s question, and we will assume it is homogeneously distributed among students. This translates in a game theory environment in which an agent´s ideal scenario takes place when the other student raises the hand and he/she doesn´t, and a disastrous (worst outcome) situation in which no one volunteers, and grade penalization takes place for everyone. Intermediate scenarios in terms of utility describe the conditions in which only one of our pupils takes a step forward, and the favorable outcome among these two situations for each of them is to be the one not raising the hand and taking advantage of the grade enhancement without incurring in any auxiliary effort.

The following pay-off matrix illustrates the previous description by assigning some numerical categories (we are working under an ordinal specification) to each of the situations when depicting the two choices our students face. The blue cells account for the location of our Nash equilibria such that no individual will have incentives to deviate unilaterally from their strategic decision. The derived utility from grade reinforcement when a student answers the professor´s question is diminished by this "cost" mentioned earlier of raising their hand, thus generating a one-point penalty for the incentives of our individual. Notice that these two equilibria pertain to the optimality criteria in terms of pareto such that, under these circumstances, no student can be made better off without making the other worse off.

Matrix Pay-off for student class interaction

	Raise hand	Not raise hand
Raise hand	2,2	2,3
Not raise hand	3,2	0,0

We will now extend the proposed analysis to a more realistic reference frame in which the enrollment to the course is of n participants in order to report a more accurate representation of in-class student habits (no possible representation since we would need as many dimensions as students). The fact that we have obtained two equilibria for our model does not pose any dilemma on our rational model (notice students are indistinguishable) when we resort to our stretched version of the model with additional students because the equilibria will adjust proportionally. For instance, the worst-case scenario will remain unchanged (0 students participating in class) and the case in which all agents raise their hands will preserve its non-equilibrium characteristic since any student would have incentives to modify its decision unilaterally. As a consequence, the only potential solution is the one in which only one student participates in the lesson because no other student will improve their payoffs by raising their hands while, at the same time, such participant has no incentives to bring down his/her hand because this would drag all the class (including him/herself) down to the worst outcome possible.

As a final remark for this section, we have seen how rational choice in a game theory context can explain why student interaction in class can report low figures and how this result does not only belong to a Nash equilibrium, but it is also efficient from the point of view of pareto. However, we are working under conditions in which individuals are assumed to proceed in an "egoistic" manner such that the only relevant motive is their self-interest, but there are other factors that could play a role in this setting. This self-seeking rationale cannot explain conducts such as the fact that individuals are motivated to participate in election voting when they find personal inconveniences even though their sole vote contribution will be negligible to the general outcome (Harsanji, 1969).

Returning to our subject of interest, current research points out that classroom dynamics reveal that chunks of engaging students are observed such that pupils involved more in class discussions are prone to sit close to each other. Yet the issue is far from black and white, one could conjecture that these students stick together beforehand (a selection effect) or, alternatively, a grounding on social influence in which participating in class holds a contagious effect. If the latter held true, then the emergence of a degree of social influence would be managing the behavior of students in this context. This is, conditional to an agent belonging to a group (e.g., a certain region within a class) in which one or more individuals are acting in a certain way (the strategic decision of raising hands), the chances of such an agent to mimic this behavior is guided by the rest of members (Young, H Peyton, 2009). This view can be extended to multiple student areas such as the activity of taking class notes in which empirical evidence has shown that when someone takes the initiative of registering by hand or laptop what the lecturer is explaining, this starts a "cascade" effect in the classroom.

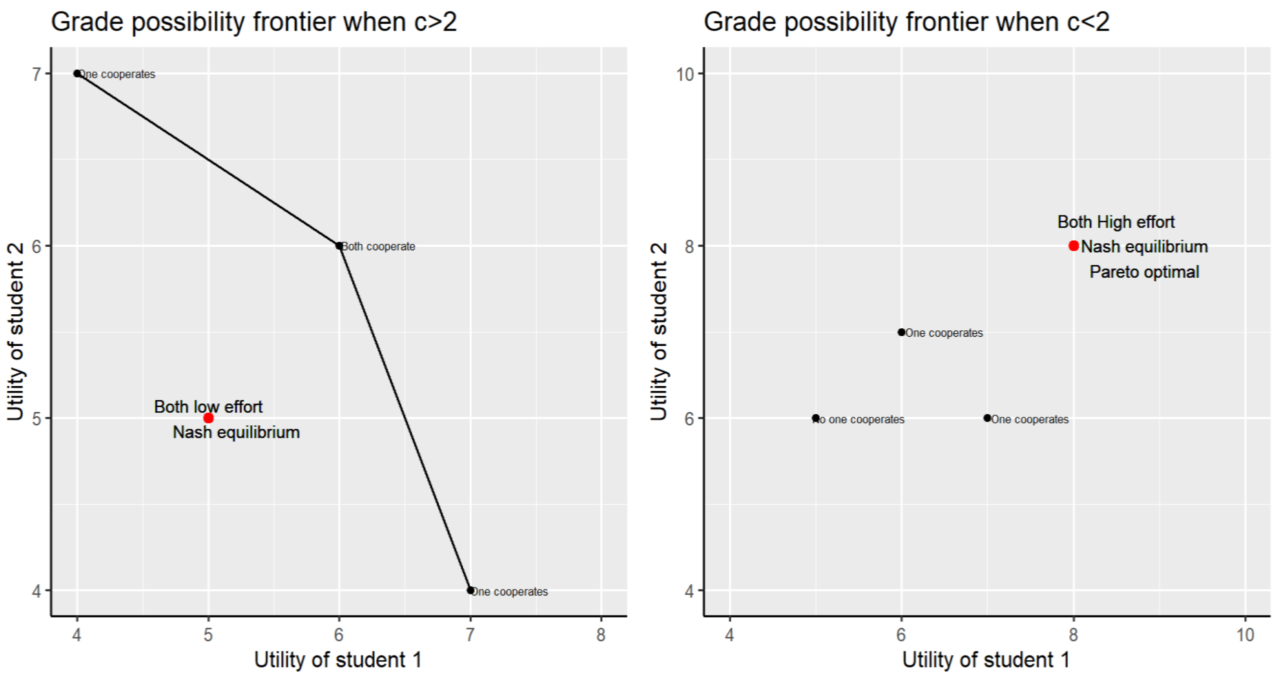
The next section of our study stems from the theory of collective action and the issues arising from the existence of public goods. A public good is such that the consumption of an increasing number of individuals does not diminish the utility derived from these agents (non-rivalry) and one cannot be prevented or excluded from its usage (non-excludability). Our proposed setting deals with a situation in which students are encouraged to arrange themselves in groups to conduct a common research project in which all of them will be homogeneously graded. The latter is key to the formulation of this part of the paper since we will assume that this collective common grade can resemble the behavior of a public good such that no individual will receive personal credit for their work, but the group will be awarded, as a whole, with a grade independent of the number of group members. In the following matrix we will portray the variety of end results that can arise associated to the degree of effort that each student adopts. We denote by 9 the grade (payoff) associated with the case in which both work hard, 7 will be the grade arrived at when only one of them submits such a high effort, and an unpleasant grade of 5 will be awarded to both individuals if none of them cooperate. Thus, the pay-off is such that, in the case of successfully acquiring their grade, we will diminish this result by the cost of submitting such an effort by an amount equal to c.

	High effort	Low effort
High effort	$9-c, 9-c$	$7-c, 7$
Low effort	$7, 7-c$	$5{,}5$

$If\ c > 2\ \rightarrow Low\ effort\ dominant\ strategy$
$If\ c < 2\ \rightarrow High\ effort\ dominant\ strategy$

We depict two cases emerging from the strategic interaction of our individuals which will behave as a function of the parameter c as the previous expression holds. For large values of the associated cost of working hard for a project, one encounters that the only conceivable dominant strategy is to input a small effort into the assignment, driving our students into the worst state of the world possible in which their grade is minimized. Unlike the previous example on class engaging, this outcome does not belong to the optimality criterion established by Pareto because both individuals can be improved in terms of utility when employing their high efforts. Notice how we come across a situation in which every outcome is pareto-efficient except for the unique Nash equilibrium since, even if only one agent works hard, it is unfeasible to make a student better off without making the other one worse off. Recalling the second case in which c is assumed to take values smaller than two, the dominant strategy under this scenario is to pursue a strategy of high effort irrespective of the other individual´s choice, being this last result the only pareto-optimal outcome (a single point). A summary of the range of discussed outcomes is supplied below in which we account with red the associated Nash equilibria. The pareto optimal outcomes in the first case correspond to the frontier line and the unique equilibrium is proven to be inefficient since it lies completely below the margin whereas the second scenario exhibits a unique Pareto optimal outcome (no possible fitted line) which matches the Nash equilibrium conditions.

Grade possibility frontier as a function of effort and cost

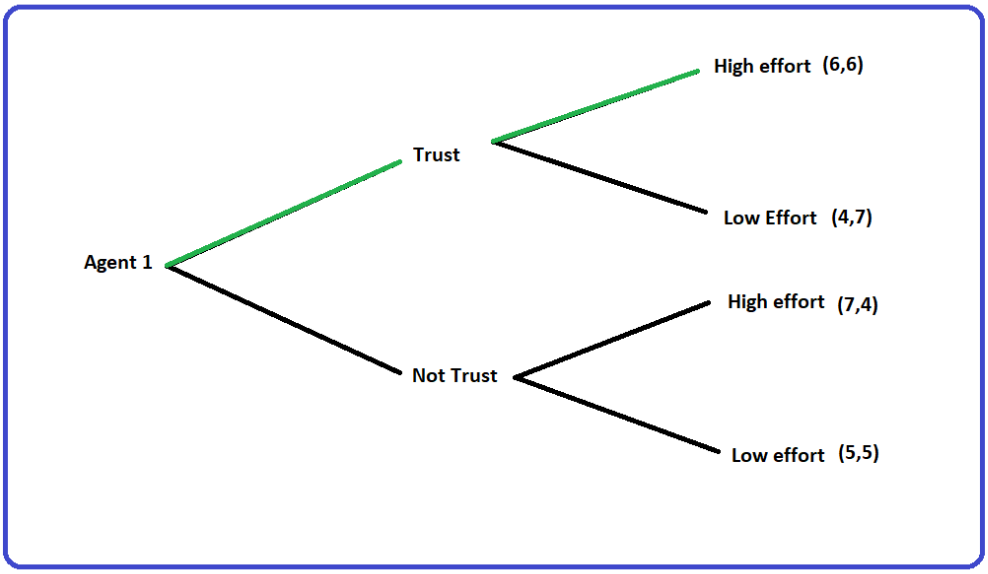


Source: Own elaboration

Intuitively, this cost associated to the development of the group project can be understood as the difficulty or tediousness faced by students on the working topic. In other words, when an assignment is demanding enough, students are better off (their dominant strategy) by relying on other team members to perform efficiently and not pushing themselves to the limit, situation which mimics the public good free rider dilemma. This takes place because one knows that the grade will be shared among all team members, and it is more attractive to take advantage of other´s hard performance rather than risking being the one undertaking all work disproportionately. One fundamental approach to overcome this dilemma stems from collective action and cooperation theory which could, conditional to certain circumstances, enable agents to arrive at the most desirable state of the world (greatest payoffs for each individual). The first constraint emerges from the group size (Olson, 1965) which will correlate negatively with the probability of individual cooperation, thus large groups (referred to as weak ties) seeking to accomplish common interests will fail unless there is some type of coercion or incentive that pushes for individual participation. Specifically, under the rationality conditions we introduced in our model, engagement of all agents will be unfeasible unless the group is sufficiently reduced in number or participants are captivated by some motivation or stimulus (selective incentives).

The other overlapping element with our predictions is manifested by the concept of trust among members of the game which enhances social cooperation and overcomes the above collaboration disincentive issue. We had already depicted in our graph how the optimal outcome in terms of pareto and grade optimization pertains to the case in which both agents set a high effort on the project, but we need the introduction of a new ingredient to the game to enable such a setting. Of course, the first factor involved in this is the degree of affinity between game players such that it is often safer to stick with one´s closest friends when performing a professional assignment than forcing teamwork and joint effort among people who barely know each other. Besides closeness, we could also model this relationship based on the interpretation of signaling and a third´s person internal evaluation of an agent due to some given characteristics. We shall distinguish between signs and signals (Gambetta, 2009), the latter referring to physical or behavioral attributes which influence the way an agent perceives another one whereas signs are implicit or "hidden" forms of signals. Operating in an academic context, the first sign that springs to our minds is the past record or achievements that a person holds (reputation as a sign of competence) which will determine the extent to which an agent can trust another one to perform well given a certain task. Other attributes such as body language or even facial signs might also play a role in group selection, but this controversy is overcome by resorting to random assignment or by allowing professors to allocate group members according to their criteria. One is able to represent the previous scenario as an extensive game such that we assume that one of the players is transparent about revealing his/her intentions. Conditional to being certain about the first player´s cooperation, our next agent´s decision revolves around the fact of complying or deceiving his/her peer (we treat individuals as identical or a symmetrical game, so we are indifferent on who moves first).

Decision tree of sequential trust game



Despite the fact that the utility is such that each player has incentives to deviate unilaterally provided that the first agent commits to trust the other one, the implicit grade behind the efforts is such that the green line pushes our agents to the maximum grade possible. Thus, this will also depend on the importance that individuals place on grade achievement or the weighting that this assignment holds on the final subject grade. Notice that we have depicted the case in which agent 2 has information about what player 1 has decided previously since he trusts that player 1 will trust him, hence we have not included a dashed line in the second section of the sequential game indicating that player two does have information about previous player´s movements (If this were the case we would arrive at similar results as in the static game). This means that player 1 trusts player 2, and that player 2 is trusting that player 1 trusts him/her, and player 1 trusts that player 2 trusts that player 1 trusts him/her…We arrive at an infinite trusting loop that can be only explained if we assume that affinity or signaling between these members is large enough to ensure that cooperation will take place. According to the previous plot, the trusting scenario will enable the game to take the path depicted in green such that common grade is optimized, and the distribution of efforts is symmetrical among our members.

This setting resembles a prisoner´s dilemma game theory scenario (when game is played once simultaneously as in the first case) in which both students hold the same dominant strategy and therefore the only equilibrium drags both to the worst scenario in contrast with the optimal situation. Triggering cooperation in a context of group assignment can be tackled by a subsequent round of questions proposed by the teacher to ensure that every member has worked accordingly or by weighting more the group project in the final course grade so that incurring in individual costs of making an effort is offset by the benefit of boosting significantly the final grade. Translating our model into a real-world context, one realizes that our predictions are, to some extent, accurate because it is of common knowledge that the emergence of free riders in group assignments is one of the main reasons why pupils resort to working with the people who they trust the most to pursue a decent task.

We will devote the last section of our study to analyze an additional setting common to many students in which the agent faces choices under conditions of uncertainty when deciding to plagiarize in an assignment or not. This issue gains especial relevance nowadays with the evolution of technologies and the ease with which students can access information and other researchers´ contributions online. The exhaustive states of the world will depict situations in which our student gets caught or not, and each of these will be associated with a probability parameter such that the summatory of these probabilistic scenarios add up to 1. We will assume that plagiarism enhances the grade of an individual by enabling the student to extract ideas, knowledge and originality from other authors and that such increment makes the individual´s mark to grow from an 8 to 9. The expected utility derived from the strategy of an agent from not plagiarizing is independent of the state of the world or the probability distribution since the grade will be a constant. The following matrix summarizes the set of premises under this setting and the associated expected utilities which we will compute as the product of each payoff with its probability.

The notion of rationality excludes any judging or subjective appraisal in the process of action selecting to maximize one´s outcome. This means that, although plagiarism might be identified with poor or unpleasant moral values, an agent is still rational if it is following a strategy based on its beliefs aiming to maximize his/her grade within the available information. In strict rationality, individual actions are based on desires and beliefs but the only assessment that should be involved is the one regarding efficiency or consistency of behaviors, not a judge of the beliefs or morals. Some research has been made on this topic suggesting that, although plagiarism examples are residual, there are some appealing driving factors that increase the likelihood of their occurrence. The first empirical observation indicates that there is a positive correlation between plagiarism and grading in the respective assignment whereas a negative association remains with final examination. This proves that one of the side-effects of adopting the copying strategy is that the individual becomes less prepared to face a final course assessment (Pierce and Zilles, 2917). Regarding socioeconomic conditions, no evidence has been found that the propensity to commit these kinds of actions is distributed differently across genders, but when researchers hypothesized the same within geographical categories, some statistically significant results were concluded. Australia, China and India pertain to the list of countries with the highest rates of plagiarism, and it was found that many students even lack an understanding of this subject. One could, therefore, detect the presence of a cultural factor fostering differences between countries in which traditions and social norms can play an important role in the way this issue is presented to students at schools and universities.

	Gets caught p	Doesn´t get caught $1-p$
Plagiarize	0	9
Not plagiarize	8	8

$$E(plagiarize) = 0 * p + 9 * (1-p) = 9 - 9p$$
$$E(not\ plagiarize) = 8 * p + 8 * (1-p) = 8$$

The results arising from our model suggest that students will be encouraged to avoid plagiarism provided that their expected utility exceeds the one that they would obtain from doing otherwise. This will only take place whenever the probability of being caught is greater than 1/9, which means that one of the tools teachers have to avoid this issue is to check on student´s work frequently. In alternative scenarios in which students are aware that the probability of being caught is small enough, the cost-benefit analysis reveals that the rational action would be to copy from other`s research work. From the professor´s point of view, an auxiliary instrument to tackle plagiarism would be to modify the first quadrant of the matrix (notice this is the only cell in which teacher manipulation can take place) such that the utility of being caught provided a student has plagiarized becomes lower than 0. This can be achieved by the implementation of more severe punishments than obtaining a 0 in the assignment such as informing the educational institution´s managing director or preventing the offender from taking place in the course no longer.

This paper aims at analyzing the rationality behind pupil behavior from the point of view of game theory and supported by some empirical evidence in order to understand the thinking process behind student´s interactions. Social complexity arises in those cases in which individual decisions are not only influenced by, but also impacts the outcome and utility that a third party could obtain. From the point of view of expected utility and social Nash equilibrium, there are incentives for an agent to let others engage in the class and participate in the lesson independent of the number of pupils enrolled in the course. In fact, the only possible equilibrium in such a case is that there is frequently one sole student who makes up for the majority of student interventions in class (see pay-off matrix above) and this is consistent with empirical observations. Evidence portrays that high rate of student interaction is seldom distributed homogeneously across pupils, but it is instead concentrated in clusters. There are additional incentives to not make enough effort in group projects unless trust or other external forces drive an individual into collaboration, and plagiarism is explained from the point of view of expected utility when institutional rules are not strict enough (one observes many distribution differences across regions). Therefore, the bad student paradox holds that, under some assumptions and setting aside moral values, low academic effort could be justified from the lens of rationality.

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