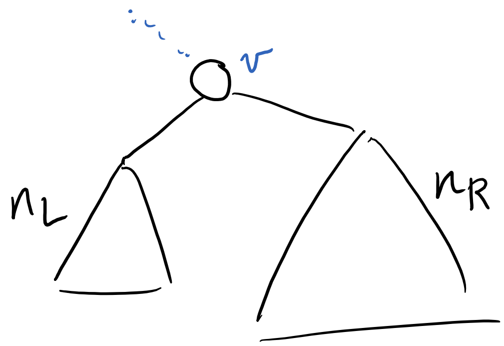
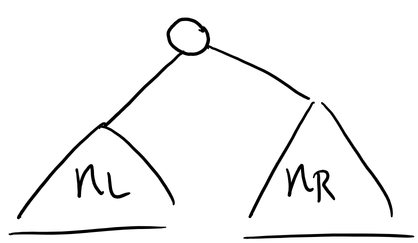
Size-Balanced Property for a node



$$\boxed{MAX(n_L, n_R) \le 2 \times \min(n_L, n_R) + 1}$$

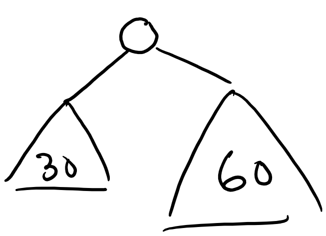
A tree is size-balanced iff all nodes ~~i~~are size-balanced. [NOT JUST GLOBAL ROOT].

Ex Suppose $N = 91$



$n_L + n_R = 90$

$60 \overset{?}{\le} \underbrace{2 \times 30 + 1}_{61}$ ✓

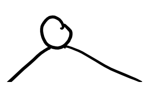


ok?

(a) YES

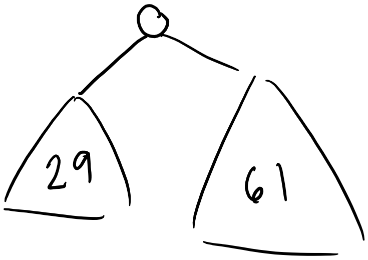
(b) NO

$61 \overset{?}{\le} 58+1$



ok? (a) YES

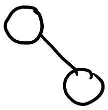
$61 \overset{?}{\leq} \underbrace{58+1}_{59}$ ✗



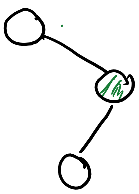
ok? (a) YES (b) NO

$1 \leq 2\times0+1$

$1 \leq 1$ ✓



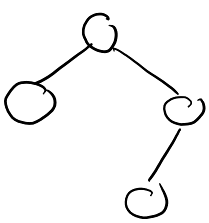
ok? (a) yes (b) No



$2 \overset{?}{\leq} 2\times0+1$

$2 \leq 1$ ✗

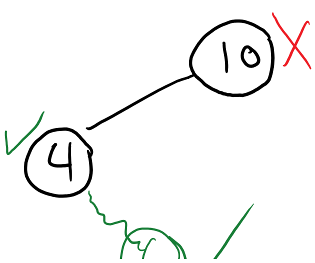
ok? (a) yes (b) No

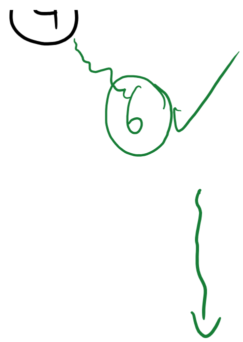


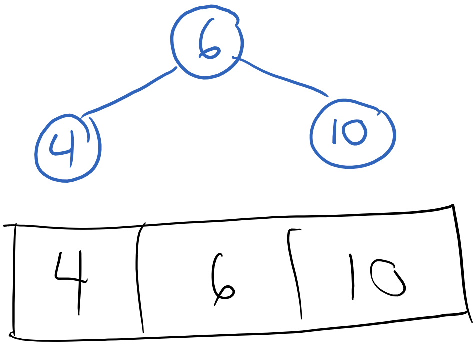
ok? (a) yes (b) No

Maintaining size balanced property...

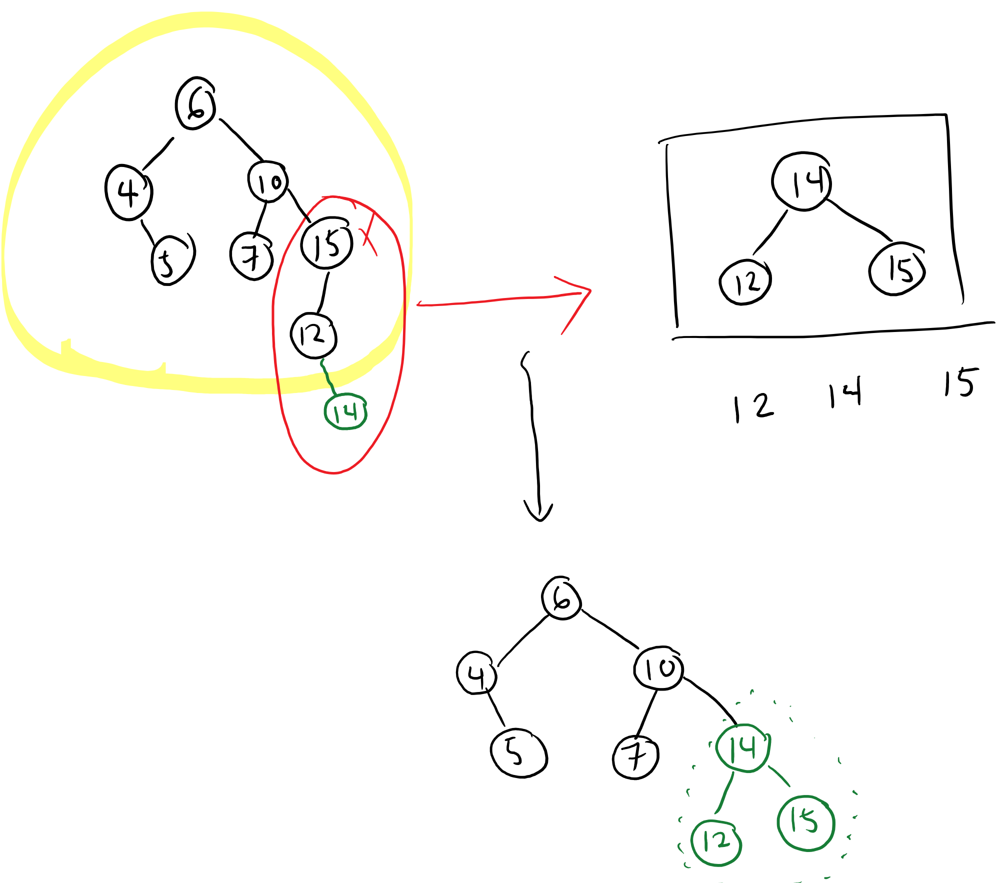
insertion seq: $\boxed{10 - 4}$ - 6





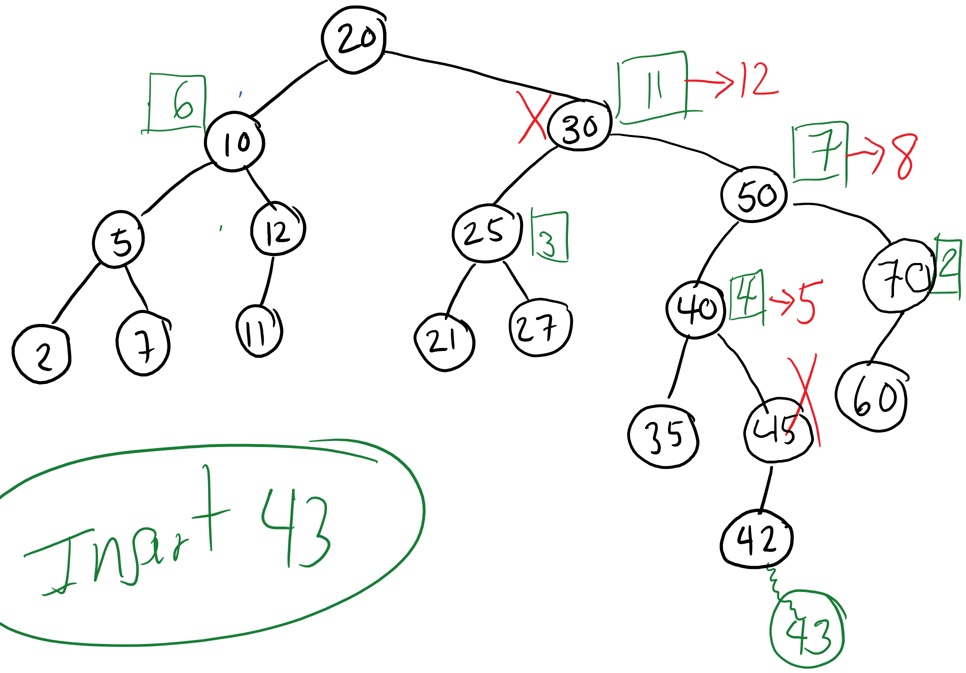


more insertions :

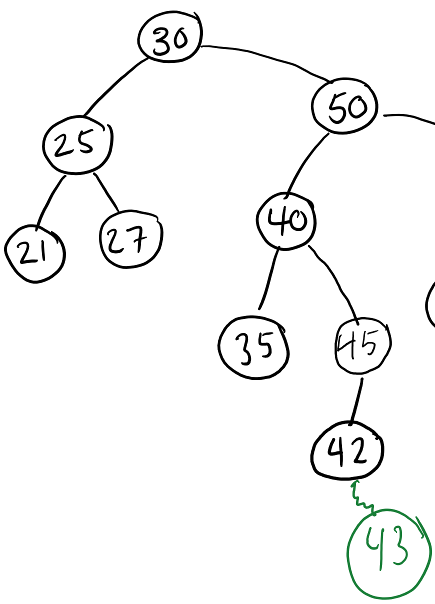


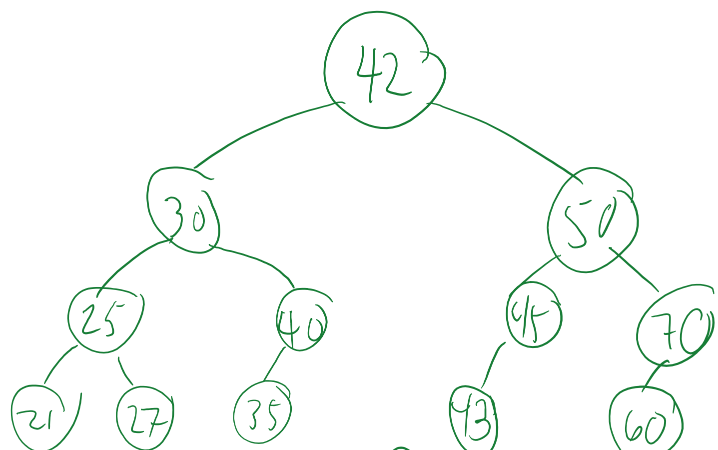
BIGGER EXAMPLE

SIZE-BALANCED ?

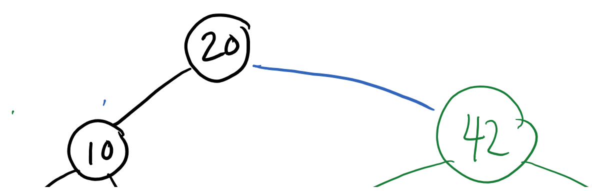


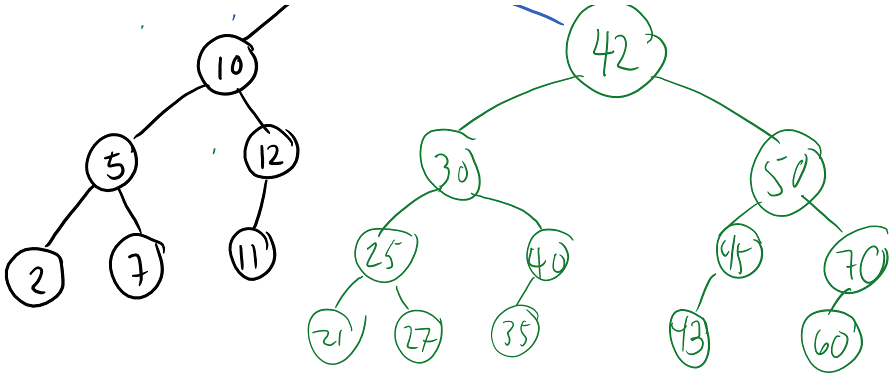
Insert 43



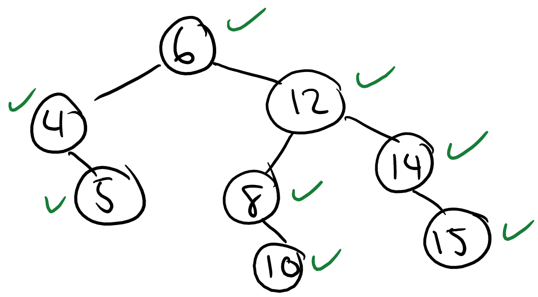


21 25 27 30 35 40 42 43 45 50 60 70





20



MAX HEIGHT OF A SIZE-BALANCED TREE W/ N NODES?

$$\approx O(\log N)$$

$$\left[\approx \log_{3/2}(n)\right]$$

Intuition of Amortized Runtime Claim...

Claim: a sequence of N insertions into an initially empty Size-Balanced BST takes

$O(N\log N)$ time

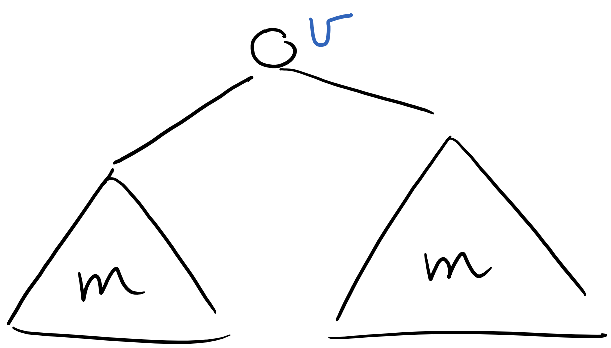
Thus, "on average" each insertion takes $O(\log N)$ time.

BUT: Any ***particular*** insertion may **not** be $O(\log N)$

Wort case for one insertion: $\Theta(N)$

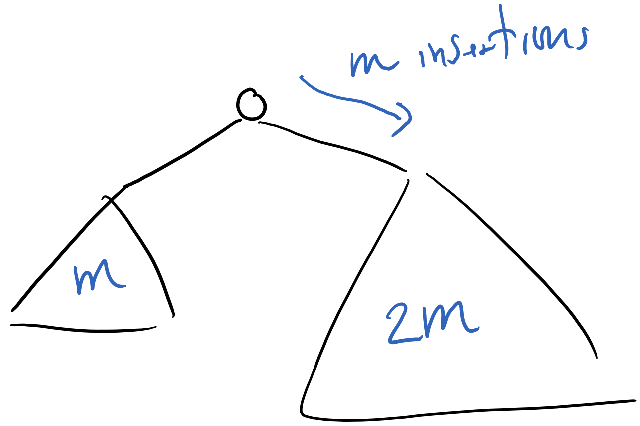
(NOT A PROOF...)

consider this config:



$N = 2m+1$

GAME: force v to be re-balanced as soon as possible via seq of insertions

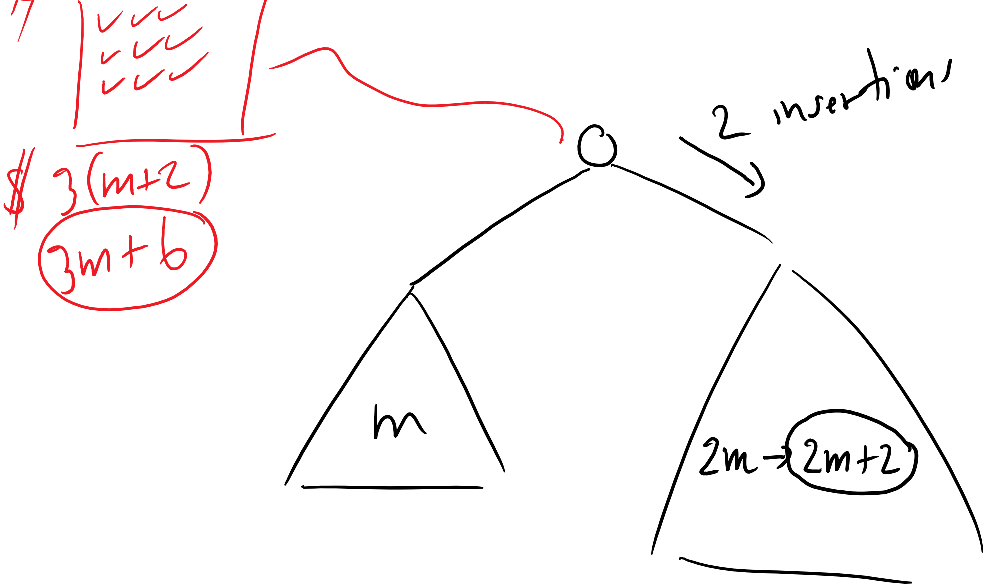


m insertions into Right-side...

still ok..

two more?

$N = m + 2m + 2 + 1$



$(2m+2) \overset{?}{\leq} 2 \times m + 1$

NOPE!

Rebalance!

$N_{start} = 2m+1$

$N_{final} = 3m+3$

Total Work?

$2m+2$ insertions $O((2m+2)\log(N_f))$

$\underline{2m+2}$ insertions $\underline{O((2m+2)\log(N_f))}$

$+$ $O(N\log N)$

Rebalance

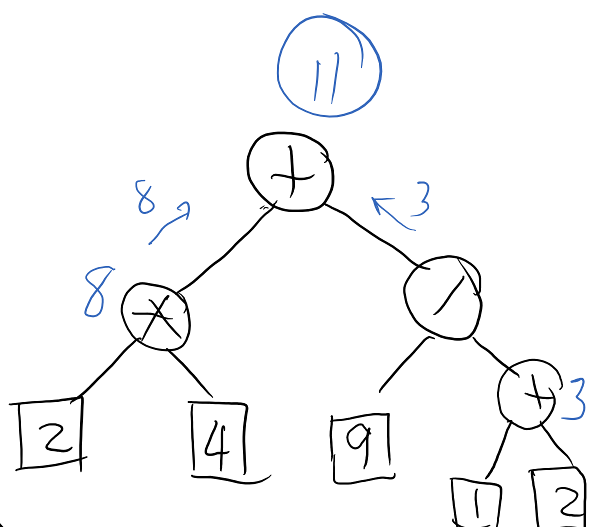
$O(N)$

Total: $N\log N + N = \Theta(N\log N)$

$2+2$	$2\ 2\ +$	$+\ 2\ 2$

$+\ \times\ 2\ 4\ /\ 9\ +\ 1\ 2$

$-$



<operator> <exprA> <exprB>

$$(2\times 4)+(9/(1+2))$$