INSTITUTO POLITÉCNICO NACIONAL

ESCUELA SUPERIOR DE CÓMPUTO

Serie Trigonométrica de Fourier

Participación 1.3

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Asignatura:

Teoría de comunicaciones y señales

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3CV17

Fecha:

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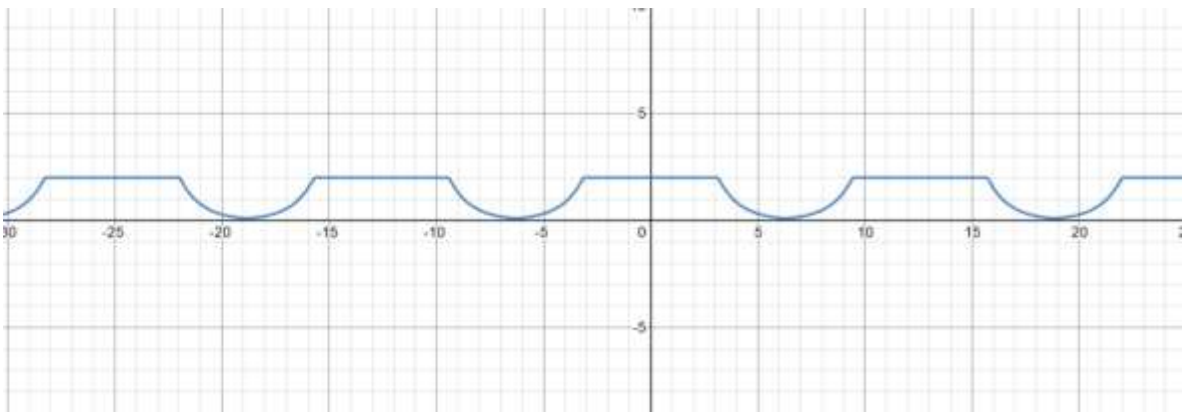
$$f(t) = 1 - \frac{\left(e^{-\pi} - 1\right)}{\pi} + \frac{4}{\pi} \sum_{n=1}^{100} \left(\frac{\sin\left(\frac{\pi n}{2}\right)}{n} + \frac{\left(2\cos\left(\frac{n\pi}{2}\right) - n\sin\left(\frac{\pi n}{2}\right) - \left(e^{-\pi} \cdot 2 \cdot (-1)^{n}\right)\right)}{n^{2}+4} \right) \cos\left(\frac{nt}{2}\right)$$



$$h(t) = \begin{cases} 2, & -\pi < t < \pi \\ 2e^{-(t-\pi)}, & \pi < t < 2\pi \\ 2e^{(t-3\pi)} & 2\pi < t < 3\pi \\ h(t) = h(t+4\pi) \end{cases}$$

Función par, $b_n = \emptyset$

$$a_0 = \frac{2}{T}\int_{\emptyset}^{T/2} f(t)\,dt$$

$$a_0 = \frac{2}{4\pi}\int_{\emptyset}^{\frac{4\pi}{2}} f(t)\,dt = \frac{1}{2\pi}\int_{\emptyset}^{2\pi} f(t)\,dt$$

$$a_0 = \frac{1}{2\pi}\left[\int_{\emptyset}^{\pi} 2\,dt + \int_{\pi}^{2\pi} 2e^{-(t-\pi)}\,dt\right]$$

$$a_0 = \frac{1}{2\pi}\left[2t\Big|_0^{\pi} + \int_{\pi}^{2\pi} 2e^{-(t-\pi)}\,dt\right]$$

$$a_0 = \frac{1}{2\pi}\left[2\pi - \emptyset + \int 2e^{-(t-\pi)}\,dt\right]$$

$$a_0 = \frac{1}{2\pi}\left[2\pi + \int_{\pi}^{2\pi} 2e^{-(t-\pi)}\,dt\right] = \frac{1}{2\pi}\left[2\pi + 2\int_{\pi}^{2\pi} e^{-(t-\pi)}\,dt\right]$$

$$a_0 = \frac{1}{2\pi}\left[2\pi + 2\int_{\pi}^{2\pi} e^{(-t+\pi)} dt\right] = \frac{1}{2\pi}\left[2\pi + 2\int_{\pi}^{2\pi} -e^{x} dt\right]$$

$x = -t+\pi \quad dt = \frac{1}{x'}dx$

$x' = 1$

$$a_0 = \frac{1}{2\pi}\left[2\pi + 2\int_{\pi}^{2\pi} e^{x} dx\right] = \frac{1}{2\pi}\left[2\pi + 2e^{x}\Big|_{\pi}^{2\pi}\right]$$

$e^{x} = e^{-t+\pi}$

$$a_0 = \frac{1}{2\pi}\left[2\pi - \left[2e^{-t+\pi}\right]\Big|_{\pi}^{2\pi}\right] = \frac{1}{2\pi}\left[2\pi - \left(2e^{-2\pi+\pi} - 2e^{-\pi+\pi}\right)\right]$$

$$a_0 = \frac{1}{2\pi}\left[2\pi - \left(2e^{-\pi} - 2\right)\right] = \frac{1}{2\pi}\left[2\pi - \left(2e^{-\pi} - 2\right)\right]$$

$$a_0 = \frac{1}{2\pi}\left(2\pi - 2\left[-e^{-\pi} - 1\right]\right) = \frac{2\pi}{2\pi} - \frac{2\left[-e^{-\pi}-1\right]}{2\pi}$$

$$a_0 = 1 - \frac{\left[-e^{-\pi}-1\right]}{\pi} = \boxed{\frac{1-e^{-\pi}-1}{\pi}}$$

$$a_n = \frac{4}{T}\int_0^{2\pi} f(t)\cos(nt/2)\,dt$$

$$a_n = \frac{1}{\pi}\int 2\cos(nt/2)\,dt + \int_{\pi}^{2\pi} 2e^{-(t-\pi)}\cos(nt/2)\,dt$$

$x = \frac{nt}{2} \quad x' = \frac{n}{2} \quad dt = \frac{1}{x'}dx$

$$a_n = \frac{1}{\pi}\left[\int_0^{\pi} \frac{4\cos(x)}{n}dx + \int_{\pi}^{2\pi} 2e^{-(t-\pi)}\cos\left(\frac{nt}{2}\right)dt\right]$$

$udv = uv - \int v\,du$

$$a_n = \frac{1}{\pi}\left[\frac{4}{n}\int_0^{\pi}\cos(x)dx + 2\int_{\pi}^{2\pi} e^{-(t-\pi)}\cos(nt/2)\,dt\right]$$

$$a_n = \frac{1}{\pi}\left[\frac{4}{n}(\text{sen}(x))\Big|_0^{\pi} + 2\int_{\pi}^{2\pi} e^{-(t-\pi)}\cos\left(\frac{nt}{2}\right)dt\right]$$

$$a_n = \frac{1}{\pi}\left[\frac{4}{n}\left(\text{sen}\left(\frac{nt}{2}\right)\Big|_0^{\pi}\right) + 2\int_{\pi}^{2\pi} e^{-(t-\pi)}\cos\left(\frac{nt}{2}\right)dt\right]$$

$$a_n = \frac{1}{\pi}\left[\frac{4}{n}\left(\text{sen}\left(\frac{\pi n}{2}\right) - \text{sen}\left(\frac{0n}{2}\right)\right) + 2\int_{\pi}^{2\pi} e^{-(t-\pi)}\cos\left(\frac{nt}{2}\right)dt\right]$$

$$a_n = \frac{1}{\pi}\left[\frac{4\,\text{sen}\left(\frac{\pi n}{2}\right)}{n} + 2\int_{\pi}^{2\pi} e^{-(t-\pi)}\cos\left(\frac{nt}{2}\right)dt\right]$$

$u = \cos\left(\frac{nt}{2}\right)$

$u' = -n\,\text{sen}\left(\frac{nt}{2}\right)/2$

$v' = e^{\pi - t}$

$v = -e^{\pi - t}$

$$a_n = \frac{1}{\pi}\left[\frac{4\,\text{sen}\left(\frac{\pi n}{2}\right)}{n} + 2\left[-e^{\pi-t}\cdot\cos\left(\frac{nt}{2}\right)\Big|_{\pi}^{2\pi} - \int_{\pi}^{2\pi} -e^{\pi-t}\cdot -n\,\text{sen}\left(\frac{nt}{2}\right)\right]\right]$$

$$a_n = \frac{1}{\pi}\left[\frac{4\,\text{sen}\left(\frac{n\pi}{2}\right)}{n} + 2\left[-e^{\pi-t}\cos\left(\frac{nt}{2}\right)\Big|_{\pi}^{2\pi} - \int_{\pi}^{2\pi} e^{\pi-t}\,\frac{n\,\text{sen}\left(\frac{nt}{2}\right)}{2}\right]\right]$$

$u = \dfrac{-n\,\text{sen}\left(\frac{nt}{2}\right)}{2}$ $\quad du' = \dfrac{-n^2\cos\left(\frac{nt}{2}\right)}{4}$ $\quad dv = -e^{\pi-t}$

$v = e^{\pi - t}$

$$a_n = \frac{1}{\pi}\left[\frac{4\,\text{sen}\left(\frac{\pi n}{2}\right)}{2} + 2\left[-e^{\pi-t}\cos\left(\frac{nt}{2}\right)\Big|_{\pi}^{2\pi} - \left(\frac{-n\,\text{sen}\left(\frac{nt}{2}\right)e^{\pi-t}}{2} - \int -e^{\pi-t}\frac{n^2}{4}\cos\left(\frac{nt}{2}\right)dt\right]\right]$$

$$a_n = \frac{1}{\pi}\left[\frac{4\,\text{sen}\left(\frac{\pi n}{2}\right)}{n} + 2\left[-e^{\pi-t}\cos\left(\frac{nt}{2}\right)\Big|_{\pi}^{2\pi} - \left(-\frac{ne^{\pi-t}\,\text{sen}\left(\frac{nt}{2}\right)}{2}\Big|_{\pi}^{2\pi} + \frac{n^2}{4}\int_{\pi}^{2\pi} e^{\pi-t}\cos\left(\frac{nt}{2}\right)dt\right]\right]$$

$$a_n = \frac{1}{\pi}\left[\frac{4\,\text{sen}\left(\frac{\pi n}{2}\right)}{n} + 2\left[\frac{\frac{ne^{\pi-t}\,\text{sen}\left(\frac{nt}{2}\right)}{2} - e^{\pi-t}\cos\left(\frac{nt}{2}\right)}{\frac{n^2}{4}+1}\right]\Big|_{\pi}^{2\pi}\right]$$

$$a_n = \frac{1}{\pi}\left[\frac{4\operatorname{sen}\left(\frac{\pi n}{2}\right)}{n} + \left(\frac{4e^{\pi - t}\left(n\operatorname{sen}\left(\frac{nt}{2}\right) - 2\cos\left(\frac{nt}{2}\right)\right)}{n^2+4}\right)\Bigg|_{\pi}^{2\pi}\right]$$

$$\frac{4e^{\pi - 2\pi}\left(n\operatorname{sen}\left(\frac{2\pi}{2}\right) - 2\cos\frac{2\pi n}{2}\right)}{n^2+4} - \frac{4e^{\pi - \pi}\left(n\operatorname{sen}\left(\frac{\pi n}{2}\right) - 2\cos\left(\frac{\pi n}{2}\right)\right)}{n^2+4}$$

$$\frac{4e^{-\pi}\left(n\operatorname{sen}(\pi n) - 2\cos(\pi n)\right)}{n^2+4} - \frac{4e^{0}\left(n\operatorname{sen}\left(\frac{\pi n}{2}\right) - 2\cos\left(\frac{\pi n}{2}\right)\right)}{n^2+4}$$

$$\frac{4e^{-\pi}\left(n\operatorname{sen}(\pi n) - 2\cos(\pi n) - e^{\pi} n\operatorname{sen}\left(\frac{\pi n}{2}\right) + 2e^{\pi}\cos\left(\frac{\pi n}{2}\right)\right)}{n^2+4}$$

$$\frac{4e^{-\pi}\left(2e^{\pi}\cos\left(\frac{n\pi}{2}\right) - e^{\pi} n\operatorname{sen}\left(\frac{n\pi}{2}\right) - 2(-1)^n\right)}{n^2+4}$$

$$a_n = \frac{1}{\pi}\left[\frac{4\operatorname{sen}\left(\frac{\pi n}{2}\right)}{n} + \frac{4e^{-\pi}\left(2e^{\pi}\cos\left(\frac{n\pi}{2}\right) - e^{\pi} n\operatorname{sen}\left(\frac{\pi n}{2}\right) - 2(-1)^n\right)}{n^2+4}\right]$$

$$\boxed{a_n = \frac{4}{\pi}\left[\frac{\operatorname{sen}\left(\frac{\pi n}{2}\right)}{n} + \frac{2\cos\left(\frac{n\pi}{2}\right) - n\operatorname{sen}\left(\frac{\pi n}{2}\right) - e^{-\pi} 2(-1)^n}{n^2+4}\right]}$$