INSTITUTO POLITÉCNICO NACIONAL

ESCUELA SUPERIOR DE CÓMPUTO

Serie Trigonométrica de Fourier

Evidencia 1.4

Integrantes:

Luis Fernando Reséndiz Chávez

Asignatura:

Teoría de Comunicaciones y Señales

Profesor:

Arzate Gordillo Jacqueline

Grupo:

3CV17

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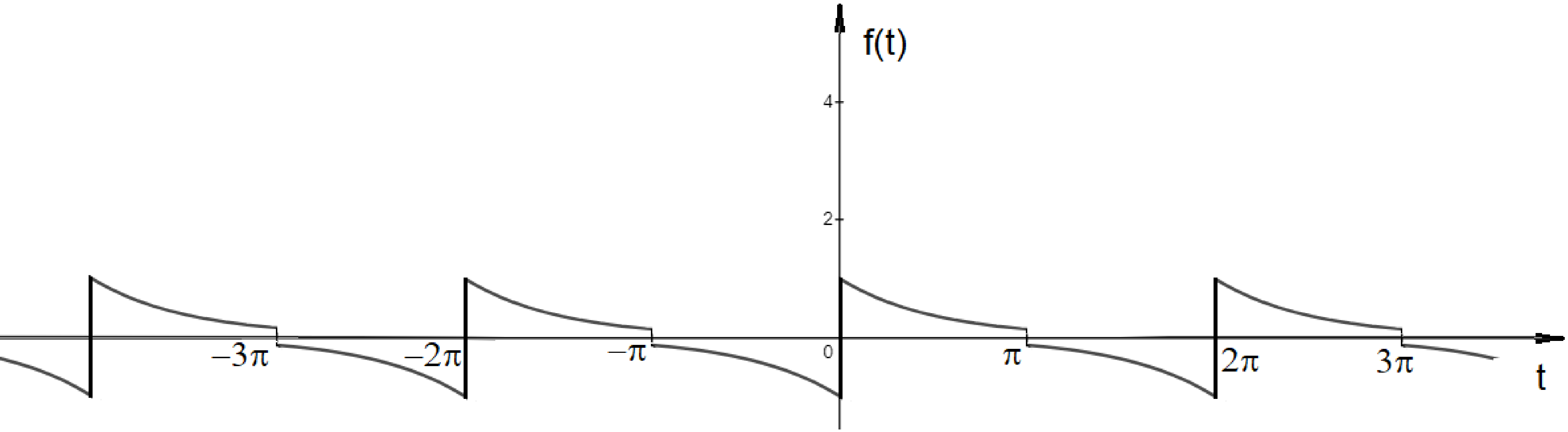
Evidencia 1.4 Encuentre y grafique la STF de *f(t)*

$$f(t) = \begin{cases} e^{-t} & 0 < t < \pi \\ -e^{t} & -\pi < t < 0 \end{cases}$$

$$f(t) = f(t + 2\pi)$$



Reséndiz Chávez Luis Fernando

STF

$$f(t) = \begin{cases} e^{-t}, & \theta < t < \pi \\ -e^{t}, & -\pi < t < \emptyset \end{cases}$$

$$b_n = \frac{2}{\pi}\int_0^{\pi} e^{-t}\,\text{sen}(nt)\,dt$$

$$= \int e^{-t}\,\text{sen}(nt)\,dt \qquad \int uv' = uv - \int u'v$$

$$u = e^{-t} \quad v' = \text{sen}(nt) \quad u' = \frac{d}{dt}(e^{-t}) = -e^{-t} \quad v = -\frac{1}{n}\cos(nt)$$

$$= e^{-t}\left(-\frac{1}{n}\cos(nt)\right) - \int (-e^{-t})\left(-\frac{1}{n}\cos(nt)\right)dt$$

$$= -e^{-t}\frac{1}{n}\cos(nt) - \int e^{-t}\frac{1}{n}\cos(nt)\,dt$$

$$= -e^{-t}\frac{1}{n}\cos(nt) - \frac{1}{n}\int e^{-t}\cos(nt)\,dt$$

$$u = e^{-t} \quad v' = \cos(nt) \quad u' = \frac{d}{dt}(e^{-t}) = -e^{-t} \quad v = \frac{1}{n}\text{sen}(nt)$$

$$= e^{-t}\frac{1}{n}\text{sen}(nt) - \int (-e^{-t})\frac{1}{n}\text{sen}(nt)\,dt$$

$$= e^{-t}\frac{1}{n}\text{sen}(nt) - \int -e^{-t}\frac{1}{n}\text{sen}(nt)\,dt$$

$\therefore$

$$= -e^{-t}\frac{1}{n}\cos(nt) - \frac{1}{n}\left(e^{-t}\frac{1}{n}\text{sen}(nt) - \int -e^{-t}\frac{1}{n}\text{sen}(nt)\,dt\right)$$

$$= -e^{-t}\frac{1}{n}\cos(nt) - \frac{1}{n}\left(e^{-t}\frac{1}{n}\text{sen}(nt) - \left(-\frac{1}{n}\int e^{-t}(nt)\,dt\right)\right)$$

$$= \frac{-ne^{-t}\cos(nt)}{n^2+1} - \frac{e^{-t}\,\text{sen}(nt)}{n^2+1} + c$$

Evaluando

$$\lim_{t\to 0^+}\left(-\frac{ne^{-t}\cos(nt)}{n^2+1}-\frac{e^{-t}\operatorname{sen}(nt)}{n^2+1}\right)$$

$$=\frac{ne^{-0}\cos(n\cdot 0)}{n^2+1}-\frac{e^{-0}\operatorname{sen}(n\cdot 0)}{n^2+1}=\boxed{-\frac{n}{n^2+1}}$$

$$\lim_{t\to \pi^-}\left(-\frac{ne^{-t}\cos(nt)}{n^2+1}-\frac{e^{-t}\operatorname{sen}(nt)}{n^2+1}\right)=$$

$$=\frac{-ne^{-\pi}\cos(n\pi)}{n^2+1}-\frac{e^{-\pi}\operatorname{sen}(n\pi)}{n^2+1}=\boxed{\frac{(-1)^n n}{e^{\pi}(n^2+1)}}$$

$$=-\frac{(-1)^n n}{e^{\pi}(n^2+1)}-\left(\frac{-n}{n^2+1}\right)=\frac{n\left(-\frac{1}{e^{\pi}}(-1)^n+1\right)}{n^2+1}$$

$$=\frac{2}{\pi}\cdot\frac{n\left(-\frac{1}{e^{\pi}}(-1)^n+1\right)}{n^2+1}=\boxed{\frac{2n\left(-\frac{1}{e^{\pi}}(-1)^n+1\right)}{\pi(n^2+1)}}$$

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos\left(\frac{n\pi t}{L}\right) + b_n \operatorname{sen}\left(\frac{n\pi}{L}\right) \right]$$

$$f(t) = \frac{0}{2} + \sum_{n=1}^{\infty} \left[(0) \cos\left(\frac{n\cancel{\pi} t}{\cancel{\pi}}\right) + \frac{2}{\pi} \left(\frac{n\left(-\frac{1}{e^{\pi}}(-1)^n + 1\right)}{n^2+1} \right) \operatorname{sen}\left(\frac{n\cancel{\pi} t}{\cancel{\pi}}\right) \right]$$

$$= \frac{2}{\pi} \sum_{n=1}^{\infty} \left[\frac{n\left(-\frac{1}{e^{\pi}}(-1)^n + 1\right) \operatorname{sen}(nt)}{n^2+1} \right]$$

$$f(t) = \frac{2}{\pi} \sum_{n=1}^{100} \left(\frac{n\left(-\frac{1}{e^{\pi}}(-1)^{n} + 1\right) \sin(nt)}{n^{2} + 1} \right)$$