Spokes

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Spokes

Sample on the $(\mathbb{S}^2)^{168}$ where $p = 2$, $r = 168$ and $n = 177$.

```
load(paste(here("data-raw", "spokes"), "spokes.RData", sep = "/"))
n <- dim(dirs)[1]
p <- dim(dirs)[3] - 1
r <- dim(dirs)[2]
spokes <- array(0, c(n, p + 1, r))
for(i in seq_len(r)) {
  spokes[, , i] <- dirs[, i, ]
}

# Plot subjects for each spok
rgl::open3d()
```

```
## glX
##   1
```

```
rgl::mfrow3d(nr = 12, nc = 14, sharedMouse = TRUE)
n <- 10
col <- rainbow(n, alpha = 0.8)
for (i in seq_len(ncol(dirs))) {
  col_i <- col # ids_labs + 1
  rgl::plot3d(dirs[1:n, i, ], xlim = c(-1, 1), ylim = c(-1, 1), zlim = c(-1, 1),
              col = col_i, axes = FALSE, box = FALSE, xlab = "",
              ylab = "", zlab = "")
  rgl::spheres3d(0, 0, 0, radius = 1, col = "lightblue", lit = FALSE,
                 alpha = 0.5)
}
```

Let's calculate the rho parameters based on a perplexity of 30:

```
rho_spokes_perp25 <- rho_optim_bst(spokes, 25)
```

```
## Time difference of 3.297134 mins
```

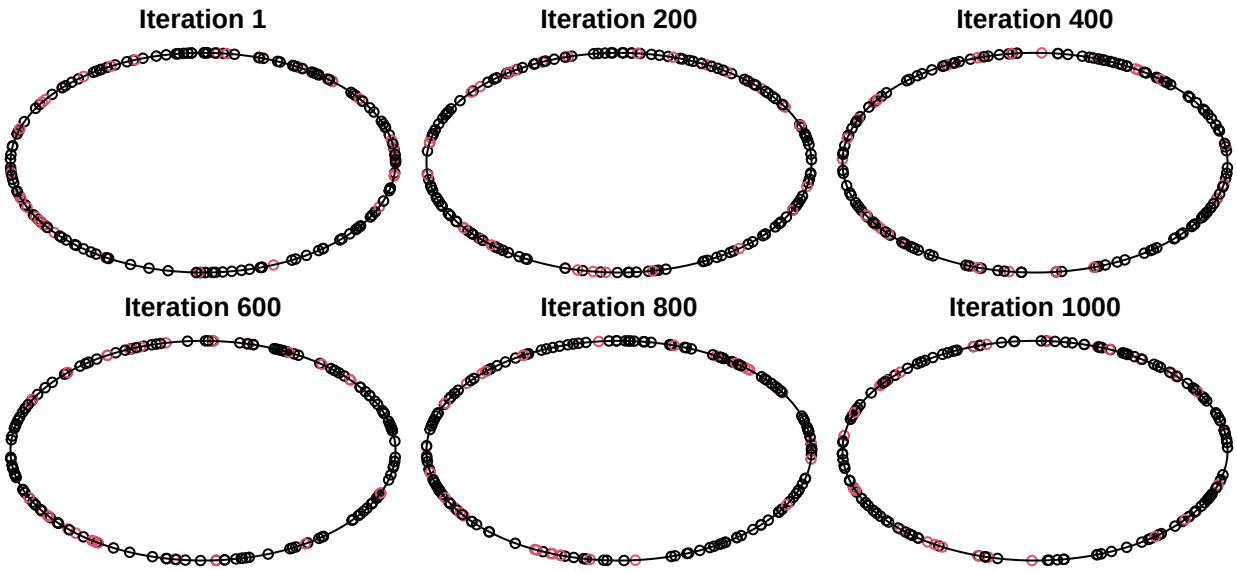
First, let's reduce to dimension $\mathbb{S}^1$ then $d = 1$ (circumference):

```
res_pscsne <- psc_sne(spokes, d = 1, rho_psc_list = rho_spokes_perp25,
             colors = ifelse(ids_labs, 2, 1), show_prog = TRUE)
```

```
## It: 1; obj: 1.306e+01; abs: 0.000e+00; rel: 0.000e+00; norm: 1.287e-01; mom: 0.000e+00;
## best it: 1; best obj: 1.306e+01
```

```
## It: 100; obj: 1.387e+01; abs: 1.431e+00; rel: 1.150e-01; norm: 3.469e-01; mom: 6.307e+00;
```

```
## best it: 61; best obj: 1.225e+01
## It: 200; obj: 1.374e+00; abs: 5.127e-03; rel: 3.718e-03; norm: 8.416e-02; mom: 3.873e+00;
## best it: 170; best obj: 1.355e+00

## It: 300; obj: 1.322e+00; abs: 1.708e-02; rel: 1.276e-02; norm: 7.758e-02; mom: 5.299e+00;
## best it: 248; best obj: 1.285e+00
## It: 400; obj: 1.341e+00; abs: 1.231e-02; rel: 9.259e-03; norm: 7.807e-02; mom: 5.196e+00;
## best it: 248; best obj: 1.285e+00

## It: 500; obj: 1.329e+00; abs: 1.672e-02; rel: 1.242e-02; norm: 7.950e-02; mom: 5.377e+00;
## best it: 248; best obj: 1.285e+00
## It: 600; obj: 1.329e+00; abs: 3.855e-04; rel: 2.900e-04; norm: 7.805e-02; mom: 5.170e+00;
## best it: 248; best obj: 1.285e+00

## It: 700; obj: 1.328e+00; abs: 1.671e-03; rel: 1.256e-03; norm: 7.688e-02; mom: 5.255e+00;
## best it: 248; best obj: 1.285e+00
## It: 800; obj: 1.321e+00; abs: 3.224e-02; rel: 2.382e-02; norm: 8.069e-02; mom: 5.667e+00;
## best it: 248; best obj: 1.285e+00

## It: 900; obj: 1.325e+00; abs: 1.917e-02; rel: 1.426e-02; norm: 7.934e-02; mom: 5.409e+00;
## best it: 248; best obj: 1.285e+00
## It: 1000; obj: 1.329e+00; abs: 7.677e-03; rel: 5.745e-03; norm: 7.803e-02; mom: 5.412e+00;
## best it: 248; best obj: 1.285e+00
```



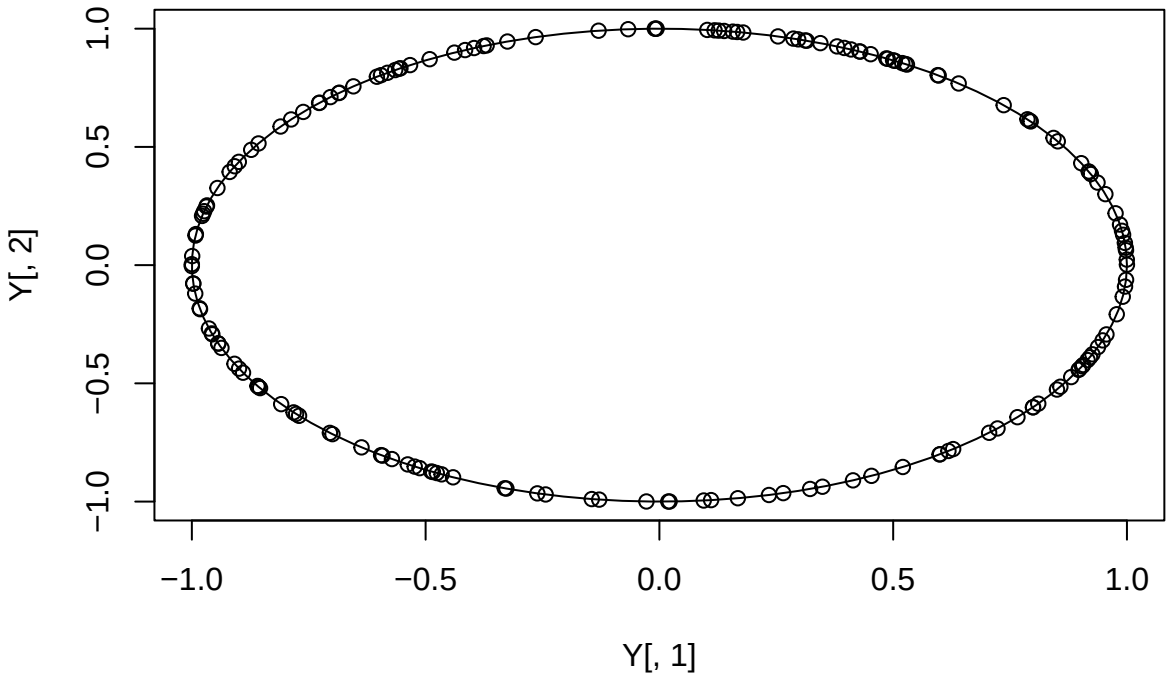

```
Y <- res_pscsne$best_Y
```

Let's apply the kernel means shift function to see what clusters identifies.

```
res_kms_dir_Y_1 <- kms_dir(x = Y, data = Y)
```

```
##   |                                                                      |
```

```
plot(Y[, 1], Y[, 2], col = res_kms_dir_Y_1$cluster, xlim = c(-1, 1),
     ylim = c(-1, 1))

polygon(x = cos(seq(0, 2 * pi, length.out = 100)),
        y = sin(seq(0, 2 * pi, length.out = 100)))
```

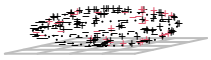


Now we are going to reduce to dimension $\mathbb{S}^2$ then $d = 2$ (sphere):

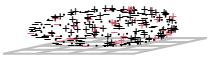
```
res_pscsne <- psc_sne(spokes, d = 2, rho_psc_list = rho_spokes_perp25,
             colors = ifelse(ids_labs, 2, 1), show_prog = TRUE)
```

```
## It: 1; obj: 1.454e+01; abs: 0.000e+00; rel: 0.000e+00; norm: 2.401e-01; mom: 0.000e+00;
## best it: 1; best obj: 1.454e+01

## It: 100; obj: 1.646e+01; abs: 3.168e+00; rel: 2.383e-01; norm: 8.409e-01; mom: 7.989e+00;
## best it: 35; best obj: 1.314e+01
## It: 200; obj: 1.547e+00; abs: 7.200e-03; rel: 4.634e-03; norm: 1.726e-01; mom: 6.153e+00;
## best it: 132; best obj: 1.488e+00

## It: 300; obj: 1.443e+00; abs: 4.417e-02; rel: 2.971e-02; norm: 1.712e-01; mom: 9.146e+00;
## best it: 248; best obj: 1.324e+00
## It: 400; obj: 1.442e+00; abs: 2.421e-02; rel: 1.651e-02; norm: 1.702e-01; mom: 8.965e+00;
## best it: 248; best obj: 1.324e+00

## It: 500; obj: 1.400e+00; abs: 1.124e-01; rel: 7.435e-02; norm: 1.688e-01; mom: 9.143e+00;
## best it: 248; best obj: 1.324e+00
## It: 600; obj: 1.520e+00; abs: 6.200e-02; rel: 4.254e-02; norm: 1.744e-01; mom: 9.095e+00;
## best it: 248; best obj: 1.324e+00

## It: 700; obj: 1.468e+00; abs: 3.149e-02; rel: 2.101e-02; norm: 1.746e-01; mom: 9.098e+00;
## best it: 248; best obj: 1.324e+00
## It: 800; obj: 1.411e+00; abs: 7.975e-02; rel: 5.349e-02; norm: 1.682e-01; mom: 9.186e+00;
## best it: 248; best obj: 1.324e+00

## It: 900; obj: 1.462e+00; abs: 1.571e-04; rel: 1.075e-04; norm: 1.708e-01; mom: 9.108e+00;
## best it: 248; best obj: 1.324e+00
## It: 1000; obj: 1.389e+00; abs: 1.173e-01; rel: 7.790e-02; norm: 1.677e-01; mom: 9.032e+00;
## best it: 248; best obj: 1.324e+00
```

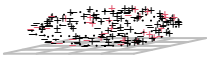
Iteration 1



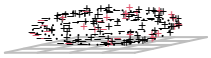
Iteration 200



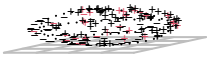
Iteration 400



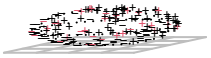
Iteration 600



Iteration 800



Iteration 1000



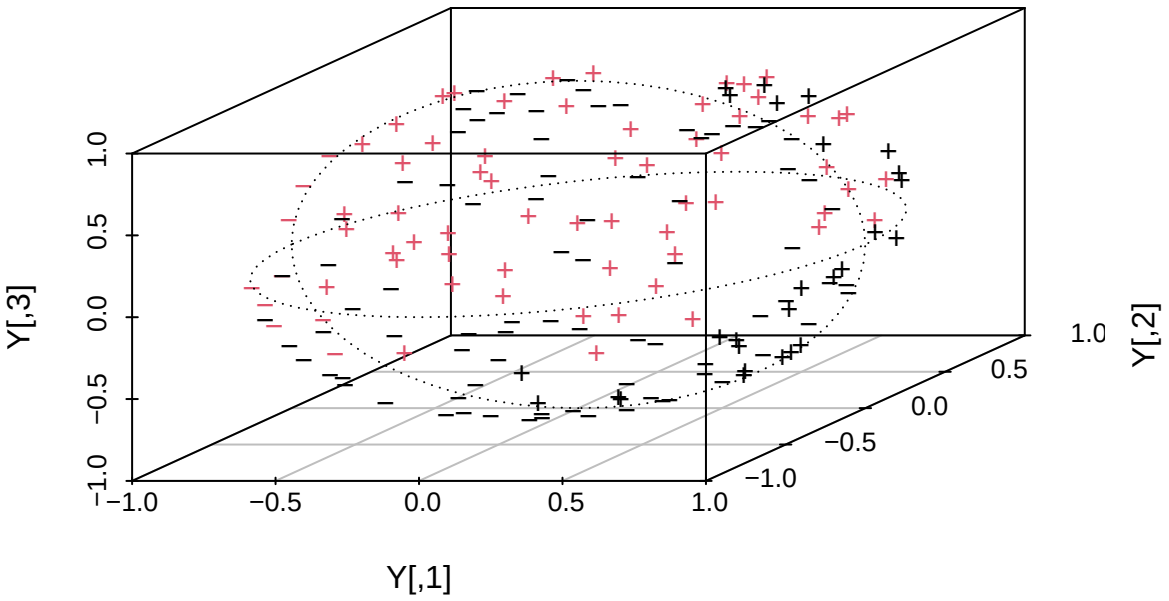
```
Y <- res_pscsne$best_Y
```

Let's do the same applying the kms onto the sphere.

```
res_kms_dir_Y <- kms_dir(x = Y, data = Y)
```

```
##   |                                                                      |
```

```
colors <- res_kms_dir_Y$cluster

# Sequence from -180 to 180 by an step of 15 in radians
seq_rad <- seq(-pi, pi, by = pi / 30)
# Meridian calculates as theta = 0 and phi = i
# where i is the radians
meridian <- do.call(rbind, lapply(seq_rad, function(i) c(0, i)))
equator <- do.call(rbind, lapply(seq_rad, function(i) c(i, pi/2)))

sd3 <- scatterplot3d::scatterplot3d(Y, xlim = c(-1, 1), ylim = c(-1, 1),
                              zlim = c(-1, 1), color = colors,
                              pch = c('+', '-')[ifelse(sign(Y[, 2]) == 1, 1, 2)])
sd3$points3d(DirStats::to_sph(th = meridian[, 1], ph = meridian[, 2]),
             type = "l", lty = 3)
sd3$points3d(DirStats::to_sph(th = equator[, 1], ph = equator[, 2]),
             type = "l", lty = 3)
```


Y[,3]
1.0
0.5
0.0
−0.5
−1.0
−1.0
−0.5
0.0
0.5
1.0
Y[,1]
1.0
0.5
0.0
−0.5
−1.0
Y[,2]