San Juan de Fuca

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```
data_ID <- "HFR/USWC/6km/hourly/RTV/HFRADAR_US_West_Coast_6km_Resolution_Hourly_RTV_best.ncd"
data_url <- paste0("http://hfrnet-tds.ucsd.edu/thredds/dodsC/", data_ID)
data <- ncdf4::nc_open(data_url)

lat <- ncdf4::ncvar_get(data,"lat")
lon <- ncdf4::ncvar_get(data,"lon")
tim <- lubridate::date("2012-01-01 00:00:00 UTC") +
  lubridate::hours(ncdf4::ncvar_get(data, "time")) # Fix weird data time

# Strait of Juan de Fuca -- just 2 months
loc <- "Strait of Juan de Fuca"
begin_lat <- 48.1
end_lat <- 48.5
begin_lon <- -124.5
end_lon <- -123
begin_tim <- lubridate::date("2021-01-01 00:00:00 UTC")
end_tim <- lubridate::date("2021-03-01 00:00:00 UTC")

# Get data function -- takes info from global environment and writes on it,
# not very elegant but does the job
get_data <- function() {

  # Obtain the data indexes associated to the given information
  begin_lat_ind <<- which(lat >= begin_lat)[1]
  begin_lon_ind <<- which(lon >= begin_lon)[1]
  begin_tim_ind <<- which(tim >= begin_tim)[1]
  end_lat_ind <<- which(lat <= end_lat)
  end_lon_ind <<- which(lon <= end_lon)
  end_tim_ind <<- which(tim <= end_tim)
  end_lat_ind <<- end_lat_ind[length(end_lat_ind)]
  end_lon_ind <<- end_lon_ind[length(end_lon_ind)]
  end_tim_ind <<- end_tim_ind[length(end_tim_ind)]

  # Download sizes
  l_lat <<- end_lat_ind - begin_lat_ind + 1
  l_lon <<- end_lon_ind - begin_lon_ind + 1
  l_tim <<- end_tim_ind - begin_tim_ind + 1
  stopifnot(l_lat > 0)
  stopifnot(l_lon > 0)
  stopifnot(l_tim > 0)

  # Upper bound on the size of one out of two objects to be downloaded
```

```
  cat("Data size:", format(object.size(rnorm(l_lat * l_lon * l_tim)),
                           units = "Mb"))

  # Download (u, v), being:
  # u (m/s) = surface_eastward_sea_water_velocity = surface_eastward_sea_water_velocity
  # v (m/s) = surface_northward_sea_water_velocity = surface_northward_sea_water_velocity
  u <<- ncdf4::ncvar_get(data, "u",
                         start = c(begin_lon_ind, begin_lat_ind, begin_tim_ind),
                         count = c(l_lon, l_lat, l_tim))
  v <<- ncdf4::ncvar_get(data, "v",
                         start = c(begin_lon_ind, begin_lat_ind, begin_tim_ind),
                         count = c(l_lon, l_lat, l_tim))
}

# Download data in a monthly loop to avoid surpassing 500Mb limit
for (year in 2020:2022) {

  month <- 1
  if (year == 2020) {
    month <- 5
  }
  while ( ( year < 2022 && month <= 12 ) || (year == 2022 && month <= 5) ) {

    # Show progress
    cat("\n", year, "-", month, "\n")

    begin_tim <- date(paste(toString(year), toString(month),
                            "01 00:00:00 UTC", sep = "-"))
    if (month != 12) {

      end_tim <- date(paste(toString(year), toString(month + 1),
                            "01 00:00:00 UTC", sep = "-"))

    } else {

      end_tim <- date(paste(toString(year + 1), toString(1),
                            "01 00:00:00 UTC", sep = "-"))

    }

    # Download data
    get_data()

    # Get data dimensions
    lat_length <- dim(u)[1]
    lon_length <- dim(u)[2]
    time_length <- dim(u)[3]

    # Find the indexes associated to dimensions
    lat_aux <- begin_lat_ind:(begin_lat_ind + lat_length - 1)
    lon_aux <- begin_lon_ind:(begin_lon_ind + lon_length - 1)
    time_aux <- begin_tim_ind:(begin_tim_ind + time_length - 1)
```

```
    # Join all the cases
    join <- merge(x = lat[lat_aux], y = lon[lon_aux])
    final_dataframe <- merge(x = join, y = tim[time_aux], by = NULL)
    colnames(final_dataframe) <- c("lat", "lon", "time")

    # Add the velocities and save the data
    final_dataframe$u <- c(u)
    final_dataframe$v <- c(v)
    save(final_dataframe, file = paste(loc, "_", toString(year), "_",
                                       toString(month), ".RData", sep = ""))

    month <- month + 1
  }

}

## Obtaining daily data for zones A, B, C, and D

# List individual RDatas
files <- list.files(pattern = "*.RData", full.names = TRUE, recursive = FALSE)

# Function that reads over all the files in the directory containing the raw
# data and return the records inside a given area delimited by longitude
# and latitude
extract_data <- function(begin_lat, end_lat, begin_lon, end_lon) {

  # Retrieve monthly data
  monthly_data <- lapply(files, function(x) {

    # Load raw data file
    load(x)
    results <- filter(final_dataframe, lat > begin_lat, lat < end_lat,
                      lon > begin_lon, lon < end_lon)

    # Calculate directions and speed
    results$d <- atan2(x = results$u, y = results$v)
    results$speed <- sqrt(results$u^2 + results$v^2)
    return(results)

  })

  # Merge available data
  total_data <- data.frame()
  for (i in seq_along(monthly_data)) {

    total_data <- rbind(total_data, monthly_data[[i]])

  }
  return(total_data)

}
```

```
results <- extract_data(begin_lat, end_lat, begin_lon, end_lon)
results <- results[order(as.Date(results$time)),]
```

```
ttt <- results %>%
  mutate(
    month = lubridate::month(time),
    year = lubridate::year(time),
    date = paste(year, month, sep="-"),
    location = paste(lat, lon, sep=",")
  ) %>%
  group_by(lat, lon, year, month, .drop=T) %>%
  na.omit %>%
  mutate(
    weights = speed / sum(speed),
    theta=circular:::WeightedMeanCircularRad(w = weights, x = d)
  ) %>%
  ungroup %>%
  select(location, date, theta) %>%
  unique %>%
  mutate(
    coord = DirStats::to_cir(theta)
  ) %>%
  select(-theta) %>%
  data.frame
ttt <- ttt %>% filter(date != "2022-6")
sanjuanfuca <- abind(split(ttt, ttt$date), along=3)
rownames(sanjuanfuca) <- sanjuanfuca[,1,1]
sanjuanfuca <- sanjuanfuca[,3:4,]
r <- dim(sanjuanfuca)[3]
sanjuanfuca <- sapply(1:r, function(k) apply(sanjuanfuca[, , k], 2, as.numeric), simplify = 'array')
save(list = "sanjuanfuca", file = "sanjuanfuca.RData")
```

```
load("~/Documents/Statistics for Data Science/TFM/psc-sne/data-raw/strait-san-juan-fuca/sanjuanfuca.rda
```

Let's calculate some rho values for a perplexity of 12, 20, 33 and 45.

```
rho_12 <- rho_optim_parallel(sanjuanfuca, 12)
```

```
## Time difference of 55.25006 secs
```

```
rho_20 <- rho_optim_parallel(sanjuanfuca, 20)
```

```
## Time difference of 50.91564 secs
```

```
rho_33 <- rho_optim_parallel(sanjuanfuca, 33)
```

```
## Time difference of 55.88984 secs
```

```
rho_45 <- rho_optim_parallel(sanjuanfuca, 45)
```

```
## Time difference of 46.01909 secs
```

Let's reduce the dimension using the poly-spherical Cauchy SNE. First, optimized rho for a perplexity of 12 and $d = 1$:

```
Y <- psc_sne(X=sanjuanfuca, d=1,
             rho_psc_list = rho_12,
             num_iteration=300)
```

```
## [1] "Iter 1, obj 9.519773, abs 0.000000, rel 0.000000, norm 1.437746"
## [1] "Iter 2, obj 9.431497, abs 0.088277, rel 0.009273, norm 1.654415"
```

```
## [1] "Iter 3, obj 9.395828, abs 0.035668, rel 0.003782, norm 1.723631"
## [1] "Iter 4, obj 9.383705, abs 0.012123, rel 0.001290, norm 1.768219"
## [1] "Iter 5, obj 9.382417, abs 0.001288, rel 0.000137, norm 1.803840"
## [1] "Iter 6, obj 9.386990, abs 0.004573, rel 0.000487, norm 1.830467"
## [1] "Iter 7, obj 9.393840, abs 0.006850, rel 0.000730, norm 1.851724"
## [1] "Iter 8, obj 9.401922, abs 0.008082, rel 0.000860, norm 1.867845"
## [1] "Iter 9, obj 9.410093, abs 0.008171, rel 0.000869, norm 1.881248"
## [1] "Iter 10, obj 9.418322, abs 0.008229, rel 0.000875, norm 1.891930"
## [1] "Iter 11, obj 9.426282, abs 0.007959, rel 0.000845, norm 1.901294"
## [1] "Iter 12, obj 9.434141, abs 0.007860, rel 0.000834, norm 1.909219"
## [1] "Iter 13, obj 9.441805, abs 0.007664, rel 0.000812, norm 1.916471"
## [1] "Iter 14, obj 9.449424, abs 0.007619, rel 0.000807, norm 1.922916"
## [1] "Iter 15, obj 9.456956, abs 0.007531, rel 0.000797, norm 1.928960"
## [1] "Iter 16, obj 9.464511, abs 0.007555, rel 0.000799, norm 1.934509"
## [1] "Iter 17, obj 9.472048, abs 0.007538, rel 0.000796, norm 1.939774"
## [1] "Iter 18, obj 9.479662, abs 0.007613, rel 0.000804, norm 1.944706"
## [1] "Iter 19, obj 9.487298, abs 0.007636, rel 0.000806, norm 1.949405"
## [1] "Iter 20, obj 9.495048, abs 0.007751, rel 0.000817, norm 1.953861"
## [1] "Iter 21, obj 9.502843, abs 0.007795, rel 0.000821, norm 1.958107"
## [1] "Iter 22, obj 9.510784, abs 0.007940, rel 0.000836, norm 1.962160"
## [1] "Iter 23, obj 9.518779, abs 0.007995, rel 0.000841, norm 1.966017"
## [1] "Iter 24, obj 9.526945, abs 0.008166, rel 0.000858, norm 1.969710"
## [1] "Iter 25, obj 9.535171, abs 0.008225, rel 0.000863, norm 1.973217"
## [1] "Iter 26, obj 9.543588, abs 0.008417, rel 0.000883, norm 1.976576"
## [1] "Iter 27, obj 9.552064, abs 0.008476, rel 0.000888, norm 1.979758"
## [1] "Iter 28, obj 9.560747, abs 0.008684, rel 0.000909, norm 1.982803"
## [1] "Iter 29, obj 9.569486, abs 0.008739, rel 0.000914, norm 1.985679"
## [1] "Iter 30, obj 9.578445, abs 0.008959, rel 0.000936, norm 1.988424"
## [1] "Iter 31, obj 9.587453, abs 0.009007, rel 0.000940, norm 1.991009"
## [1] "Iter 32, obj 9.596689, abs 0.009237, rel 0.000963, norm 1.993469"
## [1] "Iter 33, obj 9.605966, abs 0.009277, rel 0.000967, norm 1.995778"
## [1] "Iter 34, obj 9.615478, abs 0.009512, rel 0.000990, norm 1.997964"
## [1] "Iter 35, obj 9.625020, abs 0.009542, rel 0.000992, norm 2.000010"
## [1] "Iter 36, obj 9.634802, abs 0.009782, rel 0.001016, norm 2.001937"
## [1] "Iter 37, obj 9.644603, abs 0.009800, rel 0.001017, norm 2.003733"
## [1] "Iter 38, obj 9.654647, abs 0.010044, rel 0.001041, norm 2.005413"
## [1] "Iter 39, obj 9.664696, abs 0.010050, rel 0.001041, norm 2.006974"
## [1] "Iter 40, obj 9.674992, abs 0.010296, rel 0.001065, norm 2.008422"
## [1] "Iter 41, obj 9.685280, abs 0.010288, rel 0.001063, norm 2.009762"
## [1] "Iter 42, obj 9.695817, abs 0.010537, rel 0.001088, norm 2.010992"
## [1] "Iter 43, obj 9.706332, abs 0.010515, rel 0.001084, norm 2.012128"
## [1] "Iter 44, obj 9.717098, abs 0.010766, rel 0.001109, norm 2.013158"
## [1] "Iter 45, obj 9.727828, abs 0.010730, rel 0.001104, norm 2.014106"
## [1] "Iter 46, obj 9.738811, abs 0.010984, rel 0.001129, norm 2.014953"
## [1] "Iter 47, obj 9.749743, abs 0.010932, rel 0.001123, norm 2.015735"
## [1] "Iter 48, obj 9.760933, abs 0.011190, rel 0.001148, norm 2.016421"
## [1] "Iter 49, obj 9.772054, abs 0.011121, rel 0.001139, norm 2.017058"
## [1] "Iter 50, obj 9.783438, abs 0.011384, rel 0.001165, norm 2.017605"
## [1] "Iter 51, obj 9.794735, abs 0.011297, rel 0.001155, norm 2.018123"
## [1] "Iter 52, obj 9.806300, abs 0.011566, rel 0.001181, norm 2.018559"
## [1] "Iter 53, obj 9.817760, abs 0.011460, rel 0.001169, norm 2.018986"
## [1] "Iter 54, obj 9.829496, abs 0.011736, rel 0.001195, norm 2.019342"
## [1] "Iter 55, obj 9.841105, abs 0.011609, rel 0.001181, norm 2.019711"
## [1] "Iter 56, obj 9.852999, abs 0.011893, rel 0.001209, norm 2.020020"
```

```
## [1] "Iter 57, obj 9.864743, abs 0.011744, rel 0.001192, norm 2.020369"
## [1] "Iter 58, obj 9.876782, abs 0.012039, rel 0.001220, norm 2.020668"
## [1] "Iter 59, obj 9.888646, abs 0.011864, rel 0.001201, norm 2.021036"
## [1] "Iter 60, obj 9.900818, abs 0.012172, rel 0.001231, norm 2.021368"
## [1] "Iter 61, obj 9.912787, abs 0.011969, rel 0.001209, norm 2.021799"
## [1] "Iter 62, obj 9.925079, abs 0.012291, rel 0.001240, norm 2.022208"
## [1] "Iter 63, obj 9.937137, abs 0.012058, rel 0.001215, norm 2.022748"
## [1] "Iter 64, obj 9.949534, abs 0.012397, rel 0.001248, norm 2.023279"
## [1] "Iter 65, obj 9.961665, abs 0.012130, rel 0.001219, norm 2.023977"
## [1] "Iter 66, obj 9.974154, abs 0.012490, rel 0.001254, norm 2.024679"
## [1] "Iter 67, obj 9.986339, abs 0.012185, rel 0.001222, norm 2.025581"
## [1] "Iter 68, obj 9.998907, abs 0.012568, rel 0.001258, norm 2.026498"
## [1] "Iter 69, obj 10.011128, abs 0.012221, rel 0.001222, norm 2.027650"
## [1] "Iter 70, obj 10.023759, abs 0.012631, rel 0.001262, norm 2.028826"
## [1] "Iter 71, obj 10.035997, abs 0.012238, rel 0.001221, norm 2.030268"
## [1] "Iter 72, obj 10.048677, abs 0.012680, rel 0.001263, norm 2.031738"
## [1] "Iter 73, obj 10.060912, abs 0.012235, rel 0.001218, norm 2.033503"
## [1] "Iter 74, obj 10.073626, abs 0.012714, rel 0.001264, norm 2.035297"
## [1] "Iter 75, obj 10.085838, abs 0.012212, rel 0.001212, norm 2.037409"
## [1] "Iter 76, obj 10.098571, abs 0.012733, rel 0.001262, norm 2.039545"
## [1] "Iter 77, obj 10.110738, abs 0.012167, rel 0.001205, norm 2.042018"
## [1] "Iter 78, obj 10.123476, abs 0.012738, rel 0.001260, norm 2.044504"
## [1] "Iter 79, obj 10.135578, abs 0.012102, rel 0.001195, norm 2.047338"
## [1] "Iter 80, obj 10.148306, abs 0.012728, rel 0.001256, norm 2.050171"
## [1] "Iter 81, obj 10.160321, abs 0.012015, rel 0.001184, norm 2.053356"
## [1] "Iter 82, obj 10.173026, abs 0.012705, rel 0.001250, norm 2.056522"
## [1] "Iter 83, obj 10.184932, abs 0.011907, rel 0.001170, norm 2.060036"
## [1] "Iter 84, obj 10.197602, abs 0.012669, rel 0.001244, norm 2.063513"
## [1] "Iter 85, obj 10.209379, abs 0.011777, rel 0.001155, norm 2.067327"
## [1] "Iter 86, obj 10.222001, abs 0.012622, rel 0.001236, norm 2.071083"
## [1] "Iter 87, obj 10.233628, abs 0.011627, rel 0.001137, norm 2.075160"
## [1] "Iter 88, obj 10.246193, abs 0.012564, rel 0.001228, norm 2.079159"
## [1] "Iter 89, obj 10.257651, abs 0.011458, rel 0.001118, norm 2.083458"
## [1] "Iter 90, obj 10.270148, abs 0.012498, rel 0.001218, norm 2.087662"
## [1] "Iter 91, obj 10.281418, abs 0.011269, rel 0.001097, norm 2.092138"
## [1] "Iter 92, obj 10.293842, abs 0.012424, rel 0.001208, norm 2.096505"
## [1] "Iter 93, obj 10.304906, abs 0.011064, rel 0.001075, norm 2.101114"
## [1] "Iter 94, obj 10.317251, abs 0.012345, rel 0.001198, norm 2.105603"
## [1] "Iter 95, obj 10.328094, abs 0.010843, rel 0.001051, norm 2.110298"
## [1] "Iter 96, obj 10.340358, abs 0.012264, rel 0.001187, norm 2.114869"
## [1] "Iter 97, obj 10.350966, abs 0.010609, rel 0.001026, norm 2.119607"
## [1] "Iter 98, obj 10.363148, abs 0.012181, rel 0.001177, norm 2.124222"
## [1] "Iter 99, obj 10.373512, abs 0.010364, rel 0.001000, norm 2.128959"
## [1] "Iter 100, obj 10.385614, abs 0.012102, rel 0.001167, norm 2.133579"
## [1] "Iter 101, obj 1.245509, abs 9.140105, rel 0.880074, norm 0.265451"
## [1] "Iter 102, obj 1.361545, abs 0.116036, rel 0.093163, norm 0.214637"
## [1] "Iter 103, obj 1.432556, abs 0.071011, rel 0.052155, norm 0.244038"
## [1] "Iter 104, obj 1.577645, abs 0.145090, rel 0.101280, norm 0.243853"
## [1] "Iter 105, obj 1.704132, abs 0.126487, rel 0.080175, norm 0.278445"
## [1] "Iter 106, obj 1.798064, abs 0.093932, rel 0.055120, norm 0.243984"
## [1] "Iter 107, obj 1.950203, abs 0.152139, rel 0.084613, norm 0.302044"
## [1] "Iter 108, obj 2.052258, abs 0.102055, rel 0.052330, norm 0.295462"
## [1] "Iter 109, obj 2.167447, abs 0.115189, rel 0.056128, norm 0.344499"
## [1] "Iter 110, obj 2.134526, abs 0.032922, rel 0.015189, norm 0.381129"
```

```
## [1] "Iter 111, obj 2.196403, abs 0.061877, rel 0.028989, norm 0.417400"
## [1] "Iter 112, obj 2.156859, abs 0.039543, rel 0.018004, norm 0.416550"
## [1] "Iter 113, obj 2.200224, abs 0.043365, rel 0.020106, norm 0.435349"
## [1] "Iter 114, obj 2.146409, abs 0.053816, rel 0.024459, norm 0.402811"
## [1] "Iter 115, obj 2.173738, abs 0.027329, rel 0.012732, norm 0.410930"
## [1] "Iter 116, obj 2.151485, abs 0.022253, rel 0.010237, norm 0.399775"
## [1] "Iter 117, obj 2.191899, abs 0.040414, rel 0.018784, norm 0.422402"
## [1] "Iter 118, obj 2.141442, abs 0.050457, rel 0.023020, norm 0.411751"
## [1] "Iter 119, obj 2.173764, abs 0.032322, rel 0.015094, norm 0.417050"
## [1] "Iter 120, obj 2.171589, abs 0.002175, rel 0.001001, norm 0.406453"
## [1] "Iter 121, obj 2.215850, abs 0.044261, rel 0.020382, norm 0.433597"
## [1] "Iter 122, obj 2.154973, abs 0.060877, rel 0.027473, norm 0.418741"
## [1] "Iter 123, obj 2.212318, abs 0.057345, rel 0.026610, norm 0.430603"
## [1] "Iter 124, obj 2.178105, abs 0.034212, rel 0.015464, norm 0.421165"
## [1] "Iter 125, obj 2.221339, abs 0.043234, rel 0.019849, norm 0.434688"
## [1] "Iter 126, obj 2.156325, abs 0.065014, rel 0.029268, norm 0.425991"
## [1] "Iter 127, obj 2.204215, abs 0.047891, rel 0.022209, norm 0.427859"
## [1] "Iter 128, obj 2.166389, abs 0.037827, rel 0.017161, norm 0.418637"
## [1] "Iter 129, obj 2.219202, abs 0.052813, rel 0.024378, norm 0.430298"
## [1] "Iter 130, obj 2.165931, abs 0.053270, rel 0.024004, norm 0.417402"
## [1] "Iter 131, obj 2.219815, abs 0.053883, rel 0.024878, norm 0.430445"
## [1] "Iter 132, obj 2.175759, abs 0.044056, rel 0.019847, norm 0.419001"
## [1] "Iter 133, obj 2.231869, abs 0.056111, rel 0.025789, norm 0.429883"
## [1] "Iter 134, obj 2.190502, abs 0.041368, rel 0.018535, norm 0.423445"
## [1] "Iter 135, obj 2.203123, abs 0.012621, rel 0.005762, norm 0.429581"
## [1] "Iter 136, obj 2.160585, abs 0.042538, rel 0.019308, norm 0.417365"
## [1] "Iter 137, obj 2.209535, abs 0.048950, rel 0.022656, norm 0.428599"
## [1] "Iter 138, obj 2.180918, abs 0.028617, rel 0.012952, norm 0.414717"
## [1] "Iter 139, obj 2.219238, abs 0.038321, rel 0.017571, norm 0.425398"
## [1] "Iter 140, obj 2.186420, abs 0.032819, rel 0.014788, norm 0.421161"
## [1] "Iter 141, obj 2.227141, abs 0.040721, rel 0.018624, norm 0.435217"
## [1] "Iter 142, obj 2.193398, abs 0.033742, rel 0.015150, norm 0.423905"
## [1] "Iter 143, obj 2.235500, abs 0.042101, rel 0.019195, norm 0.440844"
## [1] "Iter 144, obj 2.165218, abs 0.070282, rel 0.031439, norm 0.429720"
## [1] "Iter 145, obj 2.200015, abs 0.034797, rel 0.016071, norm 0.428938"
## [1] "Iter 146, obj 2.157801, abs 0.042214, rel 0.019188, norm 0.413832"
## [1] "Iter 147, obj 2.204924, abs 0.047122, rel 0.021838, norm 0.425420"
## [1] "Iter 148, obj 2.181154, abs 0.023769, rel 0.010780, norm 0.412502"
## [1] "Iter 149, obj 2.215525, abs 0.034371, rel 0.015758, norm 0.423404"
## [1] "Iter 150, obj 2.194084, abs 0.021442, rel 0.009678, norm 0.412632"
## [1] "Iter 151, obj 2.220092, abs 0.026008, rel 0.011854, norm 0.429625"
## [1] "Iter 152, obj 2.171668, abs 0.048424, rel 0.021812, norm 0.416159"
## [1] "Iter 153, obj 2.210190, abs 0.038522, rel 0.017738, norm 0.423306"
## [1] "Iter 154, obj 2.178692, abs 0.031498, rel 0.014251, norm 0.409910"
## [1] "Iter 155, obj 2.195394, abs 0.016702, rel 0.007666, norm 0.421052"
## [1] "Iter 156, obj 2.181068, abs 0.014326, rel 0.006526, norm 0.405187"
## [1] "Iter 157, obj 2.210450, abs 0.029382, rel 0.013472, norm 0.423305"
## [1] "Iter 158, obj 2.188044, abs 0.022406, rel 0.010136, norm 0.412319"
## [1] "Iter 159, obj 2.192941, abs 0.004897, rel 0.002238, norm 0.422195"
## [1] "Iter 160, obj 2.216835, abs 0.023894, rel 0.010896, norm 0.410132"
## [1] "Iter 161, obj 2.229448, abs 0.012613, rel 0.005689, norm 0.433009"
## [1] "Iter 162, obj 2.169147, abs 0.060301, rel 0.027048, norm 0.424448"
## [1] "Iter 163, obj 2.229331, abs 0.060184, rel 0.027745, norm 0.435881"
## [1] "Iter 164, obj 2.163899, abs 0.065432, rel 0.029351, norm 0.421091"
```

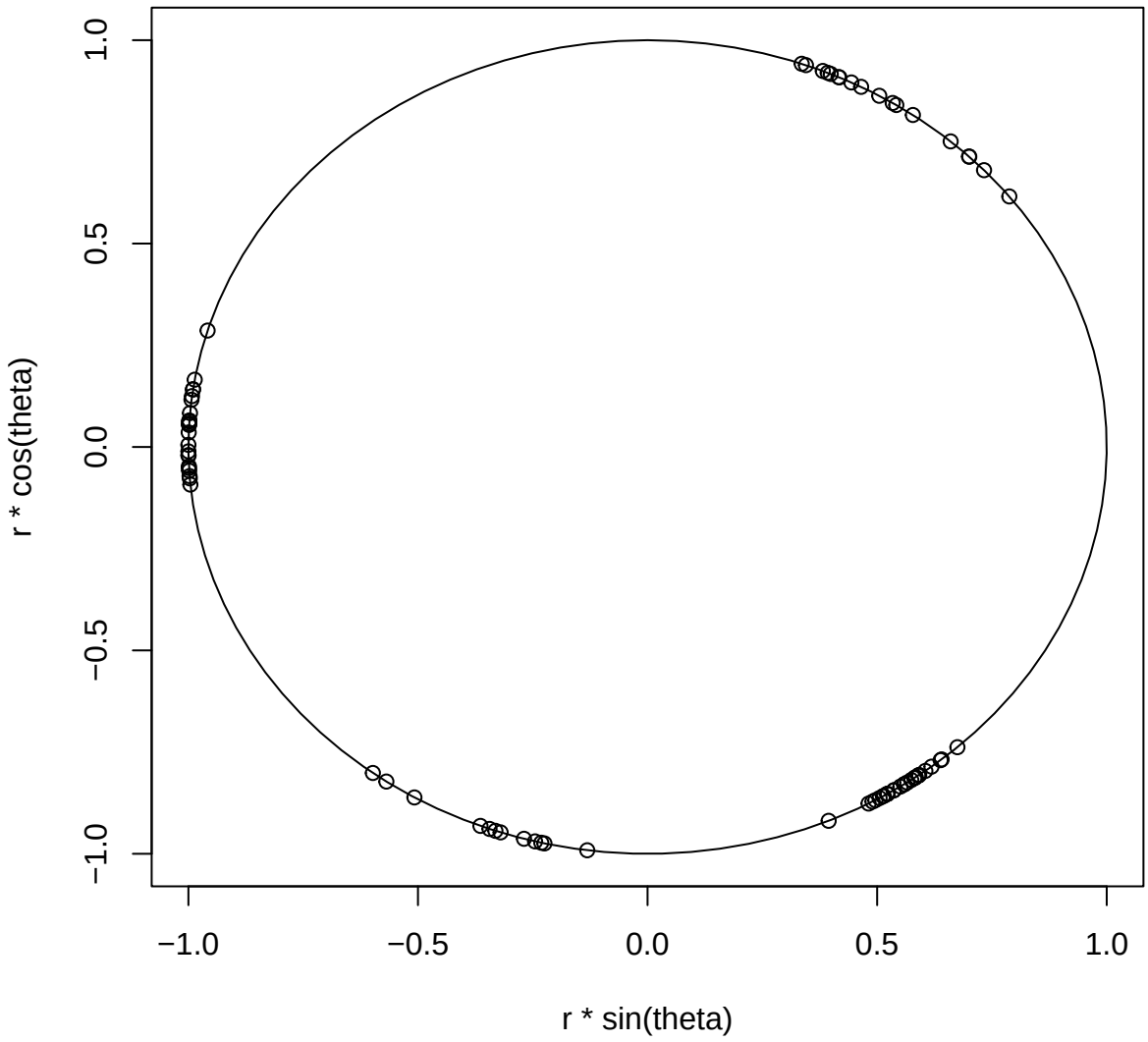
```
## [1] "Iter 165, obj 2.203806, abs 0.039907, rel 0.018442, norm 0.430322"
## [1] "Iter 166, obj 2.135645, abs 0.068161, rel 0.030929, norm 0.414929"
## [1] "Iter 167, obj 2.167176, abs 0.031531, rel 0.014764, norm 0.411535"
## [1] "Iter 168, obj 2.158181, abs 0.008995, rel 0.004151, norm 0.392810"
## [1] "Iter 169, obj 2.198478, abs 0.040298, rel 0.018672, norm 0.412379"
## [1] "Iter 170, obj 2.159476, abs 0.039003, rel 0.017741, norm 0.398767"
## [1] "Iter 171, obj 2.181505, abs 0.022030, rel 0.010202, norm 0.411861"
## [1] "Iter 172, obj 2.140967, abs 0.040539, rel 0.018583, norm 0.396368"
## [1] "Iter 173, obj 2.170749, abs 0.029782, rel 0.013911, norm 0.407629"
## [1] "Iter 174, obj 2.199349, abs 0.028600, rel 0.013175, norm 0.385703"
## [1] "Iter 175, obj 2.231515, abs 0.032167, rel 0.014626, norm 0.414163"
## [1] "Iter 176, obj 2.180322, abs 0.051194, rel 0.022941, norm 0.404399"
## [1] "Iter 177, obj 2.197326, abs 0.017004, rel 0.007799, norm 0.411185"
## [1] "Iter 178, obj 2.163747, abs 0.033579, rel 0.015282, norm 0.397109"
## [1] "Iter 179, obj 2.204574, abs 0.040827, rel 0.018869, norm 0.413801"
## [1] "Iter 180, obj 2.155248, abs 0.049326, rel 0.022374, norm 0.400944"
## [1] "Iter 181, obj 2.207549, abs 0.052300, rel 0.024267, norm 0.412758"
## [1] "Iter 182, obj 2.183243, abs 0.024306, rel 0.011010, norm 0.400133"
## [1] "Iter 183, obj 2.227055, abs 0.043812, rel 0.020067, norm 0.424634"
## [1] "Iter 184, obj 2.183670, abs 0.043385, rel 0.019481, norm 0.409786"
## [1] "Iter 185, obj 2.216937, abs 0.033267, rel 0.015235, norm 0.422464"
## [1] "Iter 186, obj 2.180273, abs 0.036664, rel 0.016538, norm 0.406859"
## [1] "Iter 187, obj 2.228343, abs 0.048071, rel 0.022048, norm 0.423347"
## [1] "Iter 188, obj 2.169674, abs 0.058669, rel 0.026329, norm 0.409412"
## [1] "Iter 189, obj 2.219370, abs 0.049696, rel 0.022905, norm 0.421289"
## [1] "Iter 190, obj 2.187014, abs 0.032356, rel 0.014579, norm 0.405146"
## [1] "Iter 191, obj 2.219534, abs 0.032520, rel 0.014870, norm 0.421028"
## [1] "Iter 192, obj 2.158460, abs 0.061075, rel 0.027517, norm 0.405663"
## [1] "Iter 193, obj 2.196332, abs 0.037872, rel 0.017546, norm 0.419416"
## [1] "Iter 194, obj 2.158126, abs 0.038205, rel 0.017395, norm 0.402248"
## [1] "Iter 195, obj 2.212112, abs 0.053986, rel 0.025015, norm 0.418291"
## [1] "Iter 196, obj 2.187461, abs 0.024651, rel 0.011144, norm 0.407859"
## [1] "Iter 197, obj 2.219397, abs 0.031936, rel 0.014599, norm 0.425865"
## [1] "Iter 198, obj 2.192096, abs 0.027301, rel 0.012301, norm 0.411896"
## [1] "Iter 199, obj 2.242570, abs 0.050475, rel 0.023026, norm 0.434883"
## [1] "Iter 200, obj 2.189136, abs 0.053434, rel 0.023827, norm 0.420742"
## [1] "Iter 201, obj 2.211343, abs 0.022207, rel 0.010144, norm 0.425207"
## [1] "Iter 202, obj 2.166922, abs 0.044422, rel 0.020088, norm 0.408675"
## [1] "Iter 203, obj 2.216294, abs 0.049373, rel 0.022785, norm 0.422259"
## [1] "Iter 204, obj 2.186215, abs 0.030079, rel 0.013572, norm 0.405922"
## [1] "Iter 205, obj 2.213489, abs 0.027274, rel 0.012475, norm 0.423536"
## [1] "Iter 206, obj 2.167096, abs 0.046393, rel 0.020959, norm 0.408069"
## [1] "Iter 207, obj 2.202808, abs 0.035712, rel 0.016479, norm 0.421411"
## [1] "Iter 208, obj 2.171295, abs 0.031513, rel 0.014306, norm 0.404426"
## [1] "Iter 209, obj 2.217734, abs 0.046439, rel 0.021388, norm 0.424961"
## [1] "Iter 210, obj 2.170231, abs 0.047503, rel 0.021420, norm 0.407558"
## [1] "Iter 211, obj 2.222398, abs 0.052167, rel 0.024038, norm 0.426008"
## [1] "Iter 212, obj 2.181855, abs 0.040543, rel 0.018243, norm 0.410046"
## [1] "Iter 213, obj 2.240087, abs 0.058232, rel 0.026689, norm 0.426525"
## [1] "Iter 214, obj 2.174980, abs 0.065107, rel 0.029065, norm 0.413829"
## [1] "Iter 215, obj 2.211092, abs 0.036112, rel 0.016603, norm 0.425633"
## [1] "Iter 216, obj 2.206447, abs 0.004644, rel 0.002101, norm 0.406948"
## [1] "Iter 217, obj 2.211974, abs 0.005527, rel 0.002505, norm 0.428663"
## [1] "Iter 218, obj 2.150055, abs 0.061919, rel 0.027993, norm 0.407678"
```

```
## [1] "Iter 219, obj 2.200543, abs 0.050488, rel 0.023482, norm 0.419872"
## [1] "Iter 220, obj 2.196206, abs 0.004337, rel 0.001971, norm 0.401646"
## [1] "Iter 221, obj 2.229769, abs 0.033562, rel 0.015282, norm 0.424387"
## [1] "Iter 222, obj 2.183675, abs 0.046093, rel 0.020672, norm 0.411381"
## [1] "Iter 223, obj 2.211614, abs 0.027939, rel 0.012794, norm 0.427713"
## [1] "Iter 224, obj 2.203548, abs 0.008066, rel 0.003647, norm 0.411143"
## [1] "Iter 225, obj 2.245582, abs 0.042035, rel 0.019076, norm 0.435635"
## [1] "Iter 226, obj 2.196704, abs 0.048878, rel 0.021766, norm 0.430239"
## [1] "Iter 227, obj 2.221628, abs 0.024924, rel 0.011346, norm 0.437563"
## [1] "Iter 228, obj 2.174779, abs 0.046849, rel 0.021088, norm 0.420048"
## [1] "Iter 229, obj 2.201618, abs 0.026838, rel 0.012341, norm 0.429888"
## [1] "Iter 230, obj 2.183011, abs 0.018607, rel 0.008452, norm 0.411918"
## [1] "Iter 231, obj 2.227802, abs 0.044791, rel 0.020518, norm 0.432346"
## [1] "Iter 232, obj 2.183663, abs 0.044139, rel 0.019813, norm 0.416889"
## [1] "Iter 233, obj 2.215563, abs 0.031900, rel 0.014609, norm 0.428373"
## [1] "Iter 234, obj 2.177287, abs 0.038276, rel 0.017276, norm 0.410532"
## [1] "Iter 235, obj 2.219738, abs 0.042451, rel 0.019497, norm 0.423879"
## [1] "Iter 236, obj 2.170526, abs 0.049212, rel 0.022170, norm 0.407900"
## [1] "Iter 237, obj 2.231482, abs 0.060956, rel 0.028083, norm 0.420216"
## [1] "Iter 238, obj 2.189207, abs 0.042274, rel 0.018944, norm 0.410988"
## [1] "Iter 239, obj 2.235226, abs 0.046019, rel 0.021021, norm 0.428989"
## [1] "Iter 240, obj 2.197002, abs 0.038224, rel 0.017101, norm 0.415359"
## [1] "Iter 241, obj 2.221828, abs 0.024825, rel 0.011300, norm 0.433979"
## [1] "Iter 242, obj 2.185735, abs 0.036093, rel 0.016245, norm 0.415978"
## [1] "Iter 243, obj 2.225407, abs 0.039672, rel 0.018150, norm 0.431472"
## [1] "Iter 244, obj 2.165487, abs 0.059919, rel 0.026925, norm 0.415944"
## [1] "Iter 245, obj 2.203086, abs 0.037599, rel 0.017363, norm 0.424932"
## [1] "Iter 246, obj 2.193109, abs 0.009977, rel 0.004529, norm 0.406648"
## [1] "Iter 247, obj 2.211851, abs 0.018742, rel 0.008546, norm 0.424302"
## [1] "Iter 248, obj 2.150034, abs 0.061817, rel 0.027948, norm 0.406753"
## [1] "Iter 249, obj 2.191119, abs 0.041085, rel 0.019109, norm 0.415834"
## [1] "Iter 250, obj 2.180626, abs 0.010492, rel 0.004789, norm 0.395669"
## [1] "Iter 251, obj 2.204204, abs 0.023578, rel 0.010812, norm 0.417688"
## [1] "Iter 252, obj 2.151778, abs 0.052426, rel 0.023785, norm 0.404088"
## [1] "Iter 253, obj 2.198456, abs 0.046678, rel 0.021693, norm 0.414229"
## [1] "Iter 254, obj 2.173214, abs 0.025242, rel 0.011482, norm 0.400307"
## [1] "Iter 255, obj 2.204672, abs 0.031458, rel 0.014475, norm 0.418885"
## [1] "Iter 256, obj 2.158889, abs 0.045783, rel 0.020766, norm 0.403970"
## [1] "Iter 257, obj 2.196378, abs 0.037489, rel 0.017365, norm 0.415189"
## [1] "Iter 258, obj 2.156987, abs 0.039391, rel 0.017934, norm 0.397110"
## [1] "Iter 259, obj 2.201575, abs 0.044588, rel 0.020671, norm 0.410747"
## [1] "Iter 260, obj 2.146698, abs 0.054877, rel 0.024926, norm 0.396619"
## [1] "Iter 261, obj 2.182426, abs 0.035727, rel 0.016643, norm 0.403160"
## [1] "Iter 262, obj 2.137102, abs 0.045324, rel 0.020768, norm 0.387408"
## [1] "Iter 263, obj 2.193378, abs 0.056277, rel 0.026333, norm 0.398550"
## [1] "Iter 264, obj 2.203932, abs 0.010553, rel 0.004811, norm 0.384126"
## [1] "Iter 265, obj 2.196936, abs 0.006995, rel 0.003174, norm 0.403713"
## [1] "Iter 266, obj 2.203188, abs 0.006251, rel 0.002845, norm 0.390084"
## [1] "Iter 267, obj 2.223780, abs 0.020592, rel 0.009347, norm 0.415262"
## [1] "Iter 268, obj 2.196734, abs 0.027046, rel 0.012162, norm 0.409147"
## [1] "Iter 269, obj 2.208797, abs 0.012063, rel 0.005491, norm 0.422122"
## [1] "Iter 270, obj 2.177233, abs 0.031564, rel 0.014290, norm 0.403850"
## [1] "Iter 271, obj 2.224162, abs 0.046928, rel 0.021554, norm 0.417163"
## [1] "Iter 272, obj 2.176590, abs 0.047571, rel 0.021388, norm 0.402915"
```

```
## [1] "Iter 273, obj 2.220009, abs 0.043419, rel 0.019948, norm 0.414690"
## [1] "Iter 274, obj 2.152746, abs 0.067263, rel 0.030298, norm 0.402340"
## [1] "Iter 275, obj 2.190807, abs 0.038061, rel 0.017680, norm 0.406077"
## [1] "Iter 276, obj 2.213837, abs 0.023030, rel 0.010512, norm 0.387241"
## [1] "Iter 277, obj 2.217146, abs 0.003309, rel 0.001494, norm 0.413956"
## [1] "Iter 278, obj 2.175475, abs 0.041670, rel 0.018795, norm 0.399172"
## [1] "Iter 279, obj 2.187932, abs 0.012456, rel 0.005726, norm 0.409438"
## [1] "Iter 280, obj 2.164421, abs 0.023510, rel 0.010745, norm 0.390642"
## [1] "Iter 281, obj 2.184722, abs 0.020300, rel 0.009379, norm 0.400391"
## [1] "Iter 282, obj 2.194464, abs 0.009742, rel 0.004459, norm 0.386063"
## [1] "Iter 283, obj 2.216325, abs 0.021861, rel 0.009962, norm 0.407707"
## [1] "Iter 284, obj 2.192387, abs 0.023938, rel 0.010801, norm 0.405025"
## [1] "Iter 285, obj 2.212133, abs 0.019746, rel 0.009006, norm 0.418044"
## [1] "Iter 286, obj 2.163277, abs 0.048856, rel 0.022085, norm 0.408797"
## [1] "Iter 287, obj 2.196703, abs 0.033426, rel 0.015451, norm 0.418715"
## [1] "Iter 288, obj 2.159329, abs 0.037374, rel 0.017014, norm 0.401873"
## [1] "Iter 289, obj 2.194175, abs 0.034846, rel 0.016137, norm 0.416548"
## [1] "Iter 290, obj 2.163255, abs 0.030920, rel 0.014092, norm 0.399655"
## [1] "Iter 291, obj 2.211211, abs 0.047956, rel 0.022168, norm 0.417521"
## [1] "Iter 292, obj 2.194073, abs 0.017137, rel 0.007750, norm 0.404723"
## [1] "Iter 293, obj 2.262976, abs 0.068903, rel 0.031404, norm 0.430352"
## [1] "Iter 294, obj 2.207234, abs 0.055742, rel 0.024632, norm 0.419100"
## [1] "Iter 295, obj 2.225517, abs 0.018283, rel 0.008283, norm 0.427364"
## [1] "Iter 296, obj 2.184598, abs 0.040919, rel 0.018386, norm 0.411772"
## [1] "Iter 297, obj 2.227325, abs 0.042727, rel 0.019558, norm 0.424486"
## [1] "Iter 298, obj 2.166805, abs 0.060519, rel 0.027171, norm 0.411247"
## [1] "Iter 299, obj 2.185358, abs 0.018553, rel 0.008562, norm 0.414029"
## [1] "Iter 300, obj 2.143729, abs 0.041629, rel 0.019049, norm 0.393814"
```

```
Y_rad <- DirStats::to_rad(Y)
r <- 1
theta <<- Y_rad
plot(r*sin(theta),
     r*cos(theta),
     xlim=c(-max(r),max(r)),
     ylim=c(-max(r),max(r)))

polygon(max(r)*sin(seq(0,2*pi,length.out=100)),max(r)*cos(seq(0,2*pi,length.out=100)))
```



Now, optimized rho for a perplexity of 12 and $d = 2$:

```
Y <- psc_sne(X=sanjuanfuca, d=2,
             rho_psc_list = rho_12,
             num_iteration=300)
```

```
## [1] "Iter 1, obj 12.347073, abs 0.000000, rel 0.000000, norm 1.161041"
## [1] "Iter 2, obj 11.542364, abs 0.804709, rel 0.065174, norm 2.019725"
## [1] "Iter 3, obj 10.765064, abs 0.777301, rel 0.067343, norm 2.910442"
## [1] "Iter 4, obj 10.283473, abs 0.481591, rel 0.044736, norm 3.390362"
## [1] "Iter 5, obj 9.872004, abs 0.411469, rel 0.040013, norm 3.679482"
## [1] "Iter 6, obj 9.700148, abs 0.171856, rel 0.017408, norm 3.773318"
## [1] "Iter 7, obj 9.522854, abs 0.177294, rel 0.018277, norm 3.853124"
## [1] "Iter 8, obj 9.425147, abs 0.097707, rel 0.010260, norm 3.869366"
## [1] "Iter 9, obj 9.297087, abs 0.128059, rel 0.013587, norm 3.879188"
## [1] "Iter 10, obj 9.198080, abs 0.099008, rel 0.010649, norm 3.868804"
## [1] "Iter 11, obj 9.078880, abs 0.119200, rel 0.012959, norm 3.856681"
## [1] "Iter 12, obj 8.984407, abs 0.094472, rel 0.010406, norm 3.838297"
## [1] "Iter 13, obj 8.890981, abs 0.093427, rel 0.010399, norm 3.819820"
## [1] "Iter 14, obj 8.829108, abs 0.061873, rel 0.006959, norm 3.801342"
## [1] "Iter 15, obj 8.777984, abs 0.051124, rel 0.005790, norm 3.786219"
```

```
## [1] "Iter 16, obj 8.754625, abs 0.023359, rel 0.002661, norm 3.774727"
## [1] "Iter 17, obj 8.739280, abs 0.015345, rel 0.001753, norm 3.767588"
## [1] "Iter 18, obj 8.741459, abs 0.002179, rel 0.000249, norm 3.764242"
## [1] "Iter 19, obj 8.747968, abs 0.006509, rel 0.000745, norm 3.764196"
## [1] "Iter 20, obj 8.764902, abs 0.016934, rel 0.001936, norm 3.767655"
## [1] "Iter 21, obj 8.784555, abs 0.019653, rel 0.002242, norm 3.773733"
## [1] "Iter 22, obj 8.810010, abs 0.025455, rel 0.002898, norm 3.783094"
## [1] "Iter 23, obj 8.837665, abs 0.027655, rel 0.003139, norm 3.794503"
## [1] "Iter 24, obj 8.867974, abs 0.030309, rel 0.003430, norm 3.808983"
## [1] "Iter 25, obj 8.900598, abs 0.032623, rel 0.003679, norm 3.824898"
## [1] "Iter 26, obj 8.933565, abs 0.032967, rel 0.003704, norm 3.843851"
## [1] "Iter 27, obj 8.969375, abs 0.035810, rel 0.004008, norm 3.863567"
## [1] "Iter 28, obj 9.003663, abs 0.034288, rel 0.003823, norm 3.886513"
## [1] "Iter 29, obj 9.041612, abs 0.037949, rel 0.004215, norm 3.909477"
## [1] "Iter 30, obj 9.076414, abs 0.034802, rel 0.003849, norm 3.935961"
## [1] "Iter 31, obj 9.115884, abs 0.039470, rel 0.004349, norm 3.961678"
## [1] "Iter 32, obj 9.150765, abs 0.034881, rel 0.003826, norm 3.991174"
## [1] "Iter 33, obj 9.191406, abs 0.040641, rel 0.004441, norm 4.019151"
## [1] "Iter 34, obj 9.226196, abs 0.034789, rel 0.003785, norm 4.051056"
## [1] "Iter 35, obj 9.267815, abs 0.041619, rel 0.004511, norm 4.080786"
## [1] "Iter 36, obj 9.302479, abs 0.034664, rel 0.003740, norm 4.114464"
## [1] "Iter 37, obj 9.344934, abs 0.042455, rel 0.004564, norm 4.145418"
## [1] "Iter 38, obj 9.379458, abs 0.034524, rel 0.003694, norm 4.180226"
## [1] "Iter 39, obj 9.422580, abs 0.043122, rel 0.004597, norm 4.211857"
## [1] "Iter 40, obj 9.456901, abs 0.034321, rel 0.003642, norm 4.247173"
## [1] "Iter 41, obj 9.500461, abs 0.043561, rel 0.004606, norm 4.278938"
## [1] "Iter 42, obj 9.534451, abs 0.033989, rel 0.003578, norm 4.314194"
## [1] "Iter 43, obj 9.578162, abs 0.043711, rel 0.004585, norm 4.345591"
## [1] "Iter 44, obj 9.611636, abs 0.033475, rel 0.003495, norm 4.380289"
## [1] "Iter 45, obj 9.655164, abs 0.043527, rel 0.004529, norm 4.410884"
## [1] "Iter 46, obj 9.687905, abs 0.032741, rel 0.003391, norm 4.444614"
## [1] "Iter 47, obj 9.730893, abs 0.042988, rel 0.004437, norm 4.474052"
## [1] "Iter 48, obj 9.762667, abs 0.031774, rel 0.003265, norm 4.506494"
## [1] "Iter 49, obj 9.804762, abs 0.042095, rel 0.004312, norm 4.534510"
## [1] "Iter 50, obj 9.835337, abs 0.030575, rel 0.003118, norm 4.565428"
## [1] "Iter 51, obj 9.876212, abs 0.040875, rel 0.004156, norm 4.591836"
## [1] "Iter 52, obj 9.905380, abs 0.029169, rel 0.002953, norm 4.621070"
## [1] "Iter 53, obj 9.944753, abs 0.039372, rel 0.003975, norm 4.645759"
## [1] "Iter 54, obj 9.972343, abs 0.027590, rel 0.002774, norm 4.673210"
## [1] "Iter 55, obj 10.009988, abs 0.037645, rel 0.003775, norm 4.696125"
## [1] "Iter 56, obj 10.035875, abs 0.025887, rel 0.002586, norm 4.721748"
## [1] "Iter 57, obj 10.071630, abs 0.035756, rel 0.003563, norm 4.742881"
## [1] "Iter 58, obj 10.095741, abs 0.024111, rel 0.002394, norm 4.766668"
## [1] "Iter 59, obj 10.129501, abs 0.033760, rel 0.003344, norm 4.786045"
## [1] "Iter 60, obj 10.151810, abs 0.022309, rel 0.002202, norm 4.808019"
## [1] "Iter 61, obj 10.183516, abs 0.031706, rel 0.003123, norm 4.825690"
## [1] "Iter 62, obj 10.204038, abs 0.020522, rel 0.002015, norm 4.845894"
## [1] "Iter 63, obj 10.233669, abs 0.029631, rel 0.002904, norm 4.861929"
## [1] "Iter 64, obj 10.252448, abs 0.018779, rel 0.001835, norm 4.880423"
## [1] "Iter 65, obj 10.280014, abs 0.027565, rel 0.002689, norm 4.894903"
## [1] "Iter 66, obj 10.297114, abs 0.017100, rel 0.001663, norm 4.911762"
## [1] "Iter 67, obj 10.322643, abs 0.025530, rel 0.002479, norm 4.924778"
## [1] "Iter 68, obj 10.338142, abs 0.015498, rel 0.001501, norm 4.940084"
## [1] "Iter 69, obj 10.361685, abs 0.023543, rel 0.002277, norm 4.951733"
```

```
## [1] "Iter 70, obj 10.375665, abs 0.013980, rel 0.001349, norm 4.965579"
## [1] "Iter 71, obj 10.397285, abs 0.021620, rel 0.002084, norm 4.975961"
## [1] "Iter 72, obj 10.409835, abs 0.012551, rel 0.001207, norm 4.988443"
## [1] "Iter 73, obj 10.429608, abs 0.019773, rel 0.001899, norm 4.997659"
## [1] "Iter 74, obj 10.440820, abs 0.011211, rel 0.001075, norm 5.008877"
## [1] "Iter 75, obj 10.458833, abs 0.018013, rel 0.001725, norm 5.017028"
## [1] "Iter 76, obj 10.468796, abs 0.009963, rel 0.000953, norm 5.027081"
## [1] "Iter 77, obj 10.485144, abs 0.016348, rel 0.001562, norm 5.034266"
## [1] "Iter 78, obj 10.493951, abs 0.008807, rel 0.000840, norm 5.043253"
## [1] "Iter 79, obj 10.508735, abs 0.014784, rel 0.001409, norm 5.049566"
## [1] "Iter 80, obj 10.516476, abs 0.007741, rel 0.000737, norm 5.057582"
## [1] "Iter 81, obj 10.529800, abs 0.013324, rel 0.001267, norm 5.063113"
## [1] "Iter 82, obj 10.536563, abs 0.006763, rel 0.000642, norm 5.070251"
## [1] "Iter 83, obj 10.548533, abs 0.011970, rel 0.001136, norm 5.075083"
## [1] "Iter 84, obj 10.554404, abs 0.005871, rel 0.000557, norm 5.081427"
## [1] "Iter 85, obj 10.565125, abs 0.010721, rel 0.001016, norm 5.085638"
## [1] "Iter 86, obj 10.570187, abs 0.005062, rel 0.000479, norm 5.091270"
## [1] "Iter 87, obj 10.579762, abs 0.009575, rel 0.000906, norm 5.094930"
## [1] "Iter 88, obj 10.584094, abs 0.004332, rel 0.000409, norm 5.099924"
## [1] "Iter 89, obj 10.592621, abs 0.008527, rel 0.000806, norm 5.103097"
## [1] "Iter 90, obj 10.596297, abs 0.003676, rel 0.000347, norm 5.107521"
## [1] "Iter 91, obj 10.603871, abs 0.007574, rel 0.000715, norm 5.110266"
## [1] "Iter 92, obj 10.606960, abs 0.003089, rel 0.000291, norm 5.114182"
## [1] "Iter 93, obj 10.613670, abs 0.006711, rel 0.000633, norm 5.116551"
## [1] "Iter 94, obj 10.616237, abs 0.002567, rel 0.000242, norm 5.120015"
## [1] "Iter 95, obj 10.622167, abs 0.005930, rel 0.000559, norm 5.122055"
## [1] "Iter 96, obj 10.624271, abs 0.002104, rel 0.000198, norm 5.125117"
## [1] "Iter 97, obj 10.629498, abs 0.005227, rel 0.000492, norm 5.126869"
## [1] "Iter 98, obj 10.631194, abs 0.001695, rel 0.000160, norm 5.129575"
## [1] "Iter 99, obj 10.635789, abs 0.004596, rel 0.000432, norm 5.131075"
## [1] "Iter 100, obj 10.637126, abs 0.001337, rel 0.000126, norm 5.133466"
## [1] "Iter 101, obj 1.338592, abs 9.298534, rel 0.874158, norm 0.413151"
## [1] "Iter 102, obj 1.937263, abs 0.598671, rel 0.447239, norm 0.300142"
## [1] "Iter 103, obj 2.796752, abs 0.859489, rel 0.443662, norm 0.656822"
## [1] "Iter 104, obj 3.002426, abs 0.205674, rel 0.073540, norm 1.120994"
## [1] "Iter 105, obj 3.038970, abs 0.036543, rel 0.012171, norm 1.346929"
## [1] "Iter 106, obj 3.071573, abs 0.032604, rel 0.010728, norm 1.398098"
## [1] "Iter 107, obj 3.140598, abs 0.069025, rel 0.022472, norm 1.440422"
## [1] "Iter 108, obj 3.119203, abs 0.021395, rel 0.006812, norm 1.435204"
## [1] "Iter 109, obj 3.114409, abs 0.004795, rel 0.001537, norm 1.460740"
## [1] "Iter 110, obj 3.113419, abs 0.000990, rel 0.000318, norm 1.458312"
## [1] "Iter 111, obj 3.136223, abs 0.022804, rel 0.007325, norm 1.456139"
## [1] "Iter 112, obj 3.114972, abs 0.021251, rel 0.006776, norm 1.459027"
## [1] "Iter 113, obj 3.117161, abs 0.002189, rel 0.000703, norm 1.465096"
## [1] "Iter 114, obj 3.084709, abs 0.032452, rel 0.010411, norm 1.460536"
## [1] "Iter 115, obj 3.073645, abs 0.011064, rel 0.003587, norm 1.503628"
## [1] "Iter 116, obj 3.042466, abs 0.031180, rel 0.010144, norm 1.492914"
## [1] "Iter 117, obj 3.025463, abs 0.017003, rel 0.005588, norm 1.503066"
## [1] "Iter 118, obj 3.033763, abs 0.008300, rel 0.002744, norm 1.483078"
## [1] "Iter 119, obj 3.020333, abs 0.013430, rel 0.004427, norm 1.496795"
## [1] "Iter 120, obj 3.032253, abs 0.011919, rel 0.003946, norm 1.479519"
## [1] "Iter 121, obj 3.017650, abs 0.014602, rel 0.004816, norm 1.493767"
## [1] "Iter 122, obj 3.030216, abs 0.012566, rel 0.004164, norm 1.477052"
## [1] "Iter 123, obj 3.012559, abs 0.017658, rel 0.005827, norm 1.491571"
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## [1] "Iter 124, obj 3.022641, abs 0.010083, rel 0.003347, norm 1.474840"
## [1] "Iter 125, obj 2.997897, abs 0.024744, rel 0.008186, norm 1.489520"
## [1] "Iter 126, obj 2.997575, abs 0.000322, rel 0.000107, norm 1.473599"
## [1] "Iter 127, obj 2.954009, abs 0.043566, rel 0.014534, norm 1.499040"
## [1] "Iter 128, obj 2.976655, abs 0.022646, rel 0.007666, norm 1.490314"
## [1] "Iter 129, obj 2.930396, abs 0.046259, rel 0.015541, norm 1.501454"
## [1] "Iter 130, obj 2.940148, abs 0.009752, rel 0.003328, norm 1.486732"
## [1] "Iter 131, obj 2.884094, abs 0.056054, rel 0.019065, norm 1.494508"
## [1] "Iter 132, obj 2.961435, abs 0.077342, rel 0.026817, norm 1.484669"
## [1] "Iter 133, obj 2.938837, abs 0.022599, rel 0.007631, norm 1.503991"
## [1] "Iter 134, obj 2.984450, abs 0.045613, rel 0.015521, norm 1.497438"
## [1] "Iter 135, obj 2.932504, abs 0.051946, rel 0.017405, norm 1.516268"
## [1] "Iter 136, obj 2.977147, abs 0.044643, rel 0.015223, norm 1.495365"
## [1] "Iter 137, obj 2.922335, abs 0.054812, rel 0.018411, norm 1.510439"
## [1] "Iter 138, obj 2.964000, abs 0.041665, rel 0.014257, norm 1.492656"
## [1] "Iter 139, obj 2.897456, abs 0.066544, rel 0.022451, norm 1.506813"
## [1] "Iter 140, obj 2.938085, abs 0.040629, rel 0.014022, norm 1.498446"
## [1] "Iter 141, obj 2.853775, abs 0.084309, rel 0.028695, norm 1.513012"
## [1] "Iter 142, obj 2.929400, abs 0.075624, rel 0.026500, norm 1.500093"
## [1] "Iter 143, obj 2.844294, abs 0.085105, rel 0.029052, norm 1.510365"
## [1] "Iter 144, obj 2.912487, abs 0.068193, rel 0.023975, norm 1.502264"
## [1] "Iter 145, obj 2.806155, abs 0.106333, rel 0.036509, norm 1.514090"
## [1] "Iter 146, obj 2.910953, abs 0.104798, rel 0.037346, norm 1.525939"
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## [1] "Iter 149, obj 2.797894, abs 0.122699, rel 0.042012, norm 1.533521"
## [1] "Iter 150, obj 2.860484, abs 0.062589, rel 0.022370, norm 1.520436"
## [1] "Iter 151, obj 2.758032, abs 0.102452, rel 0.035816, norm 1.544038"
## [1] "Iter 152, obj 2.835454, abs 0.077422, rel 0.028071, norm 1.534784"
## [1] "Iter 153, obj 2.728303, abs 0.107151, rel 0.037790, norm 1.546666"
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## [1] "Iter 165, obj 2.686820, abs 0.111131, rel 0.039719, norm 1.547938"
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## [1] "Iter 169, obj 2.689507, abs 0.103787, rel 0.037156, norm 1.552296"
## [1] "Iter 170, obj 2.791538, abs 0.102031, rel 0.037937, norm 1.544076"
## [1] "Iter 171, obj 2.690564, abs 0.100974, rel 0.036171, norm 1.553941"
## [1] "Iter 172, obj 2.790066, abs 0.099502, rel 0.036982, norm 1.545259"
## [1] "Iter 173, obj 2.691550, abs 0.098516, rel 0.035310, norm 1.555319"
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## [1] "Iter 178, obj 2.786660, abs 0.093276, rel 0.034632, norm 1.547770"
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## [1] "Iter 285, obj 2.731086, abs 0.016386, rel 0.005964, norm 1.565760"
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## [1] "Iter 293, obj 2.731111, abs 0.015967, rel 0.005812, norm 1.565410"
## [1] "Iter 294, obj 2.747014, abs 0.015904, rel 0.005823, norm 1.552702"
## [1] "Iter 295, obj 2.731106, abs 0.015908, rel 0.005791, norm 1.565328"
## [1] "Iter 296, obj 2.746962, abs 0.015856, rel 0.005806, norm 1.552554"
## [1] "Iter 297, obj 2.731099, abs 0.015862, rel 0.005775, norm 1.565248"
## [1] "Iter 298, obj 2.746919, abs 0.015819, rel 0.005792, norm 1.552416"
## [1] "Iter 299, obj 2.731091, abs 0.015827, rel 0.005762, norm 1.565169"
## [1] "Iter 300, obj 2.746883, abs 0.015792, rel 0.005782, norm 1.552288"
```
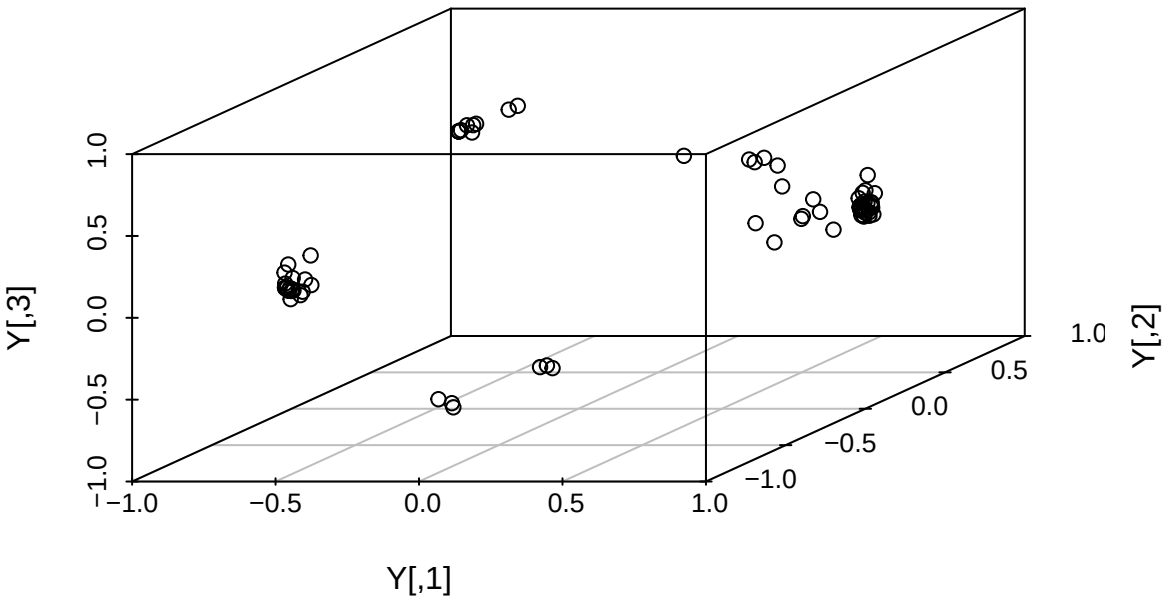

```
scatterplot3d::scatterplot3d(Y, xlim = c(-1, 1), ylim = c(-1, 1),
                             zlim = c(-1, 1), color = rep(1, nrow(Y)))
```




```
rgl::plot3d(0, 0, 0, xlim = c(-1, 1), ylim = c(-1, 1), zlim = c(-1, 1),
            radius = 1, type = "s", col = "lightblue", alpha = 0.25,
            lit = FALSE)
rgl::points3d(Y, col = rep(1, nrow(Y)))
```

Now, optimized rho for a perplexity of 20 and $d = 1$:

```
Y <- psc_sne(X=sanjuanfuca, d=1,
             rho_psc_list = rho_20,
             num_iteration=300)
```

```
## [1] "Iter 1, obj 8.075518, abs 0.000000, rel 0.000000, norm 1.221267"
## [1] "Iter 2, obj 7.982350, abs 0.093167, rel 0.011537, norm 1.421488"
## [1] "Iter 3, obj 7.945156, abs 0.037194, rel 0.004660, norm 1.507946"
## [1] "Iter 4, obj 7.934979, abs 0.010177, rel 0.001281, norm 1.572794"
## [1] "Iter 5, obj 7.938353, abs 0.003374, rel 0.000425, norm 1.627363"
## [1] "Iter 6, obj 7.948800, abs 0.010448, rel 0.001316, norm 1.671200"
```

```
## [1] "Iter 7, obj 7.962503, abs 0.013702, rel 0.001724, norm 1.707937"
## [1] "Iter 8, obj 7.978288, abs 0.015785, rel 0.001982, norm 1.738999"
## [1] "Iter 9, obj 7.995501, abs 0.017213, rel 0.002157, norm 1.766910"
## [1] "Iter 10, obj 8.014189, abs 0.018688, rel 0.002337, norm 1.792376"
## [1] "Iter 11, obj 8.034184, abs 0.019995, rel 0.002495, norm 1.816302"
## [1] "Iter 12, obj 8.055479, abs 0.021295, rel 0.002651, norm 1.838763"
## [1] "Iter 13, obj 8.077853, abs 0.022374, rel 0.002778, norm 1.859985"
## [1] "Iter 14, obj 8.101180, abs 0.023327, rel 0.002888, norm 1.879904"
## [1] "Iter 15, obj 8.125208, abs 0.024027, rel 0.002966, norm 1.898576"
## [1] "Iter 16, obj 8.149800, abs 0.024592, rel 0.003027, norm 1.915968"
## [1] "Iter 17, obj 8.174750, abs 0.024950, rel 0.003061, norm 1.932140"
## [1] "Iter 18, obj 8.199989, abs 0.025239, rel 0.003087, norm 1.947118"
## [1] "Iter 19, obj 8.225394, abs 0.025405, rel 0.003098, norm 1.960995"
## [1] "Iter 20, obj 8.250980, abs 0.025586, rel 0.003111, norm 1.973842"
## [1] "Iter 21, obj 8.276701, abs 0.025721, rel 0.003117, norm 1.985770"
## [1] "Iter 22, obj 8.302631, abs 0.025930, rel 0.003133, norm 1.996862"
## [1] "Iter 23, obj 8.328771, abs 0.026139, rel 0.003148, norm 2.007232"
## [1] "Iter 24, obj 8.355213, abs 0.026442, rel 0.003175, norm 2.016958"
## [1] "Iter 25, obj 8.381971, abs 0.026758, rel 0.003203, norm 2.026143"
## [1] "Iter 26, obj 8.409127, abs 0.027156, rel 0.003240, norm 2.034860"
## [1] "Iter 27, obj 8.436685, abs 0.027558, rel 0.003277, norm 2.043200"
## [1] "Iter 28, obj 8.464704, abs 0.028019, rel 0.003321, norm 2.051238"
## [1] "Iter 29, obj 8.493174, abs 0.028470, rel 0.003363, norm 2.059067"
## [1] "Iter 30, obj 8.522132, abs 0.028958, rel 0.003410, norm 2.066775"
## [1] "Iter 31, obj 8.551563, abs 0.029431, rel 0.003454, norm 2.074471"
## [1] "Iter 32, obj 8.581496, abs 0.029932, rel 0.003500, norm 2.082270"
## [1] "Iter 33, obj 8.611921, abs 0.030425, rel 0.003545, norm 2.090306"
## [1] "Iter 34, obj 8.642868, abs 0.030948, rel 0.003594, norm 2.098721"
## [1] "Iter 35, obj 8.674346, abs 0.031478, rel 0.003642, norm 2.107673"
## [1] "Iter 36, obj 8.706392, abs 0.032046, rel 0.003694, norm 2.117320"
## [1] "Iter 37, obj 8.739027, abs 0.032636, rel 0.003748, norm 2.127817"
## [1] "Iter 38, obj 8.772297, abs 0.033269, rel 0.003807, norm 2.139305"
## [1] "Iter 39, obj 8.806228, abs 0.033932, rel 0.003868, norm 2.151903"
## [1] "Iter 40, obj 8.840859, abs 0.034631, rel 0.003933, norm 2.165695"
## [1] "Iter 41, obj 8.876209, abs 0.035350, rel 0.003998, norm 2.180724"
## [1] "Iter 42, obj 8.912294, abs 0.036085, rel 0.004065, norm 2.196984"
## [1] "Iter 43, obj 8.949109, abs 0.036815, rel 0.004131, norm 2.214425"
## [1] "Iter 44, obj 8.986635, abs 0.037525, rel 0.004193, norm 2.232952"
## [1] "Iter 45, obj 9.024826, abs 0.038191, rel 0.004250, norm 2.252429"
## [1] "Iter 46, obj 9.063618, abs 0.038793, rel 0.004298, norm 2.272692"
## [1] "Iter 47, obj 9.102923, abs 0.039304, rel 0.004337, norm 2.293554"
## [1] "Iter 48, obj 9.142628, abs 0.039705, rel 0.004362, norm 2.314816"
## [1] "Iter 49, obj 9.182601, abs 0.039973, rel 0.004372, norm 2.336275"
## [1] "Iter 50, obj 9.222692, abs 0.040091, rel 0.004366, norm 2.357734"
## [1] "Iter 51, obj 9.262736, abs 0.040044, rel 0.004342, norm 2.379002"
## [1] "Iter 52, obj 9.302559, abs 0.039823, rel 0.004299, norm 2.399906"
## [1] "Iter 53, obj 9.341982, abs 0.039423, rel 0.004238, norm 2.420288"
## [1] "Iter 54, obj 9.380829, abs 0.038847, rel 0.004158, norm 2.440013"
## [1] "Iter 55, obj 9.418929, abs 0.038100, rel 0.004062, norm 2.458965"
## [1] "Iter 56, obj 9.456123, abs 0.037194, rel 0.003949, norm 2.477053"
## [1] "Iter 57, obj 9.492266, abs 0.036143, rel 0.003822, norm 2.494207"
## [1] "Iter 58, obj 9.527231, abs 0.034965, rel 0.003684, norm 2.510379"
## [1] "Iter 59, obj 9.560910, abs 0.033680, rel 0.003535, norm 2.525542"
## [1] "Iter 60, obj 9.593218, abs 0.032308, rel 0.003379, norm 2.539689"
```

```
## [1] "Iter 61, obj 9.624088, abs 0.030870, rel 0.003218, norm 2.552828"
## [1] "Iter 62, obj 9.653476, abs 0.029387, rel 0.003054, norm 2.564982"
## [1] "Iter 63, obj 9.681354, abs 0.027878, rel 0.002888, norm 2.576187"
## [1] "Iter 64, obj 9.707715, abs 0.026361, rel 0.002723, norm 2.586486"
## [1] "Iter 65, obj 9.732568, abs 0.024853, rel 0.002560, norm 2.595927"
## [1] "Iter 66, obj 9.755935, abs 0.023367, rel 0.002401, norm 2.604566"
## [1] "Iter 67, obj 9.777850, abs 0.021915, rel 0.002246, norm 2.612456"
## [1] "Iter 68, obj 9.798358, abs 0.020508, rel 0.002097, norm 2.619653"
## [1] "Iter 69, obj 9.817511, abs 0.019154, rel 0.001955, norm 2.626213"
## [1] "Iter 70, obj 9.835369, abs 0.017857, rel 0.001819, norm 2.632186"
## [1] "Iter 71, obj 9.851992, abs 0.016623, rel 0.001690, norm 2.637624"
## [1] "Iter 72, obj 9.867446, abs 0.015454, rel 0.001569, norm 2.642572"
## [1] "Iter 73, obj 9.881796, abs 0.014350, rel 0.001454, norm 2.647076"
## [1] "Iter 74, obj 9.895108, abs 0.013312, rel 0.001347, norm 2.651175"
## [1] "Iter 75, obj 9.907447, abs 0.012339, rel 0.001247, norm 2.654906"
## [1] "Iter 76, obj 9.918876, abs 0.011429, rel 0.001154, norm 2.658304"
## [1] "Iter 77, obj 9.929456, abs 0.010580, rel 0.001067, norm 2.661400"
## [1] "Iter 78, obj 9.939245, abs 0.009789, rel 0.000986, norm 2.664221"
## [1] "Iter 79, obj 9.948298, abs 0.009054, rel 0.000911, norm 2.666795"
## [1] "Iter 80, obj 9.956669, abs 0.008371, rel 0.000841, norm 2.669143"
## [1] "Iter 81, obj 9.964407, abs 0.007738, rel 0.000777, norm 2.671287"
## [1] "Iter 82, obj 9.971558, abs 0.007151, rel 0.000718, norm 2.673245"
## [1] "Iter 83, obj 9.978166, abs 0.006608, rel 0.000663, norm 2.675035"
## [1] "Iter 84, obj 9.984271, abs 0.006105, rel 0.000612, norm 2.676673"
## [1] "Iter 85, obj 9.989911, abs 0.005640, rel 0.000565, norm 2.678172"
## [1] "Iter 86, obj 9.995122, abs 0.005211, rel 0.000522, norm 2.679544"
## [1] "Iter 87, obj 9.999935, abs 0.004814, rel 0.000482, norm 2.680802"
## [1] "Iter 88, obj 10.004382, abs 0.004447, rel 0.000445, norm 2.681955"
## [1] "Iter 89, obj 10.008490, abs 0.004108, rel 0.000411, norm 2.683013"
## [1] "Iter 90, obj 10.012285, abs 0.003795, rel 0.000379, norm 2.683984"
## [1] "Iter 91, obj 10.015791, abs 0.003506, rel 0.000350, norm 2.684875"
## [1] "Iter 92, obj 10.019030, abs 0.003239, rel 0.000323, norm 2.685694"
## [1] "Iter 93, obj 10.022023, abs 0.002993, rel 0.000299, norm 2.686446"
## [1] "Iter 94, obj 10.024789, abs 0.002765, rel 0.000276, norm 2.687138"
## [1] "Iter 95, obj 10.027344, abs 0.002555, rel 0.000255, norm 2.687775"
## [1] "Iter 96, obj 10.029705, abs 0.002361, rel 0.000235, norm 2.688361"
## [1] "Iter 97, obj 10.031887, abs 0.002182, rel 0.000218, norm 2.688900"
## [1] "Iter 98, obj 10.033904, abs 0.002017, rel 0.000201, norm 2.689396"
## [1] "Iter 99, obj 10.035768, abs 0.001864, rel 0.000186, norm 2.689854"
## [1] "Iter 100, obj 10.037491, abs 0.001723, rel 0.000172, norm 2.690275"
## [1] "Iter 101, obj 0.971349, abs 9.066142, rel 0.903228, norm 0.185527"
## [1] "Iter 102, obj 1.030733, abs 0.059385, rel 0.061136, norm 0.120727"
## [1] "Iter 103, obj 1.252320, abs 0.221587, rel 0.214980, norm 0.170776"
## [1] "Iter 104, obj 1.468551, abs 0.216232, rel 0.172665, norm 0.197766"
## [1] "Iter 105, obj 1.601865, abs 0.133314, rel 0.090779, norm 0.267197"
## [1] "Iter 106, obj 1.718352, abs 0.116487, rel 0.072719, norm 0.300420"
## [1] "Iter 107, obj 1.800098, abs 0.081746, rel 0.047572, norm 0.425874"
## [1] "Iter 108, obj 1.830410, abs 0.030312, rel 0.016839, norm 0.473119"
## [1] "Iter 109, obj 1.843417, abs 0.013006, rel 0.007106, norm 0.490866"
## [1] "Iter 110, obj 1.863279, abs 0.019863, rel 0.010775, norm 0.488435"
## [1] "Iter 111, obj 1.862743, abs 0.000537, rel 0.000288, norm 0.491648"
## [1] "Iter 112, obj 1.867513, abs 0.004770, rel 0.002561, norm 0.494407"
## [1] "Iter 113, obj 1.852579, abs 0.014934, rel 0.007997, norm 0.490326"
## [1] "Iter 114, obj 1.811460, abs 0.041119, rel 0.022196, norm 0.494442"
```

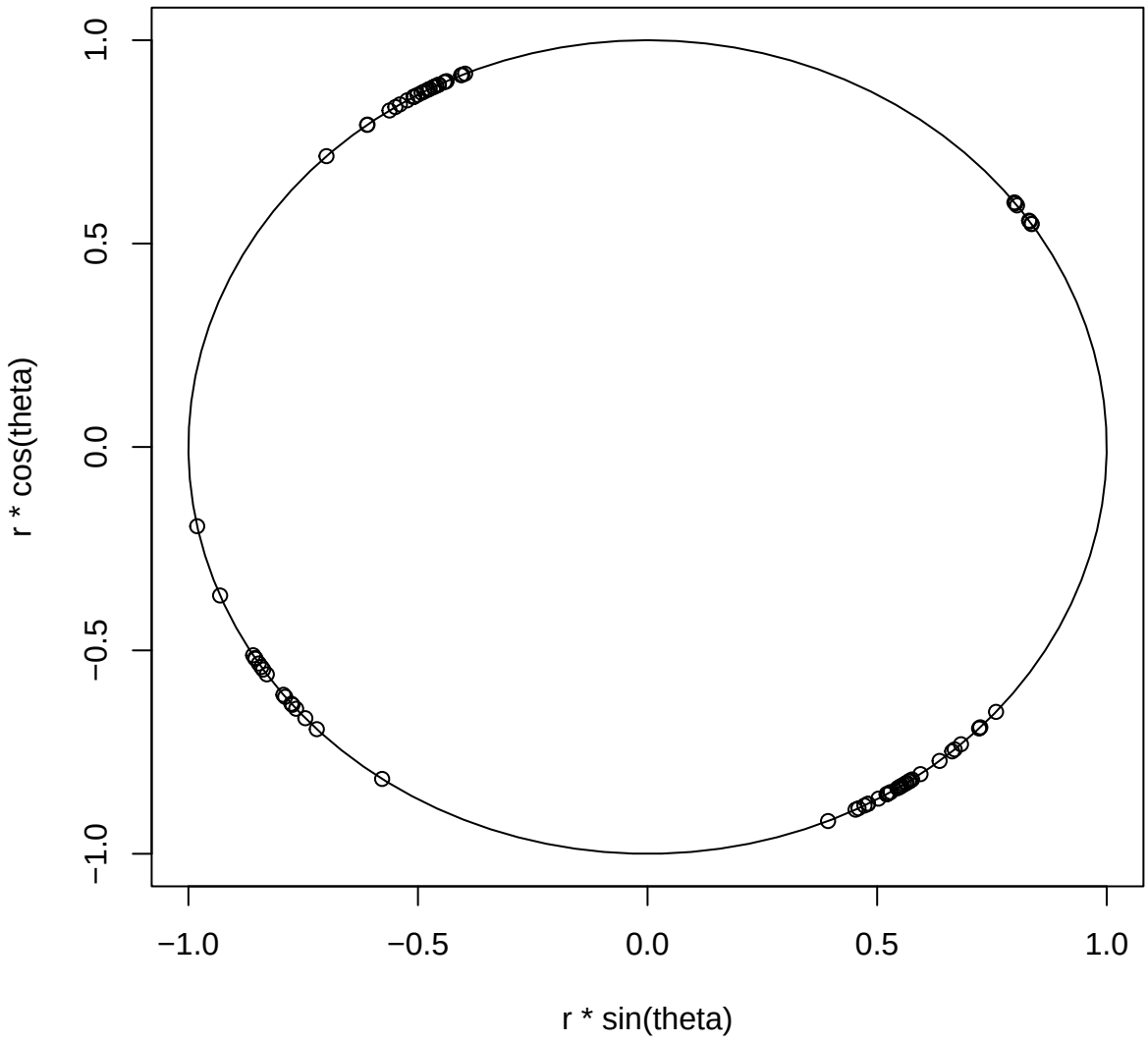
```
## [1] "Iter 115, obj 1.751204, abs 0.060256, rel 0.033264, norm 0.493205"
## [1] "Iter 116, obj 1.696765, abs 0.054440, rel 0.031087, norm 0.486817"
## [1] "Iter 117, obj 1.668955, abs 0.027810, rel 0.016390, norm 0.486791"
## [1] "Iter 118, obj 1.646500, abs 0.022455, rel 0.013455, norm 0.476857"
## [1] "Iter 119, obj 1.610625, abs 0.035875, rel 0.021788, norm 0.472463"
## [1] "Iter 120, obj 1.585063, abs 0.025562, rel 0.015871, norm 0.468156"
## [1] "Iter 121, obj 1.554055, abs 0.031008, rel 0.019562, norm 0.459257"
## [1] "Iter 122, obj 1.563741, abs 0.009686, rel 0.006233, norm 0.457186"
## [1] "Iter 123, obj 1.549436, abs 0.014305, rel 0.009148, norm 0.449167"
## [1] "Iter 124, obj 1.536521, abs 0.012915, rel 0.008335, norm 0.450827"
## [1] "Iter 125, obj 1.524972, abs 0.011549, rel 0.007516, norm 0.443109"
## [1] "Iter 126, obj 1.551524, abs 0.026553, rel 0.017412, norm 0.440869"
## [1] "Iter 127, obj 1.574016, abs 0.022492, rel 0.014497, norm 0.438883"
## [1] "Iter 128, obj 1.582337, abs 0.008320, rel 0.005286, norm 0.458621"
## [1] "Iter 129, obj 1.585291, abs 0.002954, rel 0.001867, norm 0.463928"
## [1] "Iter 130, obj 1.578650, abs 0.006641, rel 0.004189, norm 0.470267"
## [1] "Iter 131, obj 1.581755, abs 0.003105, rel 0.001967, norm 0.468052"
## [1] "Iter 132, obj 1.574078, abs 0.007677, rel 0.004853, norm 0.467708"
## [1] "Iter 133, obj 1.573037, abs 0.001041, rel 0.000661, norm 0.465438"
## [1] "Iter 134, obj 1.556729, abs 0.016308, rel 0.010367, norm 0.465681"
## [1] "Iter 135, obj 1.559635, abs 0.002906, rel 0.001867, norm 0.463008"
## [1] "Iter 136, obj 1.557538, abs 0.002097, rel 0.001345, norm 0.462119"
## [1] "Iter 137, obj 1.566579, abs 0.009041, rel 0.005805, norm 0.461810"
## [1] "Iter 138, obj 1.552763, abs 0.013816, rel 0.008819, norm 0.464462"
## [1] "Iter 139, obj 1.559328, abs 0.006565, rel 0.004228, norm 0.461496"
## [1] "Iter 140, obj 1.552812, abs 0.006516, rel 0.004179, norm 0.460311"
## [1] "Iter 141, obj 1.563337, abs 0.010525, rel 0.006778, norm 0.460535"
## [1] "Iter 142, obj 1.549797, abs 0.013540, rel 0.008661, norm 0.462378"
## [1] "Iter 143, obj 1.559797, abs 0.010000, rel 0.006452, norm 0.460624"
## [1] "Iter 144, obj 1.550594, abs 0.009203, rel 0.005900, norm 0.460261"
## [1] "Iter 145, obj 1.562951, abs 0.012358, rel 0.007970, norm 0.460216"
## [1] "Iter 146, obj 1.546951, abs 0.016000, rel 0.010237, norm 0.462152"
## [1] "Iter 147, obj 1.558547, abs 0.011596, rel 0.007496, norm 0.459986"
## [1] "Iter 148, obj 1.547763, abs 0.010784, rel 0.006919, norm 0.459365"
## [1] "Iter 149, obj 1.558497, abs 0.010734, rel 0.006935, norm 0.458616"
## [1] "Iter 150, obj 1.547102, abs 0.011396, rel 0.007312, norm 0.460828"
## [1] "Iter 151, obj 1.558199, abs 0.011097, rel 0.007173, norm 0.461277"
## [1] "Iter 152, obj 1.546194, abs 0.012005, rel 0.007704, norm 0.463245"
## [1] "Iter 153, obj 1.549400, abs 0.003206, rel 0.002073, norm 0.462280"
## [1] "Iter 154, obj 1.536471, abs 0.012929, rel 0.008345, norm 0.459925"
## [1] "Iter 155, obj 1.544501, abs 0.008030, rel 0.005226, norm 0.457309"
## [1] "Iter 156, obj 1.540168, abs 0.004333, rel 0.002805, norm 0.451409"
## [1] "Iter 157, obj 1.546047, abs 0.005880, rel 0.003817, norm 0.459313"
## [1] "Iter 158, obj 1.530348, abs 0.015699, rel 0.010155, norm 0.462733"
## [1] "Iter 159, obj 1.531436, abs 0.001088, rel 0.000711, norm 0.454210"
## [1] "Iter 160, obj 1.532868, abs 0.001432, rel 0.000935, norm 0.451033"
## [1] "Iter 161, obj 1.546163, abs 0.013295, rel 0.008673, norm 0.448858"
## [1] "Iter 162, obj 1.538809, abs 0.007354, rel 0.004756, norm 0.456274"
## [1] "Iter 163, obj 1.545230, abs 0.006421, rel 0.004173, norm 0.456647"
## [1] "Iter 164, obj 1.553904, abs 0.008674, rel 0.005613, norm 0.454145"
## [1] "Iter 165, obj 1.561840, abs 0.007935, rel 0.005107, norm 0.462908"
## [1] "Iter 166, obj 1.560411, abs 0.001428, rel 0.000914, norm 0.466530"
## [1] "Iter 167, obj 1.561705, abs 0.001293, rel 0.000829, norm 0.470914"
## [1] "Iter 168, obj 1.559713, abs 0.001992, rel 0.001275, norm 0.468673"
```

```
## [1] "Iter 169, obj 1.561100, abs 0.001387, rel 0.000889, norm 0.470593"
## [1] "Iter 170, obj 1.559520, abs 0.001580, rel 0.001012, norm 0.468479"
## [1] "Iter 171, obj 1.560688, abs 0.001169, rel 0.000749, norm 0.470585"
## [1] "Iter 172, obj 1.558727, abs 0.001961, rel 0.001257, norm 0.468193"
## [1] "Iter 173, obj 1.558029, abs 0.000698, rel 0.000448, norm 0.470260"
## [1] "Iter 174, obj 1.552906, abs 0.005123, rel 0.003288, norm 0.467204"
## [1] "Iter 175, obj 1.547530, abs 0.005376, rel 0.003462, norm 0.469213"
## [1] "Iter 176, obj 1.548551, abs 0.001021, rel 0.000660, norm 0.464113"
## [1] "Iter 177, obj 1.549652, abs 0.001101, rel 0.000711, norm 0.467127"
## [1] "Iter 178, obj 1.554977, abs 0.005325, rel 0.003436, norm 0.463453"
## [1] "Iter 179, obj 1.547549, abs 0.007428, rel 0.004777, norm 0.468808"
## [1] "Iter 180, obj 1.544958, abs 0.002591, rel 0.001674, norm 0.463812"
## [1] "Iter 181, obj 1.541677, abs 0.003281, rel 0.002123, norm 0.464998"
## [1] "Iter 182, obj 1.544658, abs 0.002981, rel 0.001934, norm 0.460954"
## [1] "Iter 183, obj 1.545143, abs 0.000484, rel 0.000313, norm 0.464846"
## [1] "Iter 184, obj 1.551035, abs 0.005893, rel 0.003814, norm 0.461975"
## [1] "Iter 185, obj 1.548470, abs 0.002566, rel 0.001654, norm 0.467093"
## [1] "Iter 186, obj 1.550177, abs 0.001708, rel 0.001103, norm 0.463206"
## [1] "Iter 187, obj 1.543030, abs 0.007147, rel 0.004610, norm 0.467149"
## [1] "Iter 188, obj 1.544637, abs 0.001607, rel 0.001041, norm 0.461743"
## [1] "Iter 189, obj 1.543812, abs 0.000824, rel 0.000534, norm 0.464634"
## [1] "Iter 190, obj 1.549598, abs 0.005785, rel 0.003747, norm 0.461484"
## [1] "Iter 191, obj 1.548251, abs 0.001347, rel 0.000869, norm 0.466413"
## [1] "Iter 192, obj 1.552535, abs 0.004284, rel 0.002767, norm 0.462756"
## [1] "Iter 193, obj 1.543777, abs 0.008758, rel 0.005641, norm 0.467569"
## [1] "Iter 194, obj 1.544299, abs 0.000522, rel 0.000338, norm 0.462070"
## [1] "Iter 195, obj 1.541436, abs 0.002863, rel 0.001854, norm 0.464249"
## [1] "Iter 196, obj 1.546457, abs 0.005020, rel 0.003257, norm 0.460620"
## [1] "Iter 197, obj 1.547354, abs 0.000898, rel 0.000581, norm 0.465069"
## [1] "Iter 198, obj 1.555885, abs 0.008531, rel 0.005513, norm 0.461906"
## [1] "Iter 199, obj 1.547029, abs 0.008856, rel 0.005692, norm 0.467801"
## [1] "Iter 200, obj 1.543551, abs 0.003477, rel 0.002248, norm 0.462984"
## [1] "Iter 201, obj 1.533859, abs 0.009693, rel 0.006279, norm 0.464089"
## [1] "Iter 202, obj 1.532694, abs 0.001164, rel 0.000759, norm 0.458018"
## [1] "Iter 203, obj 1.528059, abs 0.004635, rel 0.003024, norm 0.460611"
## [1] "Iter 204, obj 1.532835, abs 0.004776, rel 0.003125, norm 0.456016"
## [1] "Iter 205, obj 1.546070, abs 0.013236, rel 0.008635, norm 0.460253"
## [1] "Iter 206, obj 1.558229, abs 0.012158, rel 0.007864, norm 0.464326"
## [1] "Iter 207, obj 1.563567, abs 0.005338, rel 0.003425, norm 0.472405"
## [1] "Iter 208, obj 1.567035, abs 0.003468, rel 0.002218, norm 0.472378"
## [1] "Iter 209, obj 1.556291, abs 0.010743, rel 0.006856, norm 0.476954"
## [1] "Iter 210, obj 1.554896, abs 0.001395, rel 0.000897, norm 0.471651"
## [1] "Iter 211, obj 1.550346, abs 0.004550, rel 0.002926, norm 0.473141"
## [1] "Iter 212, obj 1.563900, abs 0.013554, rel 0.008742, norm 0.469040"
## [1] "Iter 213, obj 1.570773, abs 0.006873, rel 0.004395, norm 0.474940"
## [1] "Iter 214, obj 1.575323, abs 0.004550, rel 0.002896, norm 0.476008"
## [1] "Iter 215, obj 1.567410, abs 0.007913, rel 0.005023, norm 0.481580"
## [1] "Iter 216, obj 1.571164, abs 0.003754, rel 0.002395, norm 0.476859"
## [1] "Iter 217, obj 1.567251, abs 0.003913, rel 0.002491, norm 0.479922"
## [1] "Iter 218, obj 1.570143, abs 0.002892, rel 0.001846, norm 0.477076"
## [1] "Iter 219, obj 1.567554, abs 0.002589, rel 0.001649, norm 0.480106"
## [1] "Iter 220, obj 1.570344, abs 0.002790, rel 0.001780, norm 0.477499"
## [1] "Iter 221, obj 1.568613, abs 0.001731, rel 0.001102, norm 0.480400"
## [1] "Iter 222, obj 1.571055, abs 0.002442, rel 0.001557, norm 0.477869"
```

```
## [1] "Iter 223, obj 1.570038, abs 0.001018, rel 0.000648, norm 0.480612"
## [1] "Iter 224, obj 1.572036, abs 0.001999, rel 0.001273, norm 0.478109"
## [1] "Iter 225, obj 1.571364, abs 0.000672, rel 0.000428, norm 0.480785"
## [1] "Iter 226, obj 1.573085, abs 0.001720, rel 0.001095, norm 0.478196"
## [1] "Iter 227, obj 1.570147, abs 0.002938, rel 0.001867, norm 0.481032"
## [1] "Iter 228, obj 1.571944, abs 0.001797, rel 0.001144, norm 0.477911"
## [1] "Iter 229, obj 1.570159, abs 0.001785, rel 0.001136, norm 0.480455"
## [1] "Iter 230, obj 1.572266, abs 0.002107, rel 0.001342, norm 0.477801"
## [1] "Iter 231, obj 1.570611, abs 0.001655, rel 0.001053, norm 0.480541"
## [1] "Iter 232, obj 1.572645, abs 0.002034, rel 0.001295, norm 0.477842"
## [1] "Iter 233, obj 1.570269, abs 0.002376, rel 0.001511, norm 0.480659"
## [1] "Iter 234, obj 1.572302, abs 0.002033, rel 0.001295, norm 0.477749"
## [1] "Iter 235, obj 1.570256, abs 0.002046, rel 0.001301, norm 0.480487"
## [1] "Iter 236, obj 1.572374, abs 0.002118, rel 0.001349, norm 0.477697"
## [1] "Iter 237, obj 1.570387, abs 0.001987, rel 0.001264, norm 0.480511"
## [1] "Iter 238, obj 1.572466, abs 0.002079, rel 0.001324, norm 0.477694"
## [1] "Iter 239, obj 1.570324, abs 0.002142, rel 0.001362, norm 0.480547"
## [1] "Iter 240, obj 1.572372, abs 0.002048, rel 0.001304, norm 0.477657"
## [1] "Iter 241, obj 1.570336, abs 0.002036, rel 0.001295, norm 0.480510"
## [1] "Iter 242, obj 1.572370, abs 0.002033, rel 0.001295, norm 0.477631"
## [1] "Iter 243, obj 1.570393, abs 0.001976, rel 0.001257, norm 0.480519"
## [1] "Iter 244, obj 1.572378, abs 0.001984, rel 0.001264, norm 0.477619"
## [1] "Iter 245, obj 1.570405, abs 0.001973, rel 0.001255, norm 0.480536"
## [1] "Iter 246, obj 1.572337, abs 0.001932, rel 0.001230, norm 0.477600"
## [1] "Iter 247, obj 1.570433, abs 0.001903, rel 0.001210, norm 0.480534"
## [1] "Iter 248, obj 1.567927, abs 0.002507, rel 0.001596, norm 0.477585"
## [1] "Iter 249, obj 1.563665, abs 0.004262, rel 0.002718, norm 0.477092"
## [1] "Iter 250, obj 1.563931, abs 0.000266, rel 0.000170, norm 0.472540"
## [1] "Iter 251, obj 1.561856, abs 0.002075, rel 0.001327, norm 0.474064"
## [1] "Iter 252, obj 1.563059, abs 0.001204, rel 0.000771, norm 0.470802"
## [1] "Iter 253, obj 1.561627, abs 0.001432, rel 0.000916, norm 0.473282"
## [1] "Iter 254, obj 1.562823, abs 0.001196, rel 0.000766, norm 0.470254"
## [1] "Iter 255, obj 1.561718, abs 0.001105, rel 0.000707, norm 0.472892"
## [1] "Iter 256, obj 1.562841, abs 0.001123, rel 0.000719, norm 0.469968"
## [1] "Iter 257, obj 1.561730, abs 0.001111, rel 0.000711, norm 0.472711"
## [1] "Iter 258, obj 1.562772, abs 0.001042, rel 0.000667, norm 0.469762"
## [1] "Iter 259, obj 1.561587, abs 0.001185, rel 0.000758, norm 0.472563"
## [1] "Iter 260, obj 1.562602, abs 0.001015, rel 0.000650, norm 0.469592"
## [1] "Iter 261, obj 1.561467, abs 0.001135, rel 0.000726, norm 0.472428"
## [1] "Iter 262, obj 1.562481, abs 0.001013, rel 0.000649, norm 0.469466"
## [1] "Iter 263, obj 1.561443, abs 0.001037, rel 0.000664, norm 0.472347"
## [1] "Iter 264, obj 1.562423, abs 0.000979, rel 0.000627, norm 0.469384"
## [1] "Iter 265, obj 1.561459, abs 0.000963, rel 0.000617, norm 0.472307"
## [1] "Iter 266, obj 1.562377, abs 0.000918, rel 0.000588, norm 0.469323"
## [1] "Iter 267, obj 1.561470, abs 0.000907, rel 0.000581, norm 0.472281"
## [1] "Iter 268, obj 1.562319, abs 0.000849, rel 0.000544, norm 0.469269"
## [1] "Iter 269, obj 1.561479, abs 0.000840, rel 0.000538, norm 0.472259"
## [1] "Iter 270, obj 1.562258, abs 0.000779, rel 0.000499, norm 0.469221"
## [1] "Iter 271, obj 1.561501, abs 0.000757, rel 0.000485, norm 0.472246"
## [1] "Iter 272, obj 1.562203, abs 0.000702, rel 0.000450, norm 0.469181"
## [1] "Iter 273, obj 1.561535, abs 0.000668, rel 0.000428, norm 0.472242"
## [1] "Iter 274, obj 1.562150, abs 0.000616, rel 0.000394, norm 0.469148"
## [1] "Iter 275, obj 1.561573, abs 0.000577, rel 0.000369, norm 0.472247"
## [1] "Iter 276, obj 1.562095, abs 0.000522, rel 0.000334, norm 0.469119"
```

```
## [1] "Iter 277, obj 1.561615, abs 0.000480, rel 0.000307, norm 0.472257"
## [1] "Iter 278, obj 1.562036, abs 0.000421, rel 0.000270, norm 0.469094"
## [1] "Iter 279, obj 1.561662, abs 0.000375, rel 0.000240, norm 0.472273"
## [1] "Iter 280, obj 1.561975, abs 0.000314, rel 0.000201, norm 0.469072"
## [1] "Iter 281, obj 1.561713, abs 0.000262, rel 0.000168, norm 0.472295"
## [1] "Iter 282, obj 1.561911, abs 0.000198, rel 0.000127, norm 0.469053"
## [1] "Iter 283, obj 1.561769, abs 0.000143, rel 0.000091, norm 0.472324"
## [1] "Iter 284, obj 1.561843, abs 0.000074, rel 0.000048, norm 0.469038"
## [1] "Iter 285, obj 1.561827, abs 0.000016, rel 0.000010, norm 0.472359"
## [1] "Iter 286, obj 1.561769, abs 0.000058, rel 0.000037, norm 0.469025"
## [1] "Iter 287, obj 1.561889, abs 0.000120, rel 0.000077, norm 0.472401"
## [1] "Iter 288, obj 1.561689, abs 0.000200, rel 0.000128, norm 0.469015"
## [1] "Iter 289, obj 1.561954, abs 0.000264, rel 0.000169, norm 0.472450"
## [1] "Iter 290, obj 1.561602, abs 0.000352, rel 0.000225, norm 0.469008"
## [1] "Iter 291, obj 1.562021, abs 0.000419, rel 0.000268, norm 0.472508"
## [1] "Iter 292, obj 1.561505, abs 0.000516, rel 0.000330, norm 0.469005"
## [1] "Iter 293, obj 1.562089, abs 0.000584, rel 0.000374, norm 0.472576"
## [1] "Iter 294, obj 1.561396, abs 0.000693, rel 0.000444, norm 0.469005"
## [1] "Iter 295, obj 1.562158, abs 0.000762, rel 0.000488, norm 0.472656"
## [1] "Iter 296, obj 1.561271, abs 0.000887, rel 0.000567, norm 0.469009"
## [1] "Iter 297, obj 1.562224, abs 0.000953, rel 0.000610, norm 0.472752"
## [1] "Iter 298, obj 1.561125, abs 0.001099, rel 0.000703, norm 0.469019"
## [1] "Iter 299, obj 1.562284, abs 0.001158, rel 0.000742, norm 0.472869"
## [1] "Iter 300, obj 1.560948, abs 0.001336, rel 0.000855, norm 0.469036"
```

```
Y_rad <- DirStats::to_rad(Y)
r <- 1
theta <<- Y_rad
plot(r*sin(theta),
     r*cos(theta),
     xlim=c(-max(r),max(r)),
     ylim=c(-max(r),max(r)))

polygon(max(r)*sin(seq(0,2*pi,length.out=100)),max(r)*cos(seq(0,2*pi,length.out=100)))
```



Now, optimized rho for a perplexity of 12 and $d = 2$:

```
Y <- psc_sne(X=sanjuanfuca, d=2,
             rho_psc_list = rho_12,
             num_iteration=300)
```

```
## [1] "Iter 1, obj 12.347073, abs 0.000000, rel 0.000000, norm 1.161041"
## [1] "Iter 2, obj 11.542364, abs 0.804709, rel 0.065174, norm 2.019725"
## [1] "Iter 3, obj 10.765064, abs 0.777301, rel 0.067343, norm 2.910442"
## [1] "Iter 4, obj 10.283473, abs 0.481591, rel 0.044736, norm 3.390362"
## [1] "Iter 5, obj 9.872004, abs 0.411469, rel 0.040013, norm 3.679482"
## [1] "Iter 6, obj 9.700148, abs 0.171856, rel 0.017408, norm 3.773318"
## [1] "Iter 7, obj 9.522854, abs 0.177294, rel 0.018277, norm 3.853124"
## [1] "Iter 8, obj 9.425147, abs 0.097707, rel 0.010260, norm 3.869366"
## [1] "Iter 9, obj 9.297087, abs 0.128059, rel 0.013587, norm 3.879188"
## [1] "Iter 10, obj 9.198080, abs 0.099008, rel 0.010649, norm 3.868804"
## [1] "Iter 11, obj 9.078880, abs 0.119200, rel 0.012959, norm 3.856681"
## [1] "Iter 12, obj 8.984407, abs 0.094472, rel 0.010406, norm 3.838297"
## [1] "Iter 13, obj 8.890981, abs 0.093427, rel 0.010399, norm 3.819820"
## [1] "Iter 14, obj 8.829108, abs 0.061873, rel 0.006959, norm 3.801342"
## [1] "Iter 15, obj 8.777984, abs 0.051124, rel 0.005790, norm 3.786219"
```

```
## [1] "Iter 16, obj 8.754625, abs 0.023359, rel 0.002661, norm 3.774727"
## [1] "Iter 17, obj 8.739280, abs 0.015345, rel 0.001753, norm 3.767588"
## [1] "Iter 18, obj 8.741459, abs 0.002179, rel 0.000249, norm 3.764242"
## [1] "Iter 19, obj 8.747968, abs 0.006509, rel 0.000745, norm 3.764196"
## [1] "Iter 20, obj 8.764902, abs 0.016934, rel 0.001936, norm 3.767655"
## [1] "Iter 21, obj 8.784555, abs 0.019653, rel 0.002242, norm 3.773733"
## [1] "Iter 22, obj 8.810010, abs 0.025455, rel 0.002898, norm 3.783094"
## [1] "Iter 23, obj 8.837665, abs 0.027655, rel 0.003139, norm 3.794503"
## [1] "Iter 24, obj 8.867974, abs 0.030309, rel 0.003430, norm 3.808983"
## [1] "Iter 25, obj 8.900598, abs 0.032623, rel 0.003679, norm 3.824898"
## [1] "Iter 26, obj 8.933565, abs 0.032967, rel 0.003704, norm 3.843851"
## [1] "Iter 27, obj 8.969375, abs 0.035810, rel 0.004008, norm 3.863567"
## [1] "Iter 28, obj 9.003663, abs 0.034288, rel 0.003823, norm 3.886513"
## [1] "Iter 29, obj 9.041612, abs 0.037949, rel 0.004215, norm 3.909477"
## [1] "Iter 30, obj 9.076414, abs 0.034802, rel 0.003849, norm 3.935961"
## [1] "Iter 31, obj 9.115884, abs 0.039470, rel 0.004349, norm 3.961678"
## [1] "Iter 32, obj 9.150765, abs 0.034881, rel 0.003826, norm 3.991174"
## [1] "Iter 33, obj 9.191406, abs 0.040641, rel 0.004441, norm 4.019151"
## [1] "Iter 34, obj 9.226196, abs 0.034789, rel 0.003785, norm 4.051056"
## [1] "Iter 35, obj 9.267815, abs 0.041619, rel 0.004511, norm 4.080786"
## [1] "Iter 36, obj 9.302479, abs 0.034664, rel 0.003740, norm 4.114464"
## [1] "Iter 37, obj 9.344934, abs 0.042455, rel 0.004564, norm 4.145418"
## [1] "Iter 38, obj 9.379458, abs 0.034524, rel 0.003694, norm 4.180226"
## [1] "Iter 39, obj 9.422580, abs 0.043122, rel 0.004597, norm 4.211857"
## [1] "Iter 40, obj 9.456901, abs 0.034321, rel 0.003642, norm 4.247173"
## [1] "Iter 41, obj 9.500461, abs 0.043561, rel 0.004606, norm 4.278938"
## [1] "Iter 42, obj 9.534451, abs 0.033989, rel 0.003578, norm 4.314194"
## [1] "Iter 43, obj 9.578162, abs 0.043711, rel 0.004585, norm 4.345591"
## [1] "Iter 44, obj 9.611636, abs 0.033475, rel 0.003495, norm 4.380289"
## [1] "Iter 45, obj 9.655164, abs 0.043527, rel 0.004529, norm 4.410884"
## [1] "Iter 46, obj 9.687905, abs 0.032741, rel 0.003391, norm 4.444614"
## [1] "Iter 47, obj 9.730893, abs 0.042988, rel 0.004437, norm 4.474052"
## [1] "Iter 48, obj 9.762667, abs 0.031774, rel 0.003265, norm 4.506494"
## [1] "Iter 49, obj 9.804762, abs 0.042095, rel 0.004312, norm 4.534510"
## [1] "Iter 50, obj 9.835337, abs 0.030575, rel 0.003118, norm 4.565428"
## [1] "Iter 51, obj 9.876212, abs 0.040875, rel 0.004156, norm 4.591836"
## [1] "Iter 52, obj 9.905380, abs 0.029169, rel 0.002953, norm 4.621070"
## [1] "Iter 53, obj 9.944753, abs 0.039372, rel 0.003975, norm 4.645759"
## [1] "Iter 54, obj 9.972343, abs 0.027590, rel 0.002774, norm 4.673210"
## [1] "Iter 55, obj 10.009988, abs 0.037645, rel 0.003775, norm 4.696125"
## [1] "Iter 56, obj 10.035875, abs 0.025887, rel 0.002586, norm 4.721748"
## [1] "Iter 57, obj 10.071630, abs 0.035756, rel 0.003563, norm 4.742881"
## [1] "Iter 58, obj 10.095741, abs 0.024111, rel 0.002394, norm 4.766668"
## [1] "Iter 59, obj 10.129501, abs 0.033760, rel 0.003344, norm 4.786045"
## [1] "Iter 60, obj 10.151810, abs 0.022309, rel 0.002202, norm 4.808019"
## [1] "Iter 61, obj 10.183516, abs 0.031706, rel 0.003123, norm 4.825690"
## [1] "Iter 62, obj 10.204038, abs 0.020522, rel 0.002015, norm 4.845894"
## [1] "Iter 63, obj 10.233669, abs 0.029631, rel 0.002904, norm 4.861929"
## [1] "Iter 64, obj 10.252448, abs 0.018779, rel 0.001835, norm 4.880423"
## [1] "Iter 65, obj 10.280014, abs 0.027565, rel 0.002689, norm 4.894903"
## [1] "Iter 66, obj 10.297114, abs 0.017100, rel 0.001663, norm 4.911762"
## [1] "Iter 67, obj 10.322643, abs 0.025530, rel 0.002479, norm 4.924778"
## [1] "Iter 68, obj 10.338142, abs 0.015498, rel 0.001501, norm 4.940084"
## [1] "Iter 69, obj 10.361685, abs 0.023543, rel 0.002277, norm 4.951733"
```

```
## [1] "Iter 70, obj 10.375665, abs 0.013980, rel 0.001349, norm 4.965579"
## [1] "Iter 71, obj 10.397285, abs 0.021620, rel 0.002084, norm 4.975961"
## [1] "Iter 72, obj 10.409835, abs 0.012551, rel 0.001207, norm 4.988443"
## [1] "Iter 73, obj 10.429608, abs 0.019773, rel 0.001899, norm 4.997659"
## [1] "Iter 74, obj 10.440820, abs 0.011211, rel 0.001075, norm 5.008877"
## [1] "Iter 75, obj 10.458833, abs 0.018013, rel 0.001725, norm 5.017028"
## [1] "Iter 76, obj 10.468796, abs 0.009963, rel 0.000953, norm 5.027081"
## [1] "Iter 77, obj 10.485144, abs 0.016348, rel 0.001562, norm 5.034266"
## [1] "Iter 78, obj 10.493951, abs 0.008807, rel 0.000840, norm 5.043253"
## [1] "Iter 79, obj 10.508735, abs 0.014784, rel 0.001409, norm 5.049566"
## [1] "Iter 80, obj 10.516476, abs 0.007741, rel 0.000737, norm 5.057582"
## [1] "Iter 81, obj 10.529800, abs 0.013324, rel 0.001267, norm 5.063113"
## [1] "Iter 82, obj 10.536563, abs 0.006763, rel 0.000642, norm 5.070251"
## [1] "Iter 83, obj 10.548533, abs 0.011970, rel 0.001136, norm 5.075083"
## [1] "Iter 84, obj 10.554404, abs 0.005871, rel 0.000557, norm 5.081427"
## [1] "Iter 85, obj 10.565125, abs 0.010721, rel 0.001016, norm 5.085638"
## [1] "Iter 86, obj 10.570187, abs 0.005062, rel 0.000479, norm 5.091270"
## [1] "Iter 87, obj 10.579762, abs 0.009575, rel 0.000906, norm 5.094930"
## [1] "Iter 88, obj 10.584094, abs 0.004332, rel 0.000409, norm 5.099924"
## [1] "Iter 89, obj 10.592621, abs 0.008527, rel 0.000806, norm 5.103097"
## [1] "Iter 90, obj 10.596297, abs 0.003676, rel 0.000347, norm 5.107521"
## [1] "Iter 91, obj 10.603871, abs 0.007574, rel 0.000715, norm 5.110266"
## [1] "Iter 92, obj 10.606960, abs 0.003089, rel 0.000291, norm 5.114182"
## [1] "Iter 93, obj 10.613670, abs 0.006711, rel 0.000633, norm 5.116551"
## [1] "Iter 94, obj 10.616237, abs 0.002567, rel 0.000242, norm 5.120015"
## [1] "Iter 95, obj 10.622167, abs 0.005930, rel 0.000559, norm 5.122055"
## [1] "Iter 96, obj 10.624271, abs 0.002104, rel 0.000198, norm 5.125117"
## [1] "Iter 97, obj 10.629498, abs 0.005227, rel 0.000492, norm 5.126869"
## [1] "Iter 98, obj 10.631194, abs 0.001695, rel 0.000160, norm 5.129575"
## [1] "Iter 99, obj 10.635789, abs 0.004596, rel 0.000432, norm 5.131075"
## [1] "Iter 100, obj 10.637126, abs 0.001337, rel 0.000126, norm 5.133466"
## [1] "Iter 101, obj 1.338592, abs 9.298534, rel 0.874158, norm 0.413151"
## [1] "Iter 102, obj 1.937263, abs 0.598671, rel 0.447239, norm 0.300142"
## [1] "Iter 103, obj 2.796752, abs 0.859489, rel 0.443662, norm 0.656822"
## [1] "Iter 104, obj 3.002426, abs 0.205674, rel 0.073540, norm 1.120994"
## [1] "Iter 105, obj 3.038970, abs 0.036543, rel 0.012171, norm 1.346929"
## [1] "Iter 106, obj 3.071573, abs 0.032604, rel 0.010728, norm 1.398098"
## [1] "Iter 107, obj 3.140598, abs 0.069025, rel 0.022472, norm 1.440422"
## [1] "Iter 108, obj 3.119203, abs 0.021395, rel 0.006812, norm 1.435204"
## [1] "Iter 109, obj 3.114409, abs 0.004795, rel 0.001537, norm 1.460740"
## [1] "Iter 110, obj 3.113419, abs 0.000990, rel 0.000318, norm 1.458312"
## [1] "Iter 111, obj 3.136223, abs 0.022804, rel 0.007325, norm 1.456139"
## [1] "Iter 112, obj 3.114972, abs 0.021251, rel 0.006776, norm 1.459027"
## [1] "Iter 113, obj 3.117161, abs 0.002189, rel 0.000703, norm 1.465096"
## [1] "Iter 114, obj 3.084709, abs 0.032452, rel 0.010411, norm 1.460536"
## [1] "Iter 115, obj 3.073645, abs 0.011064, rel 0.003587, norm 1.503628"
## [1] "Iter 116, obj 3.042466, abs 0.031180, rel 0.010144, norm 1.492914"
## [1] "Iter 117, obj 3.025463, abs 0.017003, rel 0.005588, norm 1.503066"
## [1] "Iter 118, obj 3.033763, abs 0.008300, rel 0.002744, norm 1.483078"
## [1] "Iter 119, obj 3.020333, abs 0.013430, rel 0.004427, norm 1.496795"
## [1] "Iter 120, obj 3.032253, abs 0.011919, rel 0.003946, norm 1.479519"
## [1] "Iter 121, obj 3.017650, abs 0.014602, rel 0.004816, norm 1.493767"
## [1] "Iter 122, obj 3.030216, abs 0.012566, rel 0.004164, norm 1.477052"
## [1] "Iter 123, obj 3.012559, abs 0.017658, rel 0.005827, norm 1.491571"
```

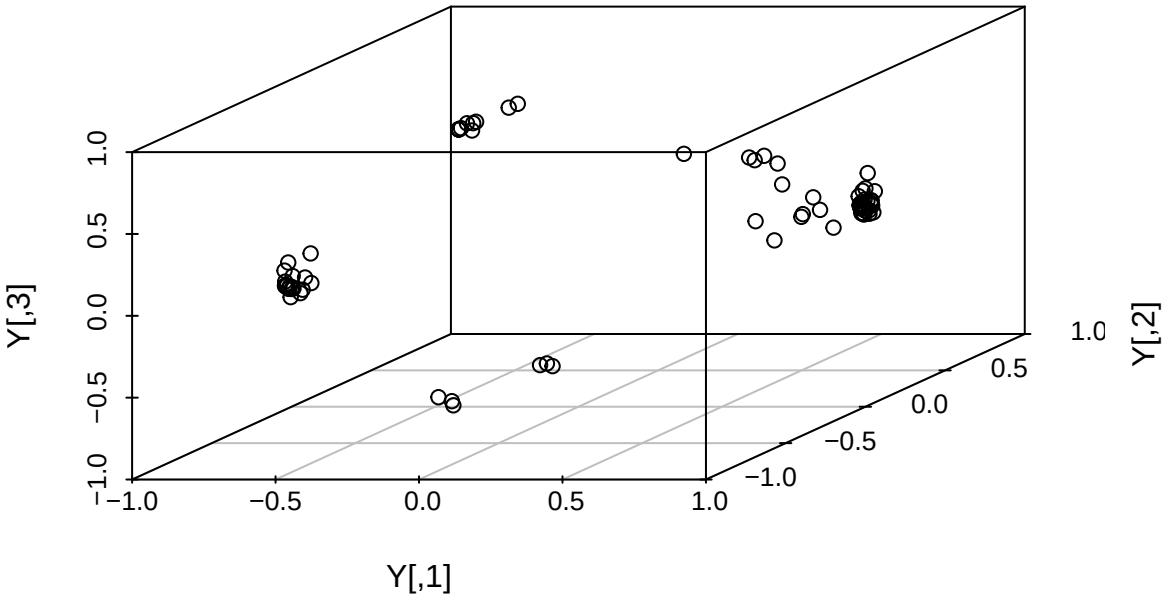
```
## [1] "Iter 124, obj 3.022641, abs 0.010083, rel 0.003347, norm 1.474840"
## [1] "Iter 125, obj 2.997897, abs 0.024744, rel 0.008186, norm 1.489520"
## [1] "Iter 126, obj 2.997575, abs 0.000322, rel 0.000107, norm 1.473599"
## [1] "Iter 127, obj 2.954009, abs 0.043566, rel 0.014534, norm 1.499040"
## [1] "Iter 128, obj 2.976655, abs 0.022646, rel 0.007666, norm 1.490314"
## [1] "Iter 129, obj 2.930396, abs 0.046259, rel 0.015541, norm 1.501454"
## [1] "Iter 130, obj 2.940148, abs 0.009752, rel 0.003328, norm 1.486732"
## [1] "Iter 131, obj 2.884094, abs 0.056054, rel 0.019065, norm 1.494508"
## [1] "Iter 132, obj 2.961435, abs 0.077342, rel 0.026817, norm 1.484669"
## [1] "Iter 133, obj 2.938837, abs 0.022599, rel 0.007631, norm 1.503991"
## [1] "Iter 134, obj 2.984450, abs 0.045613, rel 0.015521, norm 1.497438"
## [1] "Iter 135, obj 2.932504, abs 0.051946, rel 0.017405, norm 1.516268"
## [1] "Iter 136, obj 2.977147, abs 0.044643, rel 0.015223, norm 1.495365"
## [1] "Iter 137, obj 2.922335, abs 0.054812, rel 0.018411, norm 1.510439"
## [1] "Iter 138, obj 2.964000, abs 0.041665, rel 0.014257, norm 1.492656"
## [1] "Iter 139, obj 2.897456, abs 0.066544, rel 0.022451, norm 1.506813"
## [1] "Iter 140, obj 2.938085, abs 0.040629, rel 0.014022, norm 1.498446"
## [1] "Iter 141, obj 2.853775, abs 0.084309, rel 0.028695, norm 1.513012"
## [1] "Iter 142, obj 2.929400, abs 0.075624, rel 0.026500, norm 1.500093"
## [1] "Iter 143, obj 2.844294, abs 0.085105, rel 0.029052, norm 1.510365"
## [1] "Iter 144, obj 2.912487, abs 0.068193, rel 0.023975, norm 1.502264"
## [1] "Iter 145, obj 2.806155, abs 0.106333, rel 0.036509, norm 1.514090"
## [1] "Iter 146, obj 2.910953, abs 0.104798, rel 0.037346, norm 1.525939"
## [1] "Iter 147, obj 2.826548, abs 0.084404, rel 0.028995, norm 1.528673"
## [1] "Iter 148, obj 2.920594, abs 0.094046, rel 0.033272, norm 1.516407"
## [1] "Iter 149, obj 2.797894, abs 0.122699, rel 0.042012, norm 1.533521"
## [1] "Iter 150, obj 2.860484, abs 0.062589, rel 0.022370, norm 1.520436"
## [1] "Iter 151, obj 2.758032, abs 0.102452, rel 0.035816, norm 1.544038"
## [1] "Iter 152, obj 2.835454, abs 0.077422, rel 0.028071, norm 1.534784"
## [1] "Iter 153, obj 2.728303, abs 0.107151, rel 0.037790, norm 1.546666"
## [1] "Iter 154, obj 2.830832, abs 0.102530, rel 0.037580, norm 1.530648"
## [1] "Iter 155, obj 2.726281, abs 0.104551, rel 0.036933, norm 1.547481"
## [1] "Iter 156, obj 2.827836, abs 0.101555, rel 0.037250, norm 1.531838"
## [1] "Iter 157, obj 2.724380, abs 0.103456, rel 0.036585, norm 1.547787"
## [1] "Iter 158, obj 2.824673, abs 0.100293, rel 0.036813, norm 1.532482"
## [1] "Iter 159, obj 2.721705, abs 0.102968, rel 0.036453, norm 1.547744"
## [1] "Iter 160, obj 2.819212, abs 0.097507, rel 0.035826, norm 1.533723"
## [1] "Iter 161, obj 2.714231, abs 0.104981, rel 0.037238, norm 1.547844"
## [1] "Iter 162, obj 2.803385, abs 0.089153, rel 0.032847, norm 1.537967"
## [1] "Iter 163, obj 2.684562, abs 0.118822, rel 0.042385, norm 1.547829"
## [1] "Iter 164, obj 2.797951, abs 0.113389, rel 0.042237, norm 1.536759"
## [1] "Iter 165, obj 2.686820, abs 0.111131, rel 0.039719, norm 1.547938"
## [1] "Iter 166, obj 2.795417, abs 0.108598, rel 0.040419, norm 1.540336"
## [1] "Iter 167, obj 2.688316, abs 0.107101, rel 0.038313, norm 1.550321"
## [1] "Iter 168, obj 2.793294, abs 0.104978, rel 0.039050, norm 1.542535"
## [1] "Iter 169, obj 2.689507, abs 0.103787, rel 0.037156, norm 1.552296"
## [1] "Iter 170, obj 2.791538, abs 0.102031, rel 0.037937, norm 1.544076"
## [1] "Iter 171, obj 2.690564, abs 0.100974, rel 0.036171, norm 1.553941"
## [1] "Iter 172, obj 2.790066, abs 0.099502, rel 0.036982, norm 1.545259"
## [1] "Iter 173, obj 2.691550, abs 0.098516, rel 0.035310, norm 1.555319"
## [1] "Iter 174, obj 2.788799, abs 0.097248, rel 0.036131, norm 1.546227"
## [1] "Iter 175, obj 2.692486, abs 0.096313, rel 0.034536, norm 1.556483"
## [1] "Iter 176, obj 2.787676, abs 0.095190, rel 0.035354, norm 1.547050"
## [1] "Iter 177, obj 2.693384, abs 0.094292, rel 0.033825, norm 1.557476"
```

```
## [1] "Iter 178, obj 2.786660, abs 0.093276, rel 0.034632, norm 1.547770"
## [1] "Iter 179, obj 2.694256, abs 0.092404, rel 0.033159, norm 1.558332"
## [1] "Iter 180, obj 2.785723, abs 0.091467, rel 0.033949, norm 1.548413"
## [1] "Iter 181, obj 2.695114, abs 0.090609, rel 0.032526, norm 1.559080"
## [1] "Iter 182, obj 2.784846, abs 0.089732, rel 0.033294, norm 1.549002"
## [1] "Iter 183, obj 2.695966, abs 0.088880, rel 0.031915, norm 1.559741"
## [1] "Iter 184, obj 2.784013, abs 0.088047, rel 0.032659, norm 1.549553"
## [1] "Iter 185, obj 2.696821, abs 0.087192, rel 0.031319, norm 1.560336"
## [1] "Iter 186, obj 2.783214, abs 0.086393, rel 0.032035, norm 1.550078"
## [1] "Iter 187, obj 2.697685, abs 0.085529, rel 0.030730, norm 1.560878"
## [1] "Iter 188, obj 2.782440, abs 0.084755, rel 0.031418, norm 1.550588"
## [1] "Iter 189, obj 2.698563, abs 0.083877, rel 0.030145, norm 1.561381"
## [1] "Iter 190, obj 2.781683, abs 0.083120, rel 0.030801, norm 1.551089"
## [1] "Iter 191, obj 2.699458, abs 0.082225, rel 0.029559, norm 1.561852"
## [1] "Iter 192, obj 2.780937, abs 0.081479, rel 0.030183, norm 1.551586"
## [1] "Iter 193, obj 2.700374, abs 0.080562, rel 0.028970, norm 1.562301"
## [1] "Iter 194, obj 2.780197, abs 0.079823, rel 0.029560, norm 1.552083"
## [1] "Iter 195, obj 2.701314, abs 0.078883, rel 0.028373, norm 1.562732"
## [1] "Iter 196, obj 2.779462, abs 0.078148, rel 0.028929, norm 1.552582"
## [1] "Iter 197, obj 2.702280, abs 0.077182, rel 0.027769, norm 1.563152"
## [1] "Iter 198, obj 2.778727, abs 0.076447, rel 0.028290, norm 1.553085"
## [1] "Iter 199, obj 2.703274, abs 0.075452, rel 0.027154, norm 1.563562"
## [1] "Iter 200, obj 2.777991, abs 0.074717, rel 0.027639, norm 1.553591"
## [1] "Iter 201, obj 2.704299, abs 0.073692, rel 0.026527, norm 1.563968"
## [1] "Iter 202, obj 2.777253, abs 0.072954, rel 0.026977, norm 1.554103"
## [1] "Iter 203, obj 2.705355, abs 0.071898, rel 0.025888, norm 1.564371"
## [1] "Iter 204, obj 2.776513, abs 0.071158, rel 0.026303, norm 1.554618"
## [1] "Iter 205, obj 2.706444, abs 0.070069, rel 0.025236, norm 1.564772"
## [1] "Iter 206, obj 2.775770, abs 0.069326, rel 0.025615, norm 1.555137"
## [1] "Iter 207, obj 2.707567, abs 0.068204, rel 0.024571, norm 1.565174"
## [1] "Iter 208, obj 2.775025, abs 0.067458, rel 0.024915, norm 1.555657"
## [1] "Iter 209, obj 2.708722, abs 0.066303, rel 0.023893, norm 1.565576"
## [1] "Iter 210, obj 2.774277, abs 0.065555, rel 0.024201, norm 1.556179"
## [1] "Iter 211, obj 2.709911, abs 0.064367, rel 0.023201, norm 1.565979"
## [1] "Iter 212, obj 2.773528, abs 0.063617, rel 0.023476, norm 1.556700"
## [1] "Iter 213, obj 2.711130, abs 0.062398, rel 0.022498, norm 1.566382"
## [1] "Iter 214, obj 2.772777, abs 0.061647, rel 0.022739, norm 1.557217"
## [1] "Iter 215, obj 2.712378, abs 0.060400, rel 0.021783, norm 1.566784"
## [1] "Iter 216, obj 2.772026, abs 0.059648, rel 0.021991, norm 1.557727"
## [1] "Iter 217, obj 2.713651, abs 0.058375, rel 0.021059, norm 1.567183"
## [1] "Iter 218, obj 2.771274, abs 0.057623, rel 0.021235, norm 1.558227"
## [1] "Iter 219, obj 2.714944, abs 0.056330, rel 0.020326, norm 1.567576"
## [1] "Iter 220, obj 2.770522, abs 0.055578, rel 0.020471, norm 1.558713"
## [1] "Iter 221, obj 2.716253, abs 0.054269, rel 0.019588, norm 1.567959"
## [1] "Iter 222, obj 2.769770, abs 0.053517, rel 0.019702, norm 1.559180"
## [1] "Iter 223, obj 2.717570, abs 0.052199, rel 0.018846, norm 1.568330"
## [1] "Iter 224, obj 2.769017, abs 0.051447, rel 0.018931, norm 1.559622"
## [1] "Iter 225, obj 2.718889, abs 0.050128, rel 0.018103, norm 1.568684"
## [1] "Iter 226, obj 2.768264, abs 0.049375, rel 0.018160, norm 1.560035"
## [1] "Iter 227, obj 2.720201, abs 0.048063, rel 0.017362, norm 1.569017"
## [1] "Iter 228, obj 2.767511, abs 0.047310, rel 0.017392, norm 1.560413"
## [1] "Iter 229, obj 2.721499, abs 0.046013, rel 0.016626, norm 1.569326"
## [1] "Iter 230, obj 2.766758, abs 0.045260, rel 0.016630, norm 1.560749"
## [1] "Iter 231, obj 2.722773, abs 0.043986, rel 0.015898, norm 1.569607"
```

```
## [1] "Iter 232, obj 2.766006, abs 0.043233, rel 0.015878, norm 1.561040"
## [1] "Iter 233, obj 2.724014, abs 0.041992, rel 0.015181, norm 1.569856"
## [1] "Iter 234, obj 2.765254, abs 0.041240, rel 0.015140, norm 1.561281"
## [1] "Iter 235, obj 2.725214, abs 0.040040, rel 0.014480, norm 1.570073"
## [1] "Iter 236, obj 2.764504, abs 0.039291, rel 0.014417, norm 1.561469"
## [1] "Iter 237, obj 2.726364, abs 0.038140, rel 0.013796, norm 1.570254"
## [1] "Iter 238, obj 2.763757, abs 0.037393, rel 0.013715, norm 1.561599"
## [1] "Iter 239, obj 2.727458, abs 0.036300, rel 0.013134, norm 1.570401"
## [1] "Iter 240, obj 2.763015, abs 0.035557, rel 0.013037, norm 1.561672"
## [1] "Iter 241, obj 2.728488, abs 0.034527, rel 0.012496, norm 1.570512"
## [1] "Iter 242, obj 2.762279, abs 0.033791, rel 0.012385, norm 1.561687"
## [1] "Iter 243, obj 2.729449, abs 0.032830, rel 0.011885, norm 1.570589"
## [1] "Iter 244, obj 2.761551, abs 0.032102, rel 0.011761, norm 1.561644"
## [1] "Iter 245, obj 2.730336, abs 0.031214, rel 0.011303, norm 1.570634"
## [1] "Iter 246, obj 2.760834, abs 0.030497, rel 0.011170, norm 1.561547"
## [1] "Iter 247, obj 2.731149, abs 0.029685, rel 0.010752, norm 1.570647"
## [1] "Iter 248, obj 2.757112, abs 0.025963, rel 0.009506, norm 1.561398"
## [1] "Iter 249, obj 2.727828, abs 0.029284, rel 0.010621, norm 1.569195"
## [1] "Iter 250, obj 2.754777, abs 0.026949, rel 0.009879, norm 1.559416"
## [1] "Iter 251, obj 2.727741, abs 0.027036, rel 0.009814, norm 1.568323"
## [1] "Iter 252, obj 2.753648, abs 0.025907, rel 0.009498, norm 1.558896"
## [1] "Iter 253, obj 2.727946, abs 0.025702, rel 0.009334, norm 1.568017"
## [1] "Iter 254, obj 2.752827, abs 0.024881, rel 0.009121, norm 1.558424"
## [1] "Iter 255, obj 2.728324, abs 0.024503, rel 0.008901, norm 1.567762"
## [1] "Iter 256, obj 2.752165, abs 0.023841, rel 0.008738, norm 1.558019"
## [1] "Iter 257, obj 2.728715, abs 0.023450, rel 0.008521, norm 1.567557"
## [1] "Iter 258, obj 2.751584, abs 0.022869, rel 0.008381, norm 1.557630"
## [1] "Iter 259, obj 2.729093, abs 0.022491, rel 0.008174, norm 1.567371"
## [1] "Iter 260, obj 2.751060, abs 0.021967, rel 0.008049, norm 1.557257"
## [1] "Iter 261, obj 2.729439, abs 0.021621, rel 0.007859, norm 1.567202"
## [1] "Iter 262, obj 2.750580, abs 0.021141, rel 0.007745, norm 1.556894"
## [1] "Iter 263, obj 2.729747, abs 0.020833, rel 0.007574, norm 1.567044"
## [1] "Iter 264, obj 2.750138, abs 0.020391, rel 0.007470, norm 1.556539"
## [1] "Iter 265, obj 2.730014, abs 0.020124, rel 0.007317, norm 1.566897"
## [1] "Iter 266, obj 2.749731, abs 0.019717, rel 0.007222, norm 1.556193"
## [1] "Iter 267, obj 2.730243, abs 0.019488, rel 0.007087, norm 1.566757"
## [1] "Iter 268, obj 2.749358, abs 0.019115, rel 0.007001, norm 1.555856"
## [1] "Iter 269, obj 2.730435, abs 0.018923, rel 0.006883, norm 1.566625"
## [1] "Iter 270, obj 2.749019, abs 0.018583, rel 0.006806, norm 1.555531"
## [1] "Iter 271, obj 2.730595, abs 0.018423, rel 0.006702, norm 1.566500"
## [1] "Iter 272, obj 2.748712, abs 0.018116, rel 0.006635, norm 1.555217"
## [1] "Iter 273, obj 2.730726, abs 0.017986, rel 0.006543, norm 1.566380"
## [1] "Iter 274, obj 2.748436, abs 0.017710, rel 0.006486, norm 1.554916"
## [1] "Iter 275, obj 2.730831, abs 0.017605, rel 0.006405, norm 1.566266"
## [1] "Iter 276, obj 2.748190, abs 0.017359, rel 0.006357, norm 1.554629"
## [1] "Iter 277, obj 2.730914, abs 0.017276, rel 0.006286, norm 1.566157"
## [1] "Iter 278, obj 2.747973, abs 0.017059, rel 0.006247, norm 1.554356"
## [1] "Iter 279, obj 2.730978, abs 0.016995, rel 0.006185, norm 1.566052"
## [1] "Iter 280, obj 2.747783, abs 0.016804, rel 0.006153, norm 1.554099"
## [1] "Iter 281, obj 2.731027, abs 0.016756, rel 0.006098, norm 1.565952"
## [1] "Iter 282, obj 2.747616, abs 0.016590, rel 0.006074, norm 1.553857"
## [1] "Iter 283, obj 2.731062, abs 0.016554, rel 0.006025, norm 1.565854"
## [1] "Iter 284, obj 2.747472, abs 0.016411, rel 0.006009, norm 1.553629"
## [1] "Iter 285, obj 2.731086, abs 0.016386, rel 0.005964, norm 1.565760"
```

```
## [1] "Iter 286, obj 2.747349, abs 0.016263, rel 0.005955, norm 1.553417"
## [1] "Iter 287, obj 2.731101, abs 0.016247, rel 0.005914, norm 1.565669"
## [1] "Iter 288, obj 2.747243, abs 0.016142, rel 0.005910, norm 1.553219"
## [1] "Iter 289, obj 2.731110, abs 0.016133, rel 0.005873, norm 1.565580"
## [1] "Iter 290, obj 2.747153, abs 0.016044, rel 0.005874, norm 1.553034"
## [1] "Iter 291, obj 2.731112, abs 0.016041, rel 0.005839, norm 1.565494"
## [1] "Iter 292, obj 2.747078, abs 0.015965, rel 0.005846, norm 1.552862"
## [1] "Iter 293, obj 2.731111, abs 0.015967, rel 0.005812, norm 1.565410"
## [1] "Iter 294, obj 2.747014, abs 0.015904, rel 0.005823, norm 1.552702"
## [1] "Iter 295, obj 2.731106, abs 0.015908, rel 0.005791, norm 1.565328"
## [1] "Iter 296, obj 2.746962, abs 0.015856, rel 0.005806, norm 1.552554"
## [1] "Iter 297, obj 2.731099, abs 0.015862, rel 0.005775, norm 1.565248"
## [1] "Iter 298, obj 2.746919, abs 0.015819, rel 0.005792, norm 1.552416"
## [1] "Iter 299, obj 2.731091, abs 0.015827, rel 0.005762, norm 1.565169"
## [1] "Iter 300, obj 2.746883, abs 0.015792, rel 0.005782, norm 1.552288"
```

```
scatterplot3d::scatterplot3d(Y, xlim = c(-1, 1), ylim = c(-1, 1),
                             zlim = c(-1, 1), color = rep(1, nrow(Y)))
```




```
rgl::plot3d(0, 0, 0, xlim = c(-1, 1), ylim = c(-1, 1), zlim = c(-1, 1),
            radius = 1, type = "s", col = "lightblue", alpha = 0.25,
            lit = FALSE)
rgl::points3d(Y, col = rep(1, nrow(Y)))
```

Now, optimized rho for a perplexity of 33 and $d = 1$:

```
Y <- psc_sne(X=sanjuanfuca, d=1,
             rho_psc_list = rho_33,
             num_iteration=300)
```

```
## [1] "Iter 1, obj 6.901344, abs 0.000000, rel 0.000000, norm 0.965669"
## [1] "Iter 2, obj 6.793906, abs 0.107438, rel 0.015568, norm 1.138694"
## [1] "Iter 3, obj 6.748760, abs 0.045146, rel 0.006645, norm 1.255470"
## [1] "Iter 4, obj 6.741569, abs 0.007190, rel 0.001065, norm 1.360235"
## [1] "Iter 5, obj 6.759240, abs 0.017670, rel 0.002621, norm 1.457299"
## [1] "Iter 6, obj 6.794312, abs 0.035072, rel 0.005189, norm 1.544245"
```

```
## [1] "Iter 7, obj 6.841611, abs 0.047298, rel 0.006961, norm 1.622628"
## [1] "Iter 8, obj 6.896142, abs 0.054532, rel 0.007971, norm 1.692837"
## [1] "Iter 9, obj 6.952535, abs 0.056393, rel 0.008177, norm 1.754598"
## [1] "Iter 10, obj 7.006677, abs 0.054142, rel 0.007787, norm 1.807729"
## [1] "Iter 11, obj 7.056611, abs 0.049934, rel 0.007127, norm 1.853169"
## [1] "Iter 12, obj 7.102704, abs 0.046093, rel 0.006532, norm 1.892681"
## [1] "Iter 13, obj 7.146816, abs 0.044112, rel 0.006211, norm 1.928348"
## [1] "Iter 14, obj 7.191323, abs 0.044507, rel 0.006228, norm 1.961956"
## [1] "Iter 15, obj 7.238044, abs 0.046721, rel 0.006497, norm 1.994654"
## [1] "Iter 16, obj 7.287629, abs 0.049585, rel 0.006851, norm 2.026828"
## [1] "Iter 17, obj 7.339426, abs 0.051798, rel 0.007108, norm 2.058281"
## [1] "Iter 18, obj 7.391919, abs 0.052493, rel 0.007152, norm 2.088554"
## [1] "Iter 19, obj 7.443322, abs 0.051402, rel 0.006954, norm 2.117289"
## [1] "Iter 20, obj 7.492158, abs 0.048836, rel 0.006561, norm 2.144493"
## [1] "Iter 21, obj 7.537669, abs 0.045511, rel 0.006074, norm 2.170662"
## [1] "Iter 22, obj 7.580014, abs 0.042345, rel 0.005618, norm 2.196762"
## [1] "Iter 23, obj 7.620236, abs 0.040222, rel 0.005306, norm 2.224034"
## [1] "Iter 24, obj 7.660028, abs 0.039792, rel 0.005222, norm 2.253713"
## [1] "Iter 25, obj 7.701349, abs 0.041320, rel 0.005394, norm 2.286718"
## [1] "Iter 26, obj 7.745969, abs 0.044620, rel 0.005794, norm 2.323398"
## [1] "Iter 27, obj 7.795067, abs 0.049098, rel 0.006339, norm 2.363419"
## [1] "Iter 28, obj 7.849013, abs 0.053946, rel 0.006921, norm 2.405821"
## [1] "Iter 29, obj 7.907434, abs 0.058420, rel 0.007443, norm 2.449242"
## [1] "Iter 30, obj 7.969426, abs 0.061992, rel 0.007840, norm 2.492194"
## [1] "Iter 31, obj 8.033697, abs 0.064271, rel 0.008065, norm 2.533270"
## [1] "Iter 32, obj 8.098573, abs 0.064876, rel 0.008076, norm 2.571254"
## [1] "Iter 33, obj 8.162046, abs 0.063473, rel 0.007838, norm 2.605180"
## [1] "Iter 34, obj 8.222043, abs 0.059997, rel 0.007351, norm 2.634425"
## [1] "Iter 35, obj 8.276847, abs 0.054804, rel 0.006666, norm 2.658792"
## [1] "Iter 36, obj 8.325422, abs 0.048575, rel 0.005869, norm 2.678516"
## [1] "Iter 37, obj 8.367476, abs 0.042054, rel 0.005051, norm 2.694141"
## [1] "Iter 38, obj 8.403299, abs 0.035822, rel 0.004281, norm 2.706364"
## [1] "Iter 39, obj 8.433522, abs 0.030223, rel 0.003597, norm 2.715877"
## [1] "Iter 40, obj 8.458910, abs 0.025389, rel 0.003010, norm 2.723291"
## [1] "Iter 41, obj 8.480226, abs 0.021316, rel 0.002520, norm 2.729102"
## [1] "Iter 42, obj 8.498160, abs 0.017933, rel 0.002115, norm 2.733694"
## [1] "Iter 43, obj 8.513302, abs 0.015142, rel 0.001782, norm 2.737357"
## [1] "Iter 44, obj 8.526147, abs 0.012845, rel 0.001509, norm 2.740309"
## [1] "Iter 45, obj 8.537097, abs 0.010951, rel 0.001284, norm 2.742711"
## [1] "Iter 46, obj 8.546482, abs 0.009385, rel 0.001099, norm 2.744686"
## [1] "Iter 47, obj 8.554566, abs 0.008084, rel 0.000946, norm 2.746325"
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## [1] "Iter 49, obj 8.567652, abs 0.006088, rel 0.000711, norm 2.748853"
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## [1] "Iter 54, obj 8.588606, abs 0.003223, rel 0.000375, norm 2.752574"
## [1] "Iter 55, obj 8.591474, abs 0.002867, rel 0.000334, norm 2.753051"
## [1] "Iter 56, obj 8.594032, abs 0.002558, rel 0.000298, norm 2.753470"
## [1] "Iter 57, obj 8.596319, abs 0.002288, rel 0.000266, norm 2.753840"
## [1] "Iter 58, obj 8.598371, abs 0.002051, rel 0.000239, norm 2.754167"
## [1] "Iter 59, obj 8.600214, abs 0.001843, rel 0.000214, norm 2.754458"
## [1] "Iter 60, obj 8.601873, abs 0.001659, rel 0.000193, norm 2.754717"
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## [1] "Iter 61, obj 8.603370, abs 0.001497, rel 0.000174, norm 2.754948"
## [1] "Iter 62, obj 8.604722, abs 0.001352, rel 0.000157, norm 2.755155"
## [1] "Iter 63, obj 8.605946, abs 0.001224, rel 0.000142, norm 2.755341"
## [1] "Iter 64, obj 8.607055, abs 0.001109, rel 0.000129, norm 2.755508"
## [1] "Iter 65, obj 8.608061, abs 0.001006, rel 0.000117, norm 2.755659"
## [1] "Iter 66, obj 8.608975, abs 0.000914, rel 0.000106, norm 2.755795"
## [1] "Iter 67, obj 8.609806, abs 0.000831, rel 0.000097, norm 2.755918"
## [1] "Iter 68, obj 8.610563, abs 0.000757, rel 0.000088, norm 2.756029"
## [1] "Iter 69, obj 8.611253, abs 0.000690, rel 0.000080, norm 2.756130"
## [1] "Iter 70, obj 8.611882, abs 0.000629, rel 0.000073, norm 2.756222"
## [1] "Iter 71, obj 8.612456, abs 0.000574, rel 0.000067, norm 2.756305"
## [1] "Iter 72, obj 8.612980, abs 0.000524, rel 0.000061, norm 2.756381"
## [1] "Iter 73, obj 8.613460, abs 0.000479, rel 0.000056, norm 2.756450"
## [1] "Iter 74, obj 8.613898, abs 0.000438, rel 0.000051, norm 2.756513"
## [1] "Iter 75, obj 8.614299, abs 0.000401, rel 0.000047, norm 2.756570"
## [1] "Iter 76, obj 8.614667, abs 0.000367, rel 0.000043, norm 2.756623"
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## [1] "Iter 100, obj 8.618242, abs 0.000048, rel 0.000006, norm 2.757127"
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## [1] "Iter 106, obj 0.943532, abs 0.010658, rel 0.011170, norm 0.366531"
## [1] "Iter 107, obj 1.020762, abs 0.077230, rel 0.081852, norm 0.361834"
## [1] "Iter 108, obj 1.037897, abs 0.017135, rel 0.016786, norm 0.369155"
## [1] "Iter 109, obj 1.033353, abs 0.004544, rel 0.004378, norm 0.382395"
## [1] "Iter 110, obj 0.997225, abs 0.036128, rel 0.034962, norm 0.407204"
## [1] "Iter 111, obj 0.976645, abs 0.020580, rel 0.020637, norm 0.379309"
## [1] "Iter 112, obj 0.937909, abs 0.038736, rel 0.039662, norm 0.407656"
## [1] "Iter 113, obj 0.925733, abs 0.012176, rel 0.012982, norm 0.376844"
## [1] "Iter 114, obj 0.943456, abs 0.017723, rel 0.019145, norm 0.387242"
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## [1] "Iter 115, obj 0.890049, abs 0.053407, rel 0.056608, norm 0.392955"
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## [1] "Iter 263, obj 0.903783, abs 0.042020, rel 0.048761, norm 0.326011"
## [1] "Iter 264, obj 0.862439, abs 0.041344, rel 0.045745, norm 0.355146"
## [1] "Iter 265, obj 0.839081, abs 0.023358, rel 0.027084, norm 0.368256"
## [1] "Iter 266, obj 0.844188, abs 0.005107, rel 0.006086, norm 0.343230"
## [1] "Iter 267, obj 0.822661, abs 0.021526, rel 0.025500, norm 0.325992"
## [1] "Iter 268, obj 0.838217, abs 0.015556, rel 0.018909, norm 0.336959"
## [1] "Iter 269, obj 0.826998, abs 0.011219, rel 0.013385, norm 0.317906"
## [1] "Iter 270, obj 0.843005, abs 0.016007, rel 0.019356, norm 0.325554"
## [1] "Iter 271, obj 0.867853, abs 0.024848, rel 0.029476, norm 0.336569"
## [1] "Iter 272, obj 0.880144, abs 0.012290, rel 0.014162, norm 0.338251"
## [1] "Iter 273, obj 0.851761, abs 0.028383, rel 0.032248, norm 0.352469"
## [1] "Iter 274, obj 0.845219, abs 0.006542, rel 0.007680, norm 0.355081"
## [1] "Iter 275, obj 0.830691, abs 0.014528, rel 0.017188, norm 0.331962"
## [1] "Iter 276, obj 0.856814, abs 0.026123, rel 0.031447, norm 0.313308"
```
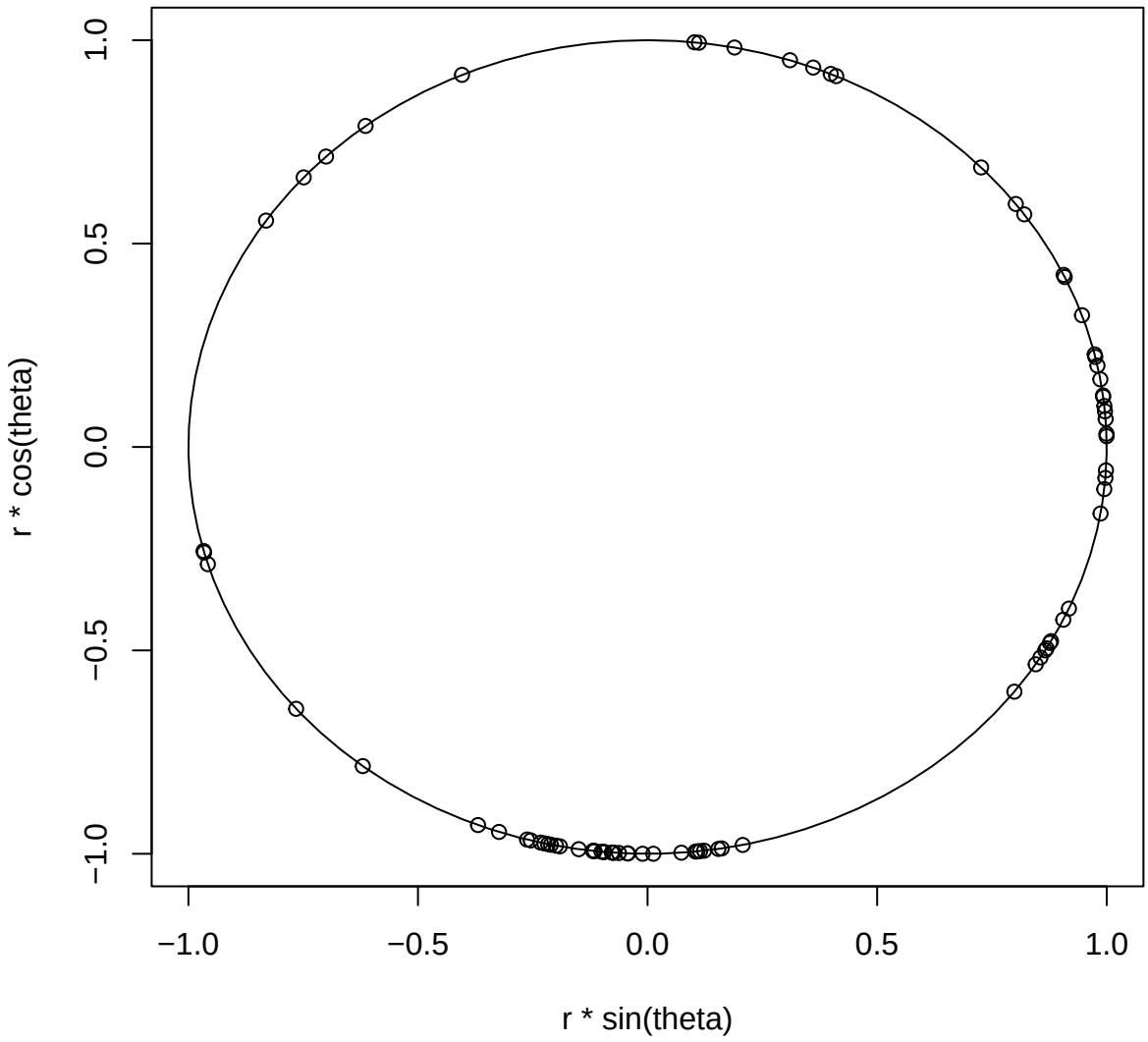

```
## [1] "Iter 277, obj 0.868982, abs 0.012168, rel 0.014201, norm 0.341365"
## [1] "Iter 278, obj 0.871234, abs 0.002252, rel 0.002591, norm 0.364242"
## [1] "Iter 279, obj 0.855829, abs 0.015405, rel 0.017682, norm 0.342609"
## [1] "Iter 280, obj 0.872196, abs 0.016367, rel 0.019124, norm 0.351651"
## [1] "Iter 281, obj 0.826302, abs 0.045894, rel 0.052619, norm 0.351195"
## [1] "Iter 282, obj 0.843261, abs 0.016959, rel 0.020524, norm 0.344902"
## [1] "Iter 283, obj 0.834702, abs 0.008558, rel 0.010149, norm 0.334559"
## [1] "Iter 284, obj 0.846794, abs 0.012091, rel 0.014486, norm 0.317286"
## [1] "Iter 285, obj 0.837683, abs 0.009110, rel 0.010759, norm 0.331616"
## [1] "Iter 286, obj 0.829910, abs 0.007773, rel 0.009280, norm 0.346342"
## [1] "Iter 287, obj 0.813684, abs 0.016226, rel 0.019552, norm 0.346245"
## [1] "Iter 288, obj 0.789403, abs 0.024281, rel 0.029841, norm 0.337255"
## [1] "Iter 289, obj 0.753548, abs 0.035855, rel 0.045421, norm 0.308603"
## [1] "Iter 290, obj 0.760171, abs 0.006623, rel 0.008789, norm 0.290448"
## [1] "Iter 291, obj 0.776808, abs 0.016637, rel 0.021886, norm 0.281671"
## [1] "Iter 292, obj 0.818647, abs 0.041839, rel 0.053860, norm 0.296183"
## [1] "Iter 293, obj 0.838874, abs 0.020227, rel 0.024708, norm 0.319742"
## [1] "Iter 294, obj 0.828611, abs 0.010263, rel 0.012235, norm 0.338617"
## [1] "Iter 295, obj 0.791240, abs 0.037370, rel 0.045100, norm 0.348111"
## [1] "Iter 296, obj 0.852571, abs 0.061331, rel 0.077512, norm 0.322870"
## [1] "Iter 297, obj 0.870818, abs 0.018247, rel 0.021403, norm 0.316889"
## [1] "Iter 298, obj 0.838754, abs 0.032065, rel 0.036821, norm 0.354614"
## [1] "Iter 299, obj 0.852280, abs 0.013526, rel 0.016127, norm 0.331487"
## [1] "Iter 300, obj 0.846895, abs 0.005385, rel 0.006318, norm 0.340075"
```

```
Y_rad <- DirStats::to_rad(Y)
r <- 1
theta <<- Y_rad
plot(r*sin(theta),
     r*cos(theta),
     xlim=c(-max(r),max(r)),
     ylim=c(-max(r),max(r)))

polygon(max(r)*sin(seq(0,2*pi,length.out=100)),max(r)*cos(seq(0,2*pi,length.out=100)))
```



Now, optimized rho for a perplexity of 33 and $d = 2$:

```
Y <- psc_sne(X=sanjuanfuca, d=2,
             rho_psc_list = rho_33,
             num_iteration=300)
```

```
## [1] "Iter 1, obj 9.253684, abs 0.000000, rel 0.000000, norm 0.762181"
## [1] "Iter 2, obj 8.985251, abs 0.268433, rel 0.029008, norm 1.706762"
## [1] "Iter 3, obj 8.986482, abs 0.001231, rel 0.000137, norm 2.979472"
## [1] "Iter 4, obj 9.012230, abs 0.025748, rel 0.002865, norm 3.513317"
## [1] "Iter 5, obj 9.055273, abs 0.043042, rel 0.004776, norm 3.711027"
## [1] "Iter 6, obj 9.080125, abs 0.024852, rel 0.002745, norm 3.837315"
## [1] "Iter 7, obj 9.066374, abs 0.013751, rel 0.001514, norm 3.902563"
## [1] "Iter 8, obj 9.009704, abs 0.056670, rel 0.006251, norm 3.912740"
## [1] "Iter 9, obj 8.933305, abs 0.076399, rel 0.008480, norm 3.893328"
## [1] "Iter 10, obj 8.834776, abs 0.098529, rel 0.011029, norm 3.861047"
## [1] "Iter 11, obj 8.721451, abs 0.113325, rel 0.012827, norm 3.819418"
## [1] "Iter 12, obj 8.583381, abs 0.138070, rel 0.015831, norm 3.770995"
## [1] "Iter 13, obj 8.422457, abs 0.160924, rel 0.018748, norm 3.716458"
## [1] "Iter 14, obj 8.226383, abs 0.196075, rel 0.023280, norm 3.659922"
## [1] "Iter 15, obj 7.999381, abs 0.227001, rel 0.027594, norm 3.609172"
```

```
## [1] "Iter 16, obj 7.752891, abs 0.246490, rel 0.030814, norm 3.583107"
## [1] "Iter 17, obj 7.547431, abs 0.205460, rel 0.026501, norm 3.611724"
## [1] "Iter 18, obj 7.421073, abs 0.126358, rel 0.016742, norm 3.712291"
## [1] "Iter 19, obj 7.352889, abs 0.068184, rel 0.009188, norm 3.850849"
## [1] "Iter 20, obj 7.324297, abs 0.028591, rel 0.003888, norm 4.003146"
## [1] "Iter 21, obj 7.333141, abs 0.008843, rel 0.001207, norm 4.167770"
## [1] "Iter 22, obj 7.370901, abs 0.037760, rel 0.005149, norm 4.337886"
## [1] "Iter 23, obj 7.432197, abs 0.061296, rel 0.008316, norm 4.507392"
## [1] "Iter 24, obj 7.513858, abs 0.081661, rel 0.010987, norm 4.672405"
## [1] "Iter 25, obj 7.613295, abs 0.099437, rel 0.013234, norm 4.828825"
## [1] "Iter 26, obj 7.726270, abs 0.112975, rel 0.014839, norm 4.971831"
## [1] "Iter 27, obj 7.846587, abs 0.120316, rel 0.015572, norm 5.096964"
## [1] "Iter 28, obj 7.966256, abs 0.119669, rel 0.015251, norm 5.201135"
## [1] "Iter 29, obj 8.077047, abs 0.110791, rel 0.013907, norm 5.283223"
## [1] "Iter 30, obj 8.173176, abs 0.096130, rel 0.011902, norm 5.344572"
## [1] "Iter 31, obj 8.252699, abs 0.079523, rel 0.009730, norm 5.388645"
## [1] "Iter 32, obj 8.316685, abs 0.063986, rel 0.007753, norm 5.419715"
## [1] "Iter 33, obj 8.367587, abs 0.050902, rel 0.006120, norm 5.441624"
## [1] "Iter 34, obj 8.408058, abs 0.040471, rel 0.004837, norm 5.457276"
## [1] "Iter 35, obj 8.440418, abs 0.032360, rel 0.003849, norm 5.468677"
## [1] "Iter 36, obj 8.466521, abs 0.026102, rel 0.003093, norm 5.477160"
## [1] "Iter 37, obj 8.487785, abs 0.021265, rel 0.002512, norm 5.483607"
## [1] "Iter 38, obj 8.505286, abs 0.017501, rel 0.002062, norm 5.488603"
## [1] "Iter 39, obj 8.519831, abs 0.014545, rel 0.001710, norm 5.492545"
## [1] "Iter 40, obj 8.532031, abs 0.012200, rel 0.001432, norm 5.495706"
## [1] "Iter 41, obj 8.542351, abs 0.010320, rel 0.001210, norm 5.498276"
## [1] "Iter 42, obj 8.551148, abs 0.008797, rel 0.001030, norm 5.500392"
## [1] "Iter 43, obj 8.558700, abs 0.007552, rel 0.000883, norm 5.502155"
## [1] "Iter 44, obj 8.565225, abs 0.006525, rel 0.000762, norm 5.503637"
## [1] "Iter 45, obj 8.570894, abs 0.005669, rel 0.000662, norm 5.504895"
## [1] "Iter 46, obj 8.575846, abs 0.004952, rel 0.000578, norm 5.505971"
## [1] "Iter 47, obj 8.580191, abs 0.004345, rel 0.000507, norm 5.506897"
## [1] "Iter 48, obj 8.584021, abs 0.003829, rel 0.000446, norm 5.507700"
## [1] "Iter 49, obj 8.587408, abs 0.003388, rel 0.000395, norm 5.508400"
## [1] "Iter 50, obj 8.590416, abs 0.003007, rel 0.000350, norm 5.509012"
## [1] "Iter 51, obj 8.593094, abs 0.002678, rel 0.000312, norm 5.509551"
## [1] "Iter 52, obj 8.595486, abs 0.002392, rel 0.000278, norm 5.510027"
## [1] "Iter 53, obj 8.597627, abs 0.002142, rel 0.000249, norm 5.510450"
## [1] "Iter 54, obj 8.599549, abs 0.001922, rel 0.000224, norm 5.510825"
## [1] "Iter 55, obj 8.601278, abs 0.001729, rel 0.000201, norm 5.511160"
## [1] "Iter 56, obj 8.602836, abs 0.001558, rel 0.000181, norm 5.511460"
## [1] "Iter 57, obj 8.604242, abs 0.001407, rel 0.000163, norm 5.511728"
## [1] "Iter 58, obj 8.605514, abs 0.001272, rel 0.000148, norm 5.511970"
## [1] "Iter 59, obj 8.606666, abs 0.001152, rel 0.000134, norm 5.512187"
## [1] "Iter 60, obj 8.607711, abs 0.001045, rel 0.000121, norm 5.512384"
## [1] "Iter 61, obj 8.608659, abs 0.000948, rel 0.000110, norm 5.512561"
## [1] "Iter 62, obj 8.609521, abs 0.000862, rel 0.000100, norm 5.512722"
## [1] "Iter 63, obj 8.610306, abs 0.000785, rel 0.000091, norm 5.512867"
## [1] "Iter 64, obj 8.611020, abs 0.000715, rel 0.000083, norm 5.512999"
## [1] "Iter 65, obj 8.611672, abs 0.000651, rel 0.000076, norm 5.513119"
## [1] "Iter 66, obj 8.612266, abs 0.000594, rel 0.000069, norm 5.513229"
## [1] "Iter 67, obj 8.612809, abs 0.000543, rel 0.000063, norm 5.513328"
## [1] "Iter 68, obj 8.613304, abs 0.000496, rel 0.000058, norm 5.513419"
## [1] "Iter 69, obj 8.613758, abs 0.000453, rel 0.000053, norm 5.513501"
```

```
## [1] "Iter 70, obj 8.614172, abs 0.000415, rel 0.000048, norm 5.513577"
## [1] "Iter 71, obj 8.614552, abs 0.000379, rel 0.000044, norm 5.513646"
## [1] "Iter 72, obj 8.614899, abs 0.000347, rel 0.000040, norm 5.513709"
## [1] "Iter 73, obj 8.615217, abs 0.000318, rel 0.000037, norm 5.513766"
## [1] "Iter 74, obj 8.615509, abs 0.000292, rel 0.000034, norm 5.513819"
## [1] "Iter 75, obj 8.615776, abs 0.000267, rel 0.000031, norm 5.513867"
## [1] "Iter 76, obj 8.616021, abs 0.000245, rel 0.000028, norm 5.513911"
## [1] "Iter 77, obj 8.616246, abs 0.000225, rel 0.000026, norm 5.513952"
## [1] "Iter 78, obj 8.616453, abs 0.000206, rel 0.000024, norm 5.513989"
## [1] "Iter 79, obj 8.616642, abs 0.000189, rel 0.000022, norm 5.514023"
## [1] "Iter 80, obj 8.616816, abs 0.000174, rel 0.000020, norm 5.514054"
## [1] "Iter 81, obj 8.616976, abs 0.000160, rel 0.000019, norm 5.514083"
## [1] "Iter 82, obj 8.617123, abs 0.000147, rel 0.000017, norm 5.514109"
## [1] "Iter 83, obj 8.617257, abs 0.000135, rel 0.000016, norm 5.514133"
## [1] "Iter 84, obj 8.617381, abs 0.000124, rel 0.000014, norm 5.514155"
## [1] "Iter 85, obj 8.617495, abs 0.000114, rel 0.000013, norm 5.514176"
## [1] "Iter 86, obj 8.617599, abs 0.000105, rel 0.000012, norm 5.514194"
## [1] "Iter 87, obj 8.617696, abs 0.000096, rel 0.000011, norm 5.514212"
## [1] "Iter 88, obj 8.617784, abs 0.000088, rel 0.000010, norm 5.514227"
## [1] "Iter 89, obj 8.617865, abs 0.000081, rel 0.000009, norm 5.514242"
## [1] "Iter 90, obj 8.617940, abs 0.000075, rel 0.000009, norm 5.514255"
## [1] "Iter 91, obj 8.618009, abs 0.000069, rel 0.000008, norm 5.514267"
## [1] "Iter 92, obj 8.618072, abs 0.000063, rel 0.000007, norm 5.514279"
## [1] "Iter 93, obj 8.618130, abs 0.000058, rel 0.000007, norm 5.514289"
## [1] "Iter 94, obj 8.618183, abs 0.000053, rel 0.000006, norm 5.514299"
## [1] "Iter 95, obj 8.618233, abs 0.000049, rel 0.000006, norm 5.514307"
## [1] "Iter 96, obj 8.618278, abs 0.000045, rel 0.000005, norm 5.514315"
## [1] "Iter 97, obj 8.618320, abs 0.000042, rel 0.000005, norm 5.514323"
## [1] "Iter 98, obj 8.618358, abs 0.000038, rel 0.000004, norm 5.514330"
## [1] "Iter 99, obj 8.618393, abs 0.000035, rel 0.000004, norm 5.514336"
## [1] "Iter 100, obj 8.618426, abs 0.000032, rel 0.000004, norm 5.514342"
## [1] "Iter 101, obj 1.319536, abs 7.298890, rel 0.846894, norm 0.379935"
## [1] "Iter 102, obj 1.922769, abs 0.603234, rel 0.457156, norm 0.677297"
## [1] "Iter 103, obj 1.952657, abs 0.029888, rel 0.015544, norm 0.871479"
## [1] "Iter 104, obj 1.946683, abs 0.005974, rel 0.003060, norm 0.960184"
## [1] "Iter 105, obj 1.916031, abs 0.030652, rel 0.015746, norm 1.042685"
## [1] "Iter 106, obj 1.889763, abs 0.026268, rel 0.013709, norm 1.158279"
## [1] "Iter 107, obj 1.826800, abs 0.062963, rel 0.033318, norm 1.199612"
## [1] "Iter 108, obj 1.834532, abs 0.007732, rel 0.004233, norm 1.203963"
## [1] "Iter 109, obj 1.792013, abs 0.042519, rel 0.023177, norm 1.176725"
## [1] "Iter 110, obj 1.765401, abs 0.026612, rel 0.014851, norm 1.202335"
## [1] "Iter 111, obj 1.783565, abs 0.018164, rel 0.010289, norm 1.179144"
## [1] "Iter 112, obj 1.793230, abs 0.009665, rel 0.005419, norm 1.206015"
## [1] "Iter 113, obj 1.806852, abs 0.013622, rel 0.007596, norm 1.200815"
## [1] "Iter 114, obj 1.774366, abs 0.032486, rel 0.017979, norm 1.237848"
## [1] "Iter 115, obj 1.820328, abs 0.045961, rel 0.025903, norm 1.233433"
## [1] "Iter 116, obj 1.717740, abs 0.102587, rel 0.056357, norm 1.267717"
## [1] "Iter 117, obj 1.877506, abs 0.159766, rel 0.093010, norm 1.234575"
## [1] "Iter 118, obj 1.720525, abs 0.156981, rel 0.083612, norm 1.256105"
## [1] "Iter 119, obj 1.810795, abs 0.090270, rel 0.052467, norm 1.270917"
## [1] "Iter 120, obj 1.446001, abs 0.364794, rel 0.201455, norm 1.272692"
## [1] "Iter 121, obj 1.731824, abs 0.285823, rel 0.197664, norm 1.138594"
## [1] "Iter 122, obj 1.647292, abs 0.084532, rel 0.048811, norm 1.139125"
## [1] "Iter 123, obj 1.689248, abs 0.041956, rel 0.025470, norm 1.240315"
```

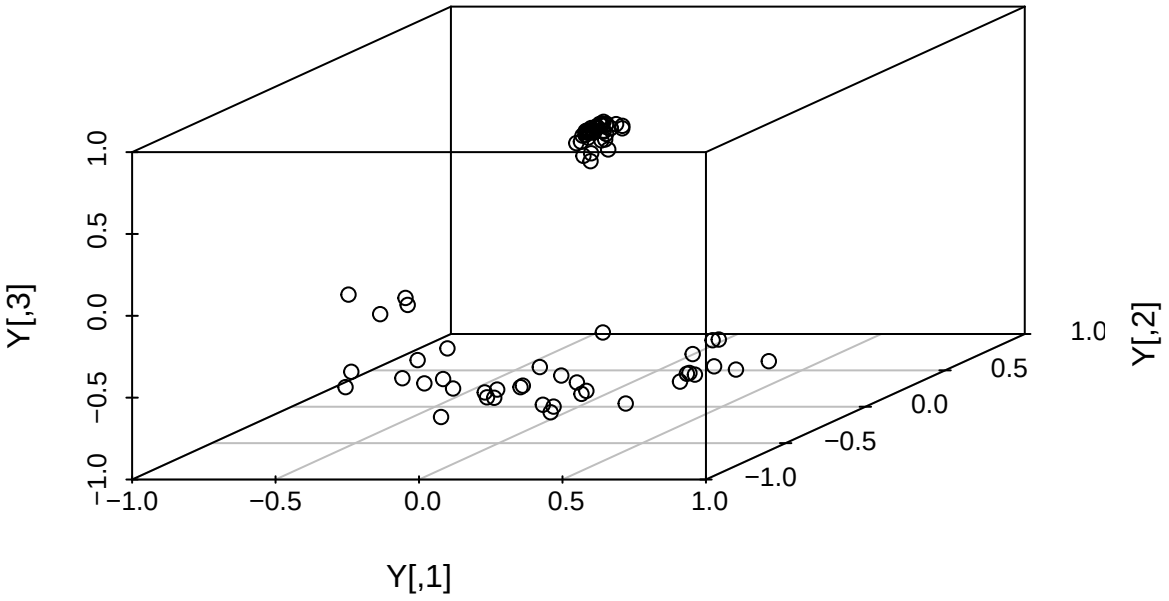
```
## [1] "Iter 124, obj 1.467393, abs 0.221855, rel 0.131333, norm 1.201884"
## [1] "Iter 125, obj 1.628392, abs 0.160999, rel 0.109718, norm 1.159355"
## [1] "Iter 126, obj 1.694597, abs 0.066205, rel 0.040656, norm 1.069635"
## [1] "Iter 127, obj 1.656574, abs 0.038023, rel 0.022438, norm 1.263004"
## [1] "Iter 128, obj 1.530916, abs 0.125658, rel 0.075854, norm 1.207987"
## [1] "Iter 129, obj 1.589332, abs 0.058416, rel 0.038157, norm 1.189396"
## [1] "Iter 130, obj 1.608165, abs 0.018833, rel 0.011850, norm 1.114365"
## [1] "Iter 131, obj 1.547046, abs 0.061118, rel 0.038005, norm 1.231322"
## [1] "Iter 132, obj 1.548582, abs 0.001536, rel 0.000993, norm 1.164763"
## [1] "Iter 133, obj 1.651986, abs 0.103404, rel 0.066773, norm 1.145838"
## [1] "Iter 134, obj 1.665367, abs 0.013381, rel 0.008100, norm 1.187800"
## [1] "Iter 135, obj 1.588222, abs 0.077145, rel 0.046323, norm 1.232698"
## [1] "Iter 136, obj 1.604104, abs 0.015882, rel 0.010000, norm 1.209862"
## [1] "Iter 137, obj 1.525552, abs 0.078552, rel 0.048970, norm 1.216631"
## [1] "Iter 138, obj 1.664071, abs 0.138519, rel 0.090799, norm 1.161231"
## [1] "Iter 139, obj 1.564406, abs 0.099665, rel 0.059892, norm 1.236525"
## [1] "Iter 140, obj 1.620920, abs 0.056513, rel 0.036125, norm 1.216337"
## [1] "Iter 141, obj 1.516323, abs 0.104597, rel 0.064529, norm 1.212493"
## [1] "Iter 142, obj 1.547358, abs 0.031036, rel 0.020468, norm 1.187715"
## [1] "Iter 143, obj 1.671029, abs 0.123671, rel 0.079924, norm 1.120341"
## [1] "Iter 144, obj 1.575466, abs 0.095563, rel 0.057188, norm 1.249342"
## [1] "Iter 145, obj 1.515453, abs 0.060013, rel 0.038092, norm 1.184670"
## [1] "Iter 146, obj 1.571840, abs 0.056386, rel 0.037208, norm 1.161795"
## [1] "Iter 147, obj 1.555155, abs 0.016684, rel 0.010615, norm 1.191342"
## [1] "Iter 148, obj 1.652264, abs 0.097108, rel 0.062443, norm 1.161869"
## [1] "Iter 149, obj 1.489194, abs 0.163070, rel 0.098695, norm 1.259346"
## [1] "Iter 150, obj 1.571947, abs 0.082754, rel 0.055569, norm 1.183235"
## [1] "Iter 151, obj 1.456181, abs 0.115766, rel 0.073645, norm 1.185170"
## [1] "Iter 152, obj 1.602525, abs 0.146344, rel 0.100498, norm 1.139504"
## [1] "Iter 153, obj 1.548182, abs 0.054343, rel 0.033911, norm 1.165164"
## [1] "Iter 154, obj 1.590371, abs 0.042189, rel 0.027250, norm 1.193747"
## [1] "Iter 155, obj 1.603782, abs 0.013411, rel 0.008432, norm 1.137800"
## [1] "Iter 156, obj 1.553810, abs 0.049972, rel 0.031159, norm 1.237486"
## [1] "Iter 157, obj 1.516028, abs 0.037781, rel 0.024315, norm 1.157318"
## [1] "Iter 158, obj 1.611334, abs 0.095305, rel 0.062865, norm 1.152673"
## [1] "Iter 159, obj 1.629914, abs 0.018580, rel 0.011531, norm 1.180409"
## [1] "Iter 160, obj 1.619056, abs 0.010858, rel 0.006662, norm 1.225229"
## [1] "Iter 161, obj 1.606193, abs 0.012863, rel 0.007945, norm 1.236669"
## [1] "Iter 162, obj 1.530371, abs 0.075822, rel 0.047206, norm 1.221794"
## [1] "Iter 163, obj 1.532716, abs 0.002346, rel 0.001533, norm 1.200887"
## [1] "Iter 164, obj 1.588658, abs 0.055942, rel 0.036498, norm 1.137108"
## [1] "Iter 165, obj 1.541048, abs 0.047610, rel 0.029969, norm 1.221421"
## [1] "Iter 166, obj 1.566678, abs 0.025630, rel 0.016632, norm 1.173239"
## [1] "Iter 167, obj 1.539715, abs 0.026963, rel 0.017210, norm 1.198113"
## [1] "Iter 168, obj 1.663923, abs 0.124208, rel 0.080670, norm 1.144142"
## [1] "Iter 169, obj 1.629099, abs 0.034824, rel 0.020929, norm 1.244908"
## [1] "Iter 170, obj 1.608524, abs 0.020575, rel 0.012630, norm 1.238956"
## [1] "Iter 171, obj 1.498078, abs 0.110445, rel 0.068662, norm 1.260339"
## [1] "Iter 172, obj 1.539800, abs 0.041722, rel 0.027850, norm 1.177016"
## [1] "Iter 173, obj 1.661147, abs 0.121346, rel 0.078807, norm 1.121246"
## [1] "Iter 174, obj 1.531595, abs 0.129552, rel 0.077989, norm 1.239092"
## [1] "Iter 175, obj 1.505487, abs 0.026108, rel 0.017046, norm 1.190095"
## [1] "Iter 176, obj 1.569400, abs 0.063912, rel 0.042453, norm 1.112111"
## [1] "Iter 177, obj 1.646277, abs 0.076877, rel 0.048985, norm 1.186945"
```

```
## [1] "Iter 178, obj 1.658634, abs 0.012357, rel 0.007506, norm 1.223614"
## [1] "Iter 179, obj 1.621211, abs 0.037423, rel 0.022562, norm 1.261771"
## [1] "Iter 180, obj 1.546479, abs 0.074733, rel 0.046097, norm 1.235676"
## [1] "Iter 181, obj 1.639274, abs 0.092796, rel 0.060005, norm 1.181757"
## [1] "Iter 182, obj 1.676434, abs 0.037159, rel 0.022668, norm 1.198655"
## [1] "Iter 183, obj 1.622604, abs 0.053830, rel 0.032110, norm 1.254646"
## [1] "Iter 184, obj 1.473636, abs 0.148968, rel 0.091808, norm 1.237517"
## [1] "Iter 185, obj 1.572543, abs 0.098907, rel 0.067118, norm 1.178992"
## [1] "Iter 186, obj 1.608411, abs 0.035868, rel 0.022809, norm 1.146284"
## [1] "Iter 187, obj 1.606063, abs 0.002348, rel 0.001460, norm 1.237059"
## [1] "Iter 188, obj 1.629773, abs 0.023710, rel 0.014763, norm 1.184173"
## [1] "Iter 189, obj 1.536668, abs 0.093104, rel 0.057127, norm 1.274575"
## [1] "Iter 190, obj 1.553197, abs 0.016529, rel 0.010756, norm 1.183127"
## [1] "Iter 191, obj 1.583965, abs 0.030768, rel 0.019809, norm 1.189010"
## [1] "Iter 192, obj 1.632486, abs 0.048521, rel 0.030633, norm 1.184345"
## [1] "Iter 193, obj 1.559936, abs 0.072550, rel 0.044441, norm 1.228758"
## [1] "Iter 194, obj 1.615749, abs 0.055814, rel 0.035779, norm 1.191699"
## [1] "Iter 195, obj 1.522581, abs 0.093168, rel 0.057662, norm 1.208951"
## [1] "Iter 196, obj 1.601850, abs 0.079269, rel 0.052062, norm 1.177537"
## [1] "Iter 197, obj 1.523029, abs 0.078821, rel 0.049206, norm 1.190605"
## [1] "Iter 198, obj 1.634429, abs 0.111400, rel 0.073144, norm 1.172470"
## [1] "Iter 199, obj 1.501758, abs 0.132671, rel 0.081173, norm 1.209649"
## [1] "Iter 200, obj 1.607997, abs 0.106239, rel 0.070743, norm 1.179043"
## [1] "Iter 201, obj 1.554676, abs 0.053321, rel 0.033160, norm 1.177481"
## [1] "Iter 202, obj 1.601357, abs 0.046681, rel 0.030026, norm 1.206184"
## [1] "Iter 203, obj 1.527239, abs 0.074119, rel 0.046285, norm 1.201900"
## [1] "Iter 204, obj 1.593873, abs 0.066634, rel 0.043631, norm 1.175432"
## [1] "Iter 205, obj 1.568504, abs 0.025369, rel 0.015916, norm 1.182203"
## [1] "Iter 206, obj 1.633363, abs 0.064859, rel 0.041351, norm 1.214388"
## [1] "Iter 207, obj 1.540744, abs 0.092619, rel 0.056705, norm 1.211735"
## [1] "Iter 208, obj 1.583902, abs 0.043158, rel 0.028011, norm 1.199864"
## [1] "Iter 209, obj 1.547560, abs 0.036342, rel 0.022945, norm 1.176957"
## [1] "Iter 210, obj 1.562424, abs 0.014864, rel 0.009605, norm 1.185809"
## [1] "Iter 211, obj 1.620445, abs 0.058021, rel 0.037135, norm 1.141647"
## [1] "Iter 212, obj 1.663377, abs 0.042932, rel 0.026494, norm 1.215452"
## [1] "Iter 213, obj 1.584346, abs 0.079030, rel 0.047512, norm 1.227299"
## [1] "Iter 214, obj 1.611367, abs 0.027020, rel 0.017055, norm 1.218021"
## [1] "Iter 215, obj 1.603218, abs 0.008149, rel 0.005057, norm 1.197894"
## [1] "Iter 216, obj 1.568502, abs 0.034716, rel 0.021654, norm 1.228124"
## [1] "Iter 217, obj 1.557423, abs 0.011079, rel 0.007063, norm 1.192566"
## [1] "Iter 218, obj 1.622287, abs 0.064864, rel 0.041648, norm 1.175727"
## [1] "Iter 219, obj 1.596255, abs 0.026032, rel 0.016046, norm 1.223460"
## [1] "Iter 220, obj 1.581100, abs 0.015155, rel 0.009494, norm 1.204035"
## [1] "Iter 221, obj 1.554295, abs 0.026806, rel 0.016954, norm 1.208766"
## [1] "Iter 222, obj 1.600465, abs 0.046170, rel 0.029705, norm 1.166424"
## [1] "Iter 223, obj 1.613414, abs 0.012949, rel 0.008091, norm 1.209575"
## [1] "Iter 224, obj 1.559374, abs 0.054041, rel 0.033495, norm 1.217556"
## [1] "Iter 225, obj 1.492266, abs 0.067107, rel 0.043035, norm 1.207346"
## [1] "Iter 226, obj 1.571086, abs 0.078820, rel 0.052819, norm 1.156374"
## [1] "Iter 227, obj 1.668164, abs 0.097078, rel 0.061790, norm 1.162299"
## [1] "Iter 228, obj 1.594448, abs 0.073716, rel 0.044190, norm 1.234225"
## [1] "Iter 229, obj 1.534220, abs 0.060228, rel 0.037774, norm 1.235302"
## [1] "Iter 230, obj 1.631009, abs 0.096789, rel 0.063087, norm 1.146765"
## [1] "Iter 231, obj 1.585561, abs 0.045449, rel 0.027865, norm 1.237539"
```

```
## [1] "Iter 232, obj 1.559948, abs 0.025613, rel 0.016154, norm 1.208301"
## [1] "Iter 233, obj 1.511803, abs 0.048145, rel 0.030863, norm 1.219799"
## [1] "Iter 234, obj 1.577055, abs 0.065252, rel 0.043162, norm 1.136397"
## [1] "Iter 235, obj 1.544796, abs 0.032259, rel 0.020455, norm 1.203560"
## [1] "Iter 236, obj 1.588877, abs 0.044081, rel 0.028535, norm 1.165468"
## [1] "Iter 237, obj 1.538843, abs 0.050034, rel 0.031490, norm 1.211460"
## [1] "Iter 238, obj 1.552096, abs 0.013253, rel 0.008612, norm 1.187088"
## [1] "Iter 239, obj 1.626277, abs 0.074181, rel 0.047794, norm 1.156472"
## [1] "Iter 240, obj 1.652794, abs 0.026517, rel 0.016305, norm 1.213986"
## [1] "Iter 241, obj 1.552842, abs 0.099952, rel 0.060475, norm 1.235986"
## [1] "Iter 242, obj 1.572145, abs 0.019303, rel 0.012431, norm 1.194549"
## [1] "Iter 243, obj 1.555247, abs 0.016898, rel 0.010748, norm 1.165596"
## [1] "Iter 244, obj 1.522926, abs 0.032321, rel 0.020782, norm 1.184489"
## [1] "Iter 245, obj 1.564764, abs 0.041837, rel 0.027472, norm 1.138952"
## [1] "Iter 246, obj 1.577786, abs 0.013022, rel 0.008322, norm 1.184075"
## [1] "Iter 247, obj 1.631444, abs 0.053658, rel 0.034008, norm 1.165973"
## [1] "Iter 248, obj 1.598054, abs 0.033390, rel 0.020466, norm 1.229269"
## [1] "Iter 249, obj 1.542662, abs 0.055392, rel 0.034662, norm 1.232664"
## [1] "Iter 250, obj 1.599723, abs 0.057061, rel 0.036988, norm 1.166441"
## [1] "Iter 251, obj 1.610035, abs 0.010312, rel 0.006446, norm 1.194134"
## [1] "Iter 252, obj 1.558291, abs 0.051744, rel 0.032138, norm 1.227385"
## [1] "Iter 253, obj 1.530603, abs 0.027688, rel 0.017768, norm 1.199521"
## [1] "Iter 254, obj 1.549517, abs 0.018914, rel 0.012357, norm 1.157838"
## [1] "Iter 255, obj 1.493273, abs 0.056244, rel 0.036298, norm 1.170807"
## [1] "Iter 256, obj 1.545010, abs 0.051737, rel 0.034646, norm 1.131019"
## [1] "Iter 257, obj 1.522298, abs 0.022711, rel 0.014700, norm 1.161730"
## [1] "Iter 258, obj 1.531549, abs 0.009251, rel 0.006077, norm 1.150757"
## [1] "Iter 259, obj 1.562087, abs 0.030537, rel 0.019939, norm 1.164926"
## [1] "Iter 260, obj 1.518418, abs 0.043668, rel 0.027955, norm 1.203684"
## [1] "Iter 261, obj 1.548725, abs 0.030306, rel 0.019959, norm 1.142794"
## [1] "Iter 262, obj 1.542541, abs 0.006183, rel 0.003993, norm 1.176767"
## [1] "Iter 263, obj 1.613410, abs 0.070869, rel 0.045943, norm 1.155229"
## [1] "Iter 264, obj 1.564029, abs 0.049381, rel 0.030606, norm 1.214303"
## [1] "Iter 265, obj 1.444754, abs 0.119275, rel 0.076261, norm 1.252467"
## [1] "Iter 266, obj 1.564865, abs 0.120111, rel 0.083136, norm 1.125755"
## [1] "Iter 267, obj 1.529761, abs 0.035104, rel 0.022432, norm 1.211873"
## [1] "Iter 268, obj 1.525762, abs 0.004000, rel 0.002615, norm 1.199295"
## [1] "Iter 269, obj 1.493758, abs 0.032004, rel 0.020976, norm 1.201510"
## [1] "Iter 270, obj 1.558754, abs 0.064997, rel 0.043512, norm 1.150433"
## [1] "Iter 271, obj 1.524384, abs 0.034370, rel 0.022050, norm 1.221318"
## [1] "Iter 272, obj 1.537855, abs 0.013471, rel 0.008837, norm 1.189463"
## [1] "Iter 273, obj 1.509074, abs 0.028781, rel 0.018715, norm 1.206050"
## [1] "Iter 274, obj 1.491083, abs 0.017992, rel 0.011922, norm 1.192341"
## [1] "Iter 275, obj 1.557559, abs 0.066476, rel 0.044582, norm 1.145474"
## [1] "Iter 276, obj 1.538329, abs 0.019229, rel 0.012346, norm 1.179618"
## [1] "Iter 277, obj 1.464167, abs 0.074163, rel 0.048210, norm 1.223494"
## [1] "Iter 278, obj 1.516378, abs 0.052211, rel 0.035659, norm 1.163001"
## [1] "Iter 279, obj 1.458899, abs 0.057479, rel 0.037906, norm 1.202412"
## [1] "Iter 280, obj 1.600263, abs 0.141364, rel 0.096898, norm 1.139383"
## [1] "Iter 281, obj 1.532283, abs 0.067980, rel 0.042480, norm 1.238429"
## [1] "Iter 282, obj 1.483763, abs 0.048520, rel 0.031665, norm 1.231316"
## [1] "Iter 283, obj 1.439256, abs 0.044508, rel 0.029996, norm 1.155200"
## [1] "Iter 284, obj 1.502654, abs 0.063398, rel 0.044049, norm 1.133618"
## [1] "Iter 285, obj 1.485288, abs 0.017365, rel 0.011557, norm 1.148014"
```

```
## [1] "Iter 286, obj 1.494532, abs 0.009244, rel 0.006224, norm 1.170798"
## [1] "Iter 287, obj 1.408441, abs 0.086091, rel 0.057604, norm 1.181544"
## [1] "Iter 288, obj 1.509063, abs 0.100622, rel 0.071442, norm 1.146921"
## [1] "Iter 289, obj 1.446317, abs 0.062746, rel 0.041579, norm 1.131609"
## [1] "Iter 290, obj 1.527856, abs 0.081539, rel 0.056377, norm 1.173167"
## [1] "Iter 291, obj 1.543618, abs 0.015762, rel 0.010316, norm 1.168769"
## [1] "Iter 292, obj 1.527496, abs 0.016122, rel 0.010444, norm 1.194703"
## [1] "Iter 293, obj 1.480001, abs 0.047495, rel 0.031093, norm 1.196168"
## [1] "Iter 294, obj 1.462095, abs 0.017906, rel 0.012099, norm 1.181595"
## [1] "Iter 295, obj 1.483259, abs 0.021164, rel 0.014475, norm 1.126318"
## [1] "Iter 296, obj 1.515899, abs 0.032641, rel 0.022006, norm 1.136412"
## [1] "Iter 297, obj 1.510562, abs 0.005338, rel 0.003521, norm 1.142563"
## [1] "Iter 298, obj 1.525917, abs 0.015355, rel 0.010165, norm 1.171577"
## [1] "Iter 299, obj 1.560328, abs 0.034411, rel 0.022551, norm 1.193906"
## [1] "Iter 300, obj 1.477260, abs 0.083068, rel 0.053238, norm 1.231123"
```

```
scatterplot3d::scatterplot3d(Y, xlim = c(-1, 1), ylim = c(-1, 1),
                             zlim = c(-1, 1), color = rep(1, nrow(Y)))
```




```
rgl::plot3d(0, 0, 0, xlim = c(-1, 1), ylim = c(-1, 1), zlim = c(-1, 1),
            radius = 1, type = "s", col = "lightblue", alpha = 0.25,
            lit = FALSE)
rgl::points3d(Y, col = rep(1, nrow(Y)))
```

Lastly, optimized rho for a perplexity of 45 and $d = 1$:

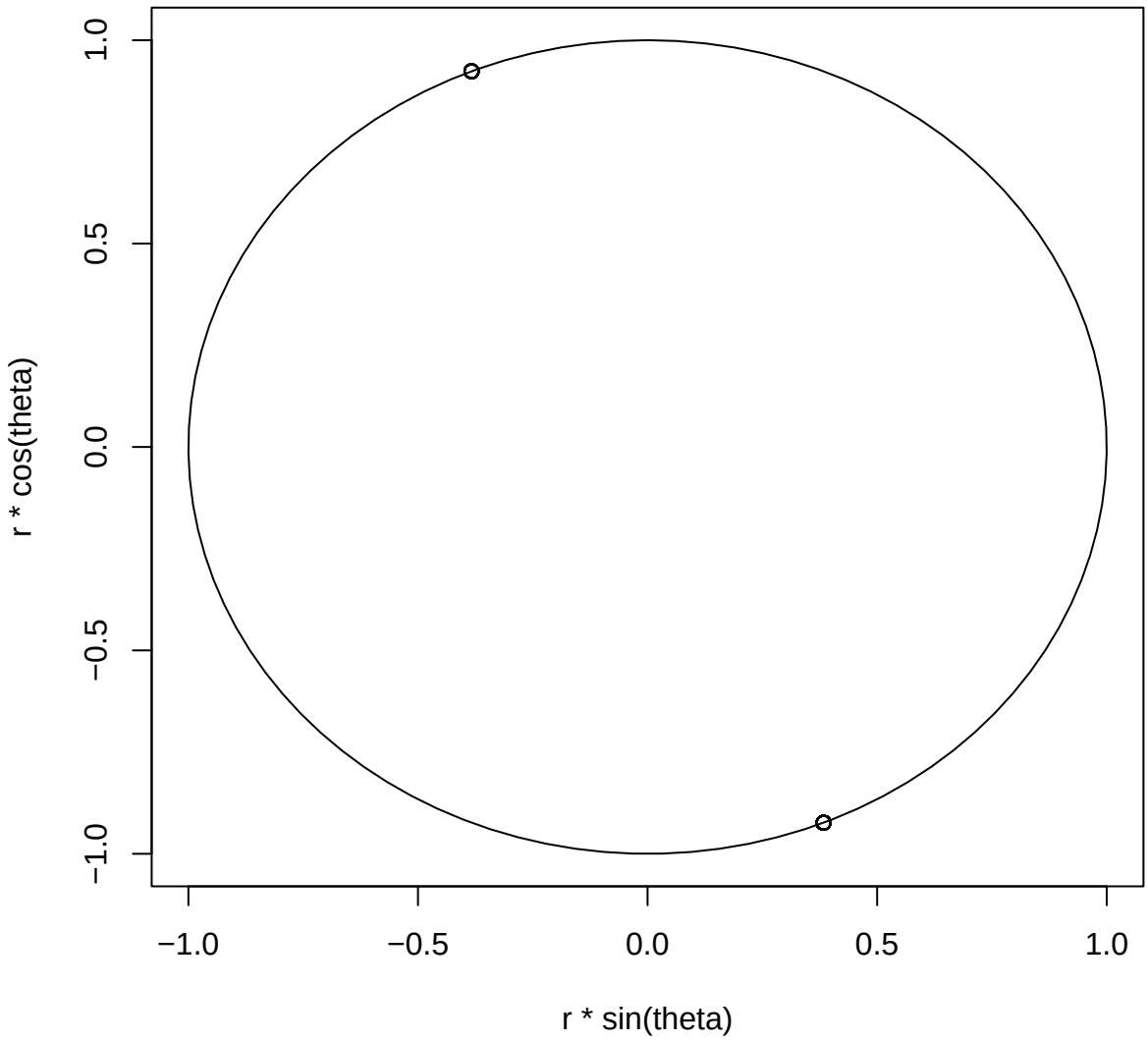
```
Y <- psc_sne(X=sanjuanfuca, d=1,
             rho_psc_list = rho_45,
             num_iteration=300)
```

```
## [1] "Iter 1, obj 6.380471, abs 0.000000, rel 0.000000, norm 0.769912"
## [1] "Iter 2, obj 6.260408, abs 0.120063, rel 0.018817, norm 0.923945"
## [1] "Iter 3, obj 6.208165, abs 0.052243, rel 0.008345, norm 1.067351"
## [1] "Iter 4, obj 6.212493, abs 0.004327, rel 0.000697, norm 1.216539"
## [1] "Iter 5, obj 6.261038, abs 0.048545, rel 0.007814, norm 1.367066"
## [1] "Iter 6, obj 6.333439, abs 0.072401, rel 0.011564, norm 1.506560"
```

```
## [1] "Iter 7, obj 6.406743, abs 0.073304, rel 0.011574, norm 1.625700"
## [1] "Iter 8, obj 6.469125, abs 0.062382, rel 0.009737, norm 1.722230"
## [1] "Iter 9, obj 6.525011, abs 0.055886, rel 0.008639, norm 1.803145"
## [1] "Iter 10, obj 6.586454, abs 0.061443, rel 0.009417, norm 1.878447"
## [1] "Iter 11, obj 6.658798, abs 0.072344, rel 0.010984, norm 1.953263"
## [1] "Iter 12, obj 6.734370, abs 0.075572, rel 0.011349, norm 2.025428"
## [1] "Iter 13, obj 6.800638, abs 0.066268, rel 0.009840, norm 2.089993"
## [1] "Iter 14, obj 6.850650, abs 0.050013, rel 0.007354, norm 2.145087"
## [1] "Iter 15, obj 6.885835, abs 0.035185, rel 0.005136, norm 2.193385"
## [1] "Iter 16, obj 6.913886, abs 0.028051, rel 0.004074, norm 2.240290"
## [1] "Iter 17, obj 6.944611, abs 0.030724, rel 0.004444, norm 2.291100"
## [1] "Iter 18, obj 6.983000, abs 0.038390, rel 0.005528, norm 2.347823"
## [1] "Iter 19, obj 7.026967, abs 0.043966, rel 0.006296, norm 2.407655"
## [1] "Iter 20, obj 7.074863, abs 0.047896, rel 0.006816, norm 2.466706"
## [1] "Iter 21, obj 7.129030, abs 0.054167, rel 0.007656, norm 2.523701"
## [1] "Iter 22, obj 7.191717, abs 0.062687, rel 0.008793, norm 2.578120"
## [1] "Iter 23, obj 7.259445, abs 0.067728, rel 0.009418, norm 2.627671"
## [1] "Iter 24, obj 7.322489, abs 0.063043, rel 0.008684, norm 2.668286"
## [1] "Iter 25, obj 7.372832, abs 0.050343, rel 0.006875, norm 2.697379"
## [1] "Iter 26, obj 7.409081, abs 0.036248, rel 0.004916, norm 2.716137"
## [1] "Iter 27, obj 7.433861, abs 0.024780, rel 0.003345, norm 2.727674"
## [1] "Iter 28, obj 7.450454, abs 0.016593, rel 0.002232, norm 2.734740"
## [1] "Iter 29, obj 7.461496, abs 0.011042, rel 0.001482, norm 2.739127"
## [1] "Iter 30, obj 7.468838, abs 0.007342, rel 0.000984, norm 2.741899"
## [1] "Iter 31, obj 7.473724, abs 0.004886, rel 0.000654, norm 2.743677"
## [1] "Iter 32, obj 7.476980, abs 0.003256, rel 0.000436, norm 2.744831"
## [1] "Iter 33, obj 7.479152, abs 0.002172, rel 0.000291, norm 2.745587"
## [1] "Iter 34, obj 7.480603, abs 0.001450, rel 0.000194, norm 2.746085"
## [1] "Iter 35, obj 7.481571, abs 0.000969, rel 0.000129, norm 2.746415"
## [1] "Iter 36, obj 7.482218, abs 0.000647, rel 0.000086, norm 2.746634"
## [1] "Iter 37, obj 7.482651, abs 0.000432, rel 0.000058, norm 2.746780"
## [1] "Iter 38, obj 7.482940, abs 0.000289, rel 0.000039, norm 2.746877"
## [1] "Iter 39, obj 7.483133, abs 0.000193, rel 0.000026, norm 2.746942"
## [1] "Iter 40, obj 7.483262, abs 0.000129, rel 0.000017, norm 2.746985"
## [1] "Iter 41, obj 7.483348, abs 0.000086, rel 0.000012, norm 2.747014"
## [1] "Iter 42, obj 7.483406, abs 0.000058, rel 0.000008, norm 2.747033"
## [1] "Iter 43, obj 7.483444, abs 0.000039, rel 0.000005, norm 2.747046"
## [1] "Iter 44, obj 7.483470, abs 0.000026, rel 0.000003, norm 2.747055"
## [1] "Iter 45, obj 7.483487, abs 0.000017, rel 0.000002, norm 2.747061"
## [1] "Iter 46, obj 7.483499, abs 0.000012, rel 0.000002, norm 2.747064"
## [1] "Iter 47, obj 7.483506, abs 0.000008, rel 0.000001, norm 2.747067"
## [1] "Iter 48, obj 7.483511, abs 0.000005, rel 0.000001, norm 2.747069"
## [1] "Iter 49, obj 7.483515, abs 0.000003, rel 0.000000, norm 2.747070"
## [1] "Iter 50, obj 7.483517, abs 0.000002, rel 0.000000, norm 2.747071"
## [1] "Iter 51, obj 7.483519, abs 0.000002, rel 0.000000, norm 2.747071"
## [1] "Iter 52, obj 7.483520, abs 0.000001, rel 0.000000, norm 2.747071"
## [1] "Iter 53, obj 7.483520, abs 0.000001, rel 0.000000, norm 2.747072"
## [1] "Iter 54, obj 7.483521, abs 0.000000, rel 0.000000, norm 2.747072"
## [1] "Iter 55, obj 7.483521, abs 0.000000, rel 0.000000, norm 2.747072"
## [1] "Iter 56, obj 7.483521, abs 0.000000, rel 0.000000, norm 2.747072"
## [1] "Iter 57, obj 7.483522, abs 0.000000, rel 0.000000, norm 2.747072"
## [1] "Iter 58, obj 7.483522, abs 0.000000, rel 0.000000, norm 2.747072"
## [1] "Iter 59, obj 7.483522, abs 0.000000, rel 0.000000, norm 2.747072"
## [1] "Iter 60, obj 7.483522, abs 0.000000, rel 0.000000, norm 2.747072"
```

```
## [1] "Iter 61, obj 7.483522, abs 0.000000, rel 0.000000, norm 2.747072"
## [1] "Iter 62, obj 7.483522, abs 0.000000, rel 0.000000, norm 2.747072"
## [1] "Iter 63, obj 7.483522, abs 0.000000, rel 0.000000, norm 2.747072"
## [1] "Iter 64, obj 7.483522, abs 0.000000, rel 0.000000, norm 2.747072"
## [1] "Iter 65, obj 7.483522, abs 0.000000, rel 0.000000, norm 2.747072"
## [1] "Iter 66, obj 7.483522, abs 0.000000, rel 0.000000, norm 2.747072"
## [1] "Iter 67, obj 7.483522, abs 0.000000, rel 0.000000, norm 2.747072"
## [1] "Iter 68, obj 7.483522, abs 0.000000, rel 0.000000, norm 2.747072"
## [1] "Iter 69, obj 7.483522, abs 0.000000, rel 0.000000, norm 2.747072"
## [1] "Iter 70, obj 7.483522, abs 0.000000, rel 0.000000, norm 2.747072"
## [1] "Iter 71, obj 7.483522, abs 0.000000, rel 0.000000, norm 2.747072"
## [1] "Iter 72, obj 7.483522, abs 0.000000, rel 0.000000, norm 2.747072"
## [1] "Iter 73, obj 7.483522, abs 0.000000, rel 0.000000, norm 2.747072"
## [1] "Iter 74, obj 7.483522, abs 0.000000, rel 0.000000, norm 2.747072"
## [1] "Iter 75, obj 7.483522, abs 0.000000, rel 0.000000, norm 2.747072"
## [1] "Iter 76, obj 7.483522, abs 0.000000, rel 0.000000, norm 2.747072"
## [1] "Iter 77, obj 7.483522, abs 0.000000, rel 0.000000, norm 2.747072"
## [1] "Iter 78, obj 7.483522, abs 0.000000, rel 0.000000, norm 2.747072"
## [1] "Iter 79, obj 7.483522, abs 0.000000, rel 0.000000, norm 2.747072"
## [1] "Iter 80, obj 7.483522, abs 0.000000, rel 0.000000, norm 2.747072"
## [1] "Iter 81, obj 7.483522, abs 0.000000, rel 0.000000, norm 2.747072"
## [1] "Iter 82, obj 7.483522, abs 0.000000, rel 0.000000, norm 2.747072"
## [1] "Iter 83, obj 7.483522, abs 0.000000, rel 0.000000, norm 2.747072"
## [1] "Iter 84, obj 7.483522, abs 0.000000, rel 0.000000, norm 2.747072"
## [1] "Iter 85, obj 7.483522, abs 0.000000, rel 0.000000, norm 2.747072"
## [1] "Iter 86, obj 7.483522, abs 0.000000, rel 0.000000, norm 2.747072"
## [1] "Iter 87, obj 7.483522, abs 0.000000, rel 0.000000, norm 2.747072"
## [1] "Iter 88, obj 7.483522, abs 0.000000, rel 0.000000, norm 2.747072"
## [1] "Iter 89, obj 7.483522, abs 0.000000, rel 0.000000, norm 2.747072"
## [1] "Iter 90, obj 7.483522, abs 0.000000, rel 0.000000, norm 2.747072"
## [1] "Iter 91, obj 7.483522, abs 0.000000, rel 0.000000, norm 2.747072"
## [1] "Iter 92, obj 7.483522, abs 0.000000, rel 0.000000, norm 2.747072"
## [1] "Iter 93, obj 7.483522, abs 0.000000, rel 0.000000, norm 2.747072"
## [1] "Iter 94, obj 7.483522, abs 0.000000, rel 0.000000, norm 2.747072"
## [1] "Iter 95, obj 7.483522, abs 0.000000, rel 0.000000, norm 2.747072"
## [1] "Iter 96, obj 7.483522, abs 0.000000, rel 0.000000, norm 2.747072"
## [1] "Iter 97, obj 7.483522, abs 0.000000, rel 0.000000, norm 2.747072"
## [1] "Iter 98, obj 7.483522, abs 0.000000, rel 0.000000, norm 2.747072"
## [1] "Iter 99, obj 7.483522, abs 0.000000, rel 0.000000, norm 2.747072"
## [1] "Iter 100, obj 7.483522, abs 0.000000, rel 0.000000, norm 2.747072"
## [1] "Iter 101, obj 0.519203, abs 6.964319, rel 0.930620, norm 0.180627"
## [1] "Iter 102, obj 0.914656, abs 0.395453, rel 0.761654, norm 0.228033"
## [1] "Iter 103, obj 0.914656, abs 0.000000, rel 0.000000, norm 0.378365"
```

```
Y_rad <- DirStats::to_rad(Y)
r <- 1
theta <<- Y_rad
plot(r*sin(theta),
     r*cos(theta),
     xlim=c(-max(r),max(r)),
     ylim=c(-max(r),max(r)))

polygon(max(r)*sin(seq(0,2*pi,length.out=100)),max(r)*cos(seq(0,2*pi,length.out=100)))
```



Now, optimized rho for a perplexity of 33 and $d = 2$:

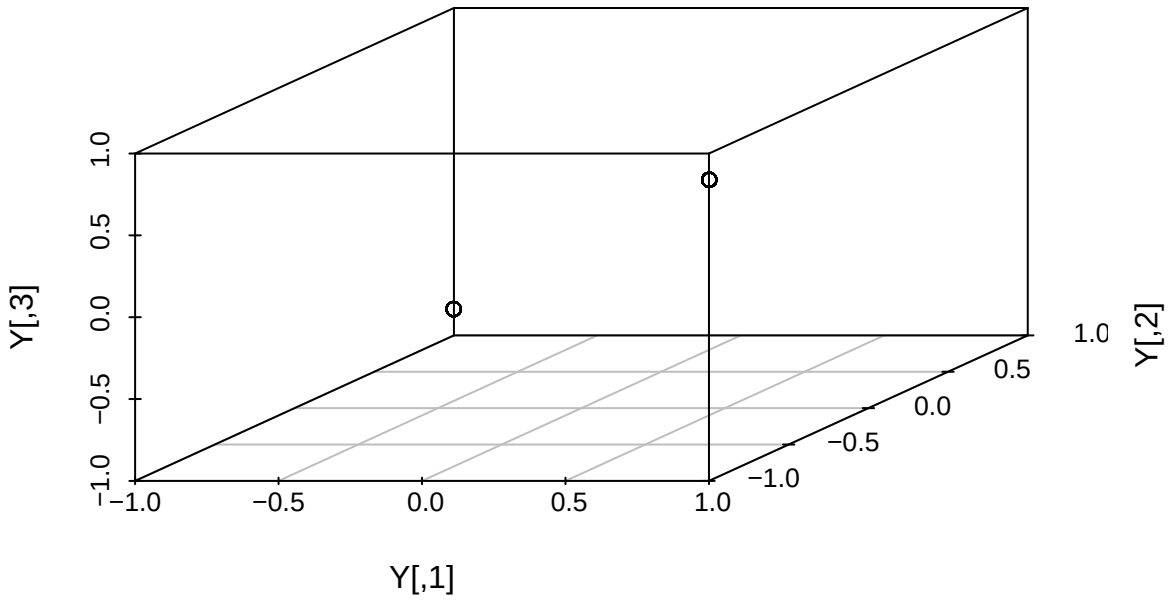
```
Y <- psc_sne(X=sanjuanfuca, d=2,
             rho_psc_list = rho_45,
             num_iteration=300)
```

```
## [1] "Iter 1, obj 8.670732, abs 0.000000, rel 0.000000, norm 0.592155"
## [1] "Iter 2, obj 8.712957, abs 0.042224, rel 0.004870, norm 1.476392"
## [1] "Iter 3, obj 8.861109, abs 0.148152, rel 0.017004, norm 2.940417"
## [1] "Iter 4, obj 8.981619, abs 0.120510, rel 0.013600, norm 3.439473"
## [1] "Iter 5, obj 9.004676, abs 0.023057, rel 0.002567, norm 3.576139"
## [1] "Iter 6, obj 8.968099, abs 0.036577, rel 0.004062, norm 3.598204"
## [1] "Iter 7, obj 8.907389, abs 0.060710, rel 0.006770, norm 3.586922"
## [1] "Iter 8, obj 8.825255, abs 0.082134, rel 0.009221, norm 3.562730"
## [1] "Iter 9, obj 8.722331, abs 0.102924, rel 0.011662, norm 3.529558"
## [1] "Iter 10, obj 8.589754, abs 0.132577, rel 0.015200, norm 3.488972"
## [1] "Iter 11, obj 8.418489, abs 0.171265, rel 0.019938, norm 3.441254"
## [1] "Iter 12, obj 8.192918, abs 0.225571, rel 0.026795, norm 3.390069"
## [1] "Iter 13, obj 7.897253, abs 0.295664, rel 0.036088, norm 3.345881"
## [1] "Iter 14, obj 7.528567, abs 0.368686, rel 0.046685, norm 3.336313"
## [1] "Iter 15, obj 7.152594, abs 0.375973, rel 0.049940, norm 3.414294"
```

```
## [1] "Iter 16, obj 6.893957, abs 0.258637, rel 0.036160, norm 3.630343"
## [1] "Iter 17, obj 6.764238, abs 0.129719, rel 0.018816, norm 3.935392"
## [1] "Iter 18, obj 6.729965, abs 0.034274, rel 0.005067, norm 4.249093"
## [1] "Iter 19, obj 6.771299, abs 0.041335, rel 0.006142, norm 4.548846"
## [1] "Iter 20, obj 6.867420, abs 0.096121, rel 0.014195, norm 4.819096"
## [1] "Iter 21, obj 6.997292, abs 0.129871, rel 0.018911, norm 5.045205"
## [1] "Iter 22, obj 7.134752, abs 0.137460, rel 0.019645, norm 5.218236"
## [1] "Iter 23, obj 7.250521, abs 0.115769, rel 0.016226, norm 5.335313"
## [1] "Iter 24, obj 7.332515, abs 0.081995, rel 0.011309, norm 5.404443"
## [1] "Iter 25, obj 7.386017, abs 0.053502, rel 0.007297, norm 5.442322"
## [1] "Iter 26, obj 7.420195, abs 0.034178, rel 0.004627, norm 5.463133"
## [1] "Iter 27, obj 7.442094, abs 0.021898, rel 0.002951, norm 5.475000"
## [1] "Iter 28, obj 7.456253, abs 0.014160, rel 0.001903, norm 5.482045"
## [1] "Iter 29, obj 7.465492, abs 0.009239, rel 0.001239, norm 5.486368"
## [1] "Iter 30, obj 7.471563, abs 0.006071, rel 0.000813, norm 5.489087"
## [1] "Iter 31, obj 7.475573, abs 0.004010, rel 0.000537, norm 5.490829"
## [1] "Iter 32, obj 7.478231, abs 0.002658, rel 0.000356, norm 5.491959"
## [1] "Iter 33, obj 7.479998, abs 0.001766, rel 0.000236, norm 5.492698"
## [1] "Iter 34, obj 7.481173, abs 0.001175, rel 0.000157, norm 5.493185"
## [1] "Iter 35, obj 7.481956, abs 0.000783, rel 0.000105, norm 5.493507"
## [1] "Iter 36, obj 7.482478, abs 0.000522, rel 0.000070, norm 5.493720"
## [1] "Iter 37, obj 7.482825, abs 0.000348, rel 0.000046, norm 5.493862"
## [1] "Iter 38, obj 7.483057, abs 0.000232, rel 0.000031, norm 5.493956"
## [1] "Iter 39, obj 7.483212, abs 0.000155, rel 0.000021, norm 5.494019"
## [1] "Iter 40, obj 7.483315, abs 0.000103, rel 0.000014, norm 5.494061"
## [1] "Iter 41, obj 7.483384, abs 0.000069, rel 0.000009, norm 5.494089"
## [1] "Iter 42, obj 7.483430, abs 0.000046, rel 0.000006, norm 5.494107"
## [1] "Iter 43, obj 7.483461, abs 0.000031, rel 0.000004, norm 5.494120"
## [1] "Iter 44, obj 7.483481, abs 0.000020, rel 0.000003, norm 5.494128"
## [1] "Iter 45, obj 7.483495, abs 0.000014, rel 0.000002, norm 5.494133"
## [1] "Iter 46, obj 7.483504, abs 0.000009, rel 0.000001, norm 5.494137"
## [1] "Iter 47, obj 7.483510, abs 0.000006, rel 0.000001, norm 5.494139"
## [1] "Iter 48, obj 7.483514, abs 0.000004, rel 0.000001, norm 5.494141"
## [1] "Iter 49, obj 7.483516, abs 0.000003, rel 0.000000, norm 5.494142"
## [1] "Iter 50, obj 7.483518, abs 0.000002, rel 0.000000, norm 5.494143"
## [1] "Iter 51, obj 7.483519, abs 0.000001, rel 0.000000, norm 5.494143"
## [1] "Iter 52, obj 7.483520, abs 0.000001, rel 0.000000, norm 5.494144"
## [1] "Iter 53, obj 7.483521, abs 0.000001, rel 0.000000, norm 5.494144"
## [1] "Iter 54, obj 7.483521, abs 0.000000, rel 0.000000, norm 5.494144"
## [1] "Iter 55, obj 7.483521, abs 0.000000, rel 0.000000, norm 5.494144"
## [1] "Iter 56, obj 7.483522, abs 0.000000, rel 0.000000, norm 5.494144"
## [1] "Iter 57, obj 7.483522, abs 0.000000, rel 0.000000, norm 5.494144"
## [1] "Iter 58, obj 7.483522, abs 0.000000, rel 0.000000, norm 5.494144"
## [1] "Iter 59, obj 7.483522, abs 0.000000, rel 0.000000, norm 5.494144"
## [1] "Iter 60, obj 7.483522, abs 0.000000, rel 0.000000, norm 5.494144"
## [1] "Iter 61, obj 7.483522, abs 0.000000, rel 0.000000, norm 5.494144"
## [1] "Iter 62, obj 7.483522, abs 0.000000, rel 0.000000, norm 5.494144"
## [1] "Iter 63, obj 7.483522, abs 0.000000, rel 0.000000, norm 5.494144"
## [1] "Iter 64, obj 7.483522, abs 0.000000, rel 0.000000, norm 5.494144"
## [1] "Iter 65, obj 7.483522, abs 0.000000, rel 0.000000, norm 5.494144"
## [1] "Iter 66, obj 7.483522, abs 0.000000, rel 0.000000, norm 5.494144"
## [1] "Iter 67, obj 7.483522, abs 0.000000, rel 0.000000, norm 5.494144"
## [1] "Iter 68, obj 7.483522, abs 0.000000, rel 0.000000, norm 5.494144"
## [1] "Iter 69, obj 7.483522, abs 0.000000, rel 0.000000, norm 5.494144"
```

```
## [1] "Iter 70, obj 7.483522, abs 0.000000, rel 0.000000, norm 5.494144"
## [1] "Iter 71, obj 7.483522, abs 0.000000, rel 0.000000, norm 5.494144"
## [1] "Iter 72, obj 7.483522, abs 0.000000, rel 0.000000, norm 5.494144"
## [1] "Iter 73, obj 7.483522, abs 0.000000, rel 0.000000, norm 5.494144"
## [1] "Iter 74, obj 7.483522, abs 0.000000, rel 0.000000, norm 5.494144"
## [1] "Iter 75, obj 7.483522, abs 0.000000, rel 0.000000, norm 5.494144"
## [1] "Iter 76, obj 7.483522, abs 0.000000, rel 0.000000, norm 5.494144"
## [1] "Iter 77, obj 7.483522, abs 0.000000, rel 0.000000, norm 5.494144"
## [1] "Iter 78, obj 7.483522, abs 0.000000, rel 0.000000, norm 5.494144"
## [1] "Iter 79, obj 7.483522, abs 0.000000, rel 0.000000, norm 5.494144"
## [1] "Iter 80, obj 7.483522, abs 0.000000, rel 0.000000, norm 5.494144"
## [1] "Iter 81, obj 7.483522, abs 0.000000, rel 0.000000, norm 5.494144"
## [1] "Iter 82, obj 7.483522, abs 0.000000, rel 0.000000, norm 5.494144"
## [1] "Iter 83, obj 7.483522, abs 0.000000, rel 0.000000, norm 5.494144"
## [1] "Iter 84, obj 7.483522, abs 0.000000, rel 0.000000, norm 5.494144"
## [1] "Iter 85, obj 7.483522, abs 0.000000, rel 0.000000, norm 5.494144"
## [1] "Iter 86, obj 7.483522, abs 0.000000, rel 0.000000, norm 5.494144"
## [1] "Iter 87, obj 7.483522, abs 0.000000, rel 0.000000, norm 5.494144"
## [1] "Iter 88, obj 7.483522, abs 0.000000, rel 0.000000, norm 5.494144"
## [1] "Iter 89, obj 7.483522, abs 0.000000, rel 0.000000, norm 5.494144"
## [1] "Iter 90, obj 7.483522, abs 0.000000, rel 0.000000, norm 5.494144"
## [1] "Iter 91, obj 7.483522, abs 0.000000, rel 0.000000, norm 5.494144"
## [1] "Iter 92, obj 7.483522, abs 0.000000, rel 0.000000, norm 5.494144"
## [1] "Iter 93, obj 7.483522, abs 0.000000, rel 0.000000, norm 5.494144"
## [1] "Iter 94, obj 7.483522, abs 0.000000, rel 0.000000, norm 5.494144"
## [1] "Iter 95, obj 7.483522, abs 0.000000, rel 0.000000, norm 5.494144"
## [1] "Iter 96, obj 7.483522, abs 0.000000, rel 0.000000, norm 5.494144"
## [1] "Iter 97, obj 7.483522, abs 0.000000, rel 0.000000, norm 5.494144"
## [1] "Iter 98, obj 7.483522, abs 0.000000, rel 0.000000, norm 5.494144"
## [1] "Iter 99, obj 7.483522, abs 0.000000, rel 0.000000, norm 5.494144"
## [1] "Iter 100, obj 7.483522, abs 0.000000, rel 0.000000, norm 5.494144"
## [1] "Iter 101, obj 1.036533, abs 6.446989, rel 0.861491, norm 0.361253"
## [1] "Iter 102, obj 1.471094, abs 0.434561, rel 0.419244, norm 0.607825"
## [1] "Iter 103, obj 1.471094, abs 0.000000, rel 0.000000, norm 0.793451"
```

```
scatterplot3d::scatterplot3d(Y, xlim = c(-1, 1), ylim = c(-1, 1),
                             zlim = c(-1, 1), color = rep(1, nrow(Y)))
```




```
rgl::plot3d(0, 0, 0, xlim = c(-1, 1), ylim = c(-1, 1), zlim = c(-1, 1),
            radius = 1, type = "s", col = "lightblue", alpha = 0.25,
            lit = FALSE)
rgl::points3d(Y, col = rep(1, nrow(Y)))
```