Stochastic Volatility of World Indexes
A comparison on the COVID crisis

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Table of Contents

The COVID Crisis

- The COVID pandemic caused an unprecedented exogenous shock to the world's financial markets
- Indexes all over the world suffered one of the largest downturns in recent history
- However, due to the nature of the shock and policy measures, the speed of recovery was also extreme, with some indexes taking only months to reach pre-crisis levels
- This period of heightened volatility has been followed by relatively stable returns, thus making it an important case study for volatility models

Stochastic Volatility

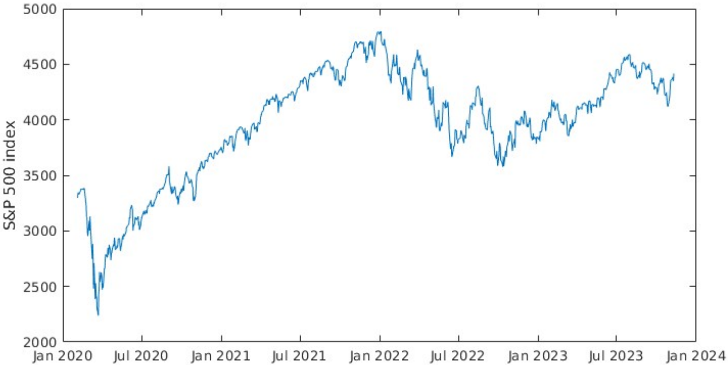
- Stochastic volatility models have been gaining traction in the academic community and industry over the recent decades
- Their theoretical underpinnings make them attractive from a risk management and asset pricing point of view
- While the estimation procedure is much more difficult and slow, the effectiveness of fit has shown that they are in general preferable to classical time series GARCH models
- In this work, we compare the performance of the two models since before the COVID crisis to better understand their behavior

Table of Contents

Datasets

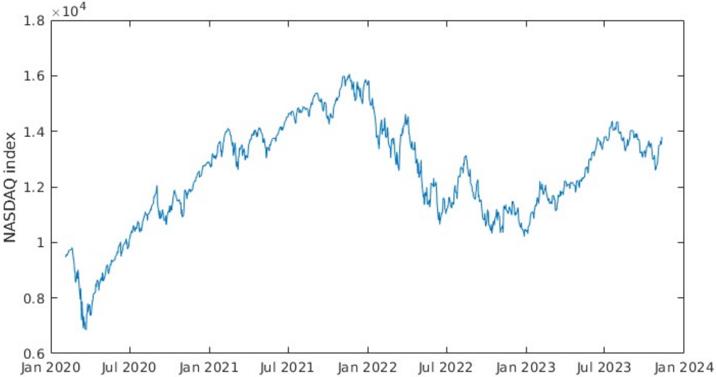
- For this study, we focus on six of the world's largest indexes
  - US: S&P 500, NASDAQ, Dow Jones Industrial Average
  - Non-US: FTSE 100 (London), Stoxx 600 (Europe), Nikkei 225 (Japan)
- Data is collected from Yahoo Finance, spanning from February 2020, right before the COVID crisis, to today
  - 951 return observations for each index
  - Collection through Python package yfinance

S&P 500



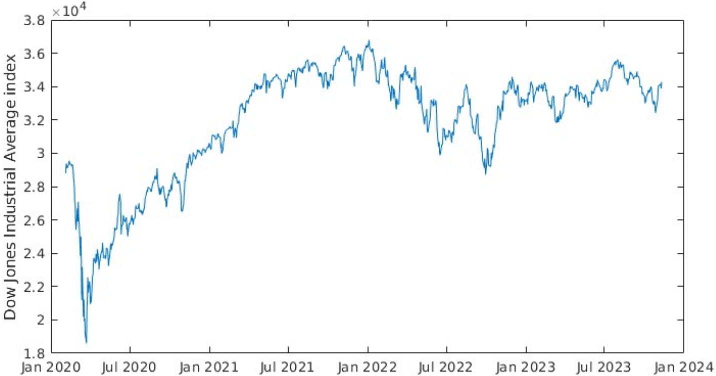
5000
4500
4000
3500
3000
2500
2000
S&P 500 index
Jan 2020
Jul 2020
Jan 2021
Jul 2021
Jan 2022
Jul 2022
Jan 2023
Jul 2023
Jan 2024

NASDAQ



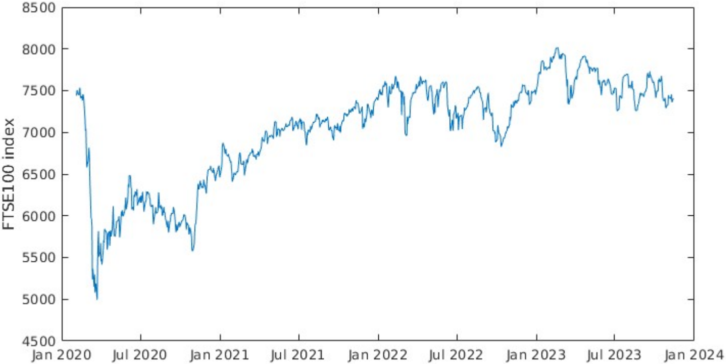
$\times 10^4$
1.8
1.6
1.4
1.2
1
0.8
0.6
NASDAQ index
Jan 2020
Jul 2020
Jan 2021
Jul 2021
Jan 2022
Jul 2022
Jan 2023
Jul 2023
Jan 2024

Dow Jones



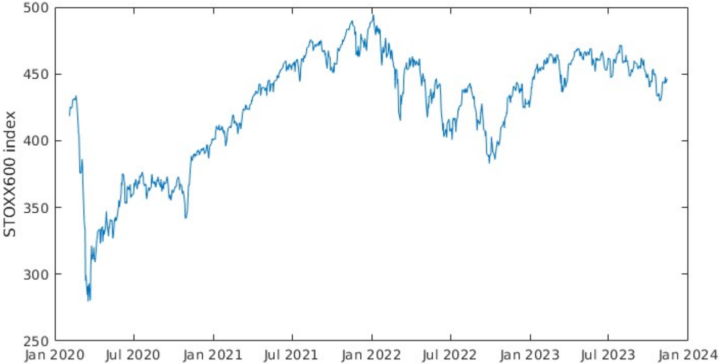
$\times 10^4$
3.8
3.6
3.4
3.2
3
2.8
2.6
2.4
2.2
2
1.8
Dow Jones Industrial Average index
Jan 2020
Jul 2020
Jan 2021
Jul 2021
Jan 2022
Jul 2022
Jan 2023
Jul 2023
Jan 2024

FTSE 100

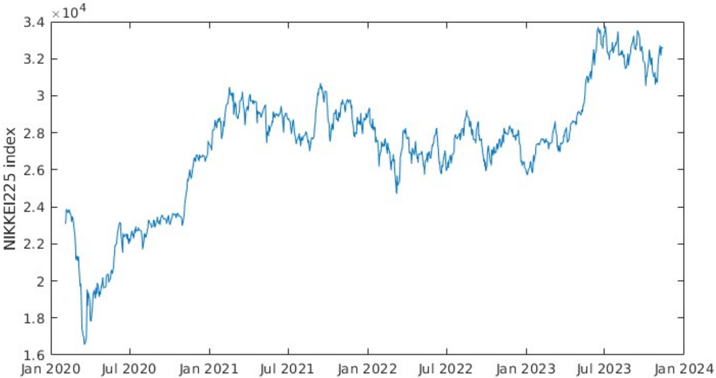


8500
8000
7500
7000
6500
6000
5500
5000
4500
FTSE100 index
Jan 2020
Jul 2020
Jan 2021
Jul 2021
Jan 2022
Jul 2022
Jan 2023
Jul 2023
Jan 2024

STOXX 600


STOXX600 index
500
450
400
350
300
250
Jan 2020
Jul 2020
Jan 2021
Jul 2021
Jan 2022
Jul 2022
Jan 2023
Jul 2023
Jan 2024

NIKKEI 225



$\times 10^4$
3.4
3.2
3
2.8
2.6
2.4
2.2
2
1.8
1.6
NIKKEI225 index
Jan 2020
Jul 2020
Jan 2021
Jul 2021
Jan 2022
Jul 2022
Jan 2023
Jul 2023
Jan 2024

Model Specification

- Discretized stochastic volatility model:

$$Y_t = \mu + \sqrt{V_t}\varepsilon_t \tag{1}$$
$$\log V_t = \phi_0 + \phi_1 \log V_{t-1} + \sigma_v \eta_t \tag{2}$$

- The parameters to be estimated are μ, ϕ_0, ϕ_1 and σ_v
- Since V_t is unobservable, we must estimate it as well

Estimation

- Estimation is done by Markov Chain Monte Carlo with the Kim, Shephard and Chib (1998) naive implementation
- Normal priors for μ, ϕ_0
- Assume Beta prior for ϕ_1 to ensure stationarity
  - Not a big assumption, since stationarity is expected from the mean reversing behavior of volatility
  - Same hyperparameters as the original paper
  - Independence Metropolis-Hastings
- σ_v is assumed to have an Inverse Gamma prior
- Log-volatilities are simulated by independence Metropolis-Hastings

Methodology

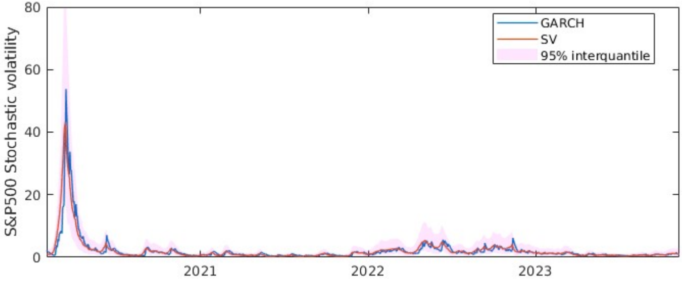
- Estimate appropriate ARMA-GARCH model for the data
- GARCH volatilities are used as an initial estimate of log-volatility, as well as other estimated parameters
- 150,000 burn-in samples, 250,000 retained samples

Table of Contents

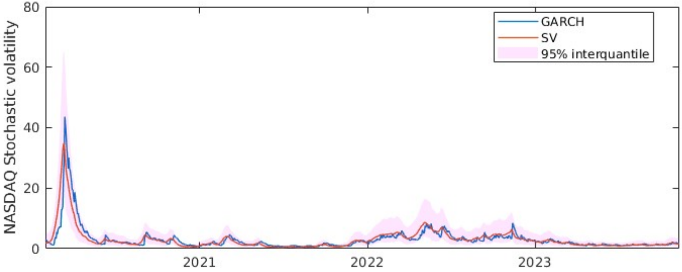
Estimation results

	S&P 500	NASDAQ	DJI	FTSE	STOXX	NIKKEI
μ	0.1057	0.1297	0.0604	0.0607	0.0921	0.0740
s.d.	0.0295	0.0396	0.0275	0.0244	0.0256	0.0357
ϕ_0	0.0037	0.0111	-0.0051	-0.0165	-0.0131	0.0120
s.d.	0.0103	0.0091	0.0108	0.0145	0.0137	0.0096
ϕ_1	0.9557	0.9823	0.9550	0.9451	0.9414	0.9535
s.d.	0.0206	0.0083	0.0206	0.0216	0.0236	0.0202
σ_v	0.0937	0.0519	0.1011	0.1577	0.1472	0.0613
s.d.	0.0250	0.0115	0.0251	0.0347	0.0348	0.0167

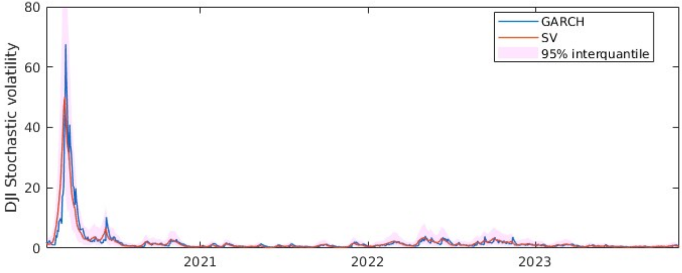
Stochastic volatility of the S&P 500



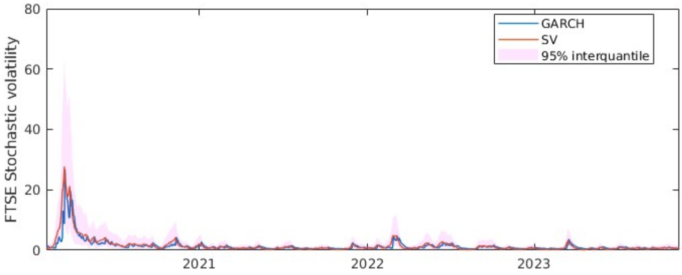
Stochastic volatility of the NASDAQ



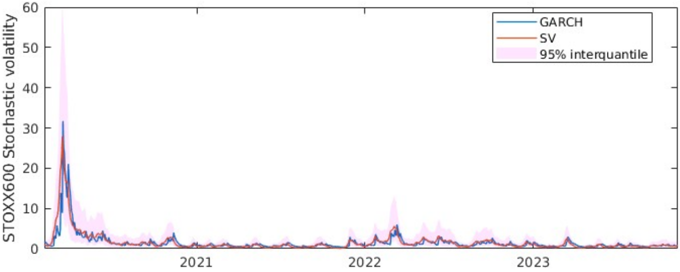
Stochastic volatility of the Dow Jones



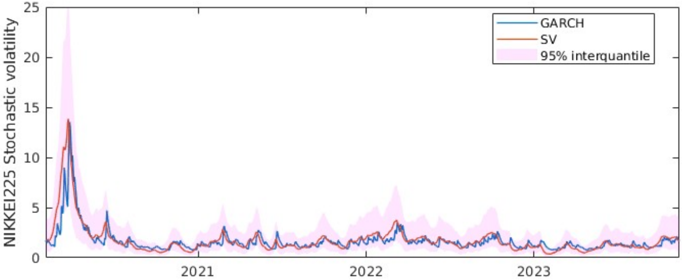
Stochastic volatility of the FTSE



Stochastic volatility of the STOXX 600

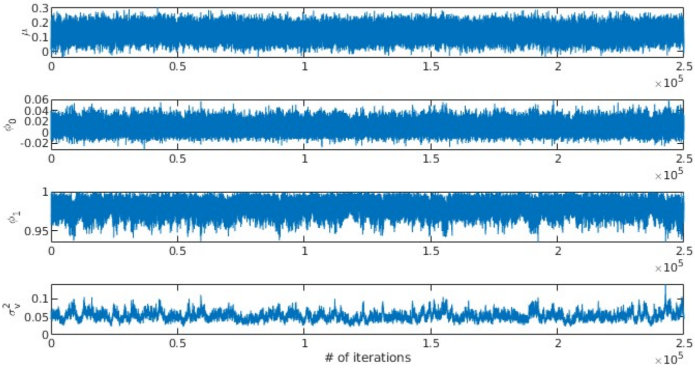


Stochastic volatility of the NIKKEI

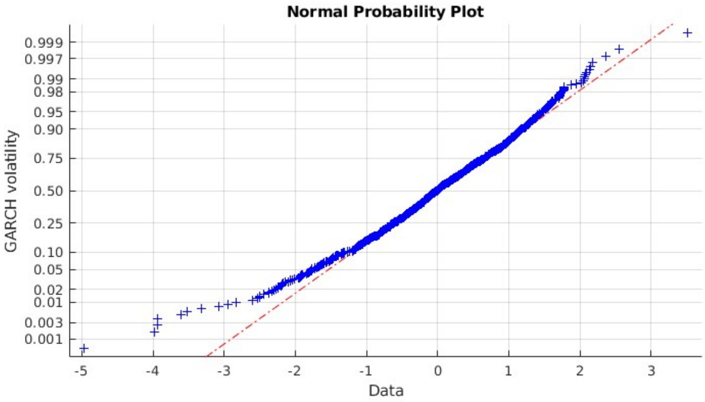


Performance

- Average run-time was of 120 seconds
- Acceptance rate of ϕ_1 candidates was around 53% for all indexes but NASDAQ, where it was of 64%

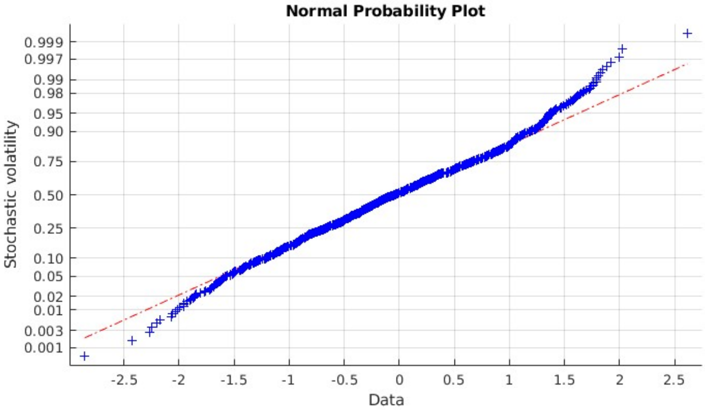


Fit: S&P 500



Normal Probability Plot
GARCH volatility
0.999
0.997
0.99
0.98
0.95
0.90
0.75
0.50
0.25
0.10
0.05
0.02
0.01
0.003
0.001
-5
-4
-3
-2
-1
0
1
2
3
Data

Fit: S&P 500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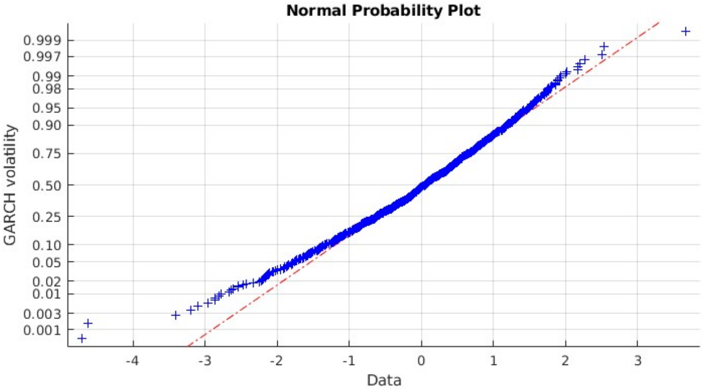
0.02
0.01
0.003
0.001
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
Data

Fit: NASDAQ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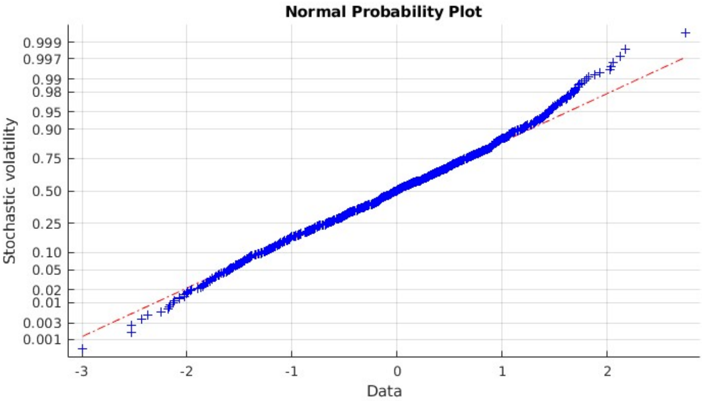
0.02
0.01
0.003
0.001
-4
-3
-2
-1
0
1
2
3
Data

Fit: NASDAQ



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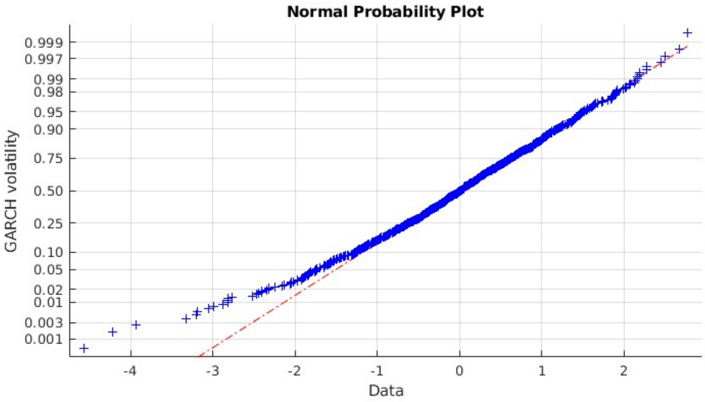
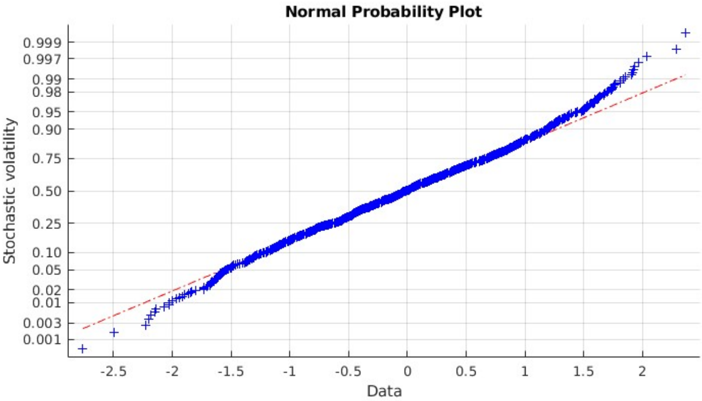
0.01
0.003
0.001
-3
-2
-1
0
1
2
Data

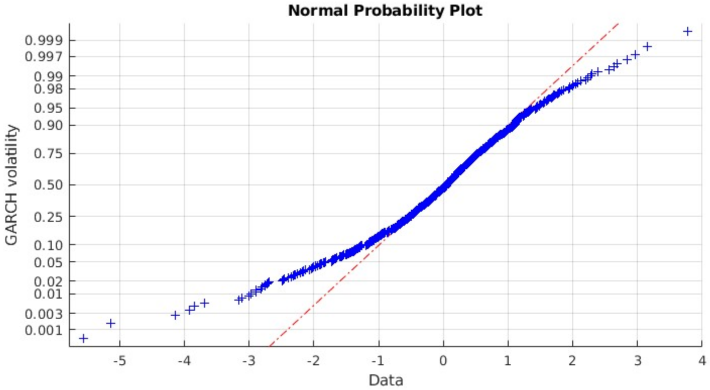
Fit: Dow Jones



Fit: Dow Jones



Fit: FTSE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0.02
0.01
0.003
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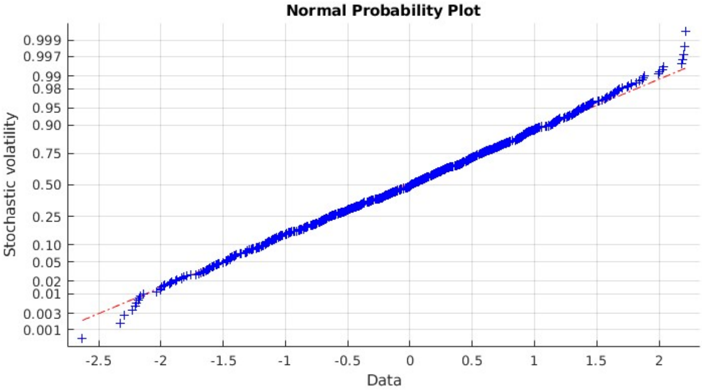
4
Data

Fit: FTSE



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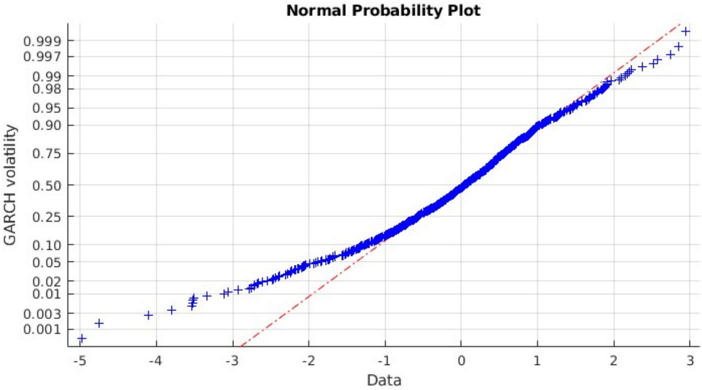
0.01
0.003
0.001
Stochastic volatility
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
Data

Fit: STOXX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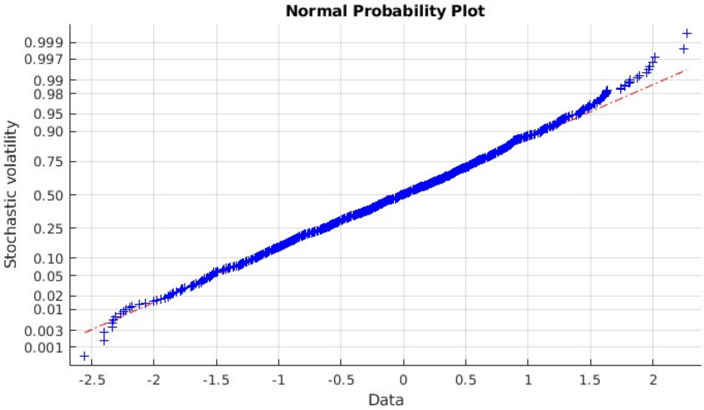
0.25
0.10
0.05
0.02
0.01
0.003
0.001
GARCH volatility
-5
-4
-3
-2
-1
0
1
2
3
Data

Fit: STOXX



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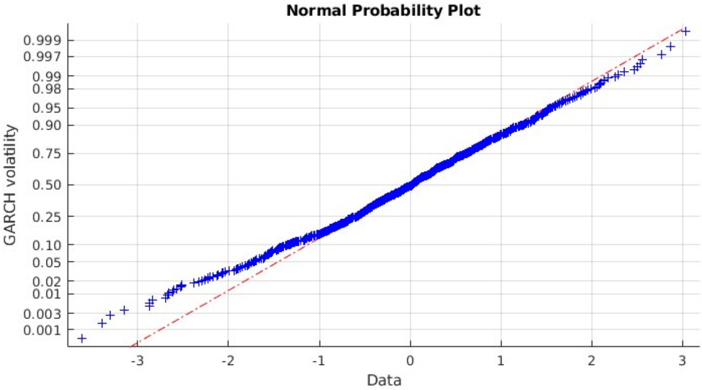
0.90
0.75
0.50
0.25
0.10
0.05
0.02
0.01
0.003
0.001
Stochastic volatility
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
Data

Fit: NIKKEI


Normal Probability Plot
GARCH volatility
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0.25
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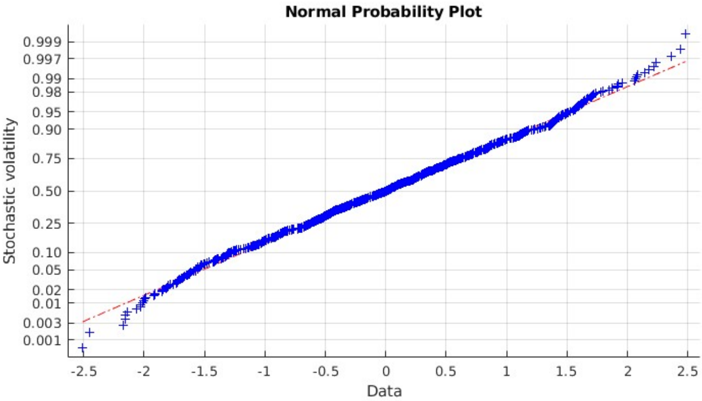
0.05
0.02
0.01
0.003
0.001
-3
-2
-1
0
1
2
3
Data

Fit: NIKKEI



Normal Probability Plot
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0.75
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0.25
0.10
0.05
0.02
0.01
0.003
0.001
Stochastic volatility
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
Data

Table of Contents

GARCH vs Stochastic Volatility

- Stochastic volatility models provide a stronger fit in general than simple GARCH models
  - GARCH volatilities appear to dominate SV in periods of higher volatility
  - However, GARCH appears to overshot most estimates
- SV vastly outperforms GARCH in Europe and somewhat in Asia
- Even though GARCH models can be estimated much faster, the estimation time of SV was relatively small
  - Acceleration methods, such as the ones suggested by KSC and other recent developments, reduce even further this problem
  - Serial correlation of the sample may also be an issue, which may be alleviated by using integration samplers, for instance

Estimates

- NASDAQ had distinctively the most persistent volatility and showed the highest average returns
  - This can be explained by the technological nature of the index
  - Most favorable environment for trading volatility derivatives and collecting volatility premiums
  - While not as popular as the S&P 500, volume has been increasing steadily over the recent years
- Europe, on the other hand, had the least persistent volatilities
  - However, we observe significantly higher vol-of-vol, which might give a clue on the difference of performance between the two models

Thank you for your attention!