

# The Implied Equity Premium

## Continuous Time Finance II

Luís Simão Ferreira

January 20, 2024

### Contents

<b>1</b>	<b>Introduction</b>	<b>2</b>
<b>2</b>	<b>Theoretical background</b>	<b>3</b>
2.1	Model setup . . . . .	3
2.2	The Carr-Madan formula . . . . .	3
2.3	Pricing kernel and risk-neutral expectations . . . . .	3
2.4	The implied risk premium (IRP) . . . . .	5
2.5	Market clearing model with heterogeneous agents . . . . .	6
<b>3</b>	<b>Data and methodology</b>	<b>7</b>
3.1	Realized variance estimation and forecasting . . . . .	7
3.2	Risk neutral market moments . . . . .	8
3.3	Variance premium and implied equity premium . . . . .	9
<b>4</b>	<b>Replication results</b>	<b>10</b>
<b>5</b>	<b>Conclusion</b>	<b>11</b>
<b>A</b>	<b>Figures</b>	<b>12</b>

# 1 Introduction

The equity risk premium, the expected excess return of the stock market over the risk-free rate, is a puzzling unobservable determinant both in the finance and economics literatures, as well as in real world policy applications.

Despite its importance in understanding stock price and firm investment fluctuations and key economic variables such as economic consumption and monetary policy and the vast research done on the topic, various methods of estimation lead to widely different results and no significant consensus has been achieved.

The paper "The Implied Equity Premium" by Paul Tetlock [1] provides a firm contribution to the literature in this direction. In the paper, a new model of the equity premium is proposed that addresses many of the current issues. On one hand, it is parsimonious in its assumptions, the main ones being that stock and derivatives markets are complete and frictionless, which we shall see that in this context are relatively natural and reflect the real financial world.

Moreover, the model has compelling financial interpretations, which make the justification of the results straightforward, and can be derived from an extremely general economic rationale, accepting both investors with rational expectations as well as general biased investors who incorporate their own beliefs into their decisions. No assumptions are made on this second group, therefore making it an acceptable model of the real world markets.

The model not only solves the problem of estimating the implied equity premium, but does so for all moments of stock market returns. A particularly important piece of this work is the variance premium, which ends up being more accessible due to the wide array of techniques available from market microstructure and econometrics in estimating realized variance forecasts.

Finally, all estimation procedures can be done using real-time data, which is essential for practical applications, either if they are for a policy making purpose or for making investment decisions.

In this replication project, we propose to replicate the results of Tetlock under considerable simplifications. We begin by exploring the theoretical setup of the model and clarifying some of the results. Later, we clearly explain the original methodology, comparing it with our own, and show our results in the end.

## 2 Theoretical background

### 2.1 Model setup

The model on which this work is based on builds up on an existing stock market and derivatives on the moments of its return.

First, assume that there exists a risk-free asset, with deterministic return  $R_{f,t}$ , and a risky market portfolio with returns  $R_{m,T}$ , with  $T > t$  denoting the investment horizon. The excess market return is then denoted

$$\tilde{R}_{m,T} = R_{m,T} - R_{f,t}. \quad (1)$$

Furthermore, assume there exist  $K - 1$  derivative securities, for  $K > 1$ , offering excess returns of

$$\tilde{R}_{m,T}^k - c_k, \quad (2)$$

for  $k = 1, 2, \dots, K$ , behaving essentially as swaps on market moments.

The constants  $c_k$  are there to ensure that the price of these derivative contracts is 0, except for the case of  $c_1$  which is set to 0 so that excess market returns take the same form as the derivatives' excess returns.

### 2.2 The Carr-Madan formula

In the options pricing theory, the Carr-Madan formula, as shown in Carr and Madan (2001) [2], equates any derivative with a non-linear payoff as a portfolio of options and forms the basis of many static hedging strategies.

Let  $g$  denote this payoff,  $S_T$  the underlying asset at time  $T$  (maturity),  $F$  a constant and  $K$  the strike price of an European option (the naming convention of  $K$  seems unfortunate but this relationship shows that the two  $K$  quantities are somewhat equivalent).

Then, the following identity holds:

$$g(S_T) = g(F) + g'(F)(S_T - F) + \int_0^F g''(K)p(S_T, K)dK + \int_F^{+\infty} g''(K)c(S_T, K)dK. \quad (3)$$

Here,  $p$  and  $c$  denote the payoffs of European put and call options, respectively, but we can obtain the relationship in terms of prices by taking risk-neutral expectations and applying Fubini's theorem.

Note that the right hand side integrals, when discretized, are equivalent to a weighted sum of puts and calls and thus form a portfolio of such assets.

Therefore, the derivatives market is complete if a continuum of strike prices is available. Since option contracts for liquid markets are available at a wide range of strikes and the value of deep OOM contracts tends to zero, this is a reasonable approximation of real markets.

In the previously described context, this is equivalent to assuming that  $K \rightarrow +\infty$ , i.e., there exist tradable derivatives on any market moment. This simplifies the theory and we shall assume it moving forward. However, for practical purposes, this economy can be well approximated by moments as low as  $K = 4$ , as is done in the paper.

### 2.3 Pricing kernel and risk-neutral expectations

Three additional assumptions are needed to completely solve the model:

- The market is arbitrage-free,

- There are no trading frictions (such as transaction costs, short sale restrictions, leverage regulations, etc.),
- Market returns are finite and strictly positive.

Before moving on, we need an additional definition.

**Definition 2.1** *The growth-optimal (GO) portfolio over the investment horizon  $T$  is the portfolio that maximizes the expected log-return over the same period, i.e., maximizes long-run growth of investor wealth.*

Under these conditions, the result by Long (1990) [3] shows the following:

**Proposition 2.1** *There exists a strictly positive pricing kernel  $M_T$ , given by the reciprocal of the return of the GO portfolio,*

$$M_T = \left[ R_{f,t} + \sum_{k=1}^{+\infty} w_{k,t,T} \left( \tilde{R}_{m,T}^k - c_k \right) \right]^{-1}, \quad (4)$$

where  $w_{k,t,T}$  denotes the weights of the GO portfolio.

The pricing kernel prices all market-related tradable securities as

$$\mathbb{E}_t[M_T R_T] = 1. \quad (5)$$

Note that, by taking  $R_T$  to be the risk-free rate, we obtain the identity

$$\mathbb{E}_t M_T = 1/R_{f,t}. \quad (6)$$

The existence of the pricing kernel then allows us to define the *risk-neutral measure*, by its Radon-Nikodym derivative with respect to the physical measure,

$$\frac{M_T}{\mathbb{E}_t M_T} = R_{f,t} M_T. \quad (7)$$

We shall denote risk-neutral expectations as  $\mathbb{E}_t^*$ . The risk-neutral expectation of any tradable return is then simply the risk-free rate,

$$\mathbb{E}_t^* R_T = R_{f,t} \mathbb{E}_t[M_T R_T] = R_{f,t} \quad (8)$$

and thus the risk-neutral expectation of any excess market return is zero,

$$\mathbb{E}_t^* \tilde{R}_T = \mathbb{E}_t^*[R_T - R_{f,t}] = 0. \quad (9)$$

Moreover, if we apply this formula to the derivative securities on market moments we deduce

$$c_k = \mathbb{E}_t^* \tilde{R}_{m,T}^k. \quad (10)$$

and these constants can be inferred from market prices of options by the Carr-Madan formula.

## 2.4 The implied risk premium (IRP)

Now, we need to relate the physical (rational) expectations of markets to risk-neutral expectations.

First, note that the real world expected excess return of any market-related security is

$$\mathbb{E}_t \tilde{R}_T = \mathbb{E}_t \left[ \frac{M_T}{\mathbb{E}_t M_T} \frac{\mathbb{E}_t M_T}{M_T} \tilde{R}_T \right] \quad (11)$$

$$= R_{f,t}^{-1} \mathbb{E}_t \left[ \frac{M_T}{\mathbb{E}_t M_T} M_T^{-1} \tilde{R}_T \right] \quad (12)$$

$$= R_{f,t}^{-1} \mathbb{E}_t^* \left[ M_T^{-1} \tilde{R}_T \right] \quad (13)$$

$$= R_{f,t}^{-1} \mathbb{E}_t^* \left[ \left[ R_{f,t} + \sum_{k=1}^{+\infty} w_{k,t,T} \left( \tilde{R}_{m,T}^k - c_k \right) \right] \tilde{R}_T \right]. \quad (14)$$

By Fubini's theorem once again, we finally get

$$\mathbb{E}_t \tilde{R}_T - \mathbb{E}_t^* \tilde{R}_T = R_{f,t}^{-1} \sum_{k=1}^{+\infty} w_{k,t,T} \left( \mathbb{E}_t^* [\tilde{R}_{m,T}^k \tilde{R}_T] - \mathbb{E}_t^* \tilde{R}_{m,T}^k \mathbb{E}_t^* \tilde{R}_T \right). \quad (15)$$

which, when applied to the tradable market securities  $\tilde{R}_T = \tilde{R}_{m,T}^k$ , gives us the *implied risk premium*:

$$\mathbb{E}_t \tilde{R}_{m,T}^n - \mathbb{E}_t^* \tilde{R}_{m,T}^n = R_{f,t}^{-1} \sum_{k=1}^{+\infty} w_{k,t,T} \left( \mathbb{E}_t^* \tilde{R}_{m,T}^{k+n} - \mathbb{E}_t^* \tilde{R}_{m,T}^k \mathbb{E}_t^* \tilde{R}_{m,T}^n \right), \quad (16)$$

for all  $n \in \mathbb{N}$ .

Thus, the risk premium of any market security is linear in the risk-neutral moments, which can be estimated from real world option data, and later the GO portfolio weights can be inferred as regression parameters.

Setting  $n = 1$ , we obtain the *implied equity premium (IEP)*

$$\mathbb{E}_t \tilde{R}_{m,T} = R_{f,t}^{-1} \sum_{k=1}^{+\infty} w_{k,t,T} \mathbb{E}_t^* \tilde{R}_{m,T}^{k+1}, \quad (17)$$

and for  $n = 2$  the *implied variance premium (IVP)*

$$\mathbb{E}_t \tilde{R}_{m,T}^2 - \mathbb{E}_t^* \tilde{R}_{m,T}^2 = R_{f,t}^{-1} \sum_{k=1}^{+\infty} w_{k,t,T} \left( \mathbb{E}_t^* \tilde{R}_{m,T}^{k+2} - \mathbb{E}_t^* \tilde{R}_{m,T}^k \mathbb{E}_t^* \tilde{R}_{m,T}^2 \right). \quad (18)$$

For implementation purposes, we must properly approximate these equations by cutting off the infinite sum.

Martin's lower bound (2007) [4] can be seen as a particular case, where  $K = 1$ ,

$$\mathbb{E}_t \tilde{R}_{m,T} = R_{f,t}^{-1} \mathbb{E}_t^* \tilde{R}_{m,T}^2, \quad (19)$$

and it is equivalent to the assumption that the GO portfolio is the stock market. According to Tetlock, however, it seems unlikely that a GO investor exploiting variation in the equity premium by timing the market would have a non-constant portfolio.

To analyze this behavior, we analyze instead a  $K$ -th order approximation of this economy:

$$\mathbb{E}_t \tilde{R}_{m,T} \approx R_{f,t}^{-1} \sum_{k=1}^K w_{k,t,T} \mathbb{E}_t^* \tilde{R}_{m,T}^{k+1}, \quad (20)$$

$$\mathbb{E}_t \tilde{R}_{m,T}^2 - \mathbb{E}_t^* \tilde{R}_{m,T}^2 \approx R_{f,t}^{-1} \sum_{k=1}^K w_{k,t,T} \left( \mathbb{E}_t^* \tilde{R}_{m,T}^{k+2} - \mathbb{E}_t^* \tilde{R}_{m,T}^k \mathbb{E}_t^* \tilde{R}_{m,T}^2 \right). \quad (21)$$

In the paper, a 4-th order approximation is chosen, as it leads to enough complexity to exhibit interesting behavior while still remaining technically tractable.

## 2.5 Market clearing model with heterogeneous agents

Finally, we show how the GO pricing kernel can be rationalized and derived from a partial equilibrium economy and explain how it prices all assets.

Suppose there are two types of investors:

- Growth-optimal investors (type  $G$ ), who maximize expected log-utility of long-run wealth in period  $T$  and have rational expectations of returns;
- Type  $B$  investors who have possibly biased expectations of returns, non-standard utilities and restraints.

While the existence of type  $B$  investors is not needed for the validity of the previously discussed model, the market clearing assumption ensures that the result remains the same.

Moreover, suppose that in period  $t$  the corresponding endowed wealths of each investor type is  $e_{G,t}$  and  $e_{B,t}$ . The supply of shares is normalized to 1 and of derivatives to 0, as usual.

Then, type  $G$  investors' portfolio is a solution to the following optimization problem:

$$\max_{w_{G,k,t}} \mathbb{E}_t [\log(e_{G,t} R_{G,p,T})], \quad \forall 1 \leq k \leq K, \quad (22)$$

$$\text{subject to } R_{G,p,T} = R_{f,t} + \sum_{k=1}^K w_{G,k,t} \left( \tilde{R}_{m,T} - \mathbb{E}_t^* \tilde{R}_{m,T}^k \right). \quad (23)$$

The first order conditions for each weight become

$$\mathbb{E}_t \left[ R_{G,p,T}^{-1} \left( \tilde{R}_{m,T}^k - \mathbb{E}_t^* \tilde{R}_{m,T}^k \right) \right] = 0, \quad (24)$$

implying that the pricing kernel  $M_T = R_{G,p,T}^{-1}$  prices all assets.

The market clearing condition then relates GO portfolio weights with the holdings of type  $B$  investors:

$$e_{G,t} w_{G,1,t} + e_{B,t} w_{B,1,t} = P_t, \quad (25)$$

$$w_{G,1,t} = \frac{P_t - w_{B,1,t} e_{B,t}}{e_{G,t}}, \quad (26)$$

where  $P_t$  is the share price of the market (or total capitalization), and a similar relationship holds for the derivatives market:

$$w_{G,k,t} = - \frac{w_{B,k,t} e_{B,t}}{e_{G,t}}. \quad (27)$$

Therefore, if type  $B$  investors hold no derivative contracts, Martin’s approximation is exact, and if they only invest in the market and variance derivatives, the equity premium will only depend on the market’s variance and skewness.

### 3 Data and methodology

We are now in conditions of explain Tetlock’s methodology. The main empirical goals of the paper were to obtain daily estimates of the equity and variance premiums for horizons of 30, 60, 90, 180 and 360 days.

For this purpose, the datasets used were the following:

- Market proxy: SPY high-frequency intraday data on market returns, from 1994 to 2021, consisting of 790 prices equally spaced in time based on filled trades;
- Option data: OptionMetrics with daily data from 1996 to 2021;
- Historical equity premium estimate: Market minus risk-free factor from Kenneth French’s website;
- Risk-free rates: constant maturity market yields from TBills of 1,3,6 and 12 months from the FRED (2 month rate is the average of 1 and 3 months rates).

Next, we describe each of the sequential steps that lead to the main results, which can be summarized as follows:

- Proxy rational expected variance  $\mathbb{E}_t \tilde{R}_{m,T}^2$  by estimating daily variances and forecasting it over the relevant horizon  $T$ ;
- Estimate risk neutral moments from daily option data using the Carr-Madan methodology;
- Compute the implied variance premium from the left hand side of equation (21);
- Use the implied variance premium equation (21) to infer the GO portfolio weights;
- Compute the implied equity premium.

#### 3.1 Realized variance estimation and forecasting

To estimate realized daily variance from intraday SPY data, the author follows common recommendations from market microstructure literature.

Using data on 30 second intervals, corresponding to 790 intraday prices, 10 staggered sets of 5 minute intervals are created, each with 79 prices and thus 78 returns.

Each of these ten sets yields a daily realized variance estimator by summing the squares of log-returns. To smooth the effects of intraday shocks and illiquidity, the final estimator is taken as the mean of these ten estimators.

The forecasts of market variance at the relevant horizons are obtained by estimating a fractionally integrated model for the daily realized variance. This choice is justified by the fact that fractionally integrated processes exhibit the well known property of long memory (and volatility clustering, in this case). Moreover, this single parameter model is simple, relatively easy to estimate and performs remarkably well, even when compared with much more convoluted models.

Thus, the daily realized variance process  $rv_t$  is assumed to satisfy

$$(1 - L)^d rv_t = \varepsilon_t, \quad (28)$$

with  $\varepsilon_t$  being white noise and  $L$  the lag operator.

By inverting the operator  $(1 - L)^d$  and Taylor expanding, we can write the process as

$$rv_t = \varepsilon_t + d\varepsilon_{t-1} + \frac{|d(d-1)|}{2!}\varepsilon_{t-2} + \dots, \quad (29)$$

which allows for recursive maximum likelihood estimation.

By applying an increasing window of estimation (and using data only up to time  $t$ ), we can obtain daily real-time estimates of this parameter  $d = d_t$ , guaranteeing that there is no look-ahead bias. Furthermore, this methodology can then be naturally used for real-time financial applications. The initial window of estimation is of about 1000 days, as suggested by the literature.

Having estimated the parameter  $d_t$  using data up to day  $t$ , we can now use the model to forecast the mean over the desired horizon  $T$  (30, 60, 90, 180 and 360 days), giving us  $\{rv_{t+h}\}_{h=1}^T$ .

The rational expected variance over these periods is then given by

$$RV_{t,T} = \sum_{h=1}^T rv_{t+h}. \quad (30)$$

At this point, we could already use

$$\mathbb{E}_t \tilde{R}_{m,T}^2 = RV_{t,T}. \quad (31)$$

However, these long-run predictions are based on daily realized variances which, on one hand, ignore overnight returns and, on the other hand, are sensitive to intraday short turn reversals which do not affect long-run performance.

Thus, the author regresses  $T$ -days squared market returns on the predicted market variances for horizon  $T$  to obtain an also real-time normalizing factor  $\kappa_t$  to account for the discussed effects, and sets

$$\mathbb{E}_t \tilde{R}_{m,T}^2 = \kappa_t RV_{t,T}. \quad (32)$$

### 3.2 Risk neutral market moments

To estimate risk-neutral moments of excess market returns, the Carr-Madan formula (3) is used.

By taking  $F = F_{t,T}$  to be the forward price of the underlying (SPY), the formula applied to the  $j$ -th moment becomes

$$R_{f,t}^{-1} \mathbb{E}_t^* \tilde{R}_{m,T}^j = \frac{j!}{S_t^j} \left[ \int_{F_{t,T}}^{+\infty} (K - F_{t,T})^{j-2} C(K) dK + \int_0^{F_{t,T}} (K - F_{t,T})^{j-2} P(K) dK \right], \quad (33)$$

with  $j = 2, 3, 4, 5, 6$ .

The author computes the discretized version of the integral using the available strike range and an extrapolation procedure as described in Chang et al. (2013) [5] to create additional data points.

This process is done for the range of available maturities on the data. To obtain the moments for the desired horizons, we must interpolate between these values.

To maximize available data for interpolation, the author proceeds as follows:



- Filter out non-negative and non-positive values for odd and even risk-neutral moments of the irregular maturities, respectively;
- Fill in missing values with lagged weekly moments using a rolling median of non-missing values of the week before;
- Linearly interpolate between the irregular maturities.

### 3.3 Variance premium and implied equity premium

The variance premium can now be directly computed as

$$VP_{t,T} = M_{2,t,T} - \kappa_t RV_{t,T}, \quad (34)$$

where  $M_{2,t,T}$  is the second risk-neutral moment obtained in the previous section.

Having obtained the remaining needed moments (up to the sixth), the GO portfolio weights are obtained as the coefficients of linear regressions as in equation (21).

There are several difficulties in this estimation. Firstly, the GO portfolio weights are time-varying, meaning that we must estimate them independently for each date. Moreover, the computed variance premiums are highly heteroskedastic, the risk-neutral moments are highly correlated and these measures are all very persistent. Finally, there is a theoretical approximation error due to the truncation at the  $K$ -th moment in the model.

To address these issues, the author proceeds as follows:

- Apply exponentially decaying weights on distant data during estimation, so that the half-life of these weights is 1000 calendar days (the same measure used in the fractional integration parameter estimation for the daily variances);
- Apply inverse variance weights to the errors in the least-squares estimations;
- Assume that the GO portfolio weights are constant across maturities, i.e.

$$w_{k,t,T_1} = w_{k,t,T_2} \quad (35)$$

for all  $T_1, T_2 = 30, 60, 90, 180, 360$ ;

- Allow for an intercept in the regressions to account for the average value of the higher order moments ignored by the  $K$ -th order approximation.
- Jointly estimate the variance premium regressions at all horizons.

For this purpose, a FGLS (feasible generalized least squares) regressor is used, with a minimum of one year of data.

Having obtained the GO portfolio weights, the Implied Equity Premium can be computed using equation (20).

## 4 Replication results

We now discuss the details of the replication and the results.

Our dataset consists of:

- 1102 daily observations of S&P500 option data from the Eikon Refinitiv database, which also includes risk-free rates for the comparable maturities, from January 2, 2018, to May 31, 2022;
- Intraday S&P500 prices on 1 minute intervals over the course of the same 1102 days.

Missing values on options data were removed. Only three consecutive datapoints of S&P500 prices were missing from the dataset, which were filled by linearly interpolating to avoid losing an entire day of data.

To replicate the methodology of the paper, we needed to make some further simplifications.

Due to us only having minute intraday data, our daily realized variance estimator consists of the average of just 5 subsamples. The MATLAB script `returns_matrix.m` performs some preparatory data treatment and formatting, returning a matrix with 1 minute intraday log-returns, which are then used in the `daily_variances.m` script to compute the daily realized variance estimates.

The fractionally integrated model estimation is then done in the R script `var_forecast.R`. We estimate the model using both a rolling time window of 252 trading days (1 year) and an expanding window, starting at 1 year of data and including more and more days as we move forward in time. This is merely for comparison purposes, as the expanding time window is used for later calculations.

In figures 1 and 2 we see how this parameter evolves over time. The parameter values of the expanding time window model follow more closely in range to those that are obtained by Tetlock in his original implementation. The effects of the COVID crisis spike on the estimation are also clearly visible. Figures 3 and 4 show the different daily estimates of expected realized variance over the annual and monthly horizons.

The MATLAB script `rn_moments.m` estimates the risk-neutral market moments from daily options data. We use only the available strikes, without any synthetic data creation, and the integral is approximated using the trapezoidal rule.

Moreover, we do not check for violations of the put-call parity, which might influence some of the results, especially around the COVID crisis, and do not filter out non-positive values for even moments or non-negative values for odd moments.

We interpolate linearly between the available maturities and extrapolate the yearly maturities, since we only have yearly options data. We also do the same procedure to obtain the necessary interest rates.

Figures 5 to 9 show the obtained risk-neutral moments over time. The possible violations of sign usually only happen at the 360-day horizon, but the collinearity of these time-series is clearly visible.

Furthermore, since these results are treated daily and independently, it is clear that the market's behavior has changed since the crisis, with the expected risk-neutral moments being much higher nowadays.

With these steps completed, we computed the variance premium, as shown in figure 12. The behavior closely mimics the one obtained by Tetlock, as seen in figure 1 of its appendix. However, the large negative value after the COVID crisis may be due to errors in variance estimation, such as the small sample available at that date, or the lack of thorough treatment of options data. Figures 10 and 11 show the two components simultaneously. The high parameter values of  $d_t$  make the

variance extremely persistent, leading to a very slow decay in expected variance until the fourth quarter of 2020.

The estimation of the GO portfolio weights was done by a pooled regression on our cross-sectional sample, without decaying weights on data, since the time span available is much shorter, and without precision adjusted errors, since it is also likely that the variance estimates aren't exact. This implementation is available on the MATLAB script `premiums.m`.

Figure 13 shows the evolution of the stock market weight on the GO portfolio for different approximations of the economy, and we obtain an interesting behavior in line with what was obtained in the original paper.

The effects of collinearity become apparent, however, in figure 14 showing the GO portfolio weights in the full ( $K = 4$ ) model.

Finally, figure 15 shows us the annualized estimated equity premiums over the different horizons. We obtain a similar pattern and strictly positive values. However, the magnitudes are much larger than expected, even if they become more sensible as the estimation moves away from 2020. This is clearly shown in figure 17, comparing our estimation of the implied equity premium with Martin's lower bound.

## 5 Conclusion

While we weren't able to exactly obtain the results of Tetlock, the implementation was successful at various steps, such as estimating risk-neutral market moments from options data and obtaining good estimates for the Implied Variance Premium.

More importantly, this project showcases the true sample size needed and the importance of proper estimation procedures when dealing with high frequency financial data.

A larger sample size would surely improve some of the results, as it is clear that the behavior improves closer to the end of the sample. However, the regressions to obtain the GO portfolio weights need to be handled with much more detail, since the pervasive issues of collinearity, heteroskedasticity and approximation error heavily bias the results.

## References

- [1] Tetlock, Paul C., The Implied Equity Premium (June 2023). Available at SSRN: <https://ssrn.com/abstract=4373579> or <http://dx.doi.org/10.2139/ssrn.4373579>
- [2] P. Carr & D. Madan (2001) Optimal positioning in derivative securities, *Quantitative Finance*, 1:1, 19-37, DOI: 10.1080/713665549
- [3] Long, John B. (1990), The numeraire portfolio, *Journal of Financial Economics* 26, 29-69.
- [4] Martin, Ian (2017), What is the expected return on the market?, *Quarterly Journal of Economics* 132, 367-433
- [5] Chang, Bo-Young, Peter Christoffersen, and Kris Jacobs (2013), Market skewness risk and the cross section of stock returns, *Journal of Financial Economics* 107, 46-68.

## A Figures

Figure 1: Parameter of fractionally integrated model of the variance on an expanding time window

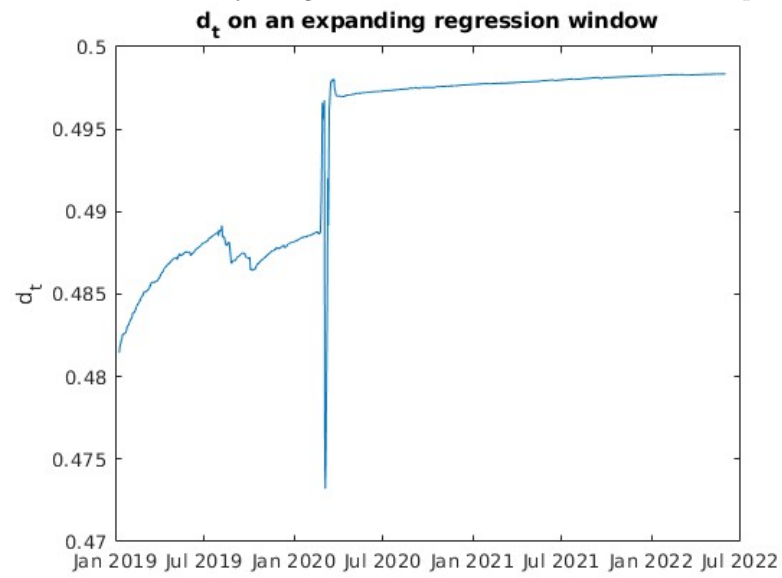


Figure 2: Parameter of fractionally integrated model of the variance on a rolling time window

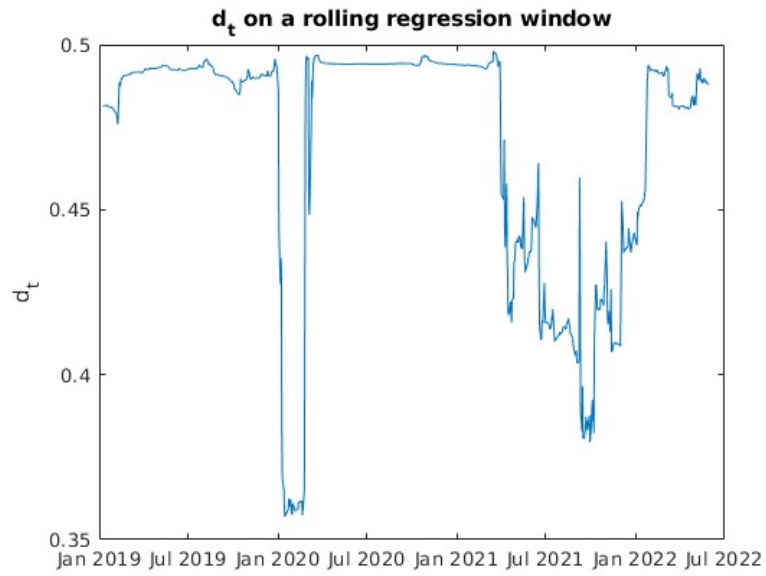


Figure 3: Forecasted annual variances with both parameter estimation methods

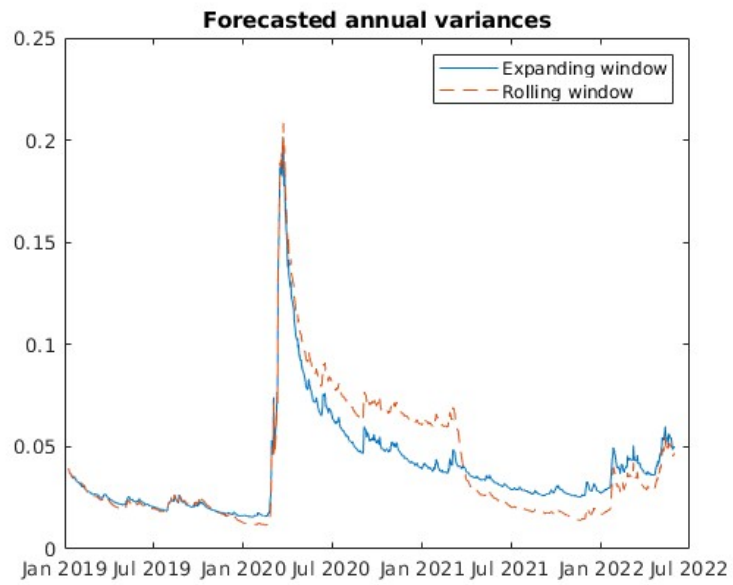


Figure 4: Forecasted annualized monthly variances with both parameter estimation methods

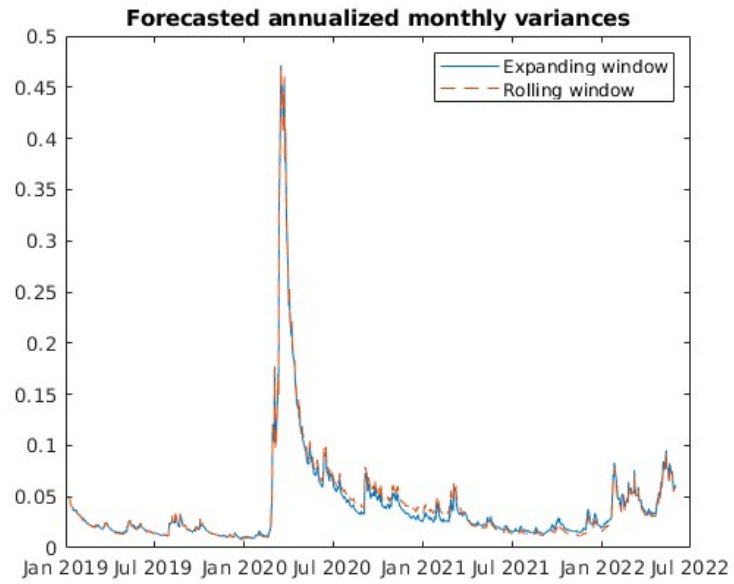


Figure 5: Annualized risk-neutral variance over different horizons

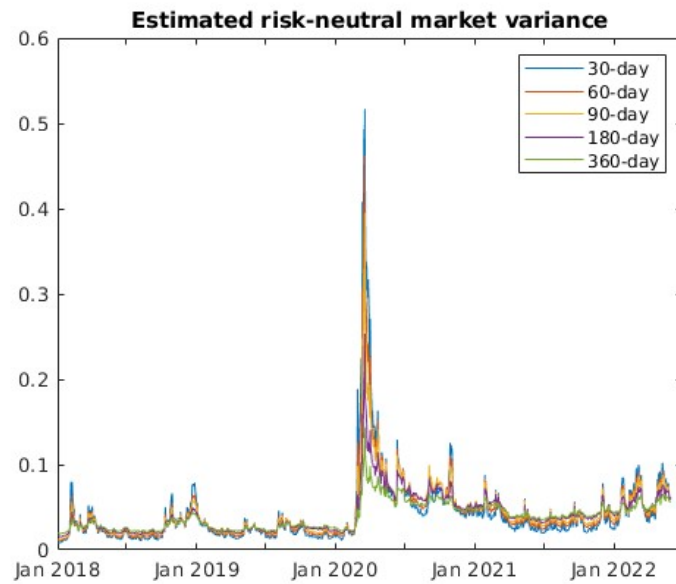


Figure 6: Annualized risk-neutral skewness over different horizons

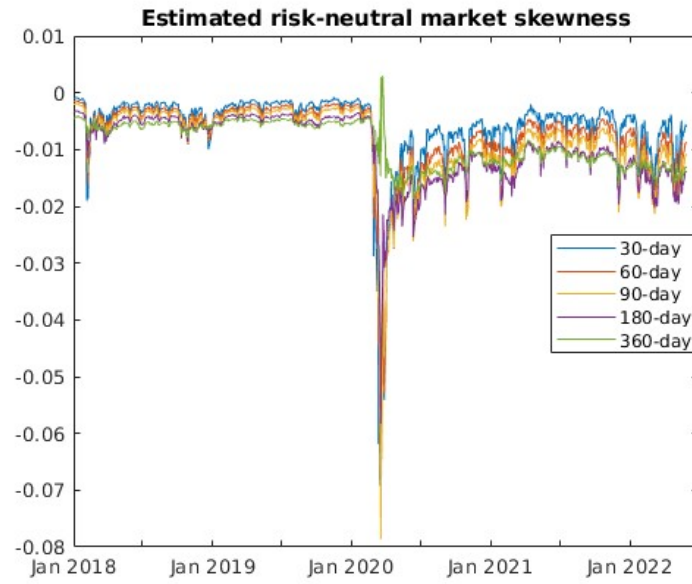


Figure 7: Annualized risk-neutral kurtosis over different horizons

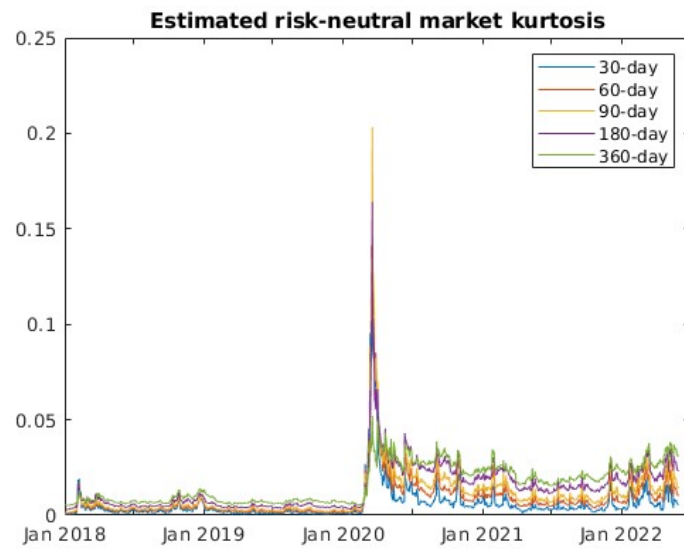


Figure 8: Annualized risk-neutral market moment of order 5 over different horizons

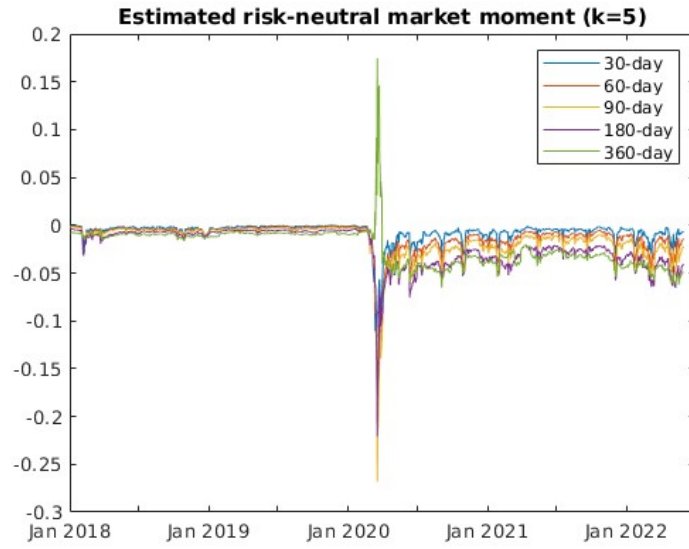


Figure 9: Annualized risk-neutral market moment of order 6 over different horizons

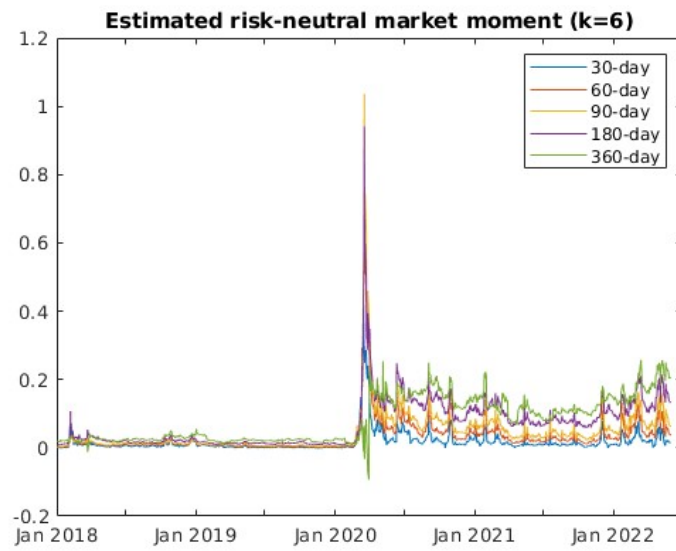




Figure 10: Risk-neutral variance vs forecasted realized variance (monthly)

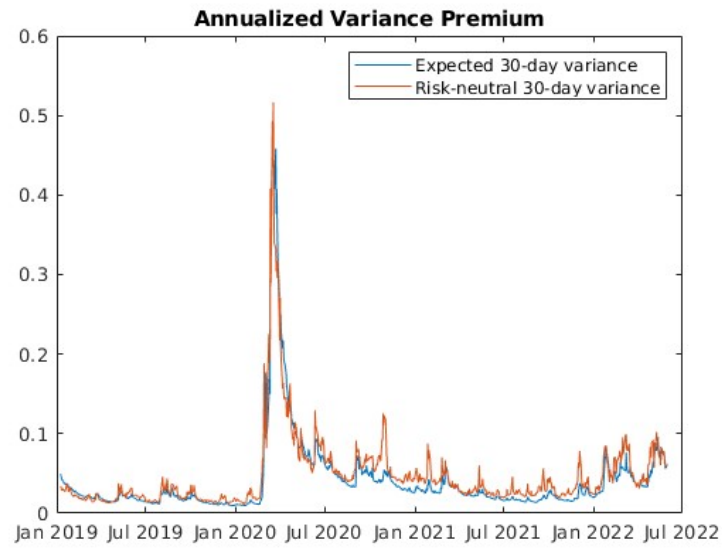


Figure 11: Risk-neutral variance vs forecasted realized variance (annual)

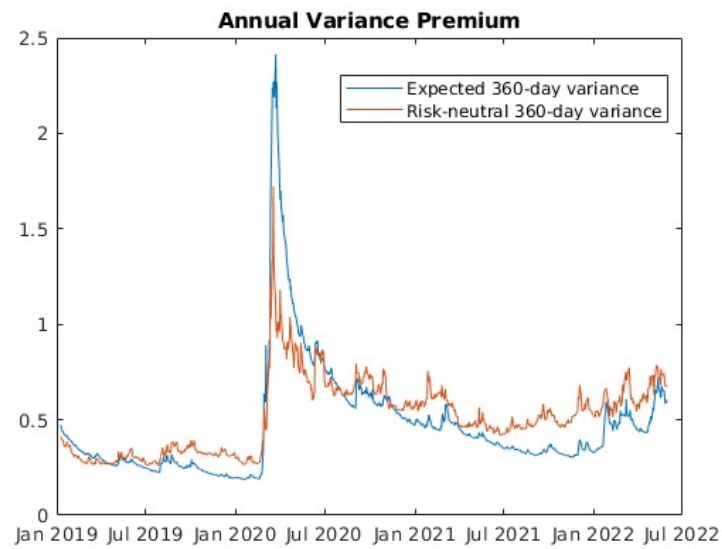


Figure 12: The Implied Variance Premium

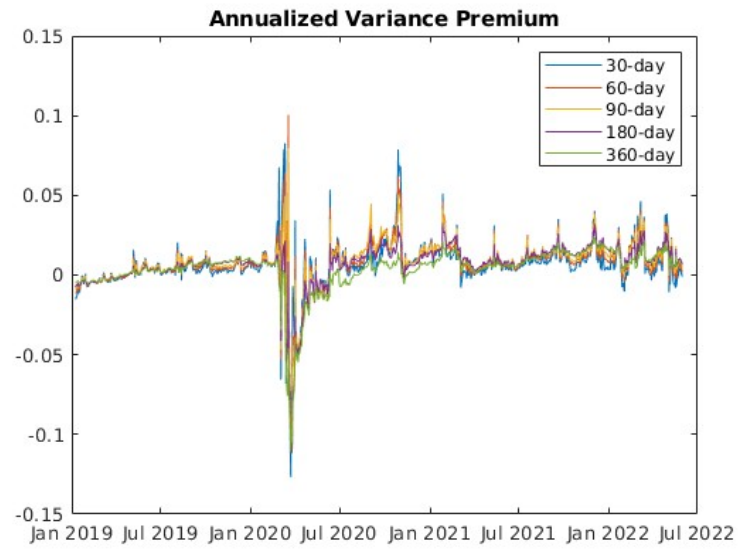


Figure 13: Market weight on the GO portfolio for different approximations

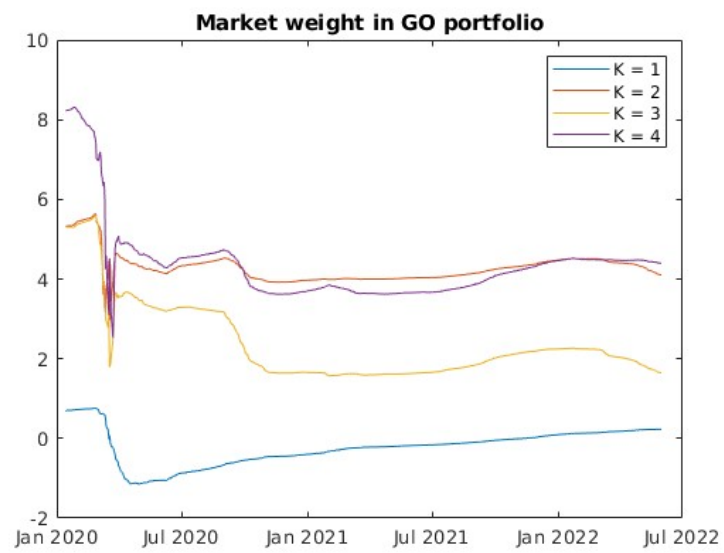


Figure 14: GO portfolio weights over time

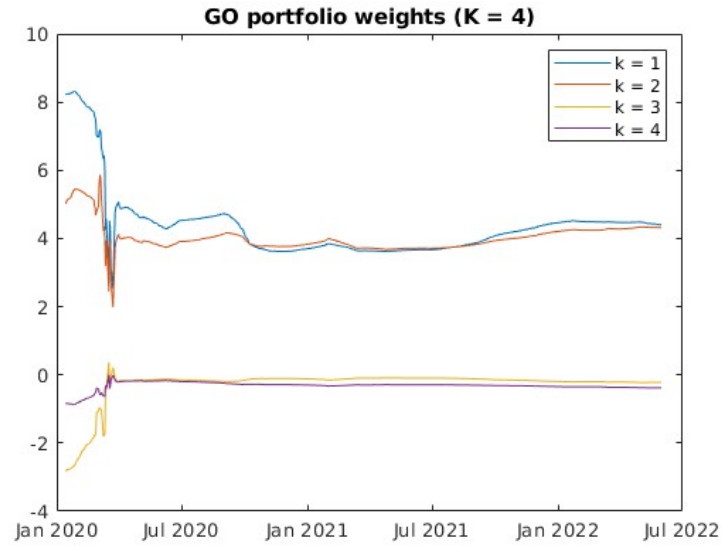


Figure 15: The Implied Equity Premium (annualized)

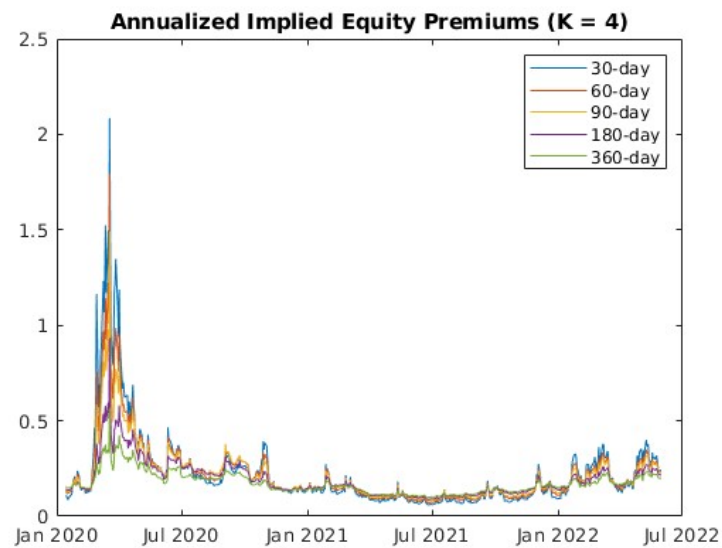


Figure 16: The Implied Equity Premium for different approximations

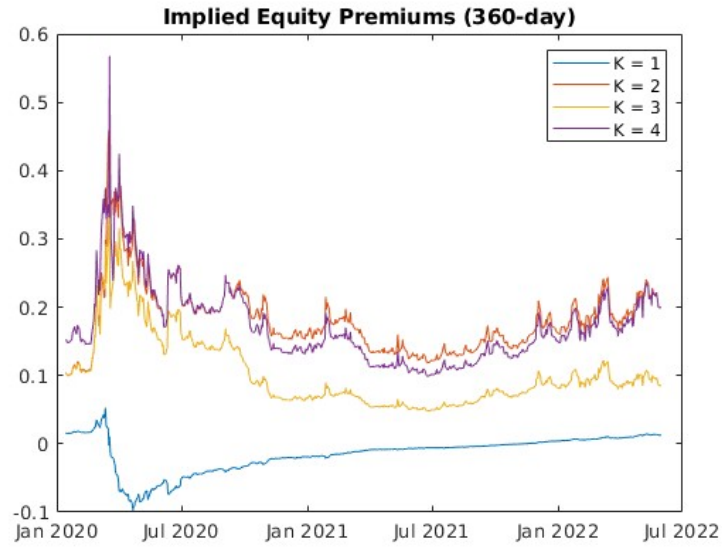


Figure 17: Martin's lower bound for the Implied Equity Premium

