

Simulation part 1

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Simulation exercise

Description

In this project the exponential distribution is investigated in R and it is compared with the Central Limit Theorem. The exponential distribution is simulated in R with `rexp(n, lambda)` where `lambda` is the rate parameter. The mean of exponential distribution is $1/\lambda$ and the standard deviation is also $1/\lambda$. Set $\lambda = 0.2$ for all of the simulations. It's investigated the distribution of averages of 40 exponentials.

Loading Libraries

The libraries needed are loaded

```
library("ggplot2")
```

Parameter definition

Reproducible seed is created. The `lambda` parameter(), the sample size (`n`), number of simulations (`simulations`) and the exponential distribution data (`df`) are created as well. Then the means for each row are calculated and saved in `means_df`

```
set.seed(10)
lambda <- 0.2
n <- 40
simulations <- 1000
df <- replicate(simulations, rexp(n, lambda))
means_df <- apply(df, 2, mean)
```

Distribution center

The simulated data center (`centers`) and theoretical center (`centert`) are calculated.

```
centert<-1/lambda
centers<-mean(means_df)
cat(sprintf("\n%f" "\n%f\n", centert, centers))
```

```
## "5.000000" "5.045060"
```

The results show that the center distribution of the data is around 5 (theoretical=5; simulated=5.04). With the simulated value really close to the theoretical value.

Variance of the distribution

The simulated data variance (variances) and theoretical variance (variancet) are calculated.

```
variancet<-1/(lambda^2*n)
variances<-var(means_df)
cat(sprintf("\n%f\n" "%f\n", variancet, variances))
```

```
## "0.625000" "0.637254"
```

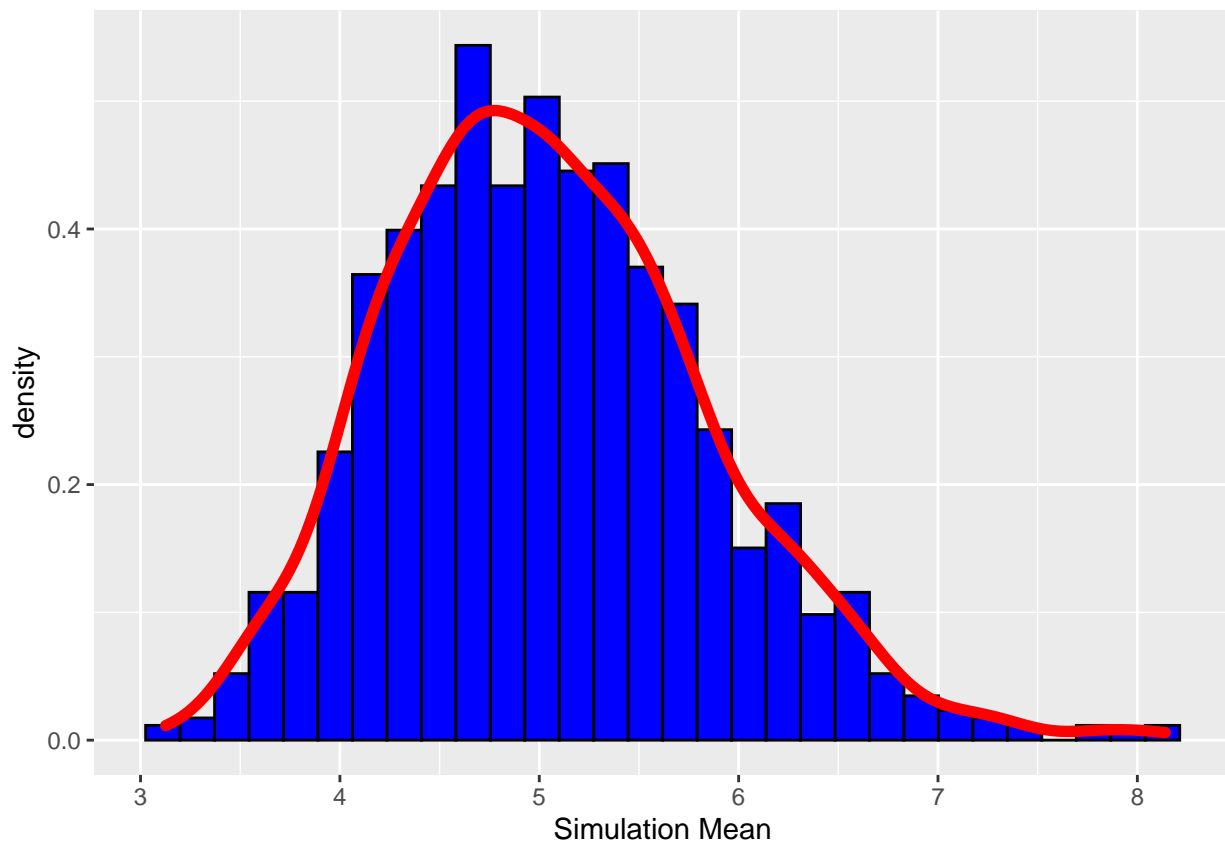
The results show that the variance of the data is around 0.625 (theoretical=0.625; simulated=0.637). With the simulated value really close to the theoretical value as well.

Normal distribution

With the values calculated before, the distribution of the data is shown

```
data<-data.frame(means_df)
hist<-ggplot(data, aes(x=means_df))+ xlab("Simulation Mean")
hist<-hist + geom_histogram(aes(y=..density..), color="black", fill="blue")
hist<-hist + geom_density(color = "red", size=2)
hist
```

```
## 'stat_bin()' using 'bins = 30'. Pick better value with 'binwidth'.
```



Due to the Central Limit Theorem (CLT), the distribution of averages of 40 exponentials is very close to a normal distribution.

Coverage for confidence interval (95% Confidence Interval)

Theoretical interval:

```
ti <- centert + c(-1,1)*1.96*sqrt(variancet)/sqrt(n)
ti
```

```
## [1] 4.755 5.245
```

Data interval:

```
di <- centers + c(-1,1)*1.96*sqrt(variances)/sqrt(n)
di
```

```
## [1] 4.797669 5.292450
```

Comparison of the distribution of averages of 40 samples with the normal distributios

```
# compare the distribution of averages of 40 exponentials to a normal distribution
qqnorm(means_df)
qqline(means_df, col = 2)
```

