

1.(a) what are the difference between DIT and DIF?

$X(k)$  computing an  $n$  points DFT requires multiplication & addition.

Q. Distinguish between DIT & DIF.

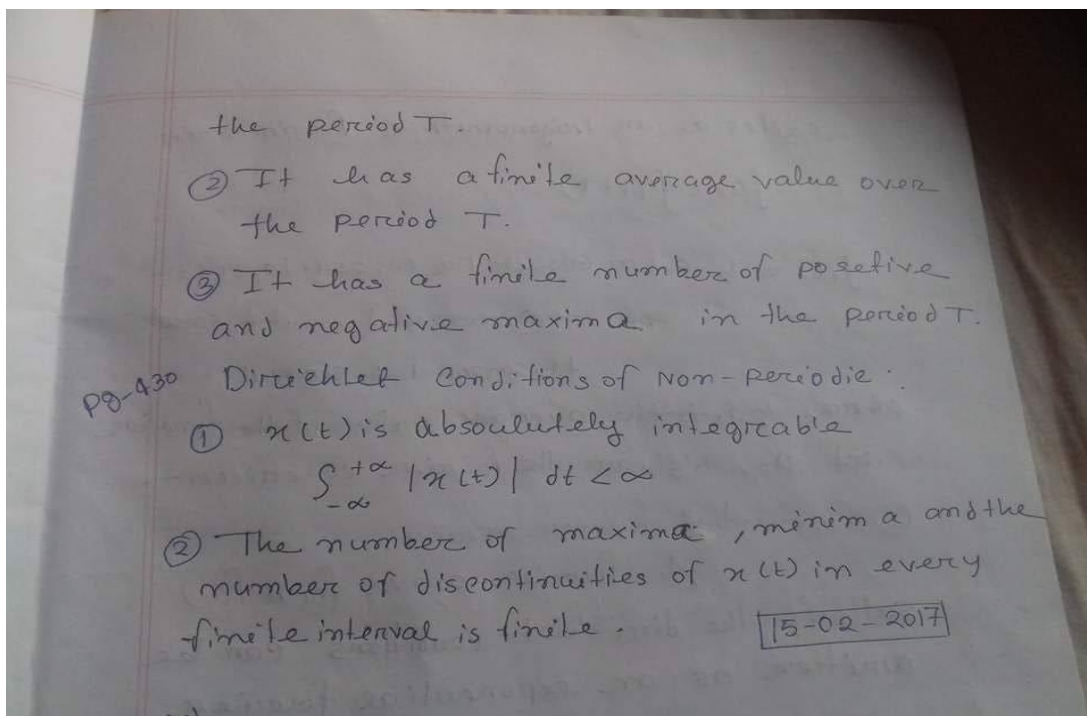
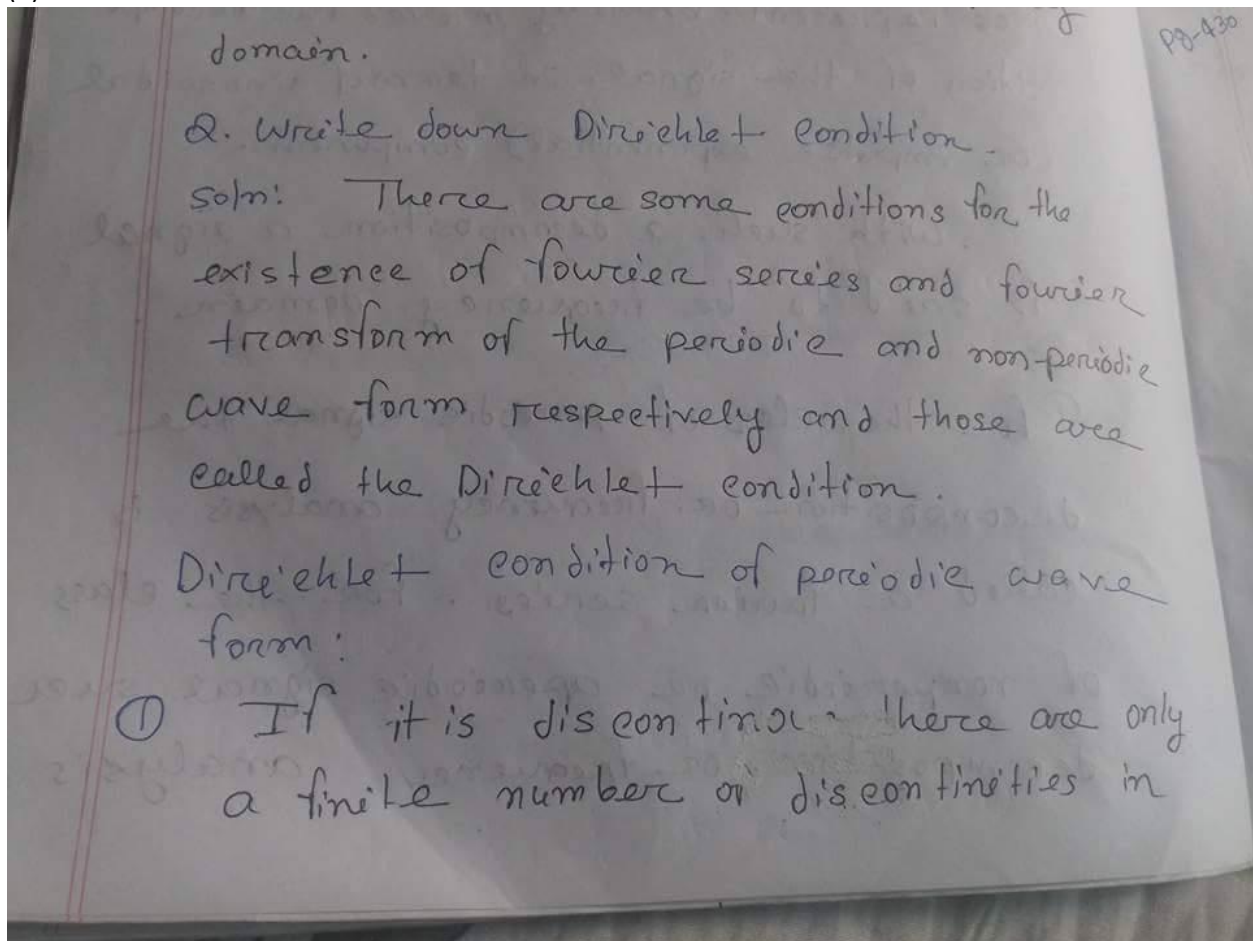
The full meaning of DIT & DIF is decimation in time and decimation in frequency respectively.

For DIT it is considered the individual samples (actually pair of samples) in time domain. And find their frequency parts and further to find FFT.

For DIF it is considered the individual samples in frequency domain. And combined them to come out with the actual FFT.

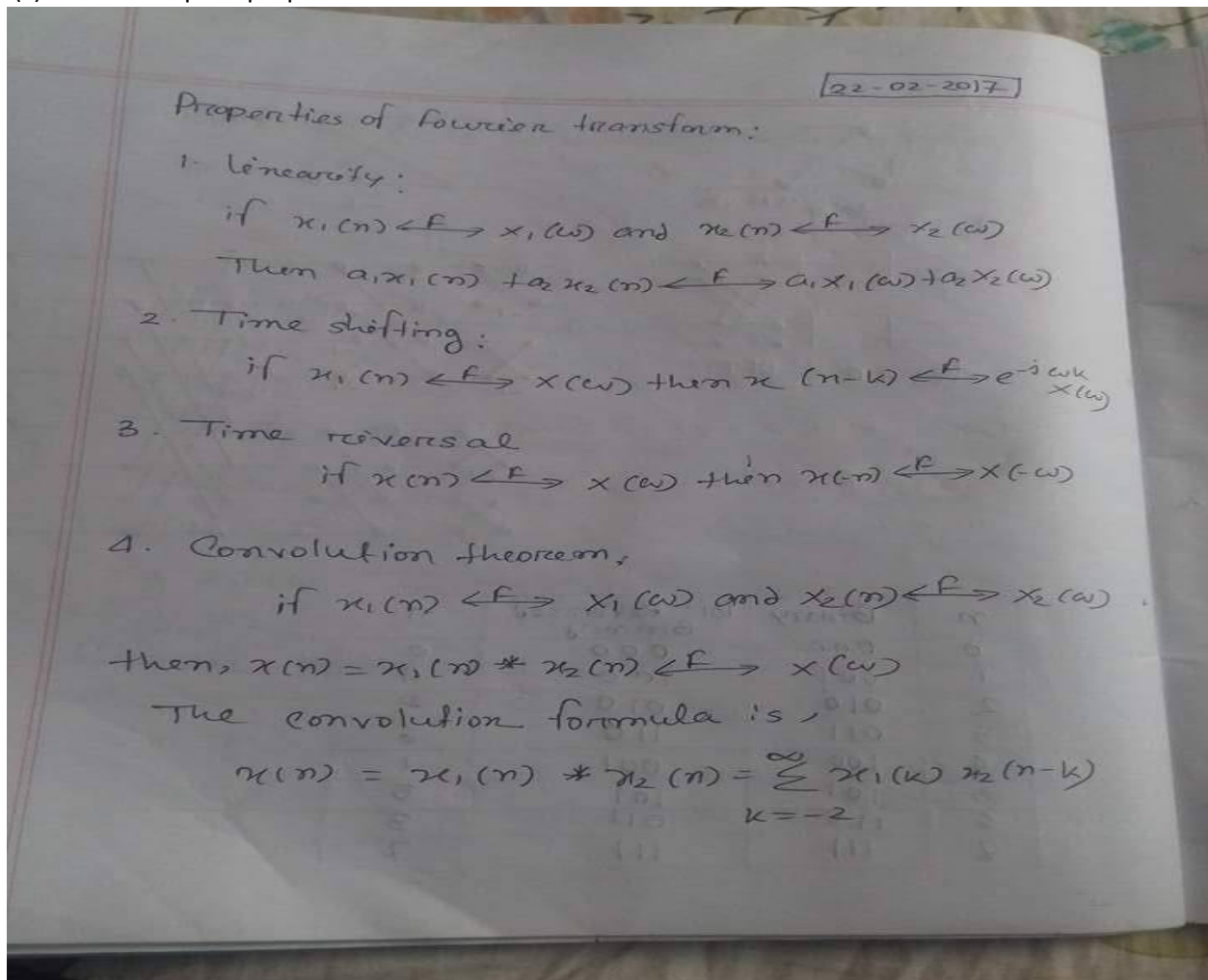
Butterfly operation: The basic computational unit of the FFT shown in fig 7.5(a) is called

(b) Write down the Dirichlet conditions.



smile

(c) State and explain properties of fourier transform



(V) Co-relation theorem:  
 If  $x_1(n) \xleftrightarrow{F} X_1(\omega)$  and  $x_2(n) \xleftrightarrow{F} X_2(\omega)$   
 Then  $r_{x_1, x_2}(n) \xleftrightarrow{F} S_{x_1, x_2}(\omega) = X_1(\omega) X_2^*(-\omega)$

In that case

$$r_{x_1, x_2}(n) = \sum_{k=-\infty}^{\infty} x_1(k) x_2^*(k-n)$$

(VI) Winerz khintcheng theorem

Let  $x(n)$  be a real integer then,

$$r_{xx}(l) \xleftrightarrow{F} S_{xx}(\omega)$$

(VII) Frequency shifting

If  $x(n) \xleftrightarrow{F} X(\omega)$  then

$$e^{j\omega_0 n} x(n) \xleftrightarrow{F} X(\omega - \omega_0)$$

(VIII) Modulation theorem:

If  $x(n) \xleftrightarrow{F} X(\omega)$

$$\text{Then } x(n) \cos \omega_0 n \xleftrightarrow{F} \frac{1}{2} [X(\omega + \omega_0) + X(\omega - \omega_0)]$$

(ix) Parseval's theorem:

If  $x_1(n) \xrightarrow{F} X_1(\omega)$  and  $x_2(n) \xrightarrow{F} X_2(\omega)$

$$\text{then, } \sum_{n=-\infty}^{\infty} x_1(n) x_2^*(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X_1(\omega) X_2^*(\omega) d\omega$$

(x) Multiplication of two sequences:

If  $x_1(n) \xrightarrow{F} X_1(\omega)$  and  $x_2(n) \xrightarrow{F} X_2(\omega)$

$$\begin{aligned} \text{Then } x_3(n) &= x_1(n) x_2(n) \xrightarrow{F} X_3(\omega) \\ &= \frac{1}{2\pi} \int_{-\pi}^{\pi} X_1(\lambda) X_2(\omega - \lambda) d\lambda \end{aligned}$$

Properties of DFD



(d) Define Butterfly operation and explain a radix-2 decimation in time FFT.

Question: How many multiplications and additions are required to compute N-point DFT using radix-2 FFT? [UNU: 2011]

Answer:

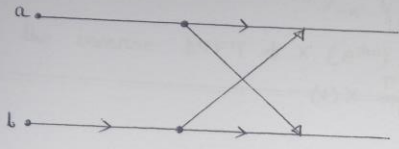


Fig: Butterfly Operation

Integ - an 8 point decimation-in-time FFT. This is so called because the alternate time sample are decimated by the process. We can see from this figure that an N-point FFT contains  $N/2$  butterflies per stage with  $\log_2 N$  stages giving a total of  $(N/2) \log_2 N$  butterflies.

This method of FFT calculation involves  $(N/2) \log_2 N$  complex multiplications and  $N \log_2 N$  complex additions providing a reduction in computations.

We observe that N-point DFT is computed using two length  $(N/2)$  DFTs and some extra operations called butterfly operations.

Following figure shows one of the butterfly operation. Each of those butterflies is a length 2-DFT breaking a length N DFT up into a two-dimensional length  $(2 \times N/2)$  DFT.

It involves one complex multiplication and two complex additions.

2(a) Describe different process of inverse Z transform.

1- The inverse z-transform By contour integration

The inverse z-transform can be directly be determined from ~~the~~ contour integral by using Cauchy-theorem.

In this method z-transform we have,

$$x(n) = \frac{1}{2\pi j} \oint_C x(z) z^{n-1} dz$$

$$= \sum_{\text{all poles } \{z_i\} \text{ inside } C} [\text{residue of } x(z) z^{n-1} \text{ at } z=z_i]$$

$$= \sum_i (z - z_i) x(z) \cdot z^{n-1} \Big|_{z=z_i}$$

We get the output from contour integration by using Cauchy's theorem.

② Inverse z-transform by power series expansion:

Let a z-transform  $x(z)$  with its corresponding ROC, we can expand  $x(z)$  into a power series of the form,

$$x(z) = \sum_{n=-\infty}^{\infty} c_n z^{-n}$$

With constants in the given ROC, then by the uniqueness of z-transform,

$$x(n) = c_n \text{ for all } n$$

When  $x(z)$  is rational the expansion can be performed by long division.



3.4.3 ③ The inverse  $z$ -transform by partial fraction expansion: / Table lookup method

In the table lookup method we attempt to express the function  $X(z)$  as a linear combination.

$$X(z) = \alpha_1 x_1(z) + \alpha_2 x_2(z) + \dots + \alpha_k x_k(z)$$

where  $x_1(z), x_2(z), \dots, x_k(z)$  are expressions with inverse  $z$  transform  $x_1(n), x_2(n), \dots, x_k(n)$  respectively available in

a table of  $z$ -transform pairs if such a decomposition is possible then  $x(n)$ , the inverse  $z$ -transform of  $X(z)$ , can easily be found using the linearity property as,

$$x(n) = \alpha_1 x_1(n) + \alpha_2 x_2(n) + \dots + \alpha_k x_k(n)$$

This approach is particularly useful if  $X(z)$  is rational function.

(b)

Define the inverse Z transform of

6

$$X(z) = \frac{1}{1 - 1.5z^{-1} + 0.5z^{-2}}$$

(a) ROC:  $|z| > 1$

(b) ROC:  $|z| < 0.5$

Answer: a) Since the ROC is the exterior of a circle, we expect  $x(n)$  to be a causal signal. Thus we seek a power series expansion in negative powers of  $z$ . By dividing the numerator of  $X(z)$  by its denominator, we obtain the power series,

$$\begin{array}{r} 1 - \frac{3}{2}z^{-1} + \frac{1}{2}z^{-2} \overline{) 1} \\ \underline{(-) 1 - \frac{3}{2}z^{-1} + \frac{1}{2}z^{-2}} \phantom{00} \\ \frac{3}{2}z^{-1} - \frac{1}{2}z^{-2} \phantom{00} \\ \underline{(-) \frac{3}{2}z^{-1} - \frac{9}{4}z^{-2} + \frac{3}{4}z^{-3}} \phantom{00} \\ \frac{7}{4}z^{-2} - \frac{3}{4}z^{-3} \phantom{00} \\ \underline{(-) \frac{7}{4}z^{-2} - \frac{21}{8}z^{-3} + \frac{7}{8}z^{-4}} \phantom{00} \\ \frac{15}{8}z^{-3} - \frac{7}{8}z^{-4} \phantom{00} \\ \underline{(-) \frac{15}{8}z^{-3} - \frac{15}{8}z^{-4} + \frac{15}{16}z^{-5}} \phantom{00} \\ \frac{7}{8}z^{-4} - \frac{15}{16}z^{-5} \phantom{00} \\ \underline{(-) \frac{7}{8}z^{-4} - \frac{14}{16}z^{-5} + \frac{7}{16}z^{-6}} \phantom{00} \\ \frac{1}{16}z^{-5} - \frac{7}{16}z^{-6} \phantom{00} \\ \underline{(-) \frac{1}{16}z^{-5} - \frac{1}{16}z^{-6} + \frac{1}{32}z^{-7}} \phantom{00} \\ \frac{1}{32}z^{-6} - \frac{1}{32}z^{-7} \phantom{00} \\ \underline{(-) \frac{1}{32}z^{-6} - \frac{1}{32}z^{-7} + \frac{1}{64}z^{-8}} \phantom{00} \\ \frac{1}{64}z^{-7} - \frac{1}{64}z^{-8} \phantom{00} \\ \underline{(-) \frac{1}{64}z^{-7} - \frac{1}{64}z^{-8} + \frac{1}{128}z^{-9}} \phantom{00} \\ \frac{1}{128}z^{-8} - \frac{1}{128}z^{-9} \phantom{00} \\ \underline{(-) \frac{1}{128}z^{-8} - \frac{1}{128}z^{-9} + \frac{1}{256}z^{-10}} \phantom{00} \\ \frac{1}{256}z^{-9} - \frac{1}{256}z^{-10} \phantom{00} \\ \underline{(-) \frac{1}{256}z^{-9} - \frac{1}{256}z^{-10} + \frac{1}{512}z^{-11}} \phantom{00} \\ \frac{1}{512}z^{-10} - \frac{1}{512}z^{-11} \phantom{00} \\ \underline{(-) \frac{1}{512}z^{-10} - \frac{1}{512}z^{-11} + \frac{1}{1024}z^{-12}} \phantom{00} \\ \frac{1}{1024}z^{-11} - \frac{1}{1024}z^{-12} \phantom{00} \\ \underline{(-) \frac{1}{1024}z^{-11} - \frac{1}{1024}z^{-12} + \frac{1}{2048}z^{-13}} \phantom{00} \\ \frac{1}{2048}z^{-12} - \frac{1}{2048}z^{-13} \phantom{00} \\ \underline{(-) \frac{1}{2048}z^{-12} - \frac{1}{2048}z^{-13} + \frac{1}{4096}z^{-14}} \phantom{00} \\ \frac{1}{4096}z^{-13} - \frac{1}{4096}z^{-14} \phantom{00} \\ \underline{(-) \frac{1}{4096}z^{-13} - \frac{1}{4096}z^{-14} + \frac{1}{8192}z^{-15}} \phantom{00} \\ \frac{1}{8192}z^{-14} - \frac{1}{8192}z^{-15} \phantom{00} \\ \underline{(-) \frac{1}{8192}z^{-14} - \frac{1}{8192}z^{-15} + \frac{1}{16384}z^{-16}} \phantom{00} \\ \frac{1}{16384}z^{-15} - \frac{1}{16384}z^{-16} \phantom{00} \\ \underline{(-) \frac{1}{16384}z^{-15} - \frac{1}{16384}z^{-16} + \frac{1}{32768}z^{-17}} \phantom{00} \\ \frac{1}{32768}z^{-16} - \frac{1}{32768}z^{-17} \phantom{00} \\ \underline{(-) \frac{1}{32768}z^{-16} - \frac{1}{32768}z^{-17} + \frac{1}{65536}z^{-18}} \phantom{00} \\ \frac{1}{65536}z^{-17} - \frac{1}{65536}z^{-18} \phantom{00} \\ \underline{(-) \frac{1}{65536}z^{-17} - \frac{1}{65536}z^{-18} + \frac{1}{131072}z^{-19}} \phantom{00} \\ \frac{1}{131072}z^{-18} - \frac{1}{131072}z^{-19} \phantom{00} \\ \underline{(-) \frac{1}{131072}z^{-18} - \frac{1}{131072}z^{-19} + \frac{1}{262144}z^{-20}} \phantom{00} \\ \frac{1}{262144}z^{-19} - \frac{1}{262144}z^{-20} \phantom{00} \\ \underline{(-) \frac{1}{262144}z^{-19} - \frac{1}{262144}z^{-20} + \frac{1}{524288}z^{-21}} \phantom{00} \\ \frac{1}{524288}z^{-20} - \frac{1}{524288}z^{-21} \phantom{00} \\ \underline{(-) \frac{1}{524288}z^{-20} - \frac{1}{524288}z^{-21} + \frac{1}{1048576}z^{-22}} \phantom{00} \\ \frac{1}{1048576}z^{-21} - \frac{1}{1048576}z^{-22} \phantom{00} \\ \underline{(-) \frac{1}{1048576}z^{-21} - \frac{1}{1048576}z^{-22} + \frac{1}{2097152}z^{-23}} \phantom{00} \\ \frac{1}{2097152}z^{-22} - \frac{1}{2097152}z^{-23} \phantom{00} \\ \underline{(-) \frac{1}{2097152}z^{-22} - \frac{1}{2097152}z^{-23} + \frac{1}{4194304}z^{-24}} \phantom{00} \\ \frac{1}{4194304}z^{-23} - \frac{1}{4194304}z^{-24} \phantom{00} \\ \underline{(-) \frac{1}{4194304}z^{-23} - \frac{1}{4194304}z^{-24} + \frac{1}{8388608}z^{-25}} \phantom{00} \\ \frac{1}{8388608}z^{-24} - \frac{1}{8388608}z^{-25} \phantom{00} \\ \underline{(-) \frac{1}{8388608}z^{-24} - \frac{1}{8388608}z^{-25} + \frac{1}{16777216}z^{-26}} \phantom{00} \\ \frac{1}{16777216}z^{-25} - \frac{1}{16777216}z^{-26} \phantom{00} \\ \underline{(-) \frac{1}{16777216}z^{-25} - \frac{1}{16777216}z^{-26} + \frac{1}{33554432}z^{-27}} \phantom{00} \\ \frac{1}{33554432}z^{-26} - \frac{1}{33554432}z^{-27} \phantom{00} \\ \underline{(-) \frac{1}{33554432}z^{-26} - \frac{1}{33554432}z^{-27} + \frac{1}{67108864}z^{-28}} \phantom{00} \\ \frac{1}{67108864}z^{-27} - \frac{1}{67108864}z^{-28} \phantom{00} \\ \underline{(-) \frac{1}{67108864}z^{-27} - \frac{1}{67108864}z^{-28} + \frac{1}{134217728}z^{-29}} \phantom{00} \\ \frac{1}{134217728}z^{-28} - \frac{1}{134217728}z^{-29} \phantom{00} \\ \underline{(-) \frac{1}{134217728}z^{-28} - \frac{1}{134217728}z^{-29} + \frac{1}{268435456}z^{-30}} \phantom{00} \\ \frac{1}{268435456}z^{-29} - \frac{1}{268435456}z^{-30} \phantom{00} \\ \underline{(-) \frac{1}{268435456}z^{-29} - \frac{1}{268435456}z^{-30} + \frac{1}{536870912}z^{-31}} \phantom{00} \\ \frac{1}{536870912}z^{-30} - \frac{1}{536870912}z^{-31} \phantom{00} \\ \underline{(-) \frac{1}{536870912}z^{-30} - \frac{1}{536870912}z^{-31} + \frac{1}{1073741824}z^{-32}} \phantom{00} \\ \frac{1}{1073741824}z^{-31} - \frac{1}{1073741824}z^{-32} \phantom{00} \\ \underline{(-) \frac{1}{1073741824}z^{-31} - \frac{1}{1073741824}z^{-32} + \frac{1}{2147483648}z^{-33}} \phantom{00} \\ \frac{1}{2147483648}z^{-32} - \frac{1}{2147483648}z^{-33} \phantom{00} \\ \underline{(-) \frac{1}{2147483648}z^{-32} - \frac{1}{2147483648}z^{-33} + \frac{1}{4294967296}z^{-34}} \phantom{00} \\ \frac{1}{4294967296}z^{-33} - \frac{1}{4294967296}z^{-34} \phantom{00} \\ \underline{(-) \frac{1}{4294967296}z^{-33} - \frac{1}{4294967296}z^{-34} + \frac{1}{8589934592}z^{-35}} \phantom{00} \\ \frac{1}{8589934592}z^{-34} - \frac{1}{8589934592}z^{-35} \phantom{00} \\ \underline{(-) \frac{1}{8589934592}z^{-34} - \frac{1}{8589934592}z^{-35} + \frac{1}{17179869184}z^{-36}} \phantom{00} \\ \frac{1}{17179869184}z^{-35} - \frac{1}{17179869184}z^{-36} \phantom{00} \\ \underline{(-) \frac{1}{17179869184}z^{-35} - \frac{1}{17179869184}z^{-36} + \frac{1}{34359738368}z^{-37}} \phantom{00} \\ \frac{1}{34359738368}z^{-36} - \frac{1}{34359738368}z^{-37} \phantom{00} \\ \underline{(-) \frac{1}{34359738368}z^{-36} - \frac{1}{34359738368}z^{-37} + \frac{1}{68719476736}z^{-38}} \phantom{00} \\ \frac{1}{68719476736}z^{-37} - \frac{1}{68719476736}z^{-38} \phantom{00} \\ \underline{(-) \frac{1}{68719476736}z^{-37} - \frac{1}{68719476736}z^{-38} + \frac{1}{137438953472}z^{-39}} \phantom{00} \\ \frac{1}{137438953472}z^{-38} - \frac{1}{137438953472}z^{-39} \phantom{00} \\ \underline{(-) \frac{1}{137438953472}z^{-38} - \frac{1}{137438953472}z^{-39} + \frac{1}{274877906944}z^{-40}} \phantom{00} \\ \frac{1}{274877906944}z^{-39} - \frac{1}{274877906944}z^{-40} \phantom{00} \\ \underline{(-) \frac{1}{274877906944}z^{-39} - \frac{1}{274877906944}z^{-40} + \frac{1}{549755813888}z^{-41}} \phantom{00} \\ \frac{1}{549755813888}z^{-40} - \frac{1}{549755813888}z^{-41} \phantom{00} \\ \underline{(-) \frac{1}{549755813888}z^{-40} - \frac{1}{549755813888}z^{-41} + \frac{1}{1099511627776}z^{-42}} \phantom{00} \\ \frac{1}{1099511627776}z^{-41} - \frac{1}{1099511627776}z^{-42} \phantom{00} \\ \underline{(-) \frac{1}{1099511627776}z^{-41} - \frac{1}{1099511627776}z^{-42} + \frac{1}{2199023255552}z^{-43}} \phantom{00} \\ \frac{1}{2199023255552}z^{-42} - \frac{1}{2199023255552}z^{-43} \phantom{00} \\ \underline{(-) \frac{1}{2199023255552}z^{-42} - \frac{1}{2199023255552}z^{-43} + \frac{1}{4398046511104}z^{-44}} \phantom{00} \\ \frac{1}{4398046511104}z^{-43} - \frac{1}{4398046511104}z^{-44} \phantom{00} \\ \underline{(-) \frac{1}{4398046511104}z^{-43} - \frac{1}{4398046511104}z^{-44} + \frac{1}{8796093022208}z^{-45}} \phantom{00} \\ \frac{1}{8796093022208}z^{-44} - \frac{1}{8796093022208}z^{-45} \phantom{00} \\ \underline{(-) \frac{1}{8796093022208}z^{-44} - \frac{1}{8796093022208}z^{-45} + \frac{1}{17592186044416}z^{-46}} \phantom{00} \\ \frac{1}{17592186044416}z^{-45} - \frac{1}{17592186044416}z^{-46} \phantom{00} \\ \underline{(-) \frac{1}{17592186044416}z^{-45} - \frac{1}{17592186044416}z^{-46} + \frac{1}{35184372088832}z^{-47}} \phantom{00} \\ \frac{1}{35184372088832}z^{-46} - \frac{1}{35184372088832}z^{-47} \phantom{00} \\ \underline{(-) \frac{1}{35184372088832}z^{-46} - \frac{1}{35184372088832}z^{-47} + \frac{1}{70368744177664}z^{-48}} \phantom{00} \\ \frac{1}{70368744177664}z^{-47} - \frac{1}{70368744177664}z^{-48} \phantom{00} \\ \underline{(-) \frac{1}{70368744177664}z^{-47} - \frac{1}{70368744177664}z^{-48} + \frac{1}{140737488355328}z^{-49}} \phantom{00} \\ \frac{1}{140737488355328}z^{-48} - \frac{1}{140737488355328}z^{-49} \phantom{00} \\ \underline{(-) \frac{1}{140737488355328}z^{-48} - \frac{1}{140737488355328}z^{-49} + \frac{1}{281474976710656}z^{-50}} \phantom{00} \\ \frac{1}{281474976710656}z^{-49} - \frac{1}{281474976710656}z^{-50} \phantom{00} \\ \underline{(-) \frac{1}{281474976710656}z^{-49} - \frac{1}{281474976710656}z^{-50} + \frac{1}{562949953421312}z^{-51}} \phantom{00} \\ \frac{1}{562949953421312}z^{-50} - \frac{1}{562949953421312}z^{-51} \phantom{00} \\ \underline{(-) \frac{1}{562949953421312}z^{-50} - \frac{1}{562949953421312}z^{-51} + \frac{1}{1125899906842624}z^{-52}} \phantom{00} \\ \frac{1}{1125899906842624}z^{-51} - \frac{1}{1125899906842624}z^{-52} \phantom{00} \\ \underline{(-) \frac{1}{1125899906842624}z^{-51} - \frac{1}{1125899906842624}z^{-52} + \frac{1}{2251799813685248}z^{-53}} \phantom{00} \\ \frac{1}{2251799813685248}z^{-52} - \frac{1}{2251799813685248}z^{-53} \phantom{00} \\ \underline{(-) \frac{1}{2251799813685248}z^{-52} - \frac{1}{2251799813685248}z^{-53} + \frac{1}{4503599627370496}z^{-54}} \phantom{00} \\ \frac{1}{4503599627370496}z^{-53} - \frac{1}{4503599627370496}z^{-54} \phantom{00} \\ \underline{(-) \frac{1}{4503599627370496}z^{-53} - \frac{1}{4503599627370496}z^{-54} + \frac{1}{9007199254740992}z^{-55}} \phantom{00} \\ \frac{1}{9007199254740992}z^{-54} - \frac{1}{9007199254740992}z^{-55} \phantom{00} \\ \underline{(-) \frac{1}{9007199254740992}z^{-54} - \frac{1}{9007199254740992}z^{-55} + \frac{1}{18014398509481984}z^{-56}} \phantom{00} \\ \frac{1}{18014398509481984}z^{-55} - \frac{1}{18014398509481984}z^{-56} \phantom{00} \\ \underline{(-) \frac{1}{18014398509481984}z^{-55} - \frac{1}{18014398509481984}z^{-56} + \frac{1}{36028797018963968}z^{-57}} \phantom{00} \\ \frac{1}{36028797018963968}z^{-56} - \frac{1}{36028797018963968}z^{-57} \phantom{00} \\ \underline{(-) \frac{1}{36028797018963968}z^{-56} - \frac{1}{36028797018963968}z^{-57} + \frac{1}{72057594037927936}z^{-58}} \phantom{00} \\ \frac{1}{72057594037927936}z^{-57} - \frac{1}{72057594037927936}z^{-58} \phantom{00} \\ \underline{(-) \frac{1}{72057594037927936}z^{-57} - \frac{1}{72057594037927936}z^{-58} + \frac{1}{144115188075855872}z^{-59}} \phantom{00} \\ \frac{1}{144115188075855872}z^{-58} - \frac{1}{144115188075855872}z^{-59} \phantom{00} \\ \underline{(-) \frac{1}{144115188075855872}z^{-58} - \frac{1}{144115188075855872}z^{-59} + \frac{1}{288230376151711744}z^{-60}} \phantom{00} \\ \frac{1}{288230376151711744}z^{-59} - \frac{1}{288230376151711744}z^{-60} \phantom{00} \\ \underline{(-) \frac{1}{288230376151711744}z^{-59} - \frac{1}{288230376151711744}z^{-60} + \frac{1}{576460752303423488}z^{-61}} \phantom{00} \\ \frac{1}{576460752303423488}z^{-60} - \frac{1}{576460752303423488}z^{-61} \phantom{00} \\ \underline{(-) \frac{1}{576460752303423488}z^{-60} - \frac{1}{576460752303423488}z^{-61} + \frac{1}{1152921504606846976}z^{-62}} \phantom{00} \\ \frac{1}{1152921504606846976}z^{-61} - \frac{1}{1152921504606846976}z^{-62} \phantom{00} \\ \underline{(-) \frac{1}{1152921504606846976}z^{-61} - \frac{1}{1152921504606846976}z^{-62} + \frac{1}{2305843009213693952}z^{-63}} \phantom{00} \\ \frac{1}{2305843009213693952}z^{-62} - \frac{1}{2305843009213693952}z^{-63} \phantom{00} \\ \underline{(-) \frac{1}{2305843009213693952}z^{-62} - \frac{1}{2305843009213693952}z^{-63} + \frac{1}{4611686018427387904}z^{-64}} \phantom{00} \\ \frac{1}{4611686018427387904}z^{-63} - \frac{1}{4611686018427387904}z^{-64} \phantom{00} \\ \underline{(-) \frac{1}{4611686018427387904}z^{-63} - \frac{1}{4611686018427387904}z^{-64} + \frac{1}{9223372036854775808}z^{-65}} \phantom{00} \\ \frac{1}{9223372036854775808}z^{-64} - \frac{1}{9223372036854775808}z^{-65} \phantom{00} \\ \underline{(-) \frac{1}{9223372036854775808}z^{-64} - \frac{1}{9223372036854775808}z^{-65} + \frac{1}{18446744073709551616}z^{-66}} \phantom{00} \\ \frac{1}{18446744073709551616}z^{-65} - \frac{1}{18446744073709551616}z^{-66} \phantom{00} \\ \underline{(-) \frac{1}{18446744073709551616}z^{-65} - \frac{1}{18446744073709551616}z^{-66} + \frac{1}{36893488147419103232}z^{-67}} \phantom{00} \\ \frac{1}{36893488147419103232}z^{-66} - \frac{1}{36893488147419103232}z^{-67} \phantom{00} \\ \underline{(-) \frac{1}{36893488147419103232}z^{-66} - \frac{1}{36893488147419103232}z^{-67} + \frac{1}{73786976294838206464}z^{-68}} \phantom{00} \\ \frac{1}{73786976294838206464}z^{-67} - \frac{1}{73786976294838206464}z^{-68} \phantom{00} \\ \underline{(-) \frac{1}{73786976294838206464}z^{-67} - \frac{1}{73786976294838206464}z^{-68} + \frac{1}{147573952589676412928}z^{-69}} \phantom{00} \\ \frac{1}{147573952589676412928}z^{-68} - \frac{1}{147573952589676412928}z^{-69} \phantom{00} \\ \underline{(-) \frac{1}{147573952589676412928}z^{-68} - \frac{1}{147573952589676412928}z^{-69} + \frac{1}{295147905179352825856}z^{-70}} \phantom{00} \\ \frac{1}{295147905179352825856}z^{-69} - \frac{1}{295147905179352825856}z^{-70} \phantom{00} \\ \underline{(-) \frac{1}{295147905179352825856}z^{-69} - \frac{1}{295147905179352825856}z^{-70} + \frac{1}{590295810358705651712}z^{-71}} \phantom{00} \\ \frac{1}{590295810358705651712}z^{-70} - \frac{1}{590295810358705651712}z^{-71} \phantom{00} \\ \underline{(-) \frac{1}{590295810358705651712}z^{-70} - \frac{1}{590295810358705651712}z^{-71} + \frac{1}{1180591620717411303424}z^{-72}} \phantom{00} \\ \frac{1}{1180591620717411303424}z^{-71} - \frac{1}{1180591620717411303424}z^{-72} \phantom{00} \\ \underline{(-) \frac{1}{1180591620717411303424}z^{-71} - \frac{1}{1180591620717411303424}z^{-72} + \frac{1}{2361183241434822606848}z^{-73}} \phantom{00} \\ \frac{1}{2361183241434822606848}z^{-72} - \frac{1}{2361183241434822606848}z^{-73} \phantom{00} \\ \underline{(-) \frac{1}{2361183241434822606848}z^{-72} - \frac{1}{2361183241434822606848}z^{-73} + \frac{1}{4722366482869645213696}z^{-74}} \phantom{00} \\ \frac{1}{4722366482869645213696}z^{-73} - \frac{1}{4722366482869645213696}z^{-74} \phantom{00} \\ \underline{(-) \frac{1}{4722366482869645213696}z^{-73} - \frac{1}{4722366482869645213696}z^{-74} + \frac{1}{9444732965739290427392}z^{-75}} \phantom{00} \\ \frac{1}{9444732965739290427392}z^{-74} - \frac{1}{9444732965739290427392}z^{-75} \phantom{00} \\ \underline{(-) \frac{1}{9444732965739290427392}z^{-74} - \frac{1}{9444732965739290427392}z^{-75} + \frac{1}{18889465931478580854784}z^{-76}} \phantom{00} \\ \frac{1}{18889465931478580854784}z^{-75} - \frac{1}{18889465931478580854784}z^{-76} \phantom{00} \\ \underline{(-) \frac{1}{18889465931478580854784}z^{-75} - \frac{1}{18889465931478580854784}z^{-76} + \frac{1}{37778931862957161709568}z^{-77}} \phantom{00} \\ \frac{1}{37778931862957161709568}z^{-76} - \frac{1}{37778931862957161709568}z^{-77} \phantom{00} \\ \underline{(-) \frac{1}{37778931862957161709568}z^{-76} - \frac{1}{37778931862957161709568}z^{-77} + \frac{1}{75557863725914323419136}z^{-78}} \phantom{00} \\ \frac{1}{75557863725914323419136}z^{-77} - \frac{1}{75557863725914323419136}z^{-78} \phantom{00} \\ \underline{(-) \frac{1}{75557863725914323419136}z^{-77} - \frac{1}{75557863725914323419136}z^{-78} + \frac{1}{151115727451828646838272}z^{-79}} \phantom{00} \\ \frac{1}{151115727451828646838272}z^{-78} - \frac{1}{151115727451828646838272}z^{-79} \phantom{00} \\ \underline{(-) \frac{1}{151115727451828646838272}z^{-78} - \frac{1}{151115727451828646838272}z^{-79} + \frac{1}{302231454903657293676544}z^{-80}} \phantom{00} \\ \frac{1}{302231454903657293676544}z^{-79} - \frac{1}{302231454903657293676544}z^{-80} \phantom{00} \\ \underline{(-) \frac{1}{302231454903657293676544}z^{-79} - \frac{1}{302231454903657293676544}z^{-80} + \frac{1}{604462909807314587353088}z^{-81}} \phantom{00} \\ \frac{1}{604462909807314587353088}z^{-80} - \frac{1}{604462909807314587353088}z^{-81} \phantom{00} \\ \underline{(-) \frac{1}{604462909807314587353088}z^{-80} - \frac{1}{604462909807314587353088}z^{-81} + \frac{1}{1208925819614629174706176}z^{-82}} \phantom{00} \\ \frac{1}{1208925819614629174706176}z^{-81} - \frac{1}{1208925819614629174706176}z^{-82} \phantom{00} \\ \underline{(-) \frac{1}{1208925819614629174706176}z^{-81} - \frac{1}{1208925819614629174706176}z^{-82} + \frac{1}{2417851639229258349412352}z^{-83}} \phantom{00} \\ \frac{1}{2417851639229258349412352}z^{-82} - \frac{1}{2417851639229258349412352}z^{-83} \phantom{00} \\ \underline{(-) \frac{1}{2417851639229258349412352}z^{-82} - \frac{1}{2417851639229258349412352}z^{-83} + \frac{1}{4835703278458516698824704}z^{-84}} \phantom{00} \\ \frac{1}{4835703278458516698824704}z^{-83} - \frac{1}{4835703278458516698824704}z^{-84} \phantom{00} \\ \underline{(-) \frac{1}{4835703278458516698824704}z^{-83} - \frac{1}{4835703278458516698824704}z^{-84} + \frac{1}{9671406556917033397649408}z^{-85}} \phantom{00} \\ \frac$$

$$\begin{array}{r}
 2z^2 + 6z^3 + 14z^4 + 30z^5 + 62z^6 + \dots \\
 \hline
 \frac{1}{2}z^{-2} - \frac{3}{2}z^{-1} + 1 \quad \bigg) 1 \\
 \underline{1 - 3z + 2z^2} \\
 3z - 2z^2 \\
 \underline{3z - 9z^2 + 6z^3} \\
 7z^2 - 6z^3 \\
 \underline{7z^2 - 21z^3 + 14z^4} \\
 15z^3 - 14z^4 \\
 \underline{15z^3 - 45z^4 + 30z^5} \\
 31z^4 - 30z^5 \\
 \dots
 \end{array}$$

Thus,

$$x(z) = \frac{1}{1 - \frac{3}{2}z^{-1} + \frac{1}{2}z^{-2}} = 2z^2 + 6z^3 + 14z^4 + 30z^5 + 62z^6 + \dots$$

In this case  $x(n) = 0$  for  $n < 0$ .

$$x(n) = \{ \dots, 62, 30, 14, 6, 2, 0, \uparrow \}$$

(c) Prove the following properties of z transform

i) The reversal property

ii) The convolution property

iii) The scaling property

Since the ROC of  $X(z)$  is  $n_1 < |z| < n_2$ , the ROC of  $X(z^{-1})$  is

$$n_1 < |z^{-1}| < n_2$$

on

$$|a|n_1 < |z| < |a|n_2$$

Time reversal: If

$$x(n) \xrightarrow{z} X(z) \quad \text{ROC: } n_1 < |z| < n_2$$

then

$$x(-n) \xrightarrow{z} X(z^{-1}) \quad \text{ROC: } \frac{1}{n_2} < |z| < \frac{1}{n_1}$$

From the equation,

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

We have

$$Z\{x(-n)\} = \sum_{n=-\infty}^{\infty} x(-n) z^{-n} = \sum_{l=-\infty}^{\infty} x(l) (z^{-1})^{-l} = X(z^{-1})$$

where the change of variable  $l = -n$  is made. The ROC of  $X(z^{-1})$  is

$$n_1 < |z^{-1}| < n_2 \text{ or equivalently } \frac{1}{n_2} < |z| < \frac{1}{n_1}$$

Differentiation in the z domain: If

$$x(n) \xrightarrow{z} X(z)$$

then

$$nx(n) \xrightarrow{z} -z \frac{dX(z)}{dz}$$

By the definition of z-transform, we have

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \quad \dots (1)$$

Now, by differentiating both sides of (1) we get,

$$\frac{dX(z)}{dz} = \sum_{n=-\infty}^{\infty} x(n) (-n) z^{-n-1} = -z^{-1} \sum_{n=-\infty}^{\infty} [nx(n)] z^{-n}$$

the ROC of  $X(z^{-1})$  is

$$r_1 < |z| < r_2$$

$$\frac{1}{r_2} < |z| < \frac{1}{r_1}$$

$$X(z^{-1})^{-1} = X(z^{-1})$$

the ROC of  $X(z^{-1})$  is

$$r_1 < |z| < \frac{1}{r_1}$$

Convolution: If  $x_1(n) \xleftrightarrow{Z} X_1(z)$   
 $x_2(n) \xleftrightarrow{Z} X_2(z)$

Then,  $x(n) = x_1(n) + x_2(n) \xleftrightarrow{Z} X(z) = X_1(z) X_2(z)$

The ROC of  $X(z)$  is at least, the intersection of that for  $X_1(z)$  and  $X_2(z)$ .

The convolution of  $x_1(n)$  and  $x_2(n)$  is defined as,

$$x(n) = \sum_{k=-\infty}^{\infty} x_1(k) x_2(n-k)$$

The z-transform of  $x(n)$  is

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} = \sum_{n=-\infty}^{\infty} \left[ \sum_{k=-\infty}^{\infty} x_1(k) x_2(n-k) \right] z^{-n}$$

Upon interchanging the order of the summations and applying the time-shifting property in  $x(n-k) \xleftrightarrow{Z} z^{-k} X(z)$

We obtain,

$$\begin{aligned} X(z) &= \sum_{k=-\infty}^{\infty} x_1(k) \left[ \sum_{n=-\infty}^{\infty} x_2(n-k) z^{-n} \right] \\ &= X_2(z) \sum_{k=-\infty}^{\infty} x_1(k) z^{-k} \\ &= X_2(z) X_1(z). \end{aligned}$$

Computation of the convolution of two signals using the z-transform requires the following steps:

1. Compute the z-transforms of the signals to be convolved,



property can easily be generalized for an arbitrary number of

Time Shifting: If

$$x(n) \xleftrightarrow{Z} X(z)$$

$$\text{then } x(n-k) \xleftrightarrow{Z} z^{-k} X(z)$$

The ROC of  $z^{-k} X(z)$  is the same as that of  $X(z)$  except for  $z=0$  if  $k > 0$  and  $z = \infty$  if  $k < 0$ .

Scaling in the z-domain: If

$$x(n) \xleftrightarrow{Z} X(z), \quad \text{ROC: } r_1 < |z| < r_2$$

$$\text{then } a^n x(n) \xleftrightarrow{Z} X(a^{-1}z), \quad \text{ROC: } |a|r_1 < |z| < |a|r_2$$

for any constant  $a$ , real or complex.

From the equation,

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

we get,

$$\begin{aligned} Z\{a^n x(n)\} &= \sum_{n=-\infty}^{\infty} a^n x(n) z^{-n} = \sum_{n=-\infty}^{\infty} x(n) (a^{-1}z)^{-n} \\ &= X(a^{-1}z) \end{aligned}$$



3(a)

Determine the response of the following system to the input signal

$$x(n) = \begin{cases} |n|, & -3 \leq n \leq 3 \\ 0, & \text{otherwise} \end{cases}$$

$$y(n) = 1/3 [x(n+1) + x(n) + x(n-1)]$$

SAPD  
Exmp 2.1

Determine the response of the following system in the input signal

$$x(n) = \begin{cases} |n| & -3 \leq n \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

$$y(n) = \frac{1}{3} [x(n+1) + x(n) + x(n-1)]$$

$$y(0) = \frac{1}{3} [x(-1) + x(0) + x(1)]$$

$$= \frac{1}{3} [1 + 0 + 1] = \frac{2}{3}$$

$$y(1) = \frac{1}{3} [x(2) + x(1) + x(0)] = \frac{1}{3} [2 + 1 + 0] = 1$$

2, 3, 4  
- 2, 3, 4  
0, 1, 2, 3, 4  
0, 1, 2, 3, 4

$$\therefore y(n) = \{ \dots, 0, 1, \frac{5}{3}, 2, 1, \frac{2}{3}, \frac{1}{2}, \frac{5}{3}, 1, 0, \dots \}$$

There are three methods that are often used for evaluation of the inversion of z-transform.

① Direct evaluation by contour integration.

smile

(b) Discuss the classification of discrete time system.

Combining as

$$x(n) = x_e(n) + x_o(n)$$

Classification of discrete time ~~signal~~ system:

Static ~~is~~ (memoryless) vs Dynamic (with memory) system:

A Discrete time signal is called static or memoryless if its output at any instant  $n$  depends at most on the input ~~sin~~ sample at the same time but not on past or future samples of the input.

$$y(n) = a x(n),$$

$$y(n) = nx(n) + bx^3(n)$$

If the output of the system depends not only the present sample of the input but also the past sample of the input is called dynamic or with memory system. Its output at time  $n$  is completely determined by the input sample in the interval, ~~determined~~   
 ~~by the input~~

$$n-N \text{ to } n (N > 0)$$

$$[n - (n-N) = N]$$

The system is said to have memory of duration  $N$ .

Example:  $y(n) = x(n) + 3x(n-1)$

$$y(n) = \sum_{k=0}^n x(n-k)$$

Time variant and time invariant systems

A system is called time invariant if its input-output characteristics output change with time. A relaxed system  $\gamma$  is time invariant or shift invariant if and only if

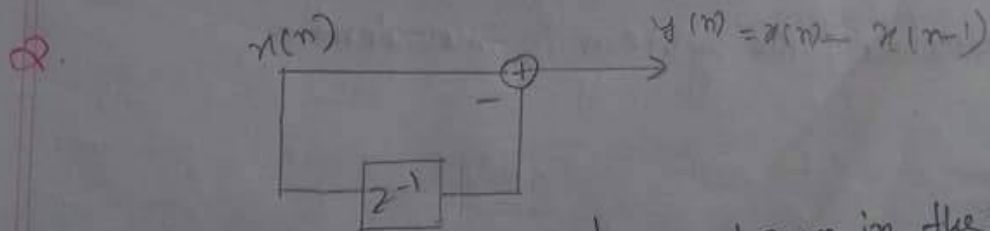
$$x(n) \xrightarrow{\gamma} y(n)$$

$$x(n, k) \xrightarrow{\gamma} y(n-k)$$

for every input signal  $x(n)$  and every time shift  $k$ .

If the output  $y(n, k) \neq y(n-k)$  at least for one value of  $k$ , is called time variant.

30-11-16



Determine if the system shown in the figure is time variant or invariant?

(c) Define the following term  
signal, system, Nyquist rate

Signal:

Signal is Defined as physical quantity that varies with time, space or any other independent variable or variables. Mathematically, It could be describe as a function of one or more independent variables.

$$S_1(t) = 5t$$

$$S_2(t) = 20t^2$$

$$S(x, y) = 3x + 2xy + 10y^2$$

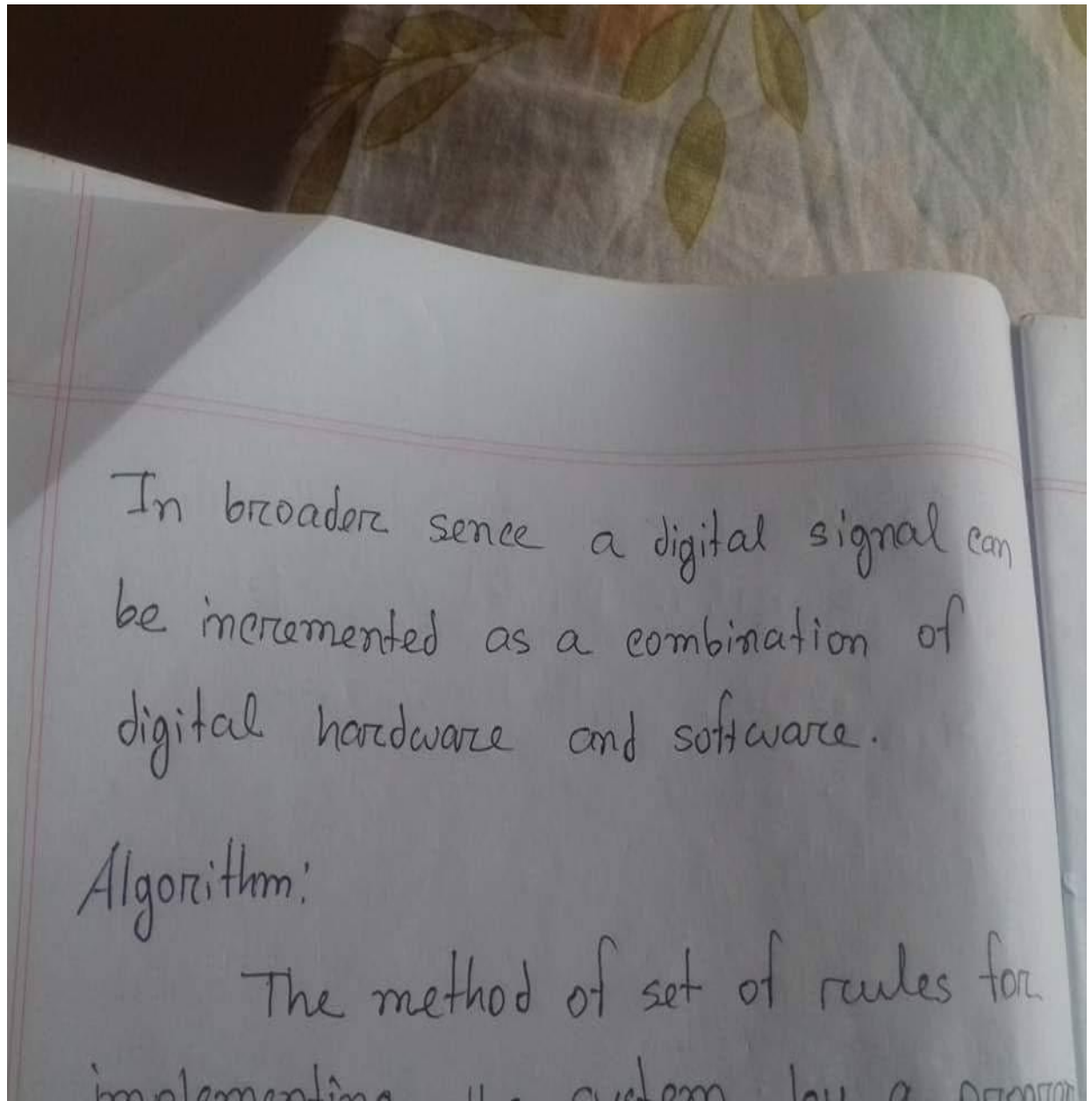
$$\sum_{i=1}^n \underbrace{A_i(t)}_{\text{amplitude}} \cdot \sin \left[ 2\pi \underbrace{f_i(t)}_{\text{frequency}} t + \underbrace{i(t)}_{\text{Phase}} \right]$$

System:

09-11-2016

A system can be define as a physical design that performs an operation on a signal or software realization of operation on a signal.



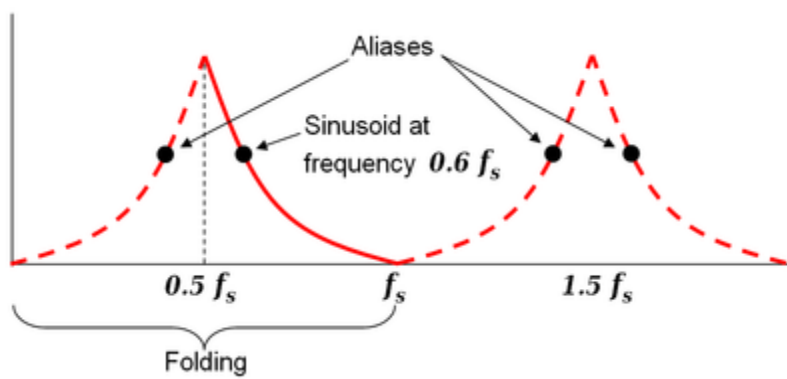


Nyquist rate:

The dashed red lines are the corresponding paths of the aliases. The **Nyquist frequency**, named after electronic engineer Harry **Nyquist**, is half of the sampling **rate** of a **discrete signal processing** system. It is sometimes known as the folding **frequency** of a sampling system.

smile





smile

(d) Consider the analog signal

$$X_a(t) = 3\cos 2000\pi t + 5\sin 6000\pi t + 10\cos 12000\pi t$$

(a) What is the Nyquist rate for this signal?

(b) Assume now that we sample this signal using a sampling rate  $F_s = 5000$  sample/s.

(c) What is the analog signal  $y_a(t)$  that we can reconstruct from the samples if we use ideal interpolation?

repeated by sampling the same discrete time signal.  
 $f_{s/2}$  is called the folding frequency.

math  
 14.4

Q-29 (a)  $x_a(t) = 3 \cos 2000\pi t + 5 \sin 6000\pi t + 10 \cos 12000\pi t$

$$f_N = 2f_{\max} = 2 \times 6000 = 12000 \text{ kHz}$$

$$x_a(t) = A \cos 2\pi f t$$

Q. What is nyquist rate?

The frequency existing in the analog signal are:

$$f_1 = 1000, f_2 = 3000, f_3 = 6000$$

$$f_N = 2f_{\max}$$

$$= 2 \times 6000 \text{ kHz}$$

$$= 12000 \text{ kHz Ans.}$$

$$= 2 \times 6000 \text{ Hz}$$

$$= 12000 \text{ Hz ans}$$

$$= 12 \text{ kHz}$$

⑥  $f_s = 5000 \text{ Hz}$  Sample/second what is the discrete time signal obtain after sampling?

$$x(n) = x_a(nT) = A \cos 2\pi f_m T$$

$$= A \cos 2\pi \left(\frac{f_i}{f_s}\right) n$$

$$f_s = 5000 \text{ Hz}$$

$$= 5 \text{ kHz}$$

$$\therefore \frac{f_s}{2} = \frac{5}{2} = 2.5 \text{ kHz}$$

$$x(n) = x_a(nT) = 3 \cos 2\pi \left(\frac{1}{5}\right)n + 5 \sin 2\pi \left(\frac{3}{5}\right)n$$

$$+ 10 \cos 2\pi \left(\frac{6}{5}\right)n$$

$$= 3 \cos 2\pi \left(\frac{1}{5}\right)n + 5 \sin 2\pi \left(1 - \frac{3}{5}\right)n$$

$$+ 10 \cos 2\pi \left(1 + \frac{1}{5}\right)n$$

$$= 3 \cos 2\pi \left(\frac{1}{5}\right)n + 5 \sin 2\pi \left(-\frac{3}{5}\right)n$$

$$+ 10 \cos 2\pi \left(\frac{1}{5}\right)n$$

$$= 13 \cos 2\pi \left(\frac{1}{5}\right)n - 5 \sin 2\pi \left(\frac{3}{5}\right)n$$

23-11-2016

What is the analog signal?  $y_a(t)$   
from the sample if we use ideal  
interpolation formula.

Here, the frequency components are 1kHz  
and 2kHz

So, the recovery signal  $y_a(t)$

$$= 13 \cos 2\pi f_1 t - 5 \sin 2\pi f_2 t$$

$$= 13 \cos (2 \times 1000) \pi t - 5 \sin (2 \times 2000) \pi t$$

$$= 13 \cos 2000 \pi t - 5 \sin 4000 \pi t$$

Block diagram Representation of discrete

4(a) what do you mean by DSP? Write down some application of DSP.

5ht

Digital signal Processing (DSP-417)

Q-1 What do you mean by dsp? List the application of DSP.

Ans: Digital Signal Processing (DSP)

Digital signal Processing is concerned with the digital representation of signals and the use of digital Processors to analyse, modify or extract information from signals. Most signals in nature are analogue in form, often meaning, that they vary continuously with time and represent the variations of Physical quantities such as sound waves.

The specific reason for Processing a digital signal may be, for example -  
To remove interference or noise from the signal, to obtain the spectrum of the data, or to transform the signal into a more suitable form.

The classification of digital signal Processing has two types. There are -

1. Analog Signal Processing (ASP)
2. Digital signal Processing (DSP)



## Application of Digital signal Processing

### ▣ Image Processing

- Pattern Recognition
- Robotic vision
- image enhancement
- Facsimile
- animation

### ▣ Instrumentation/control

- Spectrum analysis
- Position reduction
- noise reduction
- Data compression

### ▣ Speech / Audio

- Speech recognition
- Speech synthesis
- text to speech
- Digital audio

### ▣ Military

- secure communication
- radar Processing
- Sonar Processing
- missile Processing

#### ☐ Telecommunication

- echo cancellation
- adaptive equalization
- ADPCM transcoders
- video conferencing

#### ☐ Biomedical

- Patient monitoring
- scanners
- ECG brain mappers
- ECG analysis

#### ☐ Consumer application

- Digital cellular mobile phones
- Digital television
- Digital cameras
- Internet phones, music & video
- voice mail systems.

(b) Define recursive system, non-recursive system, signal, system.

smile

- (c) Consider a system whose output  $y(n]$  is related to the input  $x(n]$  by

$$y(n) = \sum_{k=-\infty}^{\infty} x(k) x(n+k)$$

Determine whether or not the system is (a) Linear (b) Shift invariant  
(c) Stable (d) Causal.

Question: Consider a system whose output  $y(n)$  is related to the input

$$x(n) \text{ by } y(n) = \sum_{k=-\infty}^{\infty} x(k) x(n+k) \quad \text{--- DE --- 4(e)}$$

Determine whether or not the system is-

- i) Linear
- ii) Shift-invariant
- iii) Stable
- iv) Causal

Answer:

$$\text{Given, } y(n) = \sum_{k=-\infty}^{\infty} x(k) x(n+k)$$

$$i) \quad y_1(n) = \sum_{k=-\infty}^{\infty} x_1^2(k)$$

$$y_2(n) = \sum_{k=-\infty}^{\infty} x_2^2(k)$$

$$y_3(n) = \sum_{k=-\infty}^{\infty} x_3^2(k)$$

$$\begin{aligned} \therefore y_3(n) &= \sum_{k=-\infty}^{\infty} [a_1 x_1(k) + a_2 x_2(k)]^2 \\ &= \sum_{k=-\infty}^{\infty} [a_1^2 x_1^2(k) + a_2^2 x_2^2(k) + 2a_1 a_2 x_1(k) x_2(k)] \end{aligned}$$

on the other hand,

$$a_1 y_1(n) + a_2 y_2(n) = a_1 \sum_{k=-\infty}^{\infty} x_1^2(k) + a_2 \sum_{k=-\infty}^{\infty} x_2^2(k)$$

$\therefore$  The system is non-linear.

$$ii) \quad \text{Given, } y(n) = \sum_{k=-\infty}^{\infty} x(k) x(n+k)$$

Output change:

$$y(n-k) = \sum_{k=-\infty}^{\infty} x(k) x(n-k+k) = \sum_{k=-\infty}^{\infty} x(k) x(n)$$

output change:

$$y(n, k) = \sum_{k=-\infty}^{\infty} x(n-k) x(n+k-k)$$

$$= \sum_{k=-\infty}^{\infty} x(n) x(n)$$

$$\therefore y(n, k) \neq y(n-k)$$

So the system is not shift.

iii) The system is not bounded, so the system is not stable or the system is unstable.

iv) Future input, not causal.



5(a) Define digital filter with simplified block diagram.

Date: 17/11/2023  
Page: 33  
City: elg

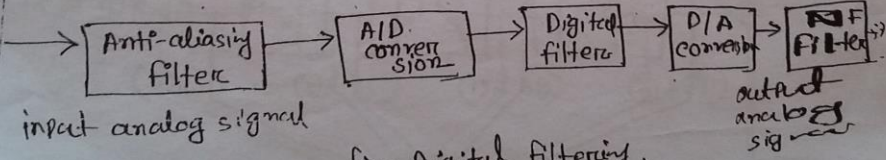
Q. What is digital filter? write down the classification of digital filter.

Digital filter:

A digital filter is a mathematical algorithm implemented in h/w and/or s/w that operates on a digital input to produce a digital output signal for the purpose of achieving a filtering objectives.

The term digital filter refers to the specific h/w or s/w routine that performs the filtering algorithm.

Digital filter often operates on digitized analog signals or just numbers, representing some variable stored in a computer memory.



```
graph LR; A[Anti-aliasing filter] --> B[A/D conversion]; B --> C[Digital filter]; C --> D[D/A conversion]; D --> E[Filter];
```

input analog signal

output analog signal

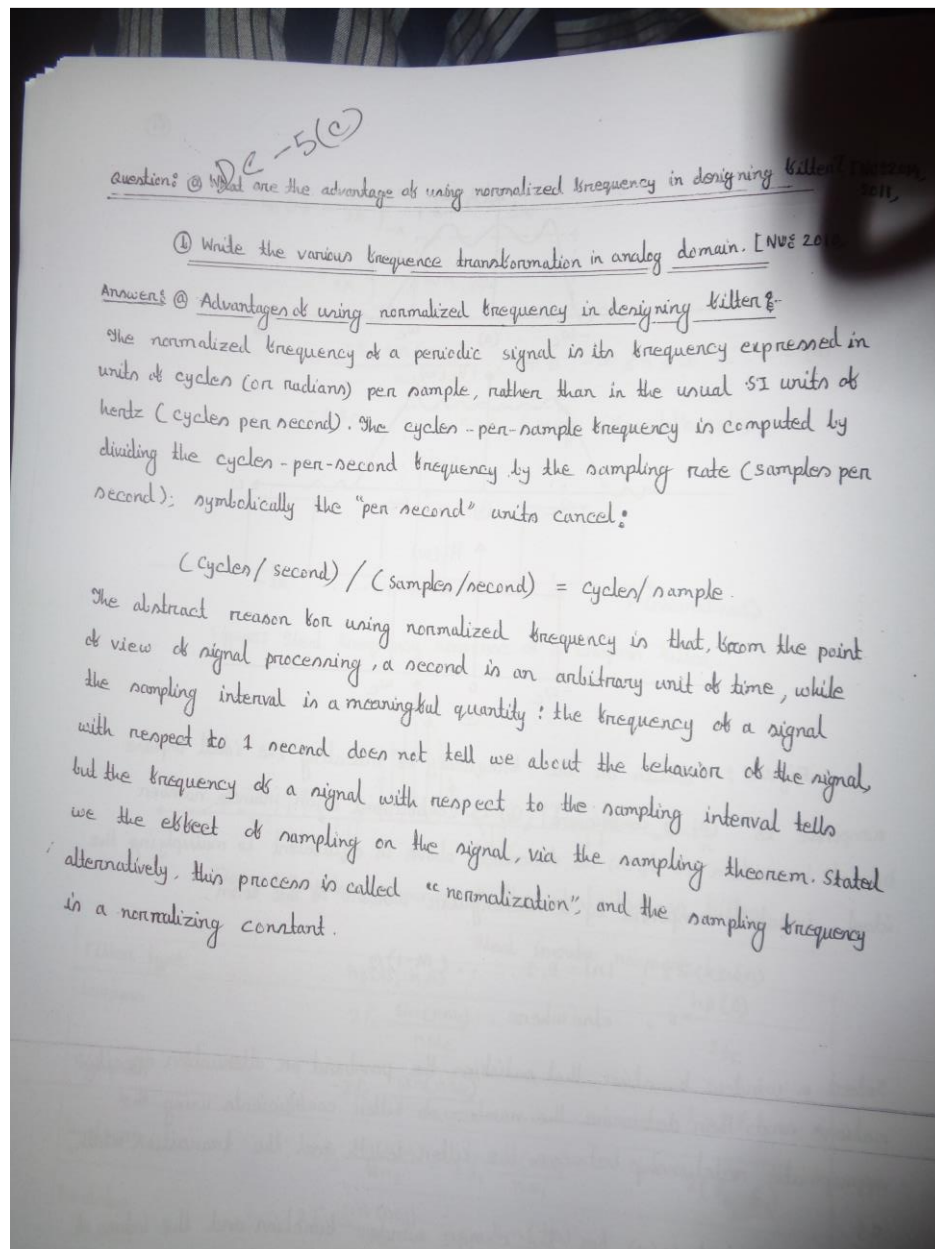
fig: Digital Filtering.

(b) Advantages of direct and cascade diagram.

smile



(c) Advantages of normalized frequency in designing filter.



smile

(d) Describe filter design step.

④ It needs much longer time to design and develop the digital sequences than analog filters. Though it can be used on other tasks or applications once developed. ordinarily good support of computer ~~aided~~ design can convert them into an enjoyable task.

- \* Describe filter design steps.  
The design of digital filters involves five steps:-
- ① specification of the filter requirement
  - ② calculation of suitable filter coefficient
  - ③ Representation of filter by a suitable structure
  - ④ Analysis of the effect of finite word length on filter performance.
  - ⑤ Implementation of filter in S/W and/or h/w

specification of the filter requirement:

Requirement specification include the following:

- (a) signal characteristics
- (b) the characteristics of the filter.
- (c) The manner of implementation.
- (d) other design constraints.

The specification often in the form of tolerance schema. The following figure shows a schema for a lowpass filter.

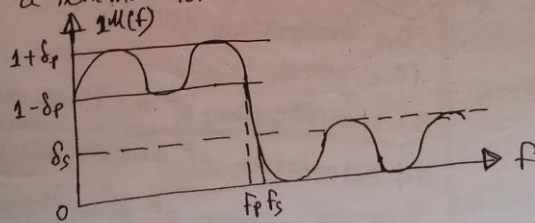


Fig: Tolerance schema for a lowpass filter.

where  $\delta_p$  = Passband deviation  
 $\delta_s$  = Stopband deviation.

$f_p$  = passband edge frequency.

$f_s$  = stopband edge

## (2) Co-efficient calculation:

Calculation of co-efficient depends on the type of the filter whether it is an IIR or FIR filter.

For IIR there are four different methods.

They are:—

- (a) pole-zero placement.
- (b) impulse invariant.
- (c) Matched z-transform.
- (d) bilinear z-transform.

For FIR filter there are several methods but most commonly used three methods are.

- (1) Window method.
- (2) Optimal sampling.

## (iii) Frequency sampling

Realization

### ③ Representation of a filter by suitable structures

Realization involves converting a given transfer function. Block diagrams are often used to depict filter structures and they show the computation procedure for implementing the DF.

For IIR filter three structures commonly used.

① direct form

② cascade "

③ parallel "

### ④ Analysis of the effect of finite wordlength

Filter performance:

In this section the arithmetic operation indicated in difference equation are performed using finite precision arithmetic. It has some features which make analysis perfect. These are



(1) A finite number of bits are to degrade the performance of the filter.

(2) It is ~~useable~~, unusable.

(3) The designer must analyse these effects and choose suitable wordlength.

The main source of performance degradation (wasting (or) errors) in digital filters are as follows.

- (1) Input/output signal quantization.
- (2) Co-efficient quantization.
- (3) Arithmetic roundoff error.
- (4) Overflow.



### ⑤ Implementation of filter in s/w and h/w:

After quantizing the co-efficients and filter variables to the selected wordlength, is acceptable, the difference equation must be implemented as a software routine or in hardware.

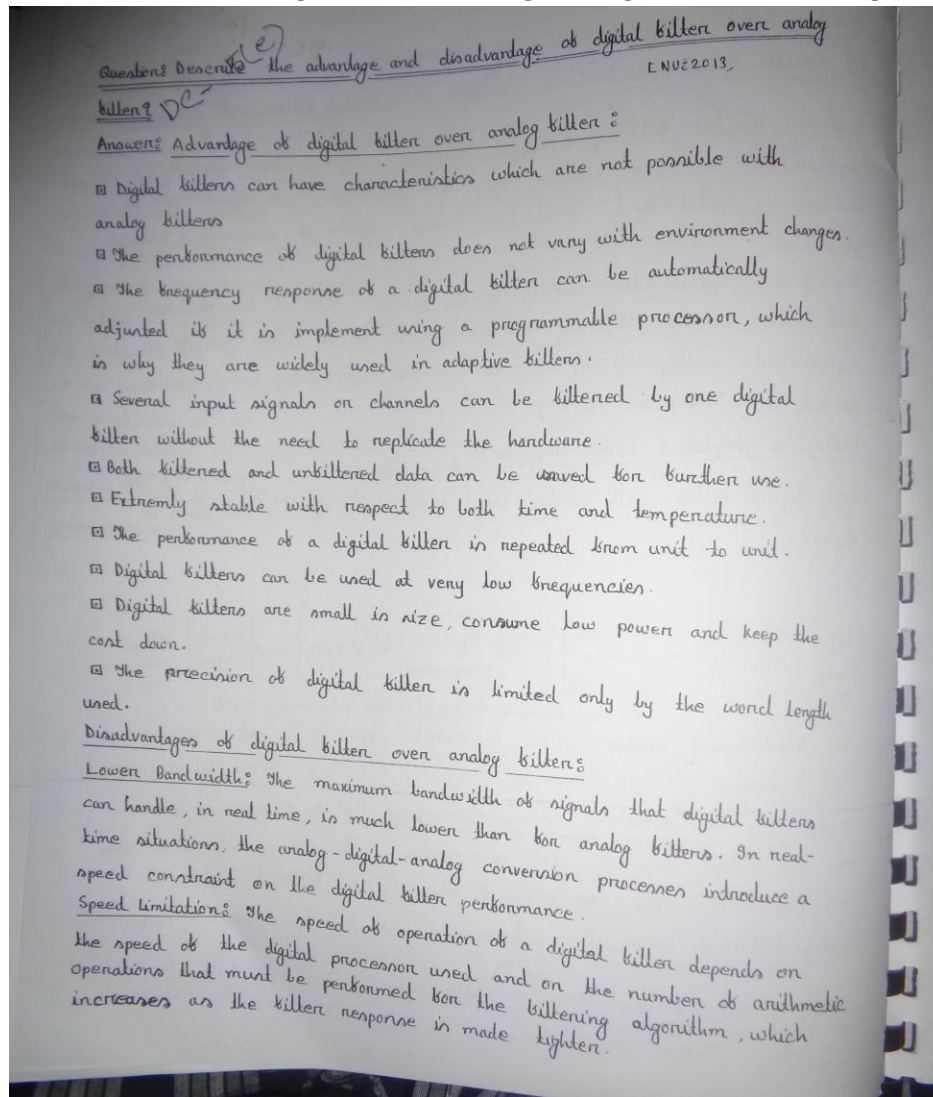
Implementation has some characteristics

There are:-

1. calculate co-efficient
2. Suitable structure defines
3. verify filter degradation
4. select wordlength.
5. Analyze co-efficient.
6. Different equation analyze.

In implementation time the designer will define which system are same where.

(e) Describe the advantages and disadvantages of digital filter over analog



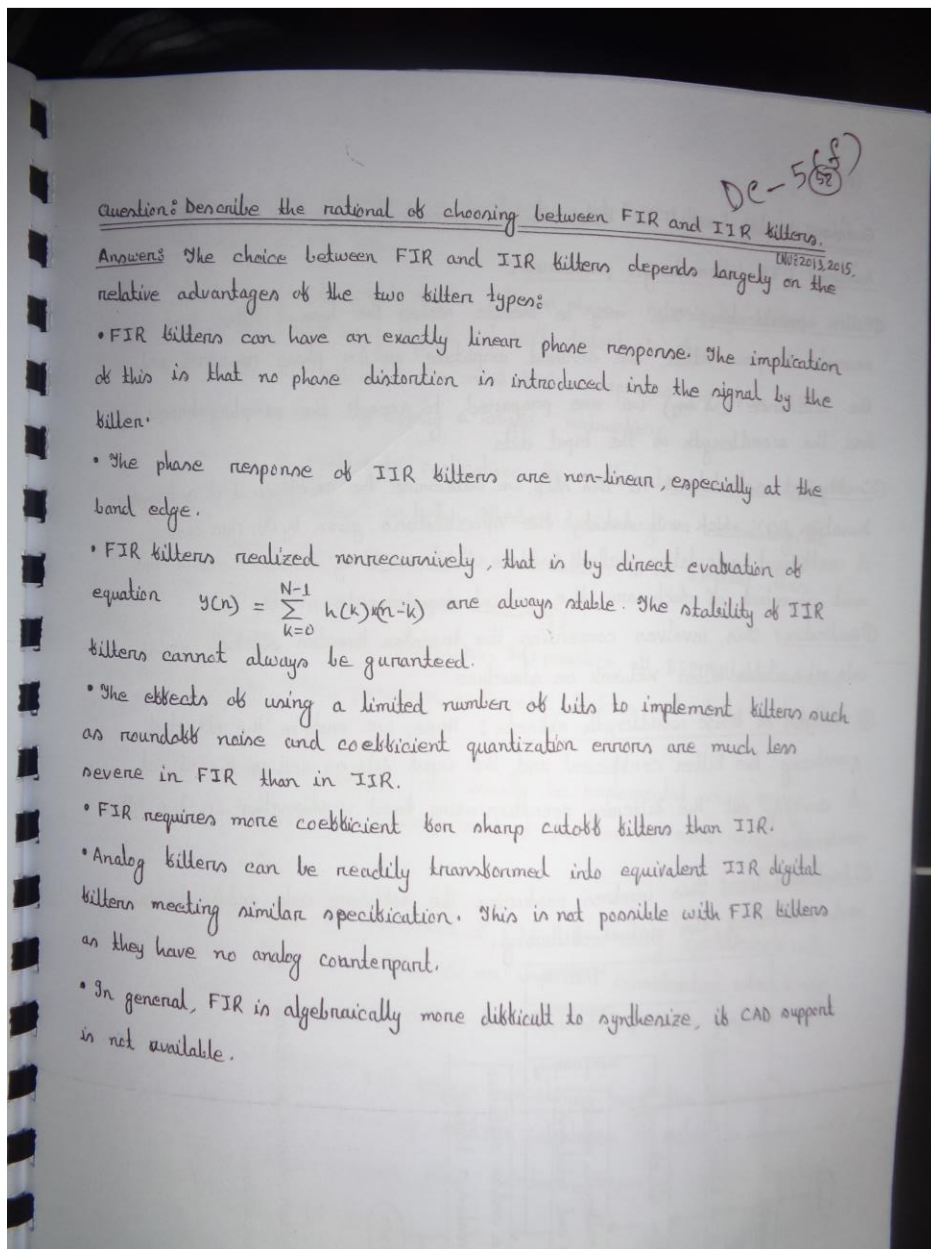
filter.

smile

Finite wordlength effects: Digital filters are subject to ADC noise resulting from quantizing a continuous signal and to roundoff noise incurred during computation. With higher order recursive filters, the accumulation of roundoff noise could lead to instability.

Long design and development times: The design and development times for digital filters, especially hardware development, can be much longer than for analog filters.

(f) Classify digital filter. Describe the rationales of choosing FIR and IIR filter.



6(a) Explain --

i) Z transform and inverse transform.

ii) FIR and IIR filter.

iii) Adaptive filter as noise canceller.

iv) DFT and IDFT