Public overspending in higher education*

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Abstract

We study the trade-off between governmental investments in pre-tertiary and tertiary education from an efficiency point of view. We develop a model containing agents with different incomes and abilities, public and private schools, and public universities that select applicants based on an admission exam. Reallocating governmental resources from tertiary to pre-tertiary education may positively affect aggregate production and human capital if per-student public spending in schools is low relative to that in universities, if there is a high proportion of credit-constrained students, or if the importance of pre-tertiary education to human capital formation is high relative to that of tertiary education.

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1 Introduction

The public education system of many developing countries is characterized by the existence of low-quality schools and elite universities that select the highest-ranked applicants, frequently benefiting students from wealthy families that can afford private schools. The Organisation for Economic Co-operation and Development (OECD) documents that countries such as Brazil, Costa Rica, and Mexico make low per-student investments in basic education, but high expenditures in tertiary education, and asserts that its governments should shift public spending from tertiary to pre-tertiary education to raise progressivity and efficiency (OECD, 2018a; OECD, 2018b; OECD, 2019).

In this paper, we study the trade-off between public investments in schools and universities from an efficiency point of view. Using a model that features heterogeneous agents concerning income and ability, public and private schools, and public universities with a limited number of vacancies, we investigate the effects of reallocating governmental resources from tertiary to pre-tertiary education on aggregate production and human capital.

In the model, public schools are tuition-free, while private schools charge a price for their services and provide a higher human capital return. To study in a public university, an agent needs to apply for admission. There is a limited number of vacancies in public universities, and only applicants with the highest grades in an admission exam are selected. An applicant's score is a stochastic function of her pre-tertiary human capital, and a cutoff score for the exam is determined in equilibrium to make the mass of admitted applicants equal to the mass of vacancies. There are credit constraints, so that high-ability poor agents cannot anticipate their future earnings to pay for a better education.

The government taxes agents to obtain revenues to fund public education. The human capital return of attending public schools and public universities depends on the amount of per-student governmental spending in pre-tertiary and tertiary education.

We study an equilibrium in which public schools are attended by low-income students, wealthy agents study in private schools, and high-ability agents apply to public universities. This equilibrium features inequality in access to public higher education because wealthy applicants have an advantage in the admission for university due to their better pre-tertiary education.

If the government reallocates resources from tertiary to pre-tertiary education, low-income students are benefited because they obtain a better quality education and have higher odds of being admitted to the public university. On the other hand, university students are harmed because the return to higher education decreases.

We use this environment to investigate conditions under which reallocating governmental resources from tertiary to pre-tertiary education results in an increase of aggregate production and human capital, a situation we call "public overspending in higher education". We highlight three conditions.

First, in accordance with OECD's policy recommendation, the existence of public overspending in higher education is favored if per-student government spending in public schools is sufficiently low relative to that in public universities. For a given educational level, there are diminishing returns from government expenditures in terms of producing education quality. Therefore, it is not optimal to allocate public resources excessively unequally across educational stages.

A well-established literature finds that earlier stages of education have a higher importance in an individual's human capital formation (Cunha et al., 2010). Our model is consistent with this mechanism: the return to public educational expenditures in a given level of education is proportional to this stage's importance to human capital formation. The higher the importance of pre-tertiary education is, the looser the conditions for the existence of public overspending in higher education in the model are.

Finally, credit constraints are an essential aspect of an educational environment (Lochner and Monge-Naranjo, 2011). We find that if the proportion of credit-constrained students is high, reallocating public expenditures towards pre-tertiary education alleviates this friction and generates an improvement in the allocation of resources, as measured by the aggregate human capital of individuals.

This paper is related to a literature that studies the trade-offs between public investments in pre-tertiary and tertiary education (Driskill and Horowitz, 2002; Restuccia and Urrutia, 2004; Su, 2004; Blankenau, 2005; Su, 2006; Blankenau et al., 2007; Arcalean and Schiopu, 2010; Sarid, 2016; Brotherhood and Delalibera, 2020; Caucutt and Lochner, 2020). We contribute to this literature by developing a simple model that delivers analytical results and a clear interpretation of several relevant mechanisms. In particular, when we analyze the effects of reallocating public resources from tertiary to non-tertiary education, the model allows us to distinguish all different effects on GDP and compare them with each other in a highly tractable way.

The rest of this paper is organized as follows. Section 2 presents the model through subsections. Subsection 2.1 describes the preliminaries of the model, subsection 2.2 presents the choices of the agents, subsection 2.3 defines and characterizes the equilibrium, subsection 2.4 studies the existence of public overspending in higher education, and subsection 2.5 introduces the government. Section 3 presents concluding comments.

2 The model

2.1 Preliminaries

There is a unitary mass of agents and two periods. Each agent is characterized by a pair of variables (w, π), where w denotes income endowment in the first period, and π is the agent's innate ability. There are two levels of income in the first period, $w_H > w_L > 0$, and two levels of innate abilities, $\pi_H > \pi_L > 0$. Denote by μ_{ij} the exogenous mass of agents of type (w_i, π_j), for $i, j \in \{L, H\}$.

Acquired ability, $\hat{\pi}$, is the human capital that an agent has immediately after completing pre-tertiary education. For an agent with innate ability π, acquired ability is given by

$$\hat{\pi} = \begin{cases} a_0\pi & \text{if agent studies in public school,} \\ a_1\pi & \text{if agent studies in private school.} \end{cases} \tag{1}$$

a_0 and a_1 reflect the human capital return of attending public and private schools, respectively, with $a_1 > a_0 > 0$. The final human capital of an agent with acquired ability $\hat{\pi}$ is

$$h = \begin{cases} \hat{\pi} & \text{if agent does not attend university,} \\ a_2\hat{\pi} & \text{if agent studies in public university,} \end{cases} \tag{2}$$

where $a_2 > 1$ denotes the human capital return of university education.

There is a limited mass of vacancies in public universities, $\lambda \in (0, 1)$. An applicant is admitted to the public university if her exam score is greater than or equal to a non-negative exam score cutoff, π^*, which is determined in equilibrium. The exam score of an applicant with acquired ability $\hat{\pi}$ is given by $\varepsilon\hat{\pi}$, where ε is an i.i.d. standard uniform random variable. Using properties of the uniform distribution, the probability of an applicant with acquired ability $\hat{\pi}$ being admitted is

$$p(\hat{\pi}|\pi^*) \equiv \Pr(\varepsilon\hat{\pi} \geq \pi^*) = \Pr\left(\varepsilon \geq \frac{\pi^*}{\hat{\pi}}\right) = \begin{cases} 0 & \text{if } 1 < \frac{\pi^*}{\hat{\pi}}, \\ 1 - \frac{\pi^*}{\hat{\pi}} & \text{if } 0 \leq \frac{\pi^*}{\hat{\pi}} \leq 1. \end{cases} \tag{3}$$

2.2 Agents' choices

In the first period, an agent chooses between public and private basic education. If she decides to study at a private school, she must pay price $q > 0$, which is exogenous. The budget constraint of an agent with income w in the first period is

$$c_1 + qs_1 = w, \tag{4}$$

where c_1 denotes consumption in period one, and s_1 is a dummy variable that indicates whether the agent chooses to study in a private school. Note that there are credit constraints: a (w_L, π_H) agent has the same funds as a (w_L, π_L) agent to invest in education, although the former has higher expected future earnings than the latter.

If an agent decides to apply to public university in the second period, she incurs a utility cost $v > 0$, which is exogenous and represents the applicant's required effort to prepare for exams. Denote by s_2 a dummy variable that indicates whether the agent applies to public university. In the second period, the agent consumes her labor earnings, which are given by her final human capital,

$$c_2 = h. \tag{5}$$

An agent takes the cutoff grade π^* as given and chooses $s = (s_1, s_2) \in \{0,1\} \times \{0,1\} \equiv S$ to maximize expected lifetime utility. The utility function is linear and there is no time discounting. The problem of an agent is

$$\begin{gathered} \max_{s \in S} u(c_1) + \mathbb{E}\left[u(c_2)\right] - v s_2 \\ \text{subject to} \\ (1),\ (2),\ (3),\ (4),\ (5),\ c_1 \geq 0,\ c_2 \geq 0, \end{gathered} \tag{6}$$

where the expectation is taken over shocks that determine an applicant's exam score.

2.3 Equilibrium

Next, we define the equilibrium in this economy.

Definition 1. *An equilibrium is given by agents' choices and a university grade point cutoff π^* such that:*

1. *Agents maximize expected lifetime utility taking π^* as given,*
2. *The mass of public university students is less than or equal to the mass of public university vacancies, with equality if $\pi^* > 0$.*

Equilibrium condition 2 states that if $\pi^* > 0$, then the mass of students in university must be exactly equal to the mass of vacancies. Additionally, we can also have the case in which there are empty vacancies and the exam cutoff score is zero in equilibrium.

We analyze an equilibrium where low-income agents study in public schools, high-income agents study in private schools, low-ability agents do not apply to public university, and high-ability agents apply.[1] The next proposition shows conditions under which the model generates such optimal choices.

Proposition 1. *Denote by s_{ij} the choice of a type-(w_i, π_j) agent. If the following conditions hold:*

1. $w_L < q < w_H$,
2. $v < \left(1 - \dfrac{\pi^*}{a_0 \pi_H}\right)(a_2 - 1)a_0\pi_H$,
3. $q < (a_1 - a_0)\pi_L$,
4. $v > \pi_L a_1(a_2 - 1)$,
5. $v - q > \pi_L(a_0 a_2 - a_1)$,
6. $q < \pi_H a_2(a_1 - a_0)$,
7. $q + v < \pi_H(a_1 - a_0)$,

then $s_{LL} = (0, 0)$, $s_{LH} = (0, 1)$, $s_{HL} = (1, 0)$, and $s_{HH} = (1, 1)$.[2]

Proof. See appendix A. □

Next, we interpret conditions in proposition 1. The first condition says that poor agents do not have funds to pay for private school, which leads them to choose public schools. Condition two induces (w_L, π_H) agents to apply for university, defining an upper bound on the cost to apply. This condition also implies that (w_H, π_H) agents apply. Condition three makes (w_H, π_L) choose private schools through restricting the price. The fourth condition defines a lower bound on v to make (w_H, π_L) agents decide not to apply. This condition also implies that (w_L, π_L) agents do not apply. The fifth condition implies that a (w_H, π_L) agent prefers $(1, 0)$ over $(0, 1)$; that is, she prefers a lower consumption in the first period and abdicating a university education in order to have better pre-tertiary education. Condition six puts an upper bound on the price of private education to make (w_H, π_H) agents choose private schools. The last condition makes (w_H, π_H) agents prefer $(1, 1)$ over $(0, 0)$.

If the conditions stated in proposition 1 hold, high-ability agents apply to college and low-ability agents do not. Suppose that the mass of applicants is greater than or

[1] If low-ability wealthy agents apply to university, the subset of the parameter space for which our main proposition holds is larger.

[2] The subset of the parameter space for which the conditions in propositions 1 and 2 hold is non-empty. For example, all conditions hold for the following parameter values: $w_L = 0.25$, $w_H = 0.75$, $\pi_L = 1$, $\pi_H = 20$, $\mu_{LL} = 0.5$, $\mu_{LH} = 0.3$, $\mu_{HL} = 0.1$, $\mu_{HH} = 0.1$, $\lambda = 0.1$, $a_0 = 1$, $a_1 = 2$, $a_2 = 1.5$, $\partial a_2/\partial a_0 = -0.01$, $q = 0.5$, and $v = 2$.

equal to the mass of vacancies, $\mu_{LH} + \mu_{HH} \geq \lambda$, so that there are no empty vacancies in equilibrium. The condition for college market clearing in this case is:

$$\mu_{LH}\ p(a_0\pi_H|\pi^*) + \mu_{HH}\ p(a_1\pi_H|\pi^*) = \lambda, \tag{7}$$

where we use the fact that $\hat{\pi}$ equals a_0 and a_1 for agents of type (π_L, w_H) and (π_H, w_H), respectively. We look for an equilibrium in which both types of high-ability individuals have positive probabilities of being admitted. From equation (3), for this to happen we must have $0 < \pi^*/(a_0\pi_H) < 1$. Assuming that this is true and substituting (3) in (7), we can solve for π^*:

$$\pi^* = \frac{a_0 a_1 \pi_H(\mu_{LH} + \mu_{HH} - \lambda)}{\mu_{LH} a_1 + \mu_{HH} a_0}. \tag{8}$$

With this closed form solution for π^*, it is straightforward to verify that a necessary and sufficient condition for $0 < \pi^*/(a_0\pi_H) < 1$ is $\lambda > \mu_{LH}(a_1 - a_0)$. This condition guarantees that (w_L, π_H) individuals have a positive probability of being admitted. It also implies that (w_H, π_H) agents have a positive probability of being admitted because they have higher acquired ability.

2.4 Public overspending in higher education

Denote by p_L (p_H) the equilibrium probabilities of a low-(high-) income, high-ability agent being admitted to university.[3] GDP in the second period is given by the sum of expected human capital of all agents weighted by their masses:

$$\begin{aligned} Y = {} & \mu_{LL} a_0 \pi_L + \mu_{LH} p_L a_0 a_2 \pi_H + \mu_{LH}(1 - p_L) a_0 \pi_H \\ & + \mu_{HL} a_1 \pi_L + \mu_{HH} p_H a_1 a_2 \pi_H + \mu_{HH}(1 - p_H) a_1 \pi_H. \end{aligned} \tag{9}$$

We define public overspending in higher education as the situation where reallocating public expenditures from tertiary towards pre-tertiary education, keeping total educational expenditures fixed, leads to higher aggregate production and human capital. For now, suppose that the return to higher education is a function of public school quality, $a_2(a_0)$. This function describes how the government produces education quality for a given fixed level of total expenditures. To increase public school quality, university quality must decrease, so that $\partial a_2/\partial a_0 < 0$. Later we show how this relationship can be generated through production functions for a_0 and a_2 and a government budget constraint.

We can use the GDP equation (9) to gain some insights into how aggregate output

[3] Which are given by $p_L = 1 - \pi^*/(a_0\pi_H)$ and $p_H = 1 - \pi^*/(a_1\pi_H)$.

reacts to an increase in the quality of public schools and a corresponding decrease in the quality of universities. A variation in a_0 and a_2 affects (i) the productivity that individuals gain from education (i.e., terms a_0 and a_2 themselves), and (ii) the distribution of agents who enter university (i.e., terms p_L and p_H). First, a higher a_0 increases the human capital of low-income agents, all else equal, but a lower a_2 decreases the human capital of high ability agents who are admitted into university. Second, such variation in a_0 and a_2 increases the probability of low-income agents entering university, p_L, since their acquired ability depends positively on a_0. At the same time, it decreases p_H because the acquired ability of (w_H, π_H) agents is kept constant at $a_1\pi_H$.

The previous discussion shows that there are some mechanisms through which a higher a_0 and lower a_2 affect GDP positively, while there are others that produce the opposite effect. The next proposition shows conditions under which the positive effects prevail over the negative effects.

Proposition 2. *If the following conditions hold, there is overspending in public universities, i.e.,* $\partial Y/\partial a_0 > 0$*:*

1. $1 > a_2 \left(\dfrac{a_1 - a_0}{a_1}\right)\left(1 - \dfrac{\mu_{LH}}{\mu_{LH} + \mu_{HH}}\right),$

2. $\mu_{LL} > \left(-\dfrac{\partial a_2}{\partial a_0}\right)\left(\dfrac{\pi_H}{\pi_L}\right)\left(\dfrac{\mu_{LH}a_0 + \mu_{HH}a_1}{\mu_{LH} + \mu_{HH}}\right).$

Proof. See appendix B. □

The two conditions in the proposition above are obtained by requiring some specific positive effects of substituting a_2 for a_0 on aggregate production to be larger than the negative effects. First, when shifting public expenditures towards pre-tertiary, more underprivileged agents will be admitted to the university. Since private schools have better quality, this substitution of admitted students will lead to a negative effect on GDP because the final human capital of newly admitted low-income students is lower than the one that wealthy students would have in case they were admitted.[4] The first condition in proposition 2 is implied by requiring this negative effect to be outweighed by the human capital gain due to higher a_0 for low-income applicants who are not admitted into university.

This condition is satisfied when a combination of the following circumstances holds: (i) a_2 is low, (ii) the gap between a_1 and a_0 is small, and (iii) the proportion of poor

[4]The existence of private universities would alleviate this negative effect of substituting university entrants, since non-admitted wealthy applicants could decide to study in private universities, mitigating their human capital loss compared to the case with no private universities. Therefore, including private universities in this model would loosen the conditions for the existence of public overspending in higher education.

agents among high-ability individuals is high. When one high-ability poor agent substitutes for one high-ability wealthy individual in university, the change in GDP is given by the difference between the human capitals of the two individuals, $a_2(a_0 - a_1)\pi_H < 0$. Note that a_2 boosts the loss of substituting a_1 for a_0. Conditions (i) and (ii) are directly associated to these terms, and together imply that this negative effect is small.

On the other hand, if the proportion of poor agents among high-ability individuals is high, the human capital gain of non-admitted low-income students outweighs the loss generated by the substitution of admitted applicants. Low-income applicants are the agents who are most affected by credit constraints in the model. Condition (iii) reflects the fact that the higher the degree of credit constraints in this economy, the higher are the gains to be made from shifting public resources from tertiary to non-tertiary education.

The second condition is obtained by requiring that the GDP gain due to higher a_0 for (w_L, π_L) agents prevails over the GDP loss driven by lower a_2 for agents admitted in university. This condition is satisfied when a combination of the following holds: (i) μ_{LL} is high, (ii) $\partial a_2/\partial a_0$ is close to zero, (iii) π_H/π_L is low, (iv) the mean pre-tertiary education quality of high-ability students is sufficiently low.

First, the higher μ_{LL} is, the larger is the number of individuals who benefit from higher a_0. Second, the closer $\partial a_2/\partial a_0$ is to zero, the smaller is the decrease in the quality of public university. Third, the GDP gain (loss) due to higher a_0 (lower a_2) is amplified by the innate ability of affected individuals. Since (w_L, π_L) agents study in public school and high-ability agents apply to university, π_H/π_L needs to be sufficiently low so that this amplification effect does not become sizable. Fourth, similar to the previous point, pre-tertiary education quality amplifies the absolute human capital decrease driven by lower a_2. Therefore, one of the negative forces produced by lower a_2 is proportional to the pre-tertiary education quality of admitted students, which is correlated with the basic education quality of high-ability students.

2.5 Government

Proposition 2 depends on the essential term $\partial a_2/\partial a_0$, which we study next. The function $a_2(a_0)$ can be generated through additional objects in the model. Suppose that the human capital return of attending public school and university are increasing and concave functions of government spending per student in school, g_0, and university, g_2, respectively:

$$a_0 = \alpha_0 g_0^{\kappa_0}, \qquad a_2 = 1 + \alpha_2 g_2^{\kappa_2}, \tag{10}$$

with slope parameters $\alpha_0, \alpha_1 > 0$ and concavity parameters $\kappa_0, \kappa_2 > 0$.

A high α_0 may represent an efficient pre-tertiary public educational system, but also an environment where pre-tertiary education is highly important for an individual's human capital formation. In an environment where low-income families who send their children to public schools don't use their private funds to complement governmental educational investments,[5] an increase in public investments will not be followed by a crowding-out of private investments. In this case, α_0 is highly proportional to the importance of pre-tertiary educational investments to an individual's human capital formation (Cunha et al., 2010).

Let G be the government's total educational spending, which is obtained from taxes in the first period. w_L and w_H can be interpreted as after-tax income because G is fixed. Total public spending is the sum of educational expenditures per student times the mass of students for each level of public education. Using agents' optimal choices,

$$G = g_0(\mu_{LL} + \mu_{LH}) + g_2\lambda. \tag{11}$$

Equations (10) and (11) link a_2 and a_0. First, invert the first equation in (10) to get $g_0(a_0)$. Second, substitute this in (11) and isolate g_2 to get $g_2(a_0)$. Finally, substituting this into the second equation in (10) gives us

$$a_2(a_0) = \alpha_2 \left\{ \frac{1}{\lambda} \left[G - \left(\frac{a_0}{\alpha_0} \right)^{\frac{1}{\kappa_0}} (\mu_{LL} + \mu_{LH}) \right] \right\}^{\kappa_2}. \tag{12}$$

Note that the interpretation of function $a_2(a_0)$ is exactly the one that we discussed before: it describes how a_2 is determined as a function of a_0, assuming that total government expenditures G are fixed. If the government increases a_0 through higher expenditures per student in public schools, g_0, then g_2 must decrease for G to remain constant, leading to a smaller a_2.

Differentiating (12) with respect to a_0,

$$\frac{\partial a_2}{\partial a_0} = -\left(\frac{\mu_{LL} + \mu_{LH}}{\lambda} \right) \left(\frac{\kappa_2 \alpha_2^{\frac{1}{\kappa_2}}}{\kappa_0 \alpha_0^{\kappa_0}} \right) \left(\frac{a_0^{\frac{1-\kappa_0}{\kappa_0}}}{a_2^{\frac{1-\kappa_2}{\kappa_2}}} \right). \tag{13}$$

This equation allows us to think more clearly about the OECD's policy recommendation that we mention in the introduction. A large amount of public resources devoted to tertiary education generates a relatively high a_2 and low a_0. In this case, a reallocation of expenditures to public schools should generate gains that compensate the loss of quality in universities because $\partial a_2 / \partial a_0$ is close to zero.

[5] We do not include this mechanism in the model for simplicity and tractability.

Also, if the importance of pre-tertiary education in human capital formation is significantly larger than that of tertiary education (high α_0 or low α_2), a reallocation of public spending from universities to schools should lead to a relatively small loss of the returns to higher education.

Finally, equation (13) also conveys a simple mechanism that is nonetheless important for thinking about public expenditures across educational stages. When switching expenditures from tertiary to pre-tertiary education, if there is a large amount of public school students or a low quantity of vacancies in public university, one monetary unit of investment made for a university student needs to be split among several public school students, resulting in a small increase of pre-tertiary education quality per student.

3 Conclusion

We develop a model in which underprivileged students attend low-quality public schools, and wealthy agents obtain better education through the private system. This fact generates inequality in admission exams to public higher education, leading to a situation in which public resources may be directed to rich agents.

The model rationalizes the intuition that shifting public resources from tertiary to pre-tertiary education may positively affect aggregate production and human capital if per-student government spending in public schools is low relative to that in public universities, if there is a sufficiently high proportion of credit-constrained students, or if pre-tertiary education has a high importance to human capital formation compared to that of tertiary education.

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Appendix

A Proof of proposition 1

First, we take as given that equilibrium π^* is the one implied by the case where high-ability agents apply to university and low-ability agents do not. Then, we show that the hypothesis in this proposition imply that those are indeed agents' optimal choices.

If high-ability agents apply and low-ability agents do not, then

$$\pi^* = \frac{a_0 a_1 \pi_H (\mu_{LH} + \mu_{HH} - \lambda)}{\mu_{LH} a_1 + \mu_{HH} a_0}. \tag{14}$$

Denote by $V_{ij}(s)$ the value function of a type-(w_i, π_j) individual choosing s. We proceed by studying optimal choices for each type of agent.

(w_L, π_L) agents. First, since $w_L < q$, consumption non-negativeness implies that this agent cannot study in private school. Second, note that a sufficient condition for this agent to prefer $(0,0)$ over $(0,1)$ is that the utility of choosing $(0,0)$ is greater than choosing $(0,1)$ and being admitted for sure:

$$\begin{gathered} V_{LL}(0,0) > V_{LL}(0,1) \\ \Longleftarrow w_L + a_0 \pi_L > w_L + a_0 a_2 \pi_L - v \iff v > a_0 \pi_L (a_2 - 1). \end{gathered} \tag{15}$$

In the case of (w_H, π_L) agents, we show that $V_{HL}(1,0) > V_{HL}(1,1)$ and that this fact implies in the last condition in the equation above.

(w_L, π_H) agents. As in the previous case, this agent cannot study in private school. Now note that

$$\begin{gathered} V_{LH}(0,1) > V_{LH}(0,0) \\ \iff w_L + \left(1 - \frac{\pi^*}{a_0 \pi_H}\right) a_0 a_2 \pi_H + \left(\frac{\pi^*}{a_0 \pi_H}\right) a_0 \pi_H - v > w_L + a_0 \pi_H \\ \iff v < \left(1 - \frac{\pi^*}{a_0 \pi_H}\right) a_0 (a_2 - 1) \pi_H. \end{gathered} \tag{16}$$

Using (14), the last expression can be written as

$$v < \left(1 - \frac{a_1 (\mu_{LH} + \mu_{HH} - \lambda)}{\mu_{LH} a_1 + \mu_{HH} a_0}\right) a_0 (a_2 - 1) \pi_H. \tag{17}$$

(w_H, π_L) **agents.** First, it is straightforward to show that

$$V_{HL}(1,0) > V_{HL}(0,0) \iff q < (a_1 - a_0)\pi_L. \tag{18}$$

Second, we have that

$$\begin{aligned} &V_{HL}(1,0) > V_{HL}(1,1) \\ \Longleftarrow\ & w_H - q + a_1\pi_L > w_H - q + a_1 a_2 \pi_L - v \\ \iff\ & v > a_1(a_2 - 1)\pi_L, \end{aligned} \tag{19}$$

which implies in (15). Third, similarly,

$$\begin{aligned} &V_{HL}(1,0) > V_{HL}(0,1) \\ \Longleftarrow\ & w_H - q + a_1\pi_L > w_H + a_0 a_2 \pi_L - v \\ \iff\ & v - q > (a_0 a_2 - a_1)\pi_L. \end{aligned} \tag{20}$$

(w_H, π_H) **agents.** First,

$$\begin{aligned} &V_{HH}(1,1) > V_{HH}(0,1) \\ \iff\ & w_H - q + \left(1 - \frac{\pi^*}{a_1\pi_H}\right) a_2 a_1 \pi_H + \left(\frac{\pi^*}{a_1\pi_H}\right) a_1\pi_H - v \\ &> w_H + \left(1 - \frac{\pi^*}{a_0\pi_H}\right) a_2 a_0 \pi_H + \left(\frac{\pi^*}{a_0\pi_H}\right) a_0\pi_H - v \\ \iff\ & q < (a_1 - a_0) a_2 \pi_H. \end{aligned} \tag{21}$$

Second, a sufficient condition for $V_{HH}(1,1) > V_{HH}(0,0)$ is that this agent's utility in the case where $s = (1,1)$ and she is not admitted to university is greater than her utility if choosing $(0,0)$. That is,

$$\begin{aligned} &V_{HH}(1,1) > V_{HH}(0,0) \\ \Longleftarrow\ & w_H - q + a_1\pi_H - v > w_H + a_0\pi_H \\ \iff\ & q + v < (a_1 - a_0)\pi_H. \end{aligned} \tag{22}$$

Finally,

$$\begin{aligned} &V_{HH}(1,1) > V_{HH}(1,0) \\ \Longleftarrow\ & w_H - q + \left(1 - \frac{\pi^*}{a_1\pi_H}\right) a_2 a_1 \pi_H + \left(\frac{\pi^*}{a_1\pi_H}\right) a_1\pi_H - v > w_H - q + a_1\pi_H \\ \iff\ & v < \left(1 - \frac{\pi^*}{a_1\pi_H}\right) a_1(a_2 - 1)\pi_H. \end{aligned} \tag{23}$$

Note that (16) implies in the condition above.

B Proof of proposition 2

The probabilities of a low- and a high-income agent entering university are given by

$$p_L = 1 - a_1 \left(\frac{\mu_{LH} + \mu_{HH} - \lambda}{\mu_{LH} a_1 + \mu_{HH} a_0} \right), \quad p_H = 1 - a_0 \left(\frac{\mu_{LH} + \mu_{HH} - \lambda}{\mu_{LH} a_1 + \mu_{HH} a_0} \right). \tag{24}$$

Define

$$\delta \equiv \mu_{LH} a_1 + \mu_{HH} a_0 \quad \text{and} \quad \psi \equiv \frac{\mu_{LH} + \mu_{HH} - \lambda}{\delta}. \tag{25}$$

Note that

$$\frac{\partial \psi}{\partial a_0} = -\frac{\psi}{\delta} \mu_{HH}. \tag{26}$$

We can write

$$p_L = 1 - a_1 \psi \quad \text{and} \quad p_H = 1 - a_0 \psi. \tag{27}$$

Using (26) and (27),

$$p_L' \equiv \frac{\partial p_L}{\partial a_0} = \frac{\psi}{\delta} \mu_{HH} a_1 = \frac{(1 - p_L)\mu_{HH}}{\delta} \tag{28}$$

and

$$p_H' \equiv \frac{\partial p_H}{\partial a_0} = -\frac{\psi}{\delta} \mu_{LH} a_1, \tag{29}$$

so that

$$p_H' = -p_L' \frac{\mu_{LH}}{\mu_{HH}}. \tag{30}$$

The derivative of GDP w.r.t. a_0 is given by

$$\begin{aligned} \frac{\partial Y}{\partial a_0} = & \underbrace{\mu_{LL}\pi_L}_{\equiv T_1} + \underbrace{\mu_{LH} p_L' a_0 a_2 \pi_H}_{\equiv T_2} + \underbrace{\mu_{LH} p_L a_2 \pi_H}_{\equiv T_3} \\ & + \underbrace{\mu_{LH} p_L a_0 a_2' \pi_H}_{\equiv T_4} - \underbrace{\mu_{LH} p_L' a_0 \pi_H}_{\equiv T_5} + \underbrace{\mu_{LH}(1 - p_L)\pi_H}_{\equiv T_6} \\ & + \underbrace{\mu_{HH} p_H' a_1 a_2 \pi_H}_{\equiv T_7} + \underbrace{\mu_{HH} p_H a_1 a_2' \pi_H}_{\equiv T_8} - \underbrace{\mu_{HH} p_H' a_1 \pi_H}_{\equiv T_9}, \end{aligned} \tag{31}$$

where $a_2' \equiv \partial a_2 / \partial a_0$.

Using (28), (30), and (31), after some algebra we get

$$T_2 + T_5 + T_7 + T_9 = \frac{(1 - p_L)}{\delta} \mu_{LH} \mu_{HH} (a_2 - 1)(a_0 - a_1)\pi_H. \tag{32}$$

Observe that $T_2 + T_5 + T_7 + T_9 < 0$ because $a_1 > a_0$. Using (32) and the definition of δ, one can show that

$$T_6 > -(T_2 + T_5 + T_7 + T_9) \iff 1 > a_2 \left(\frac{a_1 - a_0}{a_1}\right) \left(\frac{\mu_{HH}}{\mu_{LH} + \mu_{HH}}\right). \tag{33}$$

Now, apart from $T_2 + T_5 + T_7 + T_9$, the only remaining negative terms in (31) are T_4 and T_8. Note that

$$\begin{aligned}
T_1 > -(T_4 + T_8) &\iff \mu_{LL}\pi_L > \left(-\frac{\partial a_2}{\partial a_0}\right) \pi_H \left(\mu_{LH} p_L a_0 + \mu_{HH} p_H a_1\right) && (34)\\
&\Longleftarrow \mu_{LL}\pi_L > \left(-\frac{\partial a_2}{\partial a_0}\right) \pi_H \left(\mu_{LH} p_H a_0 + \mu_{HH} p_H a_1\right) && (35)\\
&\Longleftarrow p_H \mu_{LL}\pi_L > \left(-\frac{\partial a_2}{\partial a_0}\right) \pi_H \left(\mu_{LH} p_H a_0 + \mu_{HH} p_H a_1\right) && (36)\\
&\iff \mu_{LL}\pi_L > \left(-\frac{\partial a_2}{\partial a_0}\right) \pi_H \left(\mu_{LH} a_0 + \mu_{HH} a_1\right) && (37)\\
&\Longleftarrow \mu_{LL}\pi_L > \left(-\frac{\partial a_2}{\partial a_0}\right) \pi_H \left(\frac{\mu_{LH} a_0 + \mu_{HH} a_1}{\mu_{LH} + \mu_{HH}}\right), && (38)
\end{aligned}$$

where we use $p_H > p_L$ in (35), $p_H < 1$ in (36), $p_H > 0$ in (37), and $\mu_{LH} + \mu_{HH} < 1$ in (38).